TRIGONOMETRY

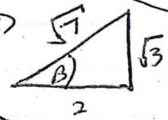
2π radians $= 360°$

$l = r\theta_{rad}$

$S^{O}_{H} \; C^{A}_{H} \; T^{O}_{A}$

Secant $= \frac{1}{\cos\theta}$, Cosecant $= \frac{1}{\sin\theta}$, Cotangent $= \frac{1}{\tan\theta}$

$\tan B = \frac{\sqrt{3}}{2} \Rightarrow$

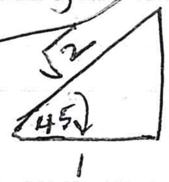


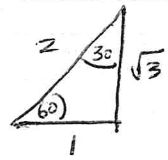
use Pythagorus
$H = \sqrt{4+3} = \sqrt{7}$
$\sin B = \sqrt{3}/\sqrt{7}$

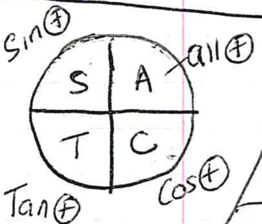
1 degree = 60 minutes
so to convert degrees to mins ×60

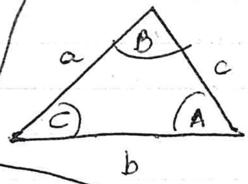
To convert minutes to deg ÷60

Q8 p40







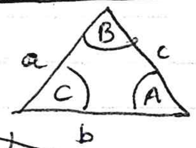


$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Always draw diagrams!

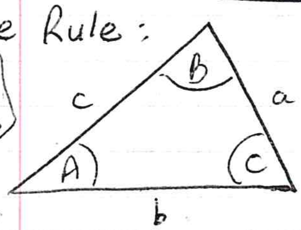
Area of $\Delta = \frac{1}{2}$ (oneside)(other side)(sin (angle between them))



$\frac{1}{2}ab\sin C$
$\frac{1}{2}bc\sin A$
$\frac{1}{2}ac\sin B$

Cosine Rule:

if don't know angles but know sides



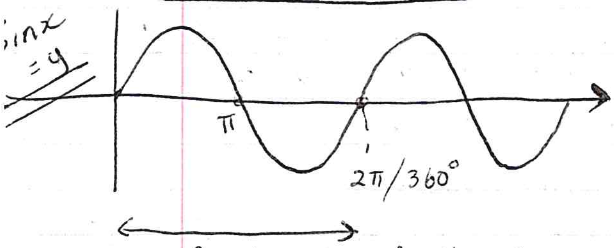
$a^2 \; b^2 + c^2 - 2bc\cos A$
$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$

Q13 p53

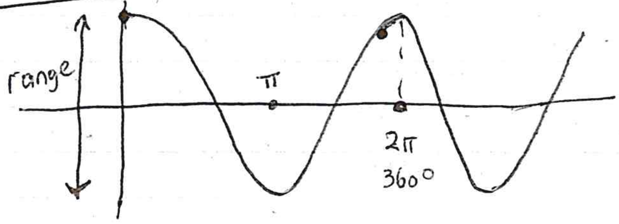
Ex 2 pg 55

Ex 3 p56

Sin x = y



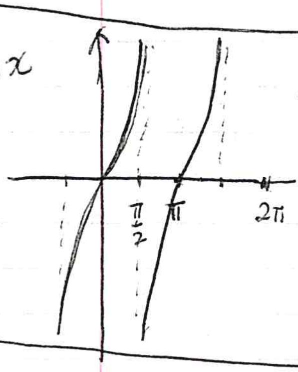
Cos x = y



range = [-1, 1]

Period = time for 1 cycle = 360° or 2π rad

y = Tan x

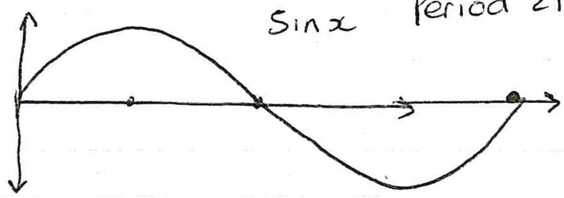


period = π or 180°

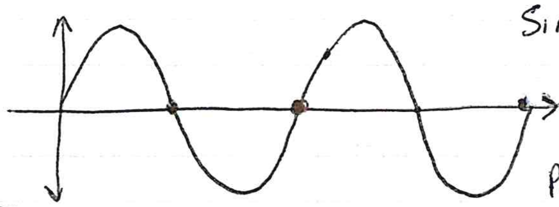
2 vertical asymtotes

Range = [-∞, ∞]

Sin x Period 2π



Sin 2x



Period $\frac{2\pi}{2}$

Sin nx has period $\frac{2\pi}{n}$ (!)

A Sin nx has range [-A, +A]

General Solutions of Trignomertric eqns:

Solns to $\cos\theta = \oplus\frac{1}{2}$ are:

$\theta = 60° + 360n°$ or $300° + 360n°$

Be careful! Tan x has period of π not 2π, 180° not 360°

so $\tan\theta = \sqrt{3}$

$\theta = 60° + 180n$

$\tan^{-1}\sqrt{3} = 60°$

!! Dont forget if you know 2 angles in a Δ you can work out the third!

Trigonometry 2

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta = 1$, $\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$

$(\tan^2\theta + 1) = \sec^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$ $\cos^2\theta = 1 - \sin^2\theta$

Prove $\sec A - \tan A \sin A = \cos A$ Choose left side and show it can be turned into Right Hand Side

EX1 P147

Identities involving Sine Rule, Cos rule : Prove $c\cos B - b\cos C = \dfrac{c^2 - b^2}{a}$ etc

Compound angles $\cos(A-B) = \cos A\cos B + \sin A \sin B$ etc

Prove $\dfrac{\sin(A+B)}{\cos A\cos B} = \tan A + \tan B$

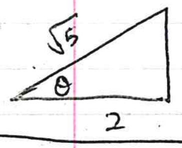
Double → half angle formulae :

$\sin(A+A) = \sin A\cos A + \cos A \sin A$

$\sin(2A) = 2\sin A\cos A$ etc

Express $\sin 3A$ interms of $\sin A$

Given that θ = acute, $\tan\theta = \frac{1}{2}$, find (i) $\sin 2\theta$, (ii) $\cos 2\theta$



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{1}{\sqrt5}\right)\left(\frac{2}{\sqrt5}\right) = \frac{4}{5}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{2}{\sqrt5}\right)^2 - \left(\frac{1}{\sqrt5}\right)^2$

$= \frac{3}{5}$

Show $\dfrac{\sin 2A}{1 + \cos 2A} = \tan A$, $\cos^4\theta + \sin^4\theta = \cos 2\theta$

Prove $\sin 2A = \dfrac{2\tan A}{1 + \tan^2 A}$

Sum, difference and Product formulae :

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$ etc..

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ etc..

Express as a sum/difference

(i) $2\cos 3x \sin x$

(ii) $\cos\theta \cos 5\theta$

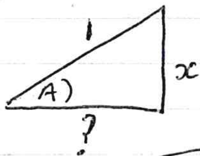
*ALWAYS put bigger angle first !

$\cos 5\theta \cos\theta$

Inverse trignometric functions :

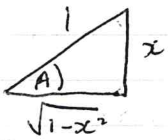
$\sin x = 3/5$ $\sin^{-1}(3/5) = x$

Express $\cos(\sin^{-1}x)$ in terms of x :



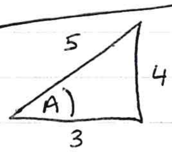
$? = \sqrt{1-x^2}$

$A = \sin^{-1}x$ $\sin A = x$ →



$\cos A = \dfrac{\sqrt{1-x^2}}{1}$

Q E



$\sin(2\underbrace{\tan^{-1}4/3}_{A})$

$\therefore \sin 2A$

$\sin 2A = 2\sin A\cos A$

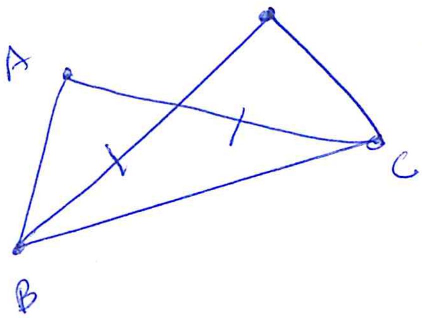
$= 2(4/5)(3/5)$

$= 24/25$

Appendix: Trigonometric Formulae

It will be assumed that these formulae are established in the order listed here. In deriving any formula, use may be made of formulae that precede it.

1. $\cos^2 A + \sin^2 A = 1$
2. sine formula: $\dfrac{a}{\text{Sin } A} = \dfrac{b}{\text{Sin } B} = \dfrac{c}{\text{Sin } C}$
3. cosine formula: $a^2 = b^2 + c^2 - 2\,bc \cos A$
4. $\cos (A-B) = \cos A \cos B + \sin A \sin B$
5. $\cos (A+B) = \cos A \cos B - \sin A \sin B$
6. $\cos 2A = \cos^2 A - \sin^2 A$
7. $\sin (A+B) = \sin A \cos B + \cos A \sin B$
8. $\sin (A-B) = \sin A \cos B - \cos A \sin B$
9. $\tan (A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
10. $\tan (A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$
11. $\sin 2A = 2 \sin A \cos A$
12. $\sin 2A = \dfrac{2 \tan A}{1 + \tan^2 A}$
13. $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$
14. $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$
15. $\cos^2 A = \tfrac{1}{2} (1 + \cos 2A)$
16. $\sin^2 A = \tfrac{1}{2} (1 - \cos 2A)$
17. $2 \cos A \cos B = \cos (A+B) + \cos (A-B)$
18. $2 \sin A \cos B = \sin (A+B) + \sin (A-B)$
19. $2 \sin A \sin B = \cos (A-B) - \cos (A+B)$
20. $2 \cos A \sin B = \sin (A+B) - \sin (A-B)$
21. $\cos A + \cos B = 2 \cos \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$
22. $\cos A - \cos B = -2 \sin \dfrac{A+B}{2} \sin \dfrac{A-B}{2}$
23. $\sin A + \sin B = 2 \sin \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$
24. $\sin A - \sin B = 2 \cos \dfrac{A+B}{2} \sin \dfrac{A-B}{2}$



then 4 points are
on circle