

SOLUTIONS

PROJECT MATHS

Text & Tests

LEAVING CERTIFICATE HIGHER LEVEL STRAND 2

4

**FULLY WORKED
SOLUTIONS
TO ALL QUESTIONS**

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The Celtic Press



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Chapter 1 Coordinate Geometry : The Line

Exercise 1.1

Q1. (i) $|AB| = \sqrt{(3+1)^2 + (-2-3)^2} = \sqrt{16+25} = \sqrt{41}$

(ii) $|BC| = \sqrt{(5-3)^2 + (2+2)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

(iii) Slope AC = $\frac{2-3}{5+1} = -\frac{1}{6}$

(iv) Midpoint = $\left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4, 0)$

Q2. Midpoint M = $\left(\frac{1-3}{2}, \frac{-6+4}{2}\right) = (-1, -1)$

$$|AM| = \sqrt{(1+1)^2 + (-6+1)^2} = \sqrt{4+25} = \sqrt{29}$$

$$|MB| = \sqrt{(-1+3)^2 + (4+1)^2} = \sqrt{4+25} = \sqrt{29}$$

Q3. (i) $\frac{3}{4}$ (ii) $\frac{-4}{3}$

Q4. Slope AB = $\frac{1-3}{-2-2} = \frac{-2}{-4} = \frac{1}{2} = m_1$

$$\text{Slope CD} = \frac{1+2}{5+1} = \frac{3}{6} = \frac{1}{2} = m_2$$

$m_1 = m_2 \Rightarrow$ Parallel lines

Q5. Slope AB = $\frac{3-1}{1+1} = \frac{2}{2} = 1 = m_1$

$$\text{Slope CD} = \frac{4-2}{4-6} = \frac{2}{-2} = -1 = m_2$$

$m_1 \cdot m_2 = (1)(-1) = -1 \Rightarrow$ perpendicular lines

Q6. Slope = $\frac{3-0}{4+2} = \frac{3}{6} = \frac{1}{2} = m_1$

$$\text{Slope} = \frac{1+1}{k-1} = \frac{2}{k-1} = m_2$$

$$\text{Parallel lines} \Rightarrow m_1 = m_2 \Rightarrow \frac{1}{2} = \frac{2}{k-1} \Rightarrow k-1 = 4$$

$$\Rightarrow k = 5$$

Q7. Slope $= \frac{13-1}{-P+1} = \frac{12}{-P+1} = \frac{2}{1}$
 $\Rightarrow -2P + 2 = 12$
 $-2P = 10$
 $\Rightarrow P = -5$

Q8. (i) Slope AB $= \frac{5-3}{2+2} = \frac{2}{4} = \frac{1}{2}$; slope BC $= \frac{1-5}{4-2} = \frac{-4}{2} = -2$; slope CA $= \frac{1-3}{4+2} = \frac{-2}{6} = \frac{-1}{3}$
(ii) Product of slopes $= \left(\frac{1}{2}\right)(-2) = -1 \Rightarrow \angle ABC = 90^\circ$

- Q9. (i) Positive slopes = a and c
(ii) Negative slopes = b and d

Q10. slope $a = \frac{2}{3}$
slope $b = \frac{3}{2}$
slope $c = \frac{4}{2} = 2$

Q11. Decreasing slope; slope $= \frac{-5}{10} = -\frac{1}{2}$

Q12. $|PQ| = \sqrt{(2-a)^2 + (3-4)^2} = \sqrt{5-4a+a^2}$
 $|RS| = \sqrt{(-2-3)^2 + (4+1)^2} = \sqrt{25+25} = \sqrt{50}$
hence, $5-4a+a^2 = 50$
 $\Rightarrow a^2 - 4a - 45 = 0$
 $\Rightarrow (a+5)(a-9) = 0$
 $\Rightarrow a = -5, a = 9$

$$\text{Q13. Slope PQ} = \frac{2-6}{k-5} = \frac{-4}{k-5}$$

$$\text{Slope QR} = \frac{-1-2}{9-k} = \frac{-3}{9-k}$$

$$\text{Perpendicular lines} \Rightarrow \frac{-4}{k-5} \cdot \frac{-3}{9-k} = \frac{-1}{1}$$

$$\Rightarrow \frac{12}{-k^2 + 14k - 45} = \frac{-1}{1}$$

$$\Rightarrow k^2 - 14k + 45 = 12$$

$$\Rightarrow k^2 - 14k + 33 = 0$$

$$\Rightarrow (k-3)(k-11) = 0$$

$$\Rightarrow k = 3, \quad k = 11$$

$$\text{Q14. (i) Slope AB} = \frac{2+2}{7+1} = \frac{4}{8} = \frac{1}{2}$$

$$\begin{aligned} \text{(ii) Slope BC} &= \frac{4-2}{k-7} = \frac{2}{k-7} = \frac{-2}{1} \Rightarrow -2k + 14 = 2 \\ &\quad -2k = -12 \\ &\quad k = 6 \end{aligned}$$

$$\text{(iii) } |AB| = \sqrt{(7+1)^2 + (2+2)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$$

$$\text{(iv) } |BC| = \sqrt{(6-7)^2 + (4-2)^2} = \sqrt{1+4} = \sqrt{5}$$

$$\text{Area } \triangle ABC = \frac{1}{2} (4\sqrt{5}) (\sqrt{5}) = 2.5 = 10$$

$$\text{Q15. (i) } |PQ| = \sqrt{(q+2)^2 + (0-2)^2} = \sqrt{q^2 + 4q + 8}$$

$$|QR| = \sqrt{(5-q)^2 + (3-0)^2} = \sqrt{q^2 - 10q + 34}$$

$$|PQ| = 2|QR|$$

$$\Rightarrow \sqrt{q^2 + 4q + 8} = 2\sqrt{q^2 - 10q + 34}$$

$$\Rightarrow q^2 + 4q + 8 = 4(q^2 - 10q + 34)$$

$$= 4q^2 - 40q + 136$$

$$\Rightarrow 3q^2 - 44q + 128 = 0$$

$$\Rightarrow (3q-32)(q-4) = 0$$

$$\Rightarrow q = \frac{32}{3}, \quad q = 4$$

$$\text{(ii) } P(-2, 2) \quad Q(4, 0) \quad R(5, 3)$$

$$\text{Slope PQ} = \frac{0-2}{4+2} = \frac{-2}{6} = \frac{-1}{3}; \quad \text{slope QR} = \frac{3-0}{5-4} = \frac{3}{1} = 3$$

$$\text{Product of slopes} = \frac{-1}{3} \cdot 3 = -1 \Rightarrow \triangle PQR \text{ is right-angled}$$

Exercise 1.2

Q1. (i) Area = $\frac{1}{2} |(2).(4) - (3).(1)| = \frac{5}{2}$ sq.units

(ii) Area = $\frac{1}{2} |(5).(6) - (3).(1)| = \frac{27}{2}$ sq.units

(iii) Area = $\frac{1}{2} |(-2).(-4) - (1).(3)| = \frac{5}{2}$ sq.units

(iv) Area = $\frac{1}{2} |(3).(-6) - (-2).(4)| = 5$ sq.units

Q2. $(2,3) \rightarrow (0,0); (-5,1) \rightarrow (-7,-2); (3,1) \rightarrow (1,-2)$; Area = $\frac{1}{2} |(-7)(-2) - (1)(-2)| = 8$ sq.units

Q3. (i) $(2,3) \rightarrow (0,0); (5,1) \rightarrow (3,-2); (2,0) \rightarrow (0,-3)$; Area = $\frac{1}{2} |(3)(-3) - (0)(-2)| = \frac{9}{2}$ sq.units

(ii) $(-2,3) \rightarrow (0,0); (4,0) \rightarrow (6,-3); (1,-4) \rightarrow (3,-7)$; Area = $\frac{1}{2} |(6)(-7) - (3)(-3)| = \frac{33}{2}$ sq.units

Q4. $\triangle ABC; (0,0)(4,-1)(2,3)$; Area = $\frac{1}{2} |(4)(3) - (2)(-1)| = 7$ sq.units

$\triangle ACD; (0,0)(2,3)(-2,4)$; Area = $\frac{1}{2} |(2)(4) - (-2)(3)| = 7$ sq.units

Area quadrilateral = $7 + 7 = 14$ sq.units

Q5. Area = $\frac{1}{2} |(1)(k+1) - (-1)(6)| = 7$

$$\Rightarrow |k+7| = 14$$

$$\Rightarrow k+7 = 14 \quad \text{OR} \quad k+7 = -14$$

$$\Rightarrow k = 7 \quad \text{OR} \quad k = -21$$

Q6. $(4,1) \rightarrow (0,0); (-1,-3) \rightarrow (-5,-4); (3,k) \rightarrow (-1,k-1)$

Area = $\frac{1}{2} |(-5)(k-1) - (-1)(-4)| = 12$

$$\Rightarrow |-5k + 5 - 4| = 24$$

$$\Rightarrow |-5k + 1| = 24$$

$$\Rightarrow -5k + 1 = 24 \quad \text{OR} \quad -5k + 1 = -24$$

$$\Rightarrow -5k = 23 \quad \text{OR} \quad -5k = -25$$

$$\Rightarrow k = \frac{-23}{5} \quad \text{OR} \quad k = 5$$

Q7. $\text{Area} = \frac{1}{2} |(4)(k) - (6)(3)| = 7$

$$\Rightarrow |4k - 18| = 14$$

$$\Rightarrow 4k - 18 = 14 \quad \text{or} \quad 4k - 18 = -14$$

$$\Rightarrow 4k = 32 \quad \text{or} \quad 4k = 4$$

$$\Rightarrow k = 8 \quad \text{or} \quad k = 1$$

Q8. $\text{Area} = \frac{1}{2} |(1)(6) - (2)(3)| = 0 \Rightarrow \text{Points are collinear}$

Q9. $(-2, -1) \rightarrow (0, 0); (1, 2) \rightarrow (3, 3); (k, 13) \rightarrow (k+2, 14)$

$$\text{Area} = \frac{1}{2} |(3)(14) - (k+2)(3)| = 6$$

$$\Rightarrow |42 - 3k - 6| = 12$$

$$\Rightarrow |-3k + 36| = 12$$

$$\Rightarrow -3k + 36 = 12 \quad \text{OR} \quad -3k + 36 = -12$$

$$\Rightarrow -3k = -24 \quad \text{OR} \quad -3k = -48$$

$$\Rightarrow k = 8 \quad \quad \quad k = 16$$

Q10. Slope AB = $\frac{3-1}{b-2} = \frac{2}{b-2}$; Slope BC = $\frac{5-3}{5-b} = \frac{2}{5-b}$

$$|\angle ABC| = 90^\circ \Rightarrow \frac{2}{b-2} \cdot \frac{2}{5-b} = -1$$

$$\Rightarrow \frac{4}{-b^2 + 7b - 10} = \frac{-1}{1}$$

$$\Rightarrow b^2 - 7b + 10 = 4$$

$$\Rightarrow b^2 - 7b + 6 = 0$$

$$\Rightarrow (b-1)(b-6) = 0$$

$$\Rightarrow b = 1, b = 6$$

$b = 6 \Rightarrow A(2, 1) \rightarrow (0, 0); B(6, 3) \rightarrow (4, 2); C(5, 5) \rightarrow (3, 4)$

$$\text{Area} = \frac{1}{2} |(4)(4) - (2)(3)| = \frac{1}{2} |16 - 6| = 5 \text{ sq. units}$$

Q11. $(2, -1) \rightarrow (0, 0); (8, k) \rightarrow (6, k+1); (11, 2) \rightarrow (9, 3)$

$$\text{Area} = \frac{1}{2} |(6)(3) - (k+1)(9)| = 0$$

$$\Rightarrow |18 - 9k - 9| = 0$$

$$\Rightarrow |-9k + 9| = 0$$

$$k = 1$$

Q12. (i) Area $\triangle ABC = \frac{1}{2}bh = \frac{1}{2}(5)(6) = 15$ sq. units

(ii) $|BC| = \sqrt{(1-3)^2 + (7-1)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$

(iii) Area $\triangle ABC = \frac{1}{2}|BC|.|AL|$
 $= \frac{1}{2} \cdot (2\sqrt{10}) \cdot |AL| = 15$
 $|AL| = \frac{15}{\sqrt{10}} = \frac{15\sqrt{10}}{\sqrt{10}\sqrt{10}} = \frac{3\sqrt{10}}{2}$

Exercise 1.3

Q1. (i) $y+1=3(x-4)$
 $\Rightarrow 3x-y-13=0$

(ii) $y+2=-2(x+5)$
 $\Rightarrow 2x+y+12=0$

Q2. $y-1=\frac{2}{3}(x+3)$
 $\Rightarrow 3y-3=2x+6$
 $\Rightarrow 2x-3y+9=0$

Q3. (i) Slope $l = \frac{-a}{b} = -\frac{1}{-3} = \frac{1}{3}$

(ii) $y+4=\frac{1}{3}(x-3)$
 $\Rightarrow 3y+12=x-3$
 $\Rightarrow x-3y-15=0$

Q4. (i) Slope AB = $\frac{5+1}{4-3} = 6$

(ii) Perpendicular slope = $-\frac{1}{6}$; C(-2,1)

Equation: $y-1=-\frac{1}{6}(x+2)$

$\Rightarrow 6y-6=-x-2$

$\Rightarrow x+6y-4=0$

Q5. $m_1 = -\frac{2}{-3} = \frac{2}{3}$; $m_2 = -\frac{3}{k}$
 Perpendicular lines $\Rightarrow \frac{2}{3} \cdot \frac{-3}{k} = \frac{-1}{1}$
 $\Rightarrow -3k = -6$
 $\Rightarrow k = 2$

Q6. $m_1 = \frac{-3}{4}$; $m_2 = -\frac{-t}{2} = \frac{t}{2}$
 Perpendicular lines $\Rightarrow \frac{-3}{4} \cdot \frac{t}{2} = \frac{-1}{1}$
 $\Rightarrow -3t = -8$
 $\Rightarrow t = \frac{8}{3}$

Q7. $2(3) + k(1) - 8 = 0$
 $6 + k - 8 = 0$
 $k = 2$

Q8. $x\text{-axis} \Rightarrow y = 0 \Rightarrow x - 6 = 0$ $y\text{-axis} \Rightarrow x = 0 \Rightarrow -3y - 6 = 0$
 $x = 6; (6, 0)$ $y = -2; (0, -2)$

Q9. Slope $h = \frac{10+2}{-4-6} = \frac{12}{-10} = \frac{-6}{5}$; Slope $k = \frac{-a}{6}$
 Perpendicular lines $\Rightarrow \frac{-6}{5} \cdot \frac{-a}{6} = -1$
 $\Rightarrow \frac{6a}{30} = \frac{-1}{1}$
 $\Rightarrow 6a = -30$
 $\Rightarrow a = -5$

Q10. (i) $x\text{-axis} \Rightarrow y = 0 \Rightarrow 2x + 6 = 0$
 $\Rightarrow x = -3 \Rightarrow C(-3, 0) \Rightarrow x = -3$

(ii) Slope $= -\frac{2}{-3} = \frac{2}{3} \Rightarrow$ Perpendicular slope $= \frac{-3}{2}$
 \Rightarrow Equation : $y - 0 = \frac{-3}{2}(x + 3)$
 $\Rightarrow 2y = -3x - 9$
 $\Rightarrow 3x + 2y + 9 = 0$

Q11. Slope = $\frac{-2}{5}$, Point = $(-2, 3)$

$$\begin{aligned}\text{Equation : } & y - 3 = \frac{-2}{5}(x + 2) \\ \Rightarrow & 5y - 15 = -2x - 4 \\ \Rightarrow & 2x + 5y - 11 = 0\end{aligned}$$

Q12. (i) $m_1 = 3, m_2 = \frac{1}{3}$; Neither

(ii) $m_1 = -2, m_2 = \frac{1}{2}$; Perpendicular

(iii) $m_1 = \frac{-2}{3}, m_2 = \frac{3}{2}$; Perpendicular

(iv) $m_1 = -\frac{1}{2}, m_2 = -\frac{1}{2}$; Parallel

(v) $m_1 = 2, m_2 = 2$; Parallel

(vi) $m_1 = -\frac{1}{3}, m_2 = 3$; Perpendicular

Q13. $x + 2y = 1$

$$2x + 3y = 4$$

$$\overline{2x + 4y = 2}$$

$$\underline{2x + 3y = 4}$$

Subtract: $y = -2 \Rightarrow x = 5; (5, -2)$

Q14. $x + y = 5$

$$2x - y = 1$$

$$\text{Add} \Rightarrow \overline{3x = 6}$$

$$x = 2 \Rightarrow y = 3 \quad (2, 3)$$

$$\text{Equation : } y - 3 = \frac{2}{3}(x - 2)$$

$$\Rightarrow 3y - 9 = 2x - 4$$

$$\Rightarrow 2x - 3y + 5 = 0$$

$$\text{Q15. } \begin{array}{r} 2x + 3y = 12 \\ 3x - 4y = 1 \\ \hline 8x + 12y = 48 \\ 9x - 12y = 3 \\ \hline \end{array}$$

$$\text{Add } \Rightarrow 17x = 51$$

$$x = 3 \Rightarrow y = 2 \quad (3, 2)$$

$$\text{Slope} = -\frac{3}{-1} = 3$$

$$\text{Equation: } y - 2 = 3(x - 3)$$

$$\Rightarrow y - 2 = 3x - 9$$

$$\Rightarrow 3x - y - 7 = 0$$

$$\text{Q16. (i) } 2 - 2(6) + 10 = 0 \dots \text{True}$$

$$\text{(ii) } 2(3) + k(2) - 12 = 0$$

$$\Rightarrow 2k = 6$$

$$\Rightarrow k = 3$$

$$\text{Q17. } \begin{array}{r} 3x - 2y = -7 \\ 5x + y = -3 \\ \hline 3x - 2y = -7 \\ 10x + 2y = -6 \\ \hline \end{array}$$

$$\text{Add } \Rightarrow 13x = -13$$

$$x = -1 \Rightarrow y = 2 \quad (-1, 2)$$

$$\text{Slope } l_2 = \frac{-5}{1} \Rightarrow \text{Perpendicular slope} = \frac{1}{5}$$

$$\text{Equation: } y - 2 = \frac{1}{5}(x + 1)$$

$$\Rightarrow 5y - 10 = x + 1$$

$$\Rightarrow x - 5y + 11 = 0$$

Q18. $x\text{-axis} \Rightarrow y = 0 \Rightarrow 3x = k$

$$\Rightarrow x = \frac{k}{3} \quad \left(\frac{k}{3}, 0 \right)$$

$y\text{-axis} \Rightarrow x = 0 \Rightarrow 4y = k$

$$y = \frac{k}{4} \quad \left(0, \frac{k}{4} \right)$$

$$\text{Area} = \frac{1}{2} \left| \left(\frac{k}{3} \right) \left(\frac{k}{4} \right) - (0)(0) \right| = 24$$

$$\Rightarrow \frac{k^2}{12} = 48$$

$$\Rightarrow k^2 = 576$$

$$\Rightarrow k = \pm 24 \Rightarrow k = 24$$

Q19. Parallel line : $2x - 3y + c = 0$

$$\text{Point}(4, 2) \Rightarrow 2(4) - 3(2) + c = 0$$

$$\Rightarrow c = -2 \Rightarrow 2x - 3y - 2 = 0$$

Q20. Parallel line : $4x + y = k$

$$x\text{-axis} \Rightarrow y = 0 \Rightarrow 4x = k$$

$$\Rightarrow x = \frac{k}{4} \quad \left(\frac{k}{4}, 0 \right)$$

$$y\text{-axis} \Rightarrow x = 0 \Rightarrow y = k \quad (0, k)$$

$$\text{Area} = \frac{1}{2} \left| \left(\frac{k}{4} \right)k - (0)(0) \right| = 18$$

$$\Rightarrow \frac{k^2}{4} = 36$$

$$\Rightarrow k^2 = 144$$

$$\Rightarrow k = \pm 12 \Rightarrow k = 12$$

$$\Rightarrow 4x + y - 12 = 0$$

Exercise 1.4

Q1. $\left[\frac{4(5)+1(-3)}{4+1}, \frac{4(-4)+1(4)}{4+1} \right] = \left(\frac{17}{5}, \frac{-12}{5} \right)$

Q2. $\left[\frac{3(3)+1(-5)}{3+1}, \frac{3(-8)+1(8)}{3+1} \right] = (1, -4)$

Q3. $\left[\frac{5(4)-2(2)}{5-2}, \frac{5(6)-2(-3)}{5-2} \right] = \left(\frac{16}{3}, 12 \right)$

Q4. (i) $\left[\frac{3(1)+2(5)}{3+2}, \frac{3(-2)+2(0)}{3+2} \right] = \left(\frac{13}{5}, \frac{-6}{5} \right)$

(ii) $\left[\frac{3(1)-2(5)}{3-2}, \frac{3(-2)-2(0)}{3-2} \right] = (-7, -6)$

Q5. (i) $\left[\frac{3(5)+1(2)}{3+1}, \frac{3(7)+1(3)}{3+1} \right] = \left(\frac{17}{4}, 6 \right)$

(ii) $\left[\frac{3(5)-1(2)}{3-1}, \frac{3(7)-1(3)}{3-1} \right] = \left(\frac{13}{2}, 9 \right)$

Q6. $C = \left[\frac{4(3)-1(-2)}{4-1}, \frac{4(4)-1(-1)}{4-1} \right] = \left(\frac{14}{3}, \frac{17}{3} \right)$

Q7. $P = \left[\frac{2(x)+1(2)}{2+1}, \frac{2(y)+1(-3)}{2+1} \right] = (6, 1)$

$$\Rightarrow 2x + 2 = 18, \quad 2y - 3 = 3$$

$$x = 8 \quad y = 3$$

Q8. $P = \left[\frac{1(x)+3(-10)}{1+3}, \frac{1(-5)+3(7)}{1+3} \right] = (-6, y)$

$$\Rightarrow x - 30 = -24, \quad -5 + 21 = 4y$$

$$x = 6 \quad y = 4$$

Q9. $C = \left[\frac{4(0)+3(x)}{4+3}, \frac{4(y)+3(0)}{4+3} \right] = (9, -8)$

$$\Rightarrow 0 + 3x = 63, \quad 4y + 0 = -56$$

$$x = 21 \quad y = -14$$

Q10. $P = \left[\frac{h(-2)+k(4)}{h+k}, \frac{h(0)+k(-3)}{h+k} \right] = (2, -2)$

$$-2h + 4k = 2h + 2k$$

$$4h = 2k$$

$$2h = 1k$$

$$\frac{h}{k} = \frac{1}{2} \Rightarrow \text{Ratio } h : k = 1 : 2$$

Exercise 1.5

Q1. (i) centroid = $\left(\frac{2+4-3}{3}, \frac{-3+0+9}{3} \right) = (1, 2)$

(ii) centroid = $\left(\frac{1+6+5}{3}, \frac{3+2-2}{3} \right) = (4, 1)$

Q2. Midpoint = $(2, 0)$; Slope x -axis = 0 \Rightarrow Perpendicular slope is undefined
 \Rightarrow Equation: $x = 2$

Midpoint = $\left(\frac{1}{2}, 1 \frac{1}{2} \right)$; Slope = $\frac{3-0}{1-0} = 3 \Rightarrow$ Perpendicular slope = $-\frac{1}{3}$

\Rightarrow Equation: $y - 1 \frac{1}{2} = -\frac{1}{3} \left(x - \frac{1}{2} \right)$

$$\Rightarrow 3y - 4 \frac{1}{2} = -x + \frac{1}{2}$$

$$\Rightarrow x + 3y = 5$$

$$x = 2 \Rightarrow 2 + 3y = 5$$

$$\Rightarrow 3y = 3$$

$$\Rightarrow y = 1 \Rightarrow \text{Circumcentre} = (2, 1)$$

Q3. Midpoint = $\left(\frac{3+6}{2}, \frac{-7+2}{2} \right) = \left(4 \frac{1}{2}, -2 \frac{1}{2} \right)$

Slope = $\frac{2+7}{6-3} = \frac{9}{3} = 3 \Rightarrow$ Perpendicular slope = $-\frac{1}{3}$

Equation: $y + 2 \frac{1}{2} = -\frac{1}{3} \left(x - 4 \frac{1}{2} \right)$

$$\Rightarrow 3y + 7 \frac{1}{2} = -x + 4 \frac{1}{2}$$

$$\Rightarrow x + 3y = -3$$

Midpoint = $\left(\frac{6+8}{2}, \frac{2-2}{2} \right) = (7, 0)$

Slope = $\frac{2+2}{6-8} = \frac{4}{-2} = -2 \Rightarrow$ Perpendicular slope = $\frac{1}{2}$

Equation: $y - 0 = \frac{1}{2} (x - 7)$

$$\Rightarrow 2y = x - 7$$

$$\Rightarrow x - 2y = 7$$

and $\underline{x + 3y = -3}$

Subtract $\Rightarrow \underline{-5y = 10}$

$$y = -2 \Rightarrow x = 3$$

$$\text{Circumcentre} = (3, -2)$$

Q4. Slope = $\frac{5-2}{-2-4} = \frac{3}{-6} = \frac{-1}{2}$ \Rightarrow Perpendicular slope = 2, Point = $(-1, -3)$

Equation : $y + 3 = 2(x + 1)$

$$y + 3 = 2x + 2$$

$$\Rightarrow 2x - y = 1$$

Slope = $\frac{2+3}{4+1} = \frac{5}{5} = 1$ \Rightarrow Perpendicular slope = -1, Point = $(-2, 5)$

Equation : $y - 5 = -1(x + 2)$

$$\Rightarrow y - 5 = -x - 2$$

$$\Rightarrow x + y = 3$$

and $\underline{2x - y = 1}$

Add $\Rightarrow 3x = 4$

$$x = \frac{4}{3} \Rightarrow y = \frac{5}{3}$$

$$\text{Orthocentre} = \left(\frac{4}{3}, \frac{5}{3} \right)$$

Q5. Slope = $\frac{4-0}{4-0} = \frac{4}{4} = 1$ \Rightarrow Perpendicular slope = -1, Point = $(4, -2)$

Equation : $y + 2 = -1(x - 4)$

$$y + 2 = -x + 4$$

$$x + y = 2$$

Slope = $\frac{-2-0}{4-0} = \frac{-2}{4} = \frac{-1}{2}$ \Rightarrow Perpendicular slope = 2, Point = $(4, 4)$

Equation : $y - 4 = 2(x - 4)$

$$\Rightarrow y - 4 = 2x - 8$$

$$\Rightarrow 2x - y = 4$$

and $\underline{x + y = 2}$

Add $\Rightarrow 3x = 6$

$$\Rightarrow x = 2 \Rightarrow y = 0$$

$$\text{Orthocentre} = (2, 0)$$

Q6.

$$\text{Midpoint} = \left(\frac{-2+5}{2}, \frac{2-5}{2} \right) = \left(1\frac{1}{2}, -1\frac{1}{2} \right)$$

$$\text{Slope} = \frac{2+5}{-2-5} = \frac{7}{-7} = -1 \Rightarrow \text{Perpendicular slope} = 1$$

$$\text{Equation : } y + 1\frac{1}{2} = 1 \left(x - 1\frac{1}{2} \right)$$

$$\Rightarrow y + 1\frac{1}{2} = x - 1\frac{1}{2}$$

$$\Rightarrow x - y = 3$$

$$\text{Midpoint} = \left(\frac{-4+5}{2}, \frac{-2-5}{2} \right) = \left(\frac{1}{2}, -3\frac{1}{2} \right)$$

$$\text{Slope} = \frac{-2+5}{-4-5} = \frac{3}{-9} = \frac{-1}{3} \Rightarrow \text{Perpendicular slope} = 3$$

$$\text{Equation : } y + 3\frac{1}{2} = 3 \left(x - \frac{1}{2} \right)$$

$$\Rightarrow y + 3\frac{1}{2} = 3x - 1\frac{1}{2}$$

$$\Rightarrow 3x - y = 5$$

and $\underline{x - y = 3}$

Subtract $\Rightarrow 2x = 2$

$$\Rightarrow x = 1 \Rightarrow y = -2$$

$$\text{Circumcentre} = (1, -2)$$

Q7. $\text{Centroid} = (-1, 3)$

$$\Rightarrow \frac{4-2+k}{3} = -1$$

$$\Rightarrow 2+k = -3$$

$$\Rightarrow k = -5$$

Exercise 1.6

Q1.

$$|\text{PD}| = \frac{|3(2) - 4(-4) - 17|}{\sqrt{(3)^2 + (-4)^2}} = \frac{|22 - 17|}{\sqrt{25}} = \frac{|5|}{5} = 1$$

Q2.

$$|\text{PD}| = \frac{|3(1) + 4(1) - 12|}{\sqrt{(3)^2 + (4)^2}} = \frac{|7 - 12|}{\sqrt{25}} = \frac{|-5|}{5} = 1$$

$$|\text{PD}| = \frac{|5(1) - 12(1) + 20|}{\sqrt{(5)^2 + (-12)^2}} = \frac{|13|}{\sqrt{169}} = \frac{|13|}{13} = 1$$

Q3. $|PD| = \frac{|5(6) - 3(2) + 10|}{\sqrt{(5)^2 + (-3)^2}} = \frac{|40 - 6|}{\sqrt{34}} = \frac{|34|}{\sqrt{34}} = \sqrt{34}$

Q4. $|PD| = \frac{|1(5) - 2(-5) + 10|}{\sqrt{(1)^2 + (-2)^2}} = \frac{|25|}{\sqrt{5}} = 5\sqrt{5}$

$$|PD| = \frac{|2(5) + 1(-5) - 30|}{\sqrt{(2)^2 + (1)^2}} = \frac{|-25|}{\sqrt{5}} = 5\sqrt{5}$$

Q5. $|PD| = \frac{|4(3) + 3(1) + c|}{\sqrt{(4)^2 + (3)^2}} = \frac{|15 + c|}{\sqrt{25}} = \frac{|15 + c|}{5}$

$$\Rightarrow \frac{15+c}{5} = +5 \quad \text{OR} \quad \frac{15+c}{5} = -5$$

$$\Rightarrow 15 + c = 25 \quad \text{OR} \quad 15 + c = -25$$

$$\Rightarrow c = 10 \quad \text{OR} \quad c = -40$$

Q6. $3(2) - (2) - 4 = 6 - 6 = 0$. True Statement.

Perpendicular distance from $(2,2)$ to line $6x - 2y + 7 = 0$:

$$|PD| = \frac{|6(2) - 2(2) + 7|}{\sqrt{(6)^2 + (-2)^2}} = \frac{|15|}{\sqrt{40}} = \frac{15}{2\sqrt{10}} = \frac{15\sqrt{10}}{20} = \frac{3\sqrt{10}}{4}$$

Q7. $|PD| = \frac{|1(1) + 7(1) - 3|}{\sqrt{(1)^2 + (7)^2}} = \frac{|5|}{\sqrt{50}} = \frac{|5|}{5\sqrt{2}} = \frac{|1|}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$$|PD| = \frac{|1(1) - 1(1) + 1|}{\sqrt{(1)^2 + (-1)^2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \text{YES.}$$

$$Q8. |PD| = \frac{|a(-2) + 1(3) - 7|}{\sqrt{(a)^2 + (1)^2}} = \frac{|-2a - 4|}{\sqrt{a^2 + 1}} = \frac{\sqrt{10}}{1}$$

$$\begin{aligned} &\Rightarrow 2a + 4 = \sqrt{10} \cdot \sqrt{a^2 + 1} \\ &\Rightarrow 4a^2 + 16a + 16 = 10(a^2 + 1) \\ &\quad = 10a^2 + 10 \\ &\Rightarrow 6a^2 - 16a - 6 = 0 \\ &\Rightarrow 3a^2 - 8a - 3 = 0 \\ &\Rightarrow (3a + 1)(a - 3) = 0 \\ &\Rightarrow a = -\frac{1}{3}, a = 3 \end{aligned}$$

$$Q9. |PD| = \frac{|4(-2) + 3(a) - 3|}{\sqrt{(4)^2 + (3)^2}} = \frac{|3a - 11|}{\sqrt{25}} = \frac{3a - 11}{5}$$

$$\begin{aligned} |PD| &= \frac{|12(-2) + 5(a) - 13|}{\sqrt{(12)^2 + (5)^2}} = \frac{|5a - 37|}{\sqrt{169}} = \frac{5a - 37}{13} \\ &\Rightarrow \frac{3a - 11}{5} = \frac{5a - 37}{13} \\ &\Rightarrow 39a - 143 = 25a - 185 \\ &\Rightarrow 14a = -42 \\ &\Rightarrow a = -3 \end{aligned}$$

$$Q10. |PD| = \frac{|3(-2) + 2(6) - 7|}{\sqrt{(3)^2 + (2)^2}} = \frac{|-13 + 12|}{\sqrt{13}} = \frac{|-1|}{\sqrt{13}} \text{ Negative side}$$

$$|PD| = \frac{|3(0) + 2(0) - 7|}{\sqrt{(3)^2 + (2)^2}} = \frac{|-7|}{\sqrt{13}} \text{ Negative side}$$

$$Q11. |PD| = \frac{|3(3) + 4(4) - 36|}{\sqrt{(3)^2 + (4)^2}} = \frac{|25 - 36|}{\sqrt{25}} = \frac{|-11|}{5} \text{ Negative side}$$

$$|PD| = \frac{|3(9) + 4(3) - 36|}{\sqrt{(3)^2 + (4)^2}} = \frac{|39 - 36|}{\sqrt{25}} = \frac{|3|}{5} \text{ Positive side}$$

$$Q12. |PD| = \frac{|2(-3) - 3(1) + 7|}{\sqrt{(2)^2 + (-3)^2}} = \frac{|-9 + 7|}{\sqrt{13}} = \frac{|-2|}{\sqrt{13}} \text{ Negative side}$$

$$|PD| = \frac{|2(3) - 3(-4) + 7|}{\sqrt{(2)^2 + (-3)^2}} = \frac{|6 + 12 + 7|}{\sqrt{13}} = \frac{|25|}{\sqrt{13}} \text{ Positive side}$$

No; Points are not on the same side of the line.

Q13. Parallel line is $4x + 3y + c = 0$.

A point on the line $4x + 3y + 1 = 0$ is $(-1, 1)$.

$$|PD| = \frac{|4(-1) + 3(1) + c|}{\sqrt{(4)^2 + (3)^2}} = 2$$

$$\Rightarrow \frac{|-1 + c|}{5} = 2$$

$$\Rightarrow -1 + c = 10 \quad \text{OR} \quad -1 + c = -10$$

$$\Rightarrow c = 11 \quad \text{OR} \quad c = -9$$

$$\Rightarrow 4x + 3y + 11 = 0 \quad \text{OR} \quad 4x + 3y - 9 = 0$$

Q14. Perpendicular line is $4x + 3y + k = 0$.

$$|PD| = \frac{|4(1) + 3(1) + k|}{\sqrt{(4)^2 + (3)^2}} = 4$$

$$\Rightarrow \frac{|7 + k|}{5} = 4$$

$$\Rightarrow 7 + k = 20 \quad \text{OR} \quad 7 + k = -20$$

$$\Rightarrow k = 13 \quad \text{OR} \quad k = -27$$

$$\Rightarrow 4x + 3y + 13 = 0 \quad \text{OR} \quad 4x + 3y - 27 = 0$$

Q15. Point $(-4, 2)$, slope $= m$

$$\text{Equation : } y - 2 = m(x + 4)$$

$$\Rightarrow y - 2 = mx + 4m$$

$$\Rightarrow mx - y + 4m + 2 = 0$$

$$|PD| = \frac{|m(0) - 1(0) + 4m + 2|}{\sqrt{(m)^2 + (-1)^2}} = 2$$

$$\Rightarrow |4m + 2| = 2\sqrt{m^2 + 1}$$

$$\Rightarrow 16m^2 + 16m + 4 = 4(m^2 + 1)$$

$$\Rightarrow 16m^2 + 16m + 4 = 4m^2 + 4$$

$$\Rightarrow 12m^2 + 16m = 0$$

$$\Rightarrow 3m^2 + 4m = 0$$

$$\Rightarrow m(3m + 4) = 0$$

$$\Rightarrow m = 0 \quad \text{OR} \quad m = -\frac{4}{3}$$

$$\Rightarrow x(0) - y + 4(0) + 2 = 0 \quad \text{OR} \quad x\left(\frac{-4}{3}\right) - y + 4\left(\frac{-4}{3}\right) + 2 = 0$$

$$\Rightarrow y - 2 = 0 \quad \text{OR} \quad 4x + 3y + 10 = 0$$

Q16. Point $(3, 5)$, slope = m

$$\text{Equation : } y - 5 = m(x - 3)$$

$$\Rightarrow y - 5 = mx - 3m$$

$$\Rightarrow mx - y - 3m + 5 = 0$$

$$|PD| = \frac{|m(0) - 1(0) - 3m + 5|}{\sqrt{(m)^2 + (-1)^2}} = 5$$

$$\Rightarrow |-3m + 5| = 5\sqrt{m^2 + 1}$$

$$\Rightarrow 9m^2 - 30m + 25 = 25(m^2 + 1)$$

$$\Rightarrow 9m^2 - 30m + 25 = 25m^2 + 25$$

$$\Rightarrow 16m^2 + 30m = 0$$

$$\Rightarrow 8m^2 + 15m = 0$$

$$\Rightarrow m(8m + 15) = 0$$

$$\Rightarrow m = 0 \text{ OR } m = \frac{-15}{8}$$

$$\Rightarrow x(0) - y - 3(0) + 5 = 0 \text{ OR } x\left(\frac{-15}{8}\right) - y - 3\left(\frac{-15}{8}\right) + 5 = 0$$

$$\Rightarrow y - 5 = 0 \text{ OR } 15x + 8y - 85 = 0$$

Q17. (i) Slope BC = $\frac{1+2}{3+1} = \frac{3}{4}$

$$\text{Equation BC : } y + 2 = \frac{3}{4}(x + 1)$$

$$\Rightarrow 4y + 8 = 3x + 3$$

$$\Rightarrow 3x - 4y - 5 = 0$$

$$|AN| = \frac{|3(1) - 4(2) - 5|}{\sqrt{(3)^2 + (-4)^2}} = \frac{|-10|}{\sqrt{25}} = \frac{|-10|}{5} = 2$$

$$\begin{aligned} \text{(ii)} \quad |BC| &= \sqrt{(3+1)^2 + (1+2)^2} \\ &= \sqrt{(4)^2 + (3)^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{Area } \triangle ABC &= \frac{1}{2}|BC| \cdot |AN| \\ &= \frac{1}{2}(5)(2) \\ &= 5 \text{ sq. units} \end{aligned}$$

Exercise 1.7

Q1. (i) Slope of $x + 2y + 4 = 0 \Rightarrow m_1 = -\frac{1}{2}$

Slope of $x - 3y + 2 = 0 \Rightarrow m_2 = -\frac{1}{-3} = \frac{1}{3}$

$$\Rightarrow \tan\theta = \pm \frac{-\frac{1}{2} - \frac{1}{3}}{1 + \left(-\frac{1}{2}\right)\left(\frac{1}{3}\right)} = \pm \frac{-\frac{5}{6}}{1 - \frac{1}{6}} = \pm \frac{-\frac{5}{6}}{\frac{5}{6}} = +1$$

(ii) Slope of $2x + 3y - 1 = 0 \Rightarrow m_1 = -\frac{2}{3}$

Slope of $x - 2y + 3 = 0 \Rightarrow m_2 = -\frac{1}{-2} = \frac{1}{2}$

$$\Rightarrow \tan\theta = \pm \frac{-\frac{2}{3} - \frac{1}{2}}{1 + \left(-\frac{2}{3}\right)\left(\frac{1}{2}\right)} = \pm \frac{-\frac{7}{6}}{1 - \frac{1}{3}} = \pm \frac{-\frac{7}{6}}{\frac{2}{3}} = +\frac{7}{4}$$

(iii) Slope of $2x + y - 6 = 0 \Rightarrow m_1 = -\frac{2}{1} = -2$

Slope of $2x - 3y + 5 = 0 \Rightarrow m_2 = \frac{-2}{-3} = \frac{2}{3}$

$$\Rightarrow \tan\theta = \pm \frac{-2 - \frac{2}{3}}{1 + (-2)\left(\frac{2}{3}\right)} = \pm \frac{-\frac{8}{3}}{1 - \frac{4}{3}} = \pm \frac{-\frac{8}{3}}{-\frac{1}{3}} = +8$$

Q2. Slope of $y = 2x + 5 \Rightarrow m_1 = 2$

Slope of $3x + y = 7 \Rightarrow m_2 = \frac{-3}{1} = -3$

$$\Rightarrow \tan\theta = \pm \frac{2 - (-3)}{1 + (2)(-3)} = \pm \frac{2 + 3}{1 - 6} = \pm \frac{5}{-5} = +1$$

$$\Rightarrow \theta = \tan^{-1}(1) = 45^\circ$$

Q3. Slope of $x - 2y - 1 = 0 \Rightarrow m_1 = -\frac{1}{-2} = \frac{1}{2}$

Slope of $3x - y + 2 = 0 \Rightarrow m_2 = \frac{-3}{-1} = 3$

$$\Rightarrow \tan\theta = \pm \frac{\frac{1}{2} - 3}{1 + \frac{1}{2}(3)} = \pm \frac{-2\frac{1}{2}}{2\frac{1}{2}} = -1$$

$$\Rightarrow \theta = \tan^{-1}(-1) = 135^\circ$$

Q4. Slope of $x - 3y + 4 = 0 \Rightarrow m_1 = -\frac{1}{-3} = \frac{1}{3}$
Slope of $2x + y - 5 = 0 \Rightarrow m_2 = -\frac{2}{1} = -2$
 $\Rightarrow \tan\theta = \pm \frac{\frac{1}{3} + 2}{1 + \left(\frac{1}{3}\right)(-2)} = \pm \frac{\frac{7}{3}}{1 - \frac{2}{3}} = \pm \frac{7}{\frac{1}{3}} = +7$
 $\Rightarrow \theta = \tan^{-1}(7) = 81.87^\circ = 82^\circ$

Q5. Slope of $x - 2y + 7 = 0 \Rightarrow m_1 = -\frac{1}{-2} = \frac{1}{2}$
Slope of $3x - y + 2 = 0 \Rightarrow m_2 = -\frac{3}{-1} = 3$
 $\Rightarrow \tan\theta = \pm \frac{\frac{1}{2} - 3}{1 + \left(\frac{1}{2}\right)(3)} = \pm \frac{-2\frac{1}{2}}{2\frac{1}{2}} = \pm 1 = -1$
 $\Rightarrow \theta = \tan^{-1}(-1) = 135^\circ$

Q6. Slope of $x - \sqrt{3}y + 4 = 0 \Rightarrow m_1 = -\frac{1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$
Slope of $\sqrt{3}x - y - 7 = 0 \Rightarrow m_2 = -\frac{\sqrt{3}}{-1} = \sqrt{3}$
 $\Rightarrow \tan\theta = \pm \frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + \left(\frac{1}{\sqrt{3}}\right)(\sqrt{3})} = \pm \frac{\frac{1-3}{\sqrt{3}}}{2} = \pm \frac{-2}{2\sqrt{3}} = \pm \frac{1}{\sqrt{3}} = +\frac{1}{\sqrt{3}}$
 $\Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$

Q7. Slope of $2x - 3y + 1 = 0 \Rightarrow m_1 = \frac{-2}{-3} = \frac{2}{3}$
 $\tan 45^\circ = \pm \frac{\frac{2}{3} - m_2}{1 + \frac{2}{3}m_2} = \pm \frac{\frac{2-3m_2}{3}}{\frac{3+2m_2}{3}} = \pm \frac{2-3m_2}{3+2m_2}$
 $\Rightarrow +\frac{2-3m_2}{3+2m_2} = \frac{1}{1} \quad \text{OR} \quad \frac{-2+3m_2}{3+2m_2} = \frac{1}{1}$
 $\Rightarrow 3+2m_2 = 2-3m_2 \quad \text{OR} \quad -2+3m_2 = 3+2m_2$
 $\Rightarrow 5m_2 = -1 \quad \text{OR} \quad m_2 = 5$
 $\Rightarrow m_2 = -\frac{1}{5}$

Q8. Slope of $2x + 3y - 4 = 0 \Rightarrow m_1 = -\frac{2}{3}$

$$\Rightarrow \tan 45^\circ = \pm \frac{\frac{-2}{3} - m_2}{1 + \left(\frac{-2}{3}\right)(m_2)} = \pm \frac{\frac{-2 - 3m_2}{3}}{\frac{3 - 2m_2}{3}} = \pm \frac{-2 - 3m_2}{3 - 2m_2}$$

$$\Rightarrow \frac{-2 - 3m_2}{3 - 2m_2} = \frac{1}{1} \quad \text{OR} \quad \frac{2 + 3m_2}{3 - 2m_2} = \frac{1}{1}$$

$$\Rightarrow 3 - 2m_2 = -2 - 3m_2 \quad \text{OR} \quad 2 + 3m_2 = 3 - 2m_2$$

$$\Rightarrow m_2 = -5 \quad \text{OR} \quad 5m_2 = 1$$

$$m_2 = \frac{1}{5}$$

Equation : $y - 0 = -5(x - 0) \quad \text{OR} \quad y - 0 = \frac{1}{5}(x - 0)$

$$\Rightarrow y = -5x \quad \Rightarrow 5y = x$$

$$\Rightarrow 5x + y = 0 \quad \Rightarrow x - 5y = 0$$

Q9. Slope of $2x + y - 2 = 0 \Rightarrow m_1 = -\frac{2}{1} = -2$

$$\Rightarrow \tan 45^\circ = \pm \frac{-2 - m_2}{1 + (-2)m_2} = \pm \frac{-2 - m_2}{1 - 2m_2}$$

$$\Rightarrow \frac{-2 - m_2}{1 - 2m_2} = \frac{1}{1} \quad \text{OR} \quad \frac{2 + m_2}{1 - 2m_2} = \frac{1}{1}$$

$$\Rightarrow -2 - m_2 = 1 - 2m_2 \quad \text{OR} \quad 2 + m_2 = 1 - 2m_2$$

$$\Rightarrow m_2 = 3 \quad \text{OR} \quad 3m_2 = -1$$

$$m_2 = \frac{-1}{3}$$

Equation : $y - 1 = 3(x + 1) \quad \text{OR} \quad y - 1 = \frac{-1}{3}(x + 1)$

$$\Rightarrow y - 1 = 3x + 3 \quad \Rightarrow 3y - 3 = -x - 1$$

$$\Rightarrow 3x - y + 4 = 0 \quad \Rightarrow x + 3y - 2 = 0$$

Q10. Slope of $x + y - 2 = 0 \Rightarrow m_1 = -\frac{1}{1} = -1$

$$\Rightarrow \tan \theta = \pm \frac{-1 - m_2}{1 + (-1)m_2} = \pm \frac{-1 - m_2}{1 - m_2} = \frac{2}{3}$$

$$\Rightarrow \frac{-1 - m_2}{1 - m_2} = \frac{2}{3} \quad \text{OR} \quad \frac{1 + m_2}{1 - m_2} = \frac{2}{3}$$

$$\Rightarrow 2 - 2m_2 = -3 - 3m_2 \quad \text{OR} \quad 3 + 3m_2 = 2 - 2m_2$$

$$\Rightarrow m_2 = -5 \quad \Rightarrow 5m_2 = -1$$

$$m_2 = -\frac{1}{5}$$

Equation: $y - 2 = -5(x - 4) \quad \text{OR} \quad y - 2 = -\frac{1}{5}(x - 4)$

$$y - 2 = -5x + 20 \quad \Rightarrow 5y - 10 = -x + 4$$

$$\Rightarrow 5x + y - 22 = 0 \quad \Rightarrow x + 5y - 14 = 0$$

Q11. Line $2x - 3y - 6 = 0$ meets the axes at $(0, -2)$ and $(3, 0)$.
 \Rightarrow Axial symmetry in the x -axis has points $(0, 2)$ and $(3, 0)$.

$$\text{Slope} = \frac{0-2}{3-0} = \frac{-2}{3}$$

$$\text{Equation : } y - 2 = -\frac{2}{3}(x - 0)$$

$$\Rightarrow 3y - 6 = -2x$$

$$\Rightarrow 2x + 3y - 6 = 0$$

Q12. (i) Slope of $tx + y - 7 = 0 \Rightarrow m_1 = -\frac{t}{1} = -t$

(ii) Slope of $y = 2x + 5 = 0 \Rightarrow m_2 = 2$

$$\Rightarrow \tan 45^\circ = \pm \frac{-t - 2}{1 + (-t)(2)} = \pm \frac{-t - 2}{1 - 2t}$$

$$\Rightarrow \frac{-t - 2}{1 - 2t} = \frac{1}{1} \quad \text{OR} \quad \frac{+t + 2}{1 - 2t} = \frac{1}{1}$$

$$\Rightarrow -t - 2 = 1 - 2t \quad \Rightarrow t + 2 = 1 - 2t$$

$$\Rightarrow t = 3 \quad \Rightarrow 3t = -1$$

$$\Rightarrow t = -\frac{1}{3}$$

Exercise 1.8

Q1. (i) (a) 95°F (b) 58°F (c) 10°C (d) 38°C

(ii) $(50, 10)(95, 35)$

$$\text{slope} = \frac{35 - 10}{95 - 50} = \frac{25}{45} = \frac{5}{9}$$

$$\text{equation : } y - 10 = \frac{5}{9}(x - 50)$$

$$\Rightarrow 9y - 90 = 5x - 250$$

$$\Rightarrow 5x - 9y - 160 = 0$$

$$(\text{iii}) \quad y = 95 \Rightarrow 5x - 9(95) - 160 = 0$$

$$\Rightarrow 5x - 855 - 160 = 0$$

$$\Rightarrow 5x = 1015$$

$$\Rightarrow x = 203^\circ\text{F}$$

Q2. $C = 20 + 4M$

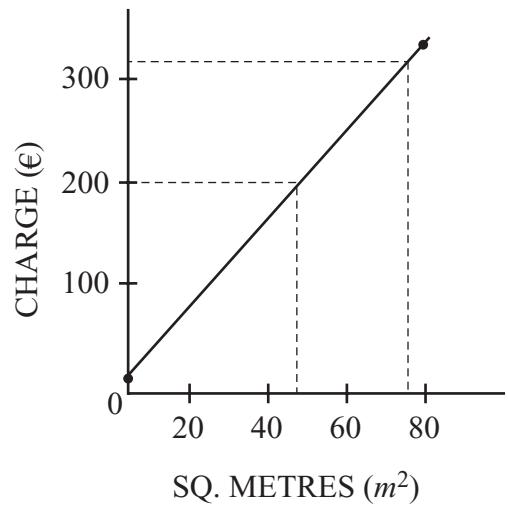
$$M = 0 \Rightarrow C = 20 + 0 = 20 \quad (0, 20)$$

$$M = 80 \Rightarrow C = 20 + 4(80) = 340 \quad (80, 340)$$

(i) $M = 75 \Rightarrow C = €320$

(ii) $C = 200 \Rightarrow M = 45m^2$

(iii) $M = 105 \Rightarrow C = 20 + 4(105) = €440$



Q3. (i) $T = 1 \Rightarrow I = 5000 \left(\frac{8}{100} \right)(1) = 400(1) = €400$

$$T = 2 \Rightarrow I = 5000 \left(\frac{8}{100} \right)(2) = 400(2) = €800$$

$$T = 3 \Rightarrow I = 5000 \left(\frac{8}{100} \right)(3) = 400(3) = €1200$$

(ii) $I = 400T$

(iii) $3500 = 400T \Rightarrow T = 8\frac{3}{4} \text{ years}$

(iv) $A = 400T + 5000$

Q4. (i) $(60, 100), (100, 50)$

(ii) slope $= \frac{50 - 100}{100 - 60} = \frac{-50}{40} = \frac{-5}{4}$

$$\text{equation : } N - 100 = \frac{-5}{4}(P - 60)$$

$$\Rightarrow 4N - 400 = -5P + 300$$

$$\Rightarrow 5P + 4N = 700$$

(iii) $N = 88 \Rightarrow 5P + 4(88) = 700$

$$\Rightarrow 5P + 352 = 700$$

$$\Rightarrow 5P = 348$$

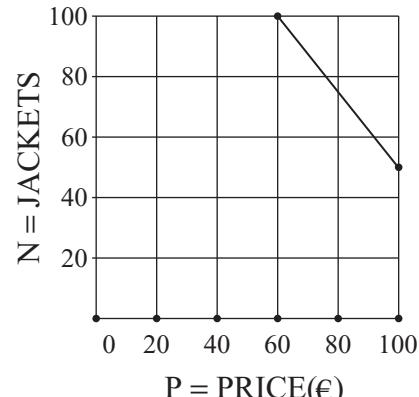
$$\Rightarrow P = €69.60$$

(iv) $P = 72 \Rightarrow 5(72) + 4N = 700$

$$\Rightarrow 360 + 4N = 700$$

$$\Rightarrow 4N = 340$$

$$\Rightarrow N = 85$$



Q5. (i) A : $P = 5 + 2D$ $(0, 5)(10, 25)$

B : $P = 2.2D$ $(0, 0)(10, 22)$

(ii) Line A: Slope = $\frac{25 - 5}{10 - 0} = \frac{20}{10} = 2$

equation : $P - 5 = 2(D - 0)$

$$\Rightarrow P - 5 = 2D$$

$$\Rightarrow P = 5 + 2D$$

Line B: Slope = $\frac{22 - 0}{10 - 0} = 2.2$

equation : $P - 0 = 2.2(D - 0)$

$$\Rightarrow P = 2.2D$$

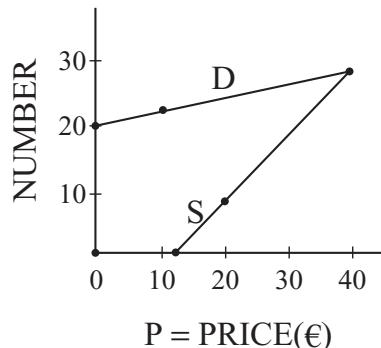
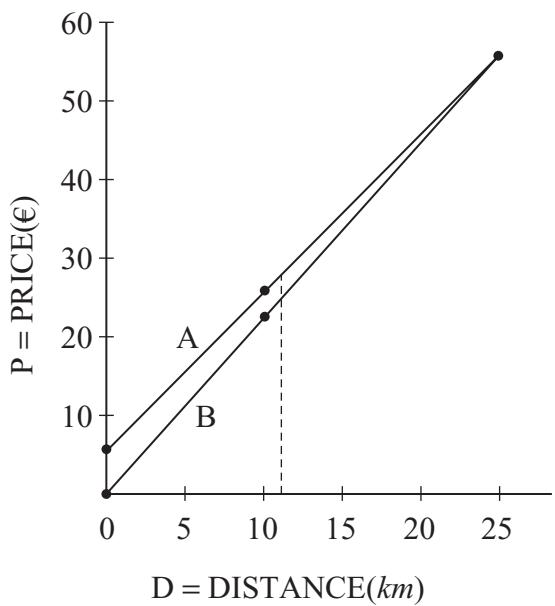
(iii) $D = 25 \text{ km}$

(iv) Firm B

Q6. (i) $D = 20 + 0.2p$ $(0, 20)(10, 22)$

$S = -12 + p$ $(12, 0)(20, 8)$

(ii) $p = €40$ and 28 articles



TEST YOURSELF 1

A Questions

Q1. slope $l = \frac{-3}{-2} = \frac{3}{2}$ \Rightarrow perpendicular slope = $-\frac{2}{3}$

equation: $y - 4 = -\frac{2}{3}(x + 1)$

$$\Rightarrow 3y - 12 = -2x - 2$$

$$\Rightarrow 2x + 3y - 10 = 0$$

Q2. Area = $\frac{1}{2}|(3)(4) - (-2)(-2)|$

$$= \frac{1}{2}|12 - 4|$$

$$= \frac{1}{2}(8)$$

$$= 4$$

Q3. slope = $\frac{6+2}{1-3} = \frac{8}{-2} = -4 = m_1$

slope = $\frac{-2}{a} = m_2$

perpendicular lines $\Rightarrow m_1 \cdot m_2 = -4 \cdot \frac{-2}{a} = \frac{8}{a} = -1$

$$\Rightarrow -a = 8$$

$$\Rightarrow a = -8$$

Q4. slope = $\frac{6-a}{-3-6} = \frac{6-a}{-9}$

$$\Rightarrow \frac{6-a}{-9} = \frac{1}{3}$$

$$\Rightarrow 18 - 3a = -9$$

$$\Rightarrow -3a = -27$$

$$\Rightarrow a = 9$$

Q5. (i) slope = $\frac{3}{2}$

(ii) $x\text{-axis} \Rightarrow y = 0 \Rightarrow \frac{3}{2}x - 2 = 0$

$$\Rightarrow 3x - 4 = 0$$

$$\Rightarrow 3x = 4$$

$$\Rightarrow x = \frac{4}{3} \quad P\left(\frac{4}{3}, 0\right)$$

$$y\text{-axis} \Rightarrow x = 0 \Rightarrow y = -2 \quad Q(0, -2)$$

(iii) Area = $\frac{1}{2} \left| \left(\frac{4}{3}\right)(-2) - (0)(0) \right|$

$$= \frac{1}{2} \left| \frac{-8}{3} \right| = \frac{4}{3}$$

Q6. (1,1) (6,3)

slope $l = \frac{3-1}{6-1} = \frac{2}{5}$

equation : $y - 1 = \frac{2}{5}(x - 1)$

$$\Rightarrow 5y - 5 = 2x - 2$$

$$\Rightarrow 2x - 5y + 3 = 0$$

Q7. (i) slope = $k^2 = m_1$
 $\text{slope} = \frac{4}{2k} = \frac{2}{k} = m_2$ Perpendicular lines $\Rightarrow m_1 \cdot m_2 = -1$
 $\Rightarrow k^2 \cdot \frac{2}{k} = -1$
 $\Rightarrow k = \frac{-1}{2}$

(ii) $l_1 : y = \frac{1}{4}x + 12$
 $l_2 : 2\left(-\frac{1}{2}\right)y = 4x + 5 \Rightarrow -y = 4x + 5$
 $\Rightarrow y = -4x - 5$

$$\begin{aligned} l_1 \cap l_2 &\Rightarrow \frac{1}{4}x + 12 = -4x - 5 \\ &\Rightarrow x + 48 = -16x - 20 \\ &\Rightarrow 17x = -68 \\ &\Rightarrow x = -4 \\ &\Rightarrow y = \frac{1}{4}(-4) + 12 = 11 \quad (-4, 11) \end{aligned}$$

Q8. (a) slope $m_1 = \frac{-2}{1} = -2$
 $\text{slope } m_2 = -\frac{1}{-2} = \frac{1}{2} \Rightarrow m_1 \cdot m_2 = (-2)\left(\frac{1}{2}\right) = -1 : \text{YES}$

(b) slope $m_1 = 3$
 $\text{slope } m_2 = -\frac{1}{3} \Rightarrow m_1 \cdot m_2 = (3)\left(-\frac{1}{3}\right) = -1 : \text{YES}$

(c) slope $m_1 = \frac{-2}{1} = -2$
 $\text{slope } m_2 = \frac{1}{2} \Rightarrow m_1 \cdot m_2 = (-2)\left(\frac{1}{2}\right) = -1 : \text{YES}$

(d) slope $m_1 = -\frac{1}{3}$
 $\text{slope } m_2 = -\frac{3}{1} = -3 \Rightarrow m_1 \cdot m_2 = \left(-\frac{1}{3}\right)(-3) = 1 \text{ (i.e. } \neq -1\text{)} : \text{NO}$

Q9. slope $= -\frac{1}{2} \Rightarrow \text{perpendicular slope} = 2$
equation : $y - 2 = 2(x - 5)$
 $\Rightarrow y - 2 = 2x - 10$
 $\Rightarrow 2x - y - 8 = 0$

Q10. midpoint = $\left(\frac{1+5}{2}, \frac{2+4}{2} \right) = (3, 3)$
 slope = $\frac{4-2}{5-1} = \frac{2}{4} = \frac{1}{2}$ \Rightarrow perpendicular slope = -2
 equation : $y - 3 = -2(x - 3)$
 $\Rightarrow y - 3 = -2x + 6$
 $\Rightarrow 2x + y = 9$
 point $(0, k) \Rightarrow 2(0) + k = 9$
 $0 + k = 9$
 $k = 9$

B Questions

Q1. (i) $|PD| = \frac{|3(-1) - 4(-5) - 2|}{\sqrt{(3)^2 + (-4)^2}} = \frac{|-3 + 20 - 2|}{\sqrt{25}} = \frac{15}{5} = 3$
 (ii) $|PD| = \frac{|3(-1) - 4(-5) + k|}{\sqrt{(3)^2 + (-4)^2}} = \frac{|-3 + 20 + k|}{\sqrt{25}} = \frac{|17 + k|}{5}$
 $\Rightarrow \frac{17 + k}{5} = \frac{3}{1}$ OR $\frac{17 + k}{5} = -3$
 $\Rightarrow 17 + k = 15$ $\Rightarrow 17 + k = -15$
 $\Rightarrow k = -2$ (not valid) $\Rightarrow k = -32$

Q2. (i) Point = $\left[\frac{2(8) + 3(-7)}{2+3}, \frac{2(-2) + 3(3)}{2+3} \right]$
 $= \left(-\frac{5}{5}, \frac{5}{5} \right) = (-1, 1)$
 (ii) (a) $x\text{-axis} \Rightarrow y = 0 \Rightarrow 2x = 6$
 $\Rightarrow x = 3 \quad (3, 0)$
 $y\text{-axis} \Rightarrow x = 0 \Rightarrow ky = 6$
 $\Rightarrow y = \frac{6}{k} \quad \left(0, \frac{6}{k} \right)$

(b) Area = $\frac{1}{2} \left| \left(3\right) \left(\frac{6}{k}\right) - (0)(0) \right| = k$
 $\Rightarrow \frac{18}{k} = \frac{2k}{1}$
 $\Rightarrow 2k^2 = 18$
 $\Rightarrow k^2 = 9$
 $\Rightarrow k = 3$

Q3. (i) slope = $\frac{-2}{1} = -2$ point $(2, 5)$

$$\text{equation : } y - 5 = -2(x - 2)$$

$$\Rightarrow y - 5 = -2x + 4$$

$$\Rightarrow 2x + y - 9 = 0$$

(ii) (a) slope = $\frac{-2}{1} = -2 \Rightarrow$ perpendicular slope = $\frac{1}{2}$ point $(1, k)$

$$\text{equation : } y - k = \frac{1}{2}(x - 1)$$

$$\Rightarrow 2y - 2k = x - 1$$

$$\Rightarrow x - 2y + 2k - 1 = 0$$

(b) Origin $(0, 0) \Rightarrow 0 - 2(0) + 2k - 1 = 0$

$$\Rightarrow 2k = 1$$

$$\Rightarrow k = \frac{1}{2}$$

Q4. Centroid = $\frac{4+1+h}{3} = 2, \quad \frac{2+7+k}{3} = 4$

$$\Rightarrow 3 + h = 6 \quad \Rightarrow 9 + k = 12$$

$$\Rightarrow h = 3 \quad \Rightarrow k = 3$$

Q5. (i) Points on ladder are : $(2.5, 0)$ and $(1.5, 2)$

$$\text{Slope} = \frac{2-0}{1.5-2.5} = \frac{2}{-1} = -2$$

Equation of line of ladder: point = $(2.5, 0)$; slope = -2

$$y - 0 = -2(x - 2.5)$$

$$y = -2x + 5$$

$\Rightarrow 2x + y - 5 = 0$ is the equation

(ii) To find the point A , let $x = 0$ in the equation $2x + y - 5 = 0$

$$x = 0 \Rightarrow y = 5$$

\therefore the height of point $A = 5$ metres

(iii) Length = $\sqrt{(2.5-0)^2 + (5-0)^2}$

$$= \sqrt{6.25 + 25}$$

$$= \sqrt{31.25}$$

$$= 5.59017 \text{ m}$$

$$= 559 \text{ cm}$$

Q6. slope = $\frac{8+2}{1-1} = \frac{10}{0}$ = undefined \Rightarrow perpendicular slope = 0, point (7,1)
 equation : $y - 1 = 0(x - 7)$
 $\Rightarrow y = 1$
 slope = $\frac{8-1}{1-7} = \frac{7}{-6} = -\frac{7}{6}$ \Rightarrow perpendicular slope = $\frac{6}{7}$, point (1,-2)
 equation : $y + 2 = \frac{6}{7}(x - 1)$
 $\Rightarrow 7y + 14 = 6x - 6$
 $\Rightarrow 6x - 7y - 20 = 0$
 $y = 1 \Rightarrow 6x - 7(1) - 20 = 0$
 $\Rightarrow 6x = 27$
 $\Rightarrow x = 4\frac{1}{2}$
 Orthocentre = $\left(4\frac{1}{2}, 1\right)$

Q7. (i) $y - 6 = m(x + 4)$
 (ii) $x\text{-axis} \Rightarrow y = 0 \Rightarrow -6 = m(x + 4)$
 $-6 = mx + 4m$
 $\Rightarrow x = \frac{-6-4m}{m} \quad \left(\frac{-6-4m}{m}, 0\right)$
 $y\text{-axis} \Rightarrow x = 0 \Rightarrow y - 6 = m(4)$
 $\Rightarrow y = 4m + 6 \quad (0, 4m + 6)$
 (iii) Area = $\frac{1}{2} \left| \left(\frac{-6-4m}{m} \right) (4m + 6) - (0)(0) \right| = 54$
 $\Rightarrow |-24m - 36 - 16m^2 - 24m| = 108m$
 $\Rightarrow 16m^2 + 48m + 36 = 108m$
 $\Rightarrow 4m^2 - 15m + 9 = 0$
 $\Rightarrow (4m - 3)(m - 3) = 0 \Rightarrow m = \frac{3}{4} \text{ OR } m = 3$

$$\begin{aligned}
 \text{Q8. (i)} \quad |\text{PD}| &= \frac{|3(3) - 4(k) + 7|}{\sqrt{(3)^2 + (-4)^2}} = 6 \\
 &\Rightarrow \frac{|16 - 4k|}{5} = 6 \\
 &\Rightarrow |16 - 4k| = 30 \\
 &\Rightarrow 16 - 4k = 30 \quad \text{OR} \quad 16 - 4k = -30 \\
 &\Rightarrow -4k = 14 \quad \Rightarrow -4k = -46 \\
 &\Rightarrow k = -3\frac{1}{2} \quad \Rightarrow k = 11\frac{1}{2}
 \end{aligned}$$

$$\text{ANS: } k = -3\frac{1}{2}$$

$$\begin{aligned}
 \text{(ii)} \quad \text{slope} &= -\frac{3}{-4} = \frac{3}{4} \quad \text{point} = \left(3, -3\frac{1}{2}\right) \\
 \text{equation: } y + 3\frac{1}{2} &= \frac{3}{4}(x - 3) \\
 &\Rightarrow 4y + 14 = 3x - 9 \\
 &\Rightarrow 3x - 4y - 23 = 0
 \end{aligned}$$

$$\begin{aligned}
 \text{Q9. (i)} \quad \text{equation: } y - 5 &= m(x - 2) \\
 &\Rightarrow y - 5 = mx - 2m \\
 &\Rightarrow mx - y + 5 - 2m = 0 \\
 \text{(ii)} \quad x\text{-axis} \Rightarrow y = 0 &\Rightarrow mx = -5 + 2m \\
 &\Rightarrow x = \frac{-5 + 2m}{m} \quad \left(\frac{-5 + 2m}{m}, 0\right) \\
 y\text{-axis} \Rightarrow x = 0 &\Rightarrow -y + 5 - 2m = 0 \\
 &\Rightarrow y = 5 - 2m \quad (0, 5 - 2m) \\
 \text{(iii)} \quad \text{Area} &= \frac{1}{2} \left| \left(\frac{-5 + 2m}{m} \right) (5 - 2m) - (0)(0) \right| = 36 \\
 &\Rightarrow |-25 + 10m + 10m - 4m^2| = 72m \\
 &\Rightarrow -4m^2 + 20m - 25 = 72m \\
 &\Rightarrow 4m^2 + 52m + 25 = 0 \\
 &\Rightarrow (2m + 1)(2m + 25) = 0 \\
 &\Rightarrow m = -\frac{1}{2}, \quad m = \frac{-25}{2}
 \end{aligned}$$

$$Q10. \text{ (i) slope } AB = \frac{k-4}{1-3} = \frac{k-4}{-2} = \frac{4-k}{2}$$

$$\text{(ii) slope } BC = \frac{-3-k}{4-1} = \frac{-3-k}{3}$$

$$\text{Perpendicular lines} \Rightarrow \left(\frac{k-4}{-2}\right) \cdot \left(\frac{-3-k}{3}\right) = -1$$

$$\Rightarrow -3k + 12 - k^2 + 4k = 6$$

$$\Rightarrow -k^2 + k + 6 = 0$$

$$\Rightarrow k^2 - k - 6 = 0$$

$$\Rightarrow (k+2)(k-3) = 0$$

$$\Rightarrow k = -2 \quad \text{OR} \quad k = 3$$

$$\text{(iii) } A(3,4) \rightarrow (0,0) \quad B(1,3) \rightarrow (-2,-1) \quad C(4,-3) \rightarrow (1,-7)$$

$$\text{Area } \Delta = \frac{1}{2} |(-2)(-7) - (1)(-1)|$$

$$= \frac{1}{2} |14 + 1|$$

$$= \frac{15}{2}$$

$$\text{Area rectangle} = 2 \left(\frac{15}{2} \right) = 15 \text{ sq.units}$$

C Questions

$$Q1. \text{ (i) } x\text{-axis} \Rightarrow Q(x_1, 0); \quad y\text{-axis} \Rightarrow R(0, y_2)$$

$$|PQ| : |PR| = 3 : 1 \Rightarrow \frac{(3)(0) + (1)(x_1)}{3+1} = 2$$

$$\Rightarrow x_1 = 8 \quad \Rightarrow Q(8, 0)$$

$$\text{and} \quad \frac{(3)(y_2) + 1(0)}{3+1} = -9$$

$$\Rightarrow 3y_2 = -36$$

$$\Rightarrow y_2 = -12 \quad \Rightarrow R(0, -12)$$

(ii) $x\text{-axis} \Rightarrow y = 0 \Rightarrow 3x = c$

$$\Rightarrow x = \frac{c}{3} \quad P\left(\frac{c}{3}, 0\right)$$

$y\text{-axis} \Rightarrow x = 0 \Rightarrow 2y = c$

$$\Rightarrow y = \frac{c}{2} \quad Q\left(0, \frac{c}{2}\right)$$

$$\text{Area} = \frac{1}{2} \left| \left(\frac{c}{3} \right) \left(\frac{c}{2} \right) - (0)(0) \right| = 24$$

$$\Rightarrow \left| \frac{c^2}{6} \right| = 48$$

$$\Rightarrow c^2 = 288$$

$$\Rightarrow c = \pm 12\sqrt{2}$$

Q2. Line $4x - 3y + 8 = 0 \Rightarrow$ Equation of parallel lines: $4x - 3y + k = 0$

$$\text{Perpendicular Distance: } \frac{|4(0) - 3(0) + k|}{\sqrt{(4)^2 + (-3)^2}} = 4$$

$$\Rightarrow \frac{|k|}{5} = 4$$

$$\Rightarrow |k| = 20$$

$$k = 20 \quad \text{OR} \quad k = -20$$

$$\text{ANS: } 4x - 3y + 20 = 0 \quad \text{OR} \quad 4x - 3y - 20 = 0$$

Q3. (i) midpoint = $\left(\frac{0-3}{2}, \frac{-9+6}{2} \right) = \left(\frac{-3}{2}, \frac{-3}{2} \right)$
 slope = $\frac{6+9}{-3-0} = -5 \Rightarrow$ Perpendicular slope = $\frac{1}{5}$
 \Rightarrow equation : $y + \frac{3}{2} = \frac{1}{5} \left(x + \frac{3}{2} \right)$
 $\Rightarrow 5y + \frac{15}{2} = x + \frac{3}{2}$
 $\Rightarrow x - 5y = 6$
 midpoint = $\left(\frac{-3+8}{2}, \frac{6+3}{2} \right) = \left(\frac{5}{2}, \frac{9}{2} \right)$
 slope = $\frac{3-6}{8+3} = \frac{-3}{11} \Rightarrow$ Perpendicular slope = $\frac{11}{3}$
 \Rightarrow equation : $y - \frac{9}{2} = \frac{11}{3} \left(x - \frac{5}{2} \right)$
 $\Rightarrow 3y - \frac{27}{2} = 11x - \frac{55}{2}$
 $\Rightarrow 11x - 3y = 14$
 and
$$\begin{array}{r} x - 5y = 6 \\ 55x - 15y = 70 \\ \hline 3x - 15y = 18 \end{array}$$

 subtract : $52x = 52$
 $\Rightarrow x = 1$
 $\Rightarrow 11 - 3y = 14$
 $\Rightarrow -3y = 3$
 $\Rightarrow y = -1$
 Circumcentre = $(1, -1)$

(ii) Radius = $\sqrt{(1-0)^2 + (-1+9)^2} = \sqrt{1+64} = \sqrt{65}$

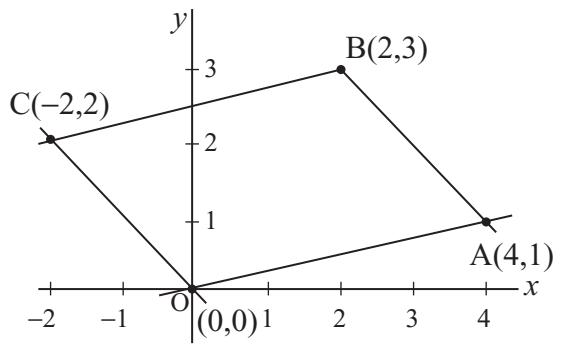
(iii) Area circle = $\pi (\sqrt{65})^2 = 65\pi$ sq.units

Q4. (i) slope $OA = \frac{1}{4} \Rightarrow$ slope $BC = \frac{1}{4}$

equation $BC: y - 3 = \frac{1}{4}(x - 2)$

$$\Rightarrow 4y - 12 = x - 2$$

$$\Rightarrow x - 4y + 10 = 0$$



(ii) slope $OC = -1$, origin $(0,0)$

equation $OC: y - 0 = -1(x - 0)$

$$\Rightarrow y = -x$$

$$\Rightarrow x + y = 0$$

and $x - 4y = -10$

subtract : $5y = +10$

$$\Rightarrow y = +2$$

$$\Rightarrow x = -2 \quad C(-2, 2)$$

midpoint $OB = \left(1, 1\frac{1}{2}\right)$

central symmetry of $C(-2, 2)$ through $\left(1, 1\frac{1}{2}\right)$ is $(4, 1)$

$$\Rightarrow A = (4, 1)$$

Q5. slope $= -\frac{1}{-2} = \frac{1}{2} = m_1$

$$\Rightarrow \tan 45^\circ = \pm \frac{\frac{1}{2} - m_2}{1 + \left(\frac{1}{2}\right)(m_2)} = \frac{1}{1}$$

$$\Rightarrow \pm \left(\frac{1}{2} - m_2\right) = 1 + \frac{1}{2}m_2$$

$$\Rightarrow \frac{1}{2} - m_2 = 1 + \frac{1}{2}m_2 \quad \text{OR} \quad -\frac{1}{2} + m_2 = 1 + \frac{1}{2}m_2$$

$$\Rightarrow 1 - 2m_2 = 2 + m_2 \quad \Rightarrow -1 + 2m_2 = 2 + m_2$$

$$\Rightarrow -3m_2 = 1 \quad \Rightarrow \quad m_2 = 3$$

$$\Rightarrow m_2 = -\frac{1}{3}$$

equation : $y - 4 = -\frac{1}{3}(x - 2)$ OR $y - 4 = 3(x - 2)$

$$\Rightarrow 3y - 12 = -x + 2$$

$$\Rightarrow x + 3y - 14 = 0$$

$$\Rightarrow y - 4 = 3x - 6$$

$$\Rightarrow 3x - y - 2 = 0$$

Q6. (i) $R(-1, -5) \rightarrow (0, 0); Q(3, -1) \rightarrow (4, 4); U(-2k, 3k) \rightarrow (-2k+1, 3k+5)$

$$\begin{aligned} \text{Area} &= \frac{1}{2} |(4)(3k+5) - (4)(-2k+1)| = 28 \\ \Rightarrow |12k + 20 + 8k - 4| &= 56 \\ \Rightarrow |20k + 16| &= 56 \\ \Rightarrow 20k + 16 &= 56 \quad \text{OR} \quad 20k + 16 = -56 \\ \Rightarrow 20k &= 40 \quad \Rightarrow 20k = -72 \\ \Rightarrow k &= 2 \quad \Rightarrow k = -3.6 \end{aligned}$$

ANS : $k = 2$

(ii) slope TS = $\frac{-3}{11}$ S(13, 9)

$$\begin{aligned} \Rightarrow \text{equation TS} : y - 9 &= \frac{-3}{11}(x - 13) \\ \Rightarrow 11y - 99 &= -3x + 39 \\ \Rightarrow 3x + 11y &= 138 \end{aligned}$$

$$\text{slope SR} = \frac{9+5}{13+1} = \frac{14}{14} = 1 \Rightarrow \text{slope TU} = 1 \text{ and } U(-4, 6)$$

$$\Rightarrow \text{equation TU} : y - 6 = 1(x + 4)$$

$$\begin{aligned} \Rightarrow y - 6 &= x + 4 \\ \Rightarrow x - y &= -10 \end{aligned}$$

and $\frac{3x + 11y = 138}{3x - 3y = -30}$

$$\begin{aligned} \frac{3x + 11y = 138}{3x - 3y = -30} \\ \hline \end{aligned}$$

subtract : $-14y = -168$

$$\Rightarrow y = 12$$

$$\Rightarrow x - 12 = -10$$

$$\Rightarrow x = 2 \Rightarrow T(2, 12)$$

Q7. (i) Graph

(ii) slope = $\frac{37 - 35}{8.5 - 7} = \frac{2}{1.5} = \frac{4}{3}$

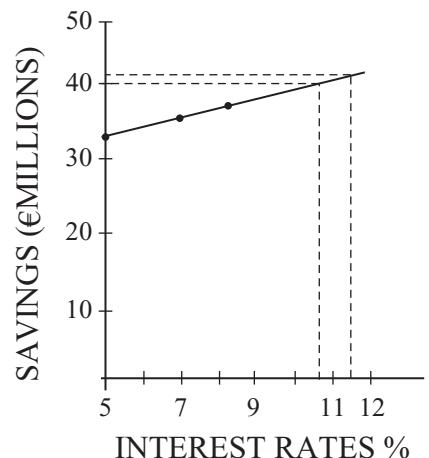
$$\text{equation} : S - 35 = \frac{4}{3}(I - 7)$$

$$\Rightarrow 3S - 105 = 4I - 28$$

$$\Rightarrow 4I - 3S + 77 = 0$$

(iii) €41 million

(iv) $10\frac{3}{4}\%$



Q8. Equation: $y + 4 = m(x - 2)$

$$\Rightarrow y + 4 = mx - 2m$$

$$\Rightarrow mx - y - 2m - 4 = 0$$

$$x\text{-axis} \Rightarrow y = 0 \quad mx = 2m + 4$$

$$\Rightarrow x = \frac{2m+4}{m} \quad \left(\frac{2m+4}{m}, 0 \right)$$

$$y\text{-axis} \Rightarrow x = 0 \quad -y = 2m + 4$$

$$\Rightarrow y = -2m - 4 \quad (0, -2m - 4)$$

$$\text{Hence, } \left(\frac{2m+4}{m} \right) + (-2m - 4) = -4$$

$$\Rightarrow 2m + 4 - 2m^2 - 4m = -4m$$

$$\Rightarrow 2m^2 - 2m - 4 = 0$$

$$\Rightarrow m^2 - m - 2 = 0$$

$$\Rightarrow (m+1)(m-2) = 0$$

$$\Rightarrow m = -1 \quad \text{OR} \quad m = 2$$

$$\tan \theta = \pm \frac{-1-2}{1+(-1)(2)} = \pm \frac{-3}{-1} = \pm 3$$

Acute $\Rightarrow \tan \theta = 3$

Q9. (i) $4(2) + 3(-1) - 5 = 0$

$$\Rightarrow 8 - 3 - 5 = 0$$

$\Rightarrow 8 - 8 = 0$ True

(ii) $4x + 3y + c = 0$

$$(iii) |\text{PD}| = \frac{|4(2) + 3(-1) + c|}{\sqrt{(4)^2 + (3)^2}} = 2$$

$$\Rightarrow \frac{|5+c|}{5} = 2$$

$$\Rightarrow |5+c| = 10$$

$$\Rightarrow 5+c = 10 \quad \text{OR} \quad 5+c = -10$$

$$\Rightarrow c = 5 \quad \text{OR} \quad c = -15$$

$$\Rightarrow 4x + 3y + 5 = 0 \quad \text{OR} \quad 4x + 3y - 15 = 0$$

$$\text{Q10. (i)} \quad \text{slope} = \frac{-t}{t+2}$$

$$\begin{aligned}\text{(ii)} \quad \text{slope } m_1 &= -\frac{1}{-2} = \frac{1}{2} \\ \tan 45^\circ &= \pm \frac{\frac{1}{2} - \left(\frac{-t}{t+2} \right)}{1 + \left(\frac{1}{2} \right) \left(\frac{-t}{t+2} \right)} \\ &\Rightarrow \pm \frac{\frac{t+2+t(2)}{2(t+2)}}{\frac{2t+4-t}{2(t+2)}} = 1 \\ &\Rightarrow \frac{3t+2}{t+4} = \pm 1 \\ &\Rightarrow 3t+2 = \pm(t+4) \\ &\Rightarrow 3t+2 = -t-4 \qquad \text{OR} \qquad 3t+2 = t+4 \\ &\Rightarrow 4t = -6 \qquad \qquad \qquad \Rightarrow 2t = 2 \\ &\Rightarrow t = -\frac{3}{2} \qquad \qquad \qquad \Rightarrow t = 1\end{aligned}$$

Chapter 2 Trigonometry 1

Exercise 2.1

Q1. (i) $30^\circ = \frac{\pi}{6}$

(ii) $45^\circ = \frac{\pi}{4}$

(iii) $150^\circ = 5(30^\circ) = \frac{5\pi}{6}$

(iv) $135^\circ = 3(45^\circ) = \frac{3\pi}{4}$

(v) $36^\circ = \frac{\pi}{180} \times 36^\circ = \frac{\pi}{5}$

(vi) $240^\circ = 4(60^\circ) = \frac{4\pi}{3}$

(vii) $390^\circ = 13(30^\circ) = \frac{13\pi}{6}$

Q2. (i) 180°

(ii) 90°

(iii) 30°

(iv) $\frac{5\pi}{6} = \frac{5 \times 180^\circ}{6} = 150^\circ$

(v) $\frac{4\pi}{9} = \frac{4 \times 180^\circ}{9} = 80^\circ$

(vi) $\frac{11\pi}{6} = \frac{11 \times 180^\circ}{6} = 330^\circ$

(vii) $\frac{5\pi}{12} = \frac{5 \times 180^\circ}{12} = 75^\circ$

Q3. (i) $l = r\theta \Rightarrow l = (4)(2) = 8 \text{ cm}$

(ii) $l = (4)(4) = 16 \text{ cm}$

(iii) $l = (4)\left(2\frac{1}{2}\right) = 10 \text{ cm}$

(iv) $l = 4\left(\frac{5}{4}\right) = 5 \text{ cm}$

Q4. (i) $r\theta = l \Rightarrow 6\theta = 6 \Rightarrow \theta = 1 \text{ radian}$

(ii) $6\theta = 12 \Rightarrow \theta = 2 \text{ radians}$

(iii) $6\theta = 3 \Rightarrow \theta = \frac{1}{2} \text{ radian}$

$$(iv) \quad 6\theta = 9 \Rightarrow \theta = 1\frac{1}{2} \text{ radians}$$

$$(v) \quad 6\theta = 7\frac{1}{2} \Rightarrow \theta = 1\frac{1}{4} \text{ radians}$$

$$Q5. \quad r\theta = l \Rightarrow 2r = 15 \Rightarrow r = 7\frac{1}{2} \text{ cm}$$

$$Q6. \quad 5\theta = 6 \Rightarrow \theta = 1\frac{1}{5} \text{ radians}$$

$$\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2}(5)^2 1\frac{1}{5} = \frac{1}{2} \cdot \frac{25}{1} \cdot \frac{6}{5} = 15 \text{ cm}^2$$

$$Q7. \quad \text{Area} = \frac{1}{2}(8)^2 \cdot \theta = 40$$

$$\Rightarrow 32\theta = 40$$

$$\Rightarrow \theta = \frac{40}{32} = \frac{5}{4} = 1\frac{1}{4} \text{ radians}$$

$$Q8. \quad l = 2\pi r \Rightarrow 2\pi r = 12\pi$$

$$\Rightarrow r = 6 \text{ cm}$$

$$\text{Area} = \frac{1}{2}(6)^2 \theta = 3\pi$$

$$\Rightarrow 18\theta = 3\pi$$

$$\Rightarrow \theta = \frac{3\pi}{18} = \frac{\pi}{6} \text{ radians}$$

$$Q9. \quad \text{Area} = \frac{1}{2}(6)^2 \theta = 27$$

$$\Rightarrow 18\theta = 27$$

$$\theta = \frac{27}{18} = \frac{3}{2} \text{ radians}$$

Q10. Shaded Area = Area large sector – Area small sector

$$= \frac{1}{2}(8)^2 \cdot \frac{\pi}{4} - \frac{1}{2}(2)^2 \cdot \frac{\pi}{4}$$

$$= 8\pi - \frac{\pi}{2}$$

$$= \frac{15\pi}{2} \text{ cm}^2$$

Q11. (i) $4\theta = 10$

$$\Rightarrow \theta = \frac{10}{4} = \frac{5}{2} \text{ radians}$$

$$(ii) \quad \theta = \frac{5}{2} \cdot \frac{180}{\pi} = 143.239^\circ = 143^\circ$$

Q12. Shaded Area = Area square – Area sector

$$= (2)(2) - \frac{1}{2}(2)^2 \cdot \frac{\pi}{2}$$

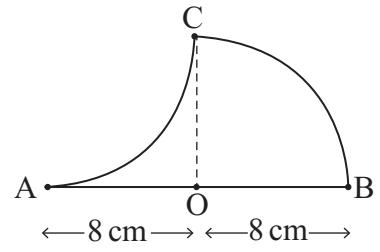
$$= (4 - \pi) \text{ cm}^2$$

Q13. Shaded region AOC = Area square – Area sector

$$= (8)^2 - \frac{1}{2}(8)^2 \cdot \frac{\pi}{2}$$

$$= (64 - 16\pi) \text{ cm}^2$$

$$\text{Shaded region COB} = \frac{1}{2}(8)^2 \cdot \frac{\pi}{2} = 16\pi \text{ cm}^2$$



Shaded figure = Shaded region AOC + Shaded region COB

$$= 64 - 16\pi + 16\pi$$

$$= 64 \text{ cm}^2$$

Q14. (i) A is a centre $\Rightarrow |AB| = 6 \text{ cm}$ (radius)

C is a centre $\Rightarrow |CB| = 6 \text{ cm}$ (radius)

$\Rightarrow \Delta ABC$ is an equilateral triangle

$$\Rightarrow |\angle ABC| = 60^\circ$$

$$(ii) \quad \text{Length arc AB} = 6 \cdot \frac{\pi}{3} = 2\pi \text{ cm}$$

$$(iii) \quad \text{Shaded region AB} = \text{Area sector} - \text{Area } \Delta ABC$$

$$= \frac{1}{2}(6)^2 \cdot \frac{\pi}{3} - \frac{1}{2}(6)(6)\sin 60^\circ$$

$$= 6\pi - 18 \cdot \frac{\sqrt{3}}{2}$$

$$= (6\pi - 9\sqrt{3}) \text{ cm}^2$$

$$\text{Total shaded region} = 2(6\pi - 9\sqrt{3}) = (12\pi - 18\sqrt{3}) \text{ cm}^2$$

$$\text{Q15. (i)} \quad \text{Length} = r\theta + 2r = 40$$

$$r\theta = 40 - 2r$$

$$\theta = \frac{40 - 2r}{r}$$

$$\text{(ii)} \quad \text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2}r^2\left(\frac{40 - 2r}{r}\right) = 100$$

$$\Rightarrow 40r - 2r^2 = 200$$

$$\Rightarrow r^2 - 20r + 100 = 0$$

$$\Rightarrow (r - 10)(r - 10) = 0$$

$$\Rightarrow r = 10 \text{ cm}$$

$$\text{(iii)} \quad \theta = \frac{40 - 2(10)}{10} = \frac{40 - 20}{10} = 2 \text{ radians}$$

Exercise 2.2

- Q1.** (i) 0.7431
 (ii) 0.2756
 (iii) 0.5407
 (iv) 0.7266
 (v) 0.5914

- Q2.** (i) 48°
 (ii) 69°
 (iii) 55°
 (iv) 78°
 (v) 42°
 (vi) 12°

Q3. (i) $\theta = \sin^{-1} \frac{2}{3} = 41.81^\circ = 42^\circ$

(ii) $\theta = \cos^{-1} \frac{3}{5} = 53.13^\circ = 53^\circ$

(iii) $\theta = \tan^{-1} \frac{7}{8} = 41.186 = 41^\circ$

(iv) $\theta = \sin^{-1} \frac{2}{5} = 23.578 = 24^\circ$

Q4. (i) $\sin^2 45^\circ + \cos^2 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$

(ii) $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$
 $= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$

(iii) $\cos^2 60^\circ + \cos 60^\circ \sin 30^\circ$
 $= \left(\frac{1}{2}\right)^2 + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Q5. $\sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{4} + \sin^2 \frac{\pi}{3}$
 $= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$
 $= \frac{1}{4} + \frac{1}{2} + \frac{3}{4} = \frac{3}{2}$

$$\begin{aligned} \text{Q6. } |XZ| &\Rightarrow \sin 30^\circ = \frac{12}{|XZ|} \\ &\Rightarrow \frac{1}{2} = \frac{12}{|XZ|} \quad \Rightarrow |XZ| = 24 \end{aligned}$$

$$\begin{aligned} |RZ| &\Rightarrow |RZ|^2 + (12)^2 = (24)^2 \\ &\Rightarrow |RZ|^2 + 144 = 576 \\ &\Rightarrow |RZ|^2 = 432 \\ &\Rightarrow |RZ| = \sqrt{432} = 12\sqrt{3} \end{aligned}$$

$$\begin{aligned} |YR| &\Rightarrow |YR|^2 + (12)^2 = (13)^2 \\ &\Rightarrow |YR|^2 + 144 = 169 \\ &\Rightarrow |YR|^2 = 25 \quad \Rightarrow |YR| = 5 \end{aligned}$$

$$\begin{aligned} \text{Hence, perimeter} &= 24 + 13 + 5 + 12\sqrt{3} \\ &= 42 + 12\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{Q7. (i)} \quad \tan 60^\circ &= \frac{x}{8} \\ &\Rightarrow x = 8 \tan 60^\circ \\ &= 13.856 \\ &= 13.9 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \sin A &= \frac{13.9}{20} = 0.695 \\ &\Rightarrow A = \sin^{-1} 0.695 \\ &= 44.02 \\ &= 44^\circ \end{aligned}$$

$$\begin{aligned} \text{Q8. (i)} \quad \sin 30^\circ &= \frac{|RT|}{\sqrt{8}} \\ &\Rightarrow \frac{1}{2} = \frac{|RT|}{\sqrt{8}} \\ &\Rightarrow 2|RT| = \sqrt{8} = 2\sqrt{2} \\ &\Rightarrow |RT| = \sqrt{2} \end{aligned}$$

$$\begin{aligned}
\text{(ii)} \quad & |PT|^2 + (\sqrt{2})^2 = (\sqrt{8})^2 \\
\Rightarrow & |PT|^2 + 2 = 8 \\
\Rightarrow & |PT|^2 = 6 \\
\Rightarrow & |PT| = \sqrt{6} \\
\Delta RTQ \text{ is isosceles} \quad & \Rightarrow |RT| = |TQ| = \sqrt{2} \\
\Rightarrow \text{Area } \Delta RPQ &= \frac{1}{2}(\sqrt{2} + \sqrt{6}).(\sqrt{2}) \\
&= \frac{1}{2}(\sqrt{4} + \sqrt{12}) \\
&= \frac{1}{2}(2 + 2\sqrt{3}) \\
&= 1 + \sqrt{3}
\end{aligned}$$

Exercise 2.3

- Q1.** (i) 0.7660
(ii) -0.7660
(iii) 0.6428
(iv) -0.6428
(v) -0.8192
(vi) 0.5736

- Q2.** (i) $0.66913 = 0.6691$
(ii) $-0.84804 = -0.8480$
(iii) $-0.900404 = -0.9004$
(iv) $-0.93358 = -0.9336$

- Q3.** (i) $\sin 50^\circ$
(ii) $-\cos 65^\circ$
(iii) $-\tan 20^\circ$
(iv) $-\cos 40^\circ$
(v) $-\sin 70^\circ$
(vi) $-\tan 60^\circ$

- Q4.** (i) $\sin 120^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$
(ii) $\cos 135^\circ = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$
(iii) $\sin 240^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$
(iv) $\sin 210^\circ = -\sin 30^\circ = -\frac{1}{2}$
(v) $\cos 330^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$
(vi) $\tan 225^\circ = \tan 45^\circ = 1$
(vii) $\cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$
(viii) $\sin 300^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

- Q5.** (i) 3rd
(ii) 1st
(iii) 2nd
(iv) 1st

Q6. (i) 124°

(ii) 68°

(iii) 240°

(iv) 345°

(v) 75°

Q7. $\sin A = 0.2167 \Rightarrow A = \sin^{-1}(0.2167)$

$$\Rightarrow A = 12.515^\circ = 13^\circ$$

$$2^{\text{nd}} \text{ quadrant} \Rightarrow A = 180^\circ - 13^\circ = 167^\circ$$

Q8. (i) $\cos A = 0.8428 \Rightarrow A = \cos^{-1}(0.8428)$

$$= 32.56^\circ = 33^\circ \text{ (reference angle)}$$

$$\cos A = -0.8428 \Rightarrow 2^{\text{nd}} \text{ quadrant} \Rightarrow A = 180^\circ - 33^\circ = 147^\circ$$

$$\text{and } 3^{\text{rd}} \text{ quadrant} \Rightarrow A = 180^\circ + 33^\circ = 213^\circ$$

(ii) $\sin B = 0.6947 \Rightarrow B = \sin^{-1}(0.6947)$

$$= 44.003^\circ = 44^\circ \text{ (reference angle)}$$

$$\sin B = -0.6947 \Rightarrow 3^{\text{rd}} \text{ quadrant} \Rightarrow B = 180^\circ + 44^\circ = 224^\circ$$

$$\text{and } 4^{\text{th}} \text{ quadrant} \Rightarrow B = 360^\circ - 44^\circ = 316^\circ$$

(iii) $\tan C = 0.9325 \Rightarrow C = \tan^{-1}(0.9325)$

$$= 42.99^\circ = 43^\circ$$

$$\text{and } 3^{\text{rd}} \text{ quadrant} \Rightarrow C = 180^\circ + 43^\circ = 223^\circ$$

Q9. $\sin \theta = \frac{1}{2} \Rightarrow \theta = \sin^{-1} \frac{1}{2} = 30^\circ$

$$\text{and } 2^{\text{nd}} \text{ quadrant} \Rightarrow \theta = 180^\circ - 30^\circ = 150^\circ$$

Q10. $\cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) = 45^\circ$

$$\text{and } 4^{\text{th}} \text{ quadrant} \Rightarrow \theta = 360^\circ - 45^\circ = 315^\circ$$

$$\Rightarrow \tan 45^\circ = 1 \quad \text{and} \quad \tan 315^\circ = -1$$

Q11. $\tan A = \frac{1}{\sqrt{3}} \Rightarrow A = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 30^\circ$

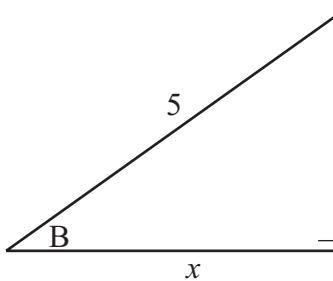
$$\text{and } 3^{\text{rd}} \text{ quadrant} = 180^\circ + 30^\circ = 210^\circ$$

$$\Rightarrow \cos 30^\circ = \frac{\sqrt{3}}{2} \quad \text{and} \quad \cos 210^\circ = -\frac{\sqrt{3}}{2}$$

Q12. $\sin \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \sin^{-1} \frac{\sqrt{3}}{2} = 60^\circ$ (reference angle)
 $\sin \theta = -\frac{\sqrt{3}}{2} \Rightarrow 3^{\text{rd}} \text{ quadrant} \Rightarrow \theta = 180^\circ + 60^\circ = 240^\circ$
and 4^{th} quadrant $\Rightarrow \theta = 360^\circ - 60^\circ = 300^\circ$
 $\Rightarrow \cos 240^\circ = -\frac{1}{2}$ and $\cos 300^\circ = \frac{1}{2}$

Q13. $\sin A = \frac{4}{5} \Rightarrow A = \sin^{-1} \frac{4}{5} = 53.13 = 53^\circ$ (reference angle)
 $\sin < 0$ and $\cos < 0 \Rightarrow 3^{\text{rd}}$ quadrant
 $\Rightarrow A = 180^\circ + 53^\circ = 233^\circ$

Q14. $\sin B = \frac{3}{5}$

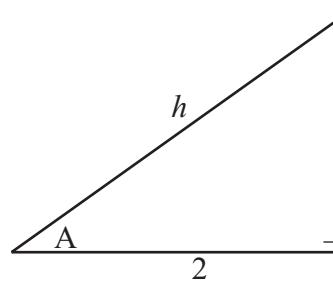


$$\begin{aligned} x^2 + 3^2 &= 5^2 \\ x^2 + 9 &= 25 \\ x^2 &= 16 \\ x &= 4 \end{aligned}$$

$\sin > 0$ and $\cos < 0 \Rightarrow 2^{\text{nd}}$ quadrant
 $\Rightarrow \tan B = -\frac{3}{4}$

Q15. $\tan B = \frac{1}{\sqrt{3}} \Rightarrow B = \tan^{-1} \frac{1}{\sqrt{3}} = 30^\circ$ (reference angle)
 $\tan > 0$ and $\sin < 0 \Rightarrow 3^{\text{rd}}$ quadrant
 $\Rightarrow \cos B = -\frac{\sqrt{3}}{2}$

Q16. $\tan A = \frac{1}{2}$



$$\begin{aligned} h^2 &= 1^2 + 2^2 \\ &= 1+4 \\ &= 5 \\ \Rightarrow h &= \sqrt{5} \end{aligned}$$

$180^\circ < A < 270^\circ \Rightarrow 3^{\text{rd}}$ quadrant

$$\Rightarrow \sin A = -\frac{1}{\sqrt{5}}$$

$$\text{Q17. (i)} \quad \sin 420^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\text{(ii)} \quad \cos 495^\circ = \cos 135^\circ = -\frac{1}{\sqrt{2}}$$

$$\text{(iii)} \quad \tan(-120^\circ) = \tan 240^\circ = \sqrt{3}$$

Exercise 2.4

$$\text{Q1.} \quad \frac{c}{\sin 42^\circ} = \frac{8}{\sin 57^\circ}$$

$$\Rightarrow c = \frac{8 \sin 42^\circ}{\sin 57^\circ} = \frac{5.353}{0.83867} = 6.38 = 6.4 \text{ m}$$

$$\frac{b}{\sin 57^\circ} = \frac{14}{\sin 39^\circ}$$

$$\Rightarrow b = \frac{14 \sin 57^\circ}{\sin 39^\circ} = \frac{11.7414}{0.62932} = 18.657 = 18.7 \text{ m}$$

$$\frac{a}{\sin 70^\circ} = \frac{7}{\sin 80^\circ}$$

$$\Rightarrow a = \frac{7 \sin 70^\circ}{\sin 80^\circ} = \frac{6.5778}{0.9848} = 6.679 = 6.7 \text{ m}$$

$$\text{Q2. (i)} \quad \frac{7}{\sin x^\circ} = \frac{10}{\sin 80^\circ} \Rightarrow 10 \sin x^\circ = 7 \sin 80^\circ$$

$$\Rightarrow \sin x^\circ = \frac{7 \sin 80^\circ}{10}$$

$$\Rightarrow \sin x^\circ = 0.6894$$

$$\Rightarrow x^\circ = \sin^{-1}(0.6894)$$

$$\Rightarrow x^\circ = 43.58^\circ$$

$$\Rightarrow x^\circ = 44^\circ$$

$$\text{(ii)} \quad \frac{12}{\sin x^\circ} = \frac{14}{\sin 42^\circ} \Rightarrow 14 \sin x^\circ = 12 \sin 42^\circ$$

$$\Rightarrow \sin x^\circ = \frac{12 \sin 42^\circ}{14}$$

$$\Rightarrow \sin x^\circ = 0.57354$$

$$\Rightarrow x^\circ = \sin^{-1}(0.57354)$$

$$\Rightarrow x^\circ = 34.997$$

$$\Rightarrow x^\circ = 35^\circ$$

$$(iii) \quad \frac{8}{\sin x^\circ} = \frac{9.5}{\sin 51^\circ} \Rightarrow 9.5 \sin x^\circ = 8 \sin 51^\circ$$

$$\Rightarrow \sin x^\circ = \frac{8 \sin 51^\circ}{9.5}$$

$$\Rightarrow \sin x^\circ = 0.65444$$

$$\Rightarrow x^\circ = \sin^{-1}(0.65444)$$

$$\Rightarrow x^\circ = 40.877$$

$$\Rightarrow x^\circ = 41^\circ$$

$$Q3. (i) \quad \frac{10}{\sin \angle BAC} = \frac{8}{\sin 48^\circ} \Rightarrow 8 \sin \angle BAC = 10 \sin 48^\circ$$

$$\Rightarrow \sin \angle BAC = \frac{10 \sin 48^\circ}{8}$$

$$\Rightarrow = 0.92893$$

$$\Rightarrow \angle BAC = \sin^{-1}(0.92893)$$

$$\Rightarrow |\angle BAC| = 68.26879^\circ$$

$$(ii) \quad 3^{\text{rd}} \text{ angle } \angle ACB = 180^\circ - (48^\circ + 68.26879^\circ) \\ = 63.7312^\circ$$

$$\text{Hence, } \frac{|AB|}{\sin 63.7312^\circ} = \frac{8}{\sin 48^\circ}$$

$$\Rightarrow |AB| = \frac{8 \sin 63.7312^\circ}{\sin 48^\circ} = \frac{7.17382}{0.743} = 9.65$$

$$\Rightarrow |AB| = 9.7$$

$$(iii) \quad \text{Area } \Delta ABC = \frac{1}{2}(10)(9.7) \sin 48^\circ \\ = 36.04 \\ = 36 \text{ sq. units}$$

$$Q4. (i) \quad \text{Area} = \frac{1}{2}(8)(10) \sin 45^\circ \\ = 28.28 \\ = 28.3 \text{ cm}^2$$

$$(ii) \quad \text{Area} = \frac{1}{2}(3.5)(5.5) \sin 100^\circ \\ = 9.478 \\ = 9.5 \text{ cm}^2$$

$$(iii) \quad \text{Area} = \frac{1}{2}(8)(7.5) \sin 80^\circ \\ = 29.544 \\ = 29.5 \text{ cm}^2$$

$$\begin{aligned}
 \text{Q5.} \quad \text{Area} &= \frac{1}{2}(11)(8)\sin A = 25 \\
 \Rightarrow \quad \sin A &= \frac{25}{44} \\
 \Rightarrow \quad A &= \sin^{-1}\left(\frac{25}{44}\right) \\
 \Rightarrow \quad A &= 34.62^\circ \\
 \Rightarrow \quad A &= 35^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{Area} &= \frac{1}{2}(8)(6.5)\sin B = 26 \\
 \Rightarrow \quad \sin B &= \left(\frac{26}{26}\right) \\
 \Rightarrow \quad B &= \sin^{-1} 1 \\
 \Rightarrow \quad B &= 90^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{Area} &= \frac{1}{2}(18)(13)\sin C = 78 \\
 \Rightarrow \quad \sin C &= \frac{78}{117} \\
 \Rightarrow \quad C &= \sin^{-1}\left(\frac{78}{117}\right) \\
 \Rightarrow \quad C &= 41.8^\circ \\
 \Rightarrow \quad C &= 42^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{Q6. (i)} \quad \text{3rd angle } \angle ACB &= 180^\circ - (46^\circ + 71^\circ) \\
 &= 63^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence, } \frac{|BC|}{\sin 71^\circ} &= \frac{22}{\sin 63^\circ} \\
 \Rightarrow |BC| &= \frac{22 \sin 71^\circ}{\sin 63^\circ} \\
 &= \frac{20.8014}{0.891} \\
 |BC| &= 23.34 \\
 &= 23 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \text{Area } \Delta ABC &= \frac{1}{2}(22)(23)\sin 46^\circ \\
 &= 181.99 \\
 &= 182 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Q7. (i)} \quad \frac{|RQ|}{\sin 30^\circ} &= \frac{\sqrt{8}}{\sin 45^\circ} \\
 \Rightarrow |RQ| &= \frac{\sqrt{8} \sin 30^\circ}{\sin 45^\circ} = 2
 \end{aligned}$$

$$\text{(ii) Third angle } \angle PRQ = 180^\circ - (30^\circ + 45^\circ) \\ = 105^\circ$$

$$\text{Hence, area } \Delta PQR = \frac{1}{2}(\sqrt{8})(2)\sin 105^\circ \\ = 2.732 \\ = 2.7 \text{ m}^2$$

$$\text{Q8. Area } \Delta ABC = \frac{1}{2}(x)(x+2)\sin 150^\circ = 6$$

$$\Rightarrow (x^2 + 2x)\left(\frac{1}{2}\right) = 12 \\ \Rightarrow x^2 + 2x = 24 \\ \Rightarrow x^2 + 2x - 24 = 0 \\ \Rightarrow (x+6)(x-4) = 0 \\ \Rightarrow x = -6 \text{ or } x = 4$$

Answer: $x = 4$

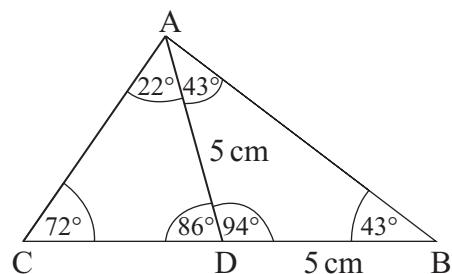
$$\text{Q9. } \frac{5.4}{\sin \angle ACB} = \frac{3}{\sin 32^\circ}$$

$$\Rightarrow (3)(\sin \angle ACB) = (5.4)(\sin 32^\circ) \\ \Rightarrow \sin(\angle ACB) = \frac{(5.4)(\sin 32^\circ)}{3} \\ \Rightarrow \sin(\angle ACB) = 0.953855 \\ \Rightarrow 1^{\text{st}} \text{ Quadrant} \Rightarrow \angle AC_1B = 72.526^\circ \\ \Rightarrow \angle AC_1B = 72.5^\circ \\ \Rightarrow 2^{\text{nd}} \text{ Quadrant} \Rightarrow \angle AC_2B = 180^\circ - 72.5^\circ \\ = 107.5^\circ$$

$$\text{Q10. (i) } |\angle DAB| = 43^\circ \\ \Rightarrow |\angle ADB| = 180^\circ - (43^\circ + 43^\circ) = 94^\circ$$

Hence, $\frac{|AB|}{\sin 94^\circ} = \frac{5}{\sin 43^\circ}$

$$\Rightarrow |AB| = \frac{5 \sin 94^\circ}{\sin 43^\circ} = 7.31 = 7.3 \text{ cm}$$

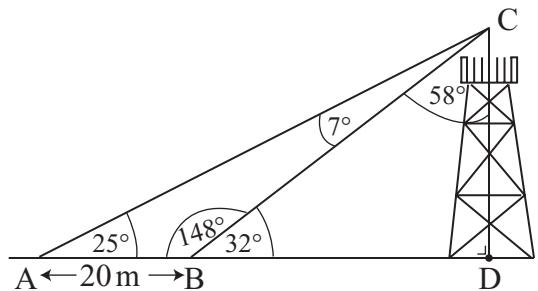


$$\begin{aligned}
 \text{(ii)} \quad & |\angle ADC| = 180^\circ - 94^\circ = 86^\circ \\
 \Rightarrow & |\angle CAD| = 180^\circ - (86^\circ + 72^\circ) = 22^\circ \\
 \text{Hence, } & \frac{|CD|}{\sin 22^\circ} = \frac{5}{\sin 72^\circ} \\
 \Rightarrow & |CD| = \frac{5 \sin 22^\circ}{\sin 72^\circ} = 1.96 = 2.0 \text{ cm}
 \end{aligned}$$

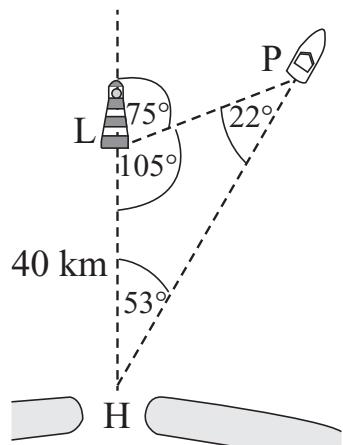
$$\begin{aligned}
 \text{Q11.} \quad & |\angle ABC| = 180^\circ - 32^\circ = 148^\circ \\
 \Rightarrow & |\angle ACB| = 180^\circ - (25^\circ + 148^\circ) = 7^\circ \\
 \text{Hence, } & \frac{|BC|}{\sin 25^\circ} = \frac{20}{\sin 7^\circ} \\
 \Rightarrow & |BC| = \frac{(20)(\sin 25^\circ)}{\sin 7^\circ} \\
 & = 69.356 \text{ m}
 \end{aligned}$$

ΔABC is right-angled Δ .

$$\begin{aligned}
 \Rightarrow & \sin 32^\circ = \frac{|CD|}{69.356} \\
 \Rightarrow & |CD| = (69.356)(\sin 32^\circ) \\
 & = 36.75 \\
 & = 36.8 \text{ m}
 \end{aligned}$$



$$\begin{aligned}
 \text{Q12.} \quad & |\angle HLP| = 180^\circ - 75^\circ = 105^\circ \\
 & |\angle HPL| = 180^\circ - (105^\circ + 53^\circ) \\
 & = 22^\circ \\
 \text{Hence, } & \frac{|PH|}{\sin 105^\circ} = \frac{40}{\sin 22^\circ} \\
 \Rightarrow & |PH| = \frac{(40)(\sin 105^\circ)}{\sin 22^\circ} \\
 & = 103.14 \\
 & = 103 \text{ km}
 \end{aligned}$$



Exercise 2.5

Q1. (i) $x^2 = (8)^2 + (5)^2 - 2(8)(5)\cos 62^\circ$

$$\Rightarrow x^2 = 89 - 80(0.46947)$$

$$\Rightarrow x^2 = 51.4423$$

$$\Rightarrow x = \sqrt{51.4423}$$

$$\Rightarrow x = 7.17$$

$$\Rightarrow x = 7.2 \text{ cm}$$

(ii) $x^2 = (14)^2 + (11)^2 - 2(14)(11)\cos 38^\circ$

$$x^2 = 317 - (308)(0.788)$$

$$x^2 = 74.296$$

$$\Rightarrow x = \sqrt{74.296}$$

$$\Rightarrow x = 8.61$$

$$\Rightarrow x = 8.6 \text{ cm}$$

(iii) $x^2 = (5)^2 + (6.8)^2 - 2(5)(6.8)\cos 105^\circ$

$$\Rightarrow x^2 = 71.24 - 68(-0.2588)$$

$$\Rightarrow x^2 = 88.8397$$

$$\Rightarrow x = \sqrt{88.8397}$$

$$\Rightarrow x = 9.42$$

$$\Rightarrow x = 9.4 \text{ cm}$$

Q2. $(12)^2 = (7)^2 + (8)^2 - 2(7)(8)\cos A$

$$\Rightarrow 144 = 49 + 64 - 112\cos A$$

$$\Rightarrow 112\cos A = 113 - 144$$

$$\Rightarrow \cos A = -\frac{31}{112} = -0.2768$$

$$\Rightarrow A = 106.07^\circ$$

$$\Rightarrow A = 106^\circ$$

$$(14)^2 = (20)^2 + (16)^2 - 2(20)(16)\cos B$$

$$\Rightarrow 196 = 400 + 256 - 640\cos B$$

$$\Rightarrow 640\cos B = 656 - 196 = 460$$

$$\Rightarrow \cos B = \frac{460}{640} = 0.71875$$

$$\Rightarrow B = \cos^{-1}(0.71875)$$

$$\Rightarrow B = 44.05$$

$$\Rightarrow B = 44^\circ$$

$$\begin{aligned}
(18)^2 &= (9)^2 + (13)^2 - 2(9)(13)\cos C \\
\Rightarrow 324 &= 81 + 169 - (234)\cos C \\
\Rightarrow 234 \cos C &= 250 - 324 \\
\Rightarrow \cos C &= -\frac{74}{234} = -0.31624 \\
\Rightarrow C &= \cos^{-1}(-0.31624) \\
\Rightarrow C &= 108.4^\circ \\
\Rightarrow C &= 108^\circ
\end{aligned}$$

Q3.

$$\begin{aligned}
(7)^2 &= (3)^2 + (5)^2 - 2(3)(5)\cos A \\
\Rightarrow 49 &= 9 + 25 - 30\cos A \\
\Rightarrow 30\cos A &= 34 - 49 = -15 \\
\Rightarrow \cos A &= -\frac{15}{30} = -0.5 \\
\Rightarrow A &= \cos^{-1}(-0.5) = 120^\circ
\end{aligned}$$

Q4.

$$\begin{aligned}
(10)^2 &= (4)^2 + (8)^2 - 2(4)(8)\cos A \\
\Rightarrow 100 &= 16 + 64 - 64\cos A \\
\Rightarrow 64\cos A &= 80 - 100 = -20 \\
\Rightarrow \cos A &= -\frac{20}{64} = -0.3125 \\
\Rightarrow A &= \cos^{-1}(-0.3125) = 108.21^\circ \\
\Rightarrow \text{Area} &= \frac{1}{2}(4)(8)\sin(108.21^\circ) \\
&= 16(0.9499) = 15.19 = 15.2 \text{ sq. units}
\end{aligned}$$

Q5. (i)

$$\begin{aligned}
|\angle QRS| &= 180^\circ - (30^\circ + 52^\circ) = 98^\circ \\
\text{Hence, } \frac{|QS|}{\sin 98^\circ} &= \frac{2}{\sin 30^\circ} \\
\Rightarrow |QS| &= \frac{2(\sin 98^\circ)}{\sin 30^\circ} = 3.96 = 4.0 \text{ units}
\end{aligned}$$

(ii) $\angle PQS \Rightarrow (6.5)^2 = (3.5)^2 + (4.0)^2 - 2(3.5)(4.0)\cos \angle PQS$

$$\begin{aligned}
\Rightarrow 42.25 &= 12.25 + 16 - 28\cos \angle PQS \\
\Rightarrow 28\cos \angle PQS &= 28.25 - 42.25 = -14 \\
\Rightarrow \cos \angle PQS &= -\frac{14}{28} = -0.5 \\
\Rightarrow |\angle PQS| &= \cos^{-1}(-0.5) = 120^\circ
\end{aligned}$$

Q6. $|QR|^2 = (42)^2 + (50)^2 - 2(42)(50)\cos 72^\circ$
 $= 1764 + 2500 - 4200(0.309)$
 $= 4264 - 1297.87$
 $= 2966.13$

$\Rightarrow |QR| = \sqrt{2966.13} = 54.46 = 54.5 \text{ m}^2$

$\Rightarrow \text{Length of rope} = 42 + 50 + 54.5$
 $= 146.5 \text{ m}$

Q7. (i) Area $\Delta ABC = \frac{1}{2}(12)(15)\sin \angle A = 65$

$\Rightarrow 90 \sin \angle A = 65$

$\Rightarrow \sin \angle A = \frac{65}{90} = 0.7222$

$\Rightarrow \angle A = \sin^{-1}(0.7222)$

$\Rightarrow \angle A = 46.2^\circ = 46^\circ$

(ii) $|BC|^2 = (12)^2 + (15)^2 - 2(12)(15)\cos 46^\circ$
 $= 144 + 225 - 360(0.69466)$
 $= 369 - 250.08$
 $= 118.92$

$\Rightarrow |BC| = \sqrt{118.92} = 10.905 = 10.9 \text{ cm}$

Q8. (i) $(6)^2 = (4)^2 + (5)^2 - 2(4)(5)\cos \theta$

$\Rightarrow 36 = 16 + 25 - 40 \cos \theta$

$\Rightarrow 40 \cos \theta = 41 - 36 = 5$

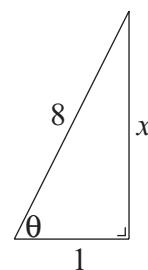
$\Rightarrow \cos \theta = \frac{5}{40} = \frac{1}{8}$

(ii) $x^2 + (1)^2 = (8)^2$

$\Rightarrow x^2 = 64 - 1 = 63$

$\Rightarrow x = \sqrt{63} = \sqrt{9}\sqrt{7} = 3\sqrt{7}$

Hence, $\sin \theta = \frac{3\sqrt{7}}{8} \Rightarrow a = 3, b = 8.$



Q9. $\text{Area } \Delta = \frac{1}{2}(2)(\sqrt{2})\sin A = 1$

$$\Rightarrow \sqrt{2} \sin A = 1$$

$$\Rightarrow \sin A = \frac{1}{\sqrt{2}}$$

$$\Rightarrow A = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

Hence, $a^2 = (2)^2 + (\sqrt{2})^2 - 2(2)(\sqrt{2})\cos 45^\circ$

$$= 4 + 2 - 4\sqrt{2} \cdot \frac{1}{\sqrt{2}}$$

$$= 2$$

$$\Rightarrow a = \sqrt{2}$$

Pythagoras' th. $\Rightarrow 2^2 = (\sqrt{2})^2 + (\sqrt{2})^2$

$$\Rightarrow 4 = 2 + 2$$

True; hence Δ is right-angled.

2 sides have length $\sqrt{2}$ each; hence Δ is isosceles.

Q10. $\text{Area } \Delta ABC = \left(\frac{1}{2}\right)(3.2)(8.4)\sin \angle B = 10$

$$\Rightarrow 13.44 \sin \angle B = 10$$

$$\Rightarrow \sin \angle B = \frac{10}{13.44} = 0.744$$

$$\Rightarrow \angle B = \sin^{-1}(0.744) = 48.07^\circ$$

$$|AC|^2 = (3.2)^2 + (8.4)^2 - 2(3.2)(8.4)\cos 48.07^\circ$$

$$= 10.24 + 70.56 - 53.76(0.668)$$

$$= 44.888$$

$$\Rightarrow |AC| = \sqrt{44.888} = 6.69 = 6.7$$

$$\text{Perimeter} = 6.7 + 3.2 + 8.4 = 18.3 \text{ cm}$$

Q11. (i) $(2x-1)^2 = (x-1)^2 + (x+1)^2 - 2(x-1)(x+1)\cos 120^\circ$

$$\Rightarrow 4x^2 - 4x + 1 = x^2 - 2x + 1 + x^2 + 2x + 1 - 2(x^2 - 1)\left(-\frac{1}{2}\right)$$

$$\Rightarrow 4x^2 - 4x + 1 = 2x^2 + 2 + x^2 - 1$$

$$\Rightarrow x^2 - 4x = 0$$

$$\Rightarrow x(x-4) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 4$$

$$\Rightarrow \text{Answer: } x = 4$$

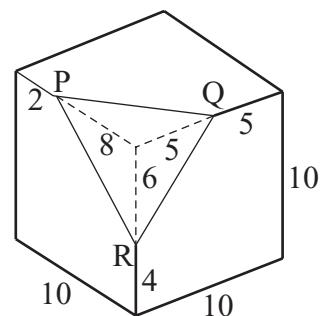
(ii) $\text{Area } \Delta = \frac{1}{2}(3)(5)\sin 120^\circ$

$$= \left(\frac{15}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{15\sqrt{3}}{4}$$

Q12. (i) $|FG|^2 = (50)^2 + (60)^2 - 2(50)(60)\cos 20^\circ$
 $= 2500 + 3600 - 6000(0.9397)$
 $= 6100 - 5638.2$
 $= 461.8$
 $\Rightarrow |FG| = \sqrt{461.8} = 21.4895 = 21.5 \text{ cm}$

(ii) $\cos 20^\circ = \frac{120}{|CB|}$
 $\Rightarrow |CB| = \frac{120}{\cos 20^\circ} = 127.7 \text{ cm}$
 $\Rightarrow |AC| = 127.7 \text{ cm}$
 $\tan 20^\circ = \frac{|CE|}{120}$
 $\Rightarrow |CE| = 120(\tan 20^\circ) = 43.676 = 43.7$
 $\Rightarrow \text{Total length} = 4(60) + 2(127.7) + 2(21.5) + 43.7$
 $= 582.1 = 582 \text{ cm}$

Q13. $|PR|^2 = (8)^2 + (6)^2 = 64 + 36 = 100$
 $\Rightarrow |PR| = \sqrt{100} = 10$
 $|PQ|^2 = (8)^2 + (5)^2 = 64 + 25 = 89$
 $\Rightarrow |PQ| = \sqrt{89}$
 $|RQ|^2 = (6)^2 + (5)^2 = 36 + 25 = 61$
 $\Rightarrow |RQ| = \sqrt{61}$
Hence, $(10)^2 = (\sqrt{89})^2 + (\sqrt{61})^2 - 2\sqrt{89}\sqrt{61} \cos \angle Q$
 $\Rightarrow 100 = 89 + 61 - (147.36)(\cos \angle Q)$
 $\Rightarrow (147.36)(\cos \angle Q) = 150 - 100 = 50$
 $\Rightarrow \cos(\angle Q) = \frac{50}{147.36} = 0.339$
 $\Rightarrow \angle PQR = \cos^{-1}(0.339)$
 $= 70.18^\circ$
 $= 70^\circ$



Exercise 2.6

Q1. (i) $|HB|^2 = (5)^2 + (10)^2 = 25 + 100 = 125$

$$\Rightarrow |HB| = \sqrt{125} = 5\sqrt{5}$$

Hence, $|GH|^2 = (5\sqrt{5})^2 + (4)^2 = 125 + 16 = 141$

$$\Rightarrow |GH| = \sqrt{141} = 11.8749 = 11.87 \text{ cm}$$

(ii) $\sin(\angle GHB) = \frac{4}{11.87} = 0.337$

$$\Rightarrow |\angle GHB| = \sin^{-1}(0.337) = 19.69^\circ = 19.7^\circ$$

Q2. (i) $\tan 25^\circ = \frac{12}{|AB|}$

$$\Rightarrow |AB| = \frac{12}{\tan 25^\circ} = 25.73 = 25.7 \text{ m}$$

(ii) $\tan(\angle TCB) = \frac{12}{15} = 0.8$

$$\Rightarrow |\angle TCB| = \tan^{-1}(0.8) = 38.657^\circ = 38.7^\circ$$

(iii) $|DB|^2 = (15)^2 + (25.7)^2 = 225 + 660.49 = 885.79$

$$\Rightarrow |DB| = \sqrt{885.79} = 29.757 = 29.8 \text{ m}$$

(iv) $\tan(\angle TDB) = \frac{12}{29.8} = 0.4027$

$$\Rightarrow |\angle TDB| = \tan^{-1}(0.4027) \\ = 21.93^\circ = 21.9^\circ$$

Q3. (i) $\tan 48^\circ = \frac{200}{|PY|} \Rightarrow |PY| = \frac{200}{\tan 48^\circ} = 180.08 = 180 \text{ m}$

$$\tan 34^\circ = \frac{200}{|QY|} \Rightarrow |QY| = \frac{200}{\tan 34^\circ} = 296.5 = 297 \text{ m}$$

(ii) $|PQ|^2 = (180)^2 + (297)^2 - 2(180)(297)\cos 84^\circ$

$$= 32400 + 88209 - 106920(0.104528)$$

$$= 120608 - 11176.18$$

$$= 109431.82$$

$$\Rightarrow |PQ| = \sqrt{109431.82} = 330.8 = 331 \text{ m}$$

$$\begin{aligned} \text{Q4. (i)} \quad & \tan(\angle ABF) = \frac{9}{27} = \frac{1}{3} = 0.3333 \\ & \Rightarrow |\angle ABF| = \tan^{-1}(0.3333) = 18.43 = 18.4^\circ \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & |AC|^2 = (17)^2 + (27)^2 \\ & = 289 + 729 \\ & = 1018 \\ & \Rightarrow |AC| = \sqrt{1018} = 31.906 = 31.9 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad & \tan(\angle ACF) = \frac{9}{31.9} = 0.2821 \\ & \Rightarrow |\angle ACF| = \tan^{-1}(0.2821) = 15.75^\circ = 15.8^\circ \end{aligned}$$

$$\begin{aligned} \text{Q5. (i)} \quad & |PR|^2 = (3)^2 + (5)^2 - 2(3)(5)\cos 120^\circ \\ & = 9 + 25 - 30(-0.5) \\ & = 34 + 15 \\ & = 49 \\ & \Rightarrow |PR| = \sqrt{49} = 7 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & \tan 60^\circ = \frac{|DP|}{7} \\ & \Rightarrow |DP| = 7 \tan 60^\circ = 12.12 \\ & \text{Hence, } |DQ|^2 = (12.12)^2 + 5^2 \\ & = 146.89 + 25 \\ & = 171.89 \\ & \Rightarrow |DQ| = \sqrt{171.89} = 13.11 = 13 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Q6. (i)} \quad & |AO| = |OB| = x \\ & \Rightarrow x^2 + x^2 = 3^2 \\ & \Rightarrow 2x^2 = 9 \\ & \Rightarrow x^2 = \frac{9}{2} = 4.5 \\ & \Rightarrow x = \sqrt{4.5} \\ & \text{Hence, } |AE|^2 = (\sqrt{4.5})^2 + (2.5)^2 \\ & = 4.5 + 6.25 = 10.75 \\ & \Rightarrow |AE| = \sqrt{10.75} = 3.278 = 3.3 \text{ m} \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & (3)^2 = (3.3)^2 + (3.3)^2 - 2(3.3)(3.3)(\cos \angle AEB) \\
 \Rightarrow & 9 = 10.89 + 10.89 - 21.78(\cos \angle AEB) \\
 \Rightarrow & 21.78(\cos \angle AEB) = 21.78 - 9 = 12.78
 \end{aligned}$$

$$\begin{aligned}
 \cos \angle AEB &= \frac{12.78}{21.78} = 0.58677 \\
 \Rightarrow \quad & \angle AEB = \cos^{-1}(0.58677) \\
 & = 54.07^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence, area } \Delta AEB &= \frac{1}{2}(3.3)(3.3)\sin 54.07^\circ \\
 &= (5.445)(0.8097) \\
 &= 4.4088
 \end{aligned}$$

$$\begin{aligned}
 \text{Total Area} &= 4(4.4088) \\
 &= 17.6352 \\
 &= 17.6 \text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Q7.} \quad & \tan 60^\circ = \frac{36}{|AD|} \\
 \Rightarrow \quad & |AD| = \frac{36}{\tan 60^\circ} = 20.7846 = 20.78 \text{ m} \\
 \Delta BCD \text{ is isosceles} \Rightarrow & |BD| = |CD| = 36 \text{ m} \\
 \text{Hence, } & |AB|^2 + (20.78)^2 = (36)^2 \\
 \Rightarrow \quad & |AB|^2 = 1296 - 431.8 = 864.2 \\
 \Rightarrow \quad & |AB| = \sqrt{864.2} = 29.39 = 29 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Q8. (i)} \quad & |DB|^2 = (7)^2 + (12)^2 = 49 + 144 = 193 \\
 \Rightarrow \quad & |DB| = \sqrt{193} \\
 \text{Hence, } & |DF|^2 = (\sqrt{193})^2 + (5)^2 = 193 + 25 = 218 \\
 \Rightarrow \quad & |DF| = \sqrt{218} = 14.76 = 14.8 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \sin \angle BDF = \frac{5}{14.8} = 0.3378 \\
 \Rightarrow \quad & |\angle BDF| = \sin^{-1}(0.3378) \\
 & = 19.745^\circ \\
 & = 19.7^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & |MB|^2 = (6)^2 + (7)^2 = 36 + 49 = 85 \\
 \Rightarrow \quad & |MB| = \sqrt{85} = 9.2195 \\
 \text{Hence, } & \tan \angle FMB = \frac{5}{9.2195} = 0.5423 \\
 \Rightarrow \quad & |\angle FMB| = \tan^{-1}(0.5423) \\
 & = 28.47^\circ \\
 & = 28.5^\circ
 \end{aligned}$$

Q9. $|OA| = |OB| = y$

Hence, $y^2 + y^2 = (2x)^2$

$$\Rightarrow 2y^2 = 4x^2$$

$$\Rightarrow y^2 = 2x^2$$

$$\Rightarrow y = \sqrt{2} \cdot x$$

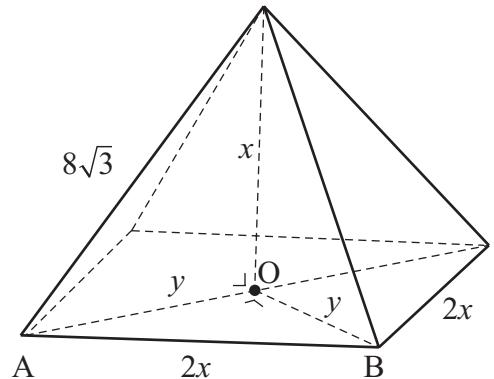
ΔAOE is right-angled $\Rightarrow y^2 + x^2 = (8\sqrt{3})^2$

$$\Rightarrow 2x^2 + x^2 = 192$$

$$\Rightarrow 3x^2 = 192$$

$$\Rightarrow x^2 = 64$$

$$\Rightarrow x = 8$$



Q10. $\Delta ABCD$ is right-angled.

$$\Rightarrow |DB|^2 = (5)^2 + (10)^2$$

$$= 25 + 100$$

$$= 125$$

$$\Rightarrow |DB| = \sqrt{125} = 5\sqrt{5}$$

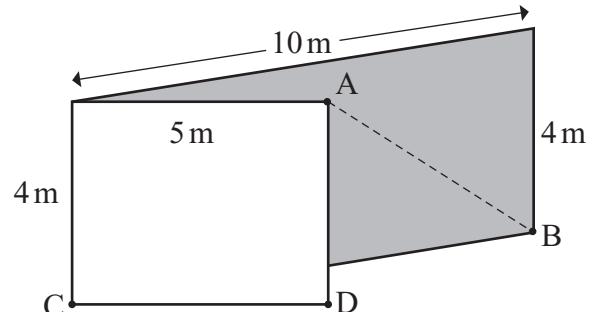
ΔADB is right-angled.

$$\Rightarrow |AB|^2 = (5\sqrt{5})^2 + (4)^2$$

$$= 125 + 16$$

$$= 141$$

$$\Rightarrow |AB| = \sqrt{141} = 11.87 = 11.9 \text{ m}$$



Q11. $\tan 30^\circ = \frac{4}{|SO|}$

$$\Rightarrow |SO| = \frac{4}{\tan 30^\circ} = \frac{4}{\frac{1}{\sqrt{3}}} = 4\sqrt{3}$$

$$\tan 60^\circ = \frac{4}{|OV|}$$

$$\Rightarrow |OV| = \frac{4}{\tan 60^\circ} = \frac{4}{\sqrt{3}}$$

$$|SV|^2 = (4\sqrt{3})^2 + \left(\frac{4}{\sqrt{3}}\right)^2 - 2(4\sqrt{3}) \cdot \left(\frac{4}{\sqrt{3}}\right) \cos 60^\circ$$

$$= 48 + \frac{16}{3} - 2 \cdot (4\sqrt{3}) \cdot \left(\frac{4}{\sqrt{3}}\right) \cdot \frac{1}{2}$$

$$= 53\frac{1}{3} - 16$$

$$= 37\frac{1}{3}$$

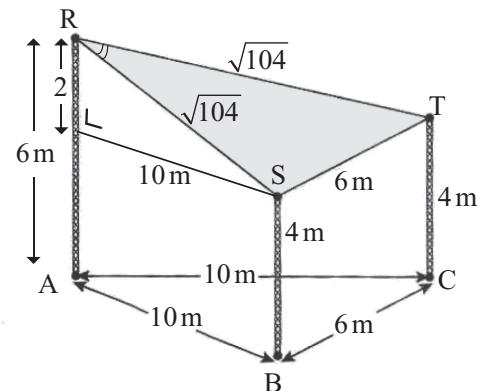
$$\Rightarrow |SV| = \sqrt{37\frac{1}{3}}$$

$$= 6.11$$

$$= 6.1 \text{ m}$$

$$\begin{aligned}
\text{Q12. (i)} \quad \angle BAC &\Rightarrow (6)^2 = (10)^2 + (10)^2 - 2(10)(10) \cos \angle BAC \\
&\Rightarrow 36 = 100 + 100 - 200 \cos \angle BAC \\
&\Rightarrow 200 \cos \angle BAC = 200 - 36 = 164 \\
&\Rightarrow \cos \angle BAC = \frac{164}{200} = 0.82 \\
&\Rightarrow |\angle BAC| = \cos^{-1}(0.82) = 34.9152^\circ
\end{aligned}$$

$$\begin{aligned}
\text{Hence, area } ABC &= \frac{1}{2}(10)(10) \sin(34.9152^\circ) \\
&= 50(0.57236) = 28.618 = 28.6 \text{ m}^2
\end{aligned}$$



$$\begin{aligned}
\text{(ii)} \quad |RS|^2 &= (10)^2 + (2)^2 = 100 + 4 = 104 \\
\Rightarrow |RS| &= \sqrt{104} \Rightarrow |RT| = \sqrt{104} \\
\text{Hence, } (6)^2 &= (\sqrt{104})^2 + (\sqrt{104})^2 - 2(\sqrt{104})(\sqrt{104}) \cos \angle SRT \\
\Rightarrow 36 &= 104 + 104 - 208 \cos \angle SRT \\
\Rightarrow 208 \cos \angle SRT &= 208 - 36 = 172 \\
\Rightarrow \cos \angle SRT &= \frac{172}{208} = 0.8269 \\
\Rightarrow |\angle SRT| &= \cos^{-1}(0.8269) = 34.2184^\circ \\
\text{Hence, area } \Delta RST &= \frac{1}{2}(\sqrt{104})(\sqrt{104})(\sin 34.2184^\circ) \\
&= (52)(0.5623) = 29.23 = 29.2 \text{ m}^2
\end{aligned}$$

$$\begin{aligned}
\text{Q13. (i)} \quad \sin 41^\circ &= \frac{105}{|TA|} \\
\Rightarrow |TA| &= \frac{105}{\sin 41^\circ} = 160.04 = 160 \text{ m}
\end{aligned}$$

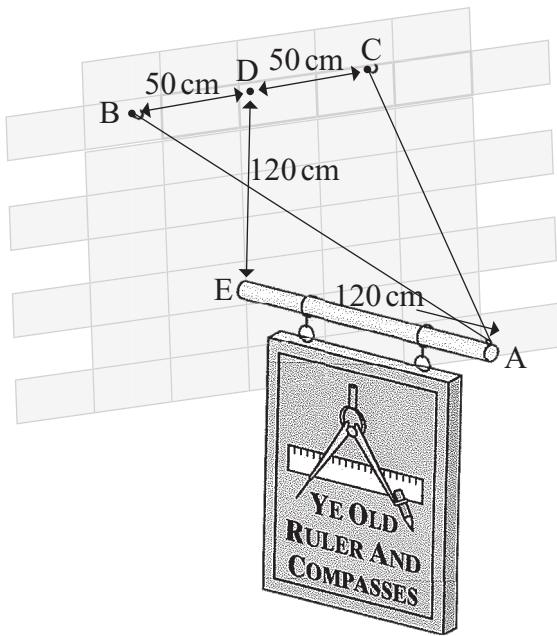
$$\begin{aligned}
\text{(ii)} \quad |BT|^2 &= (300)^2 + (160)^2 = 90,000 + 25,600 = 115,600 \\
\Rightarrow |BT| &= \sqrt{115,600} = 340 \text{ m}
\end{aligned}$$

$$\begin{aligned}
\text{(iii)} \quad \text{Vertical} &= 105 \text{ m} \\
\text{Length of Road} &= 5 \cdot 105 \\
&= 525 \text{ m}
\end{aligned}$$

$$\begin{aligned}
\text{(iv)} \quad |AC|^2 &= (525)^2 - (160)^2 \\
&= 275625 - 25600 \\
&= 250028 \\
\Rightarrow |AC| &= \sqrt{250028} \\
&= 500 \text{ m}
\end{aligned}$$

$$\begin{aligned}
\text{Hence, } |BC| &= 500 - 300 \\
&= 200 \text{ m}
\end{aligned}$$

Q14. (i) $|DA|^2 = (120)^2 + (120)^2$
 $= 14400 + 14400$
 $= 28800$
 $\Rightarrow |DA| = \sqrt{28800} = 169.7 \text{ cm}$



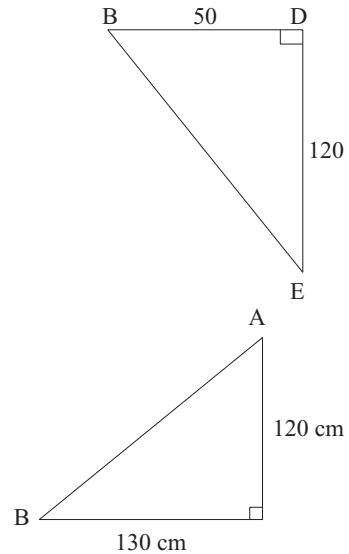
(ii) $|BE|^2 = \sqrt{120^2 + 50^2}$
 $= \sqrt{16900}$
 $|BE| = 130 \text{ cm}$

$$|AB|^2 = \sqrt{120^2 + 130^2}$$

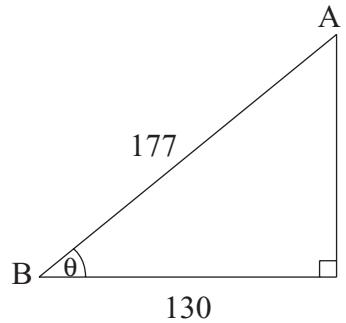
$$= \sqrt{31300}$$

$$|AB| = 176.9$$

$$= 177 \text{ cm}$$



(iii) $\cos \theta = \frac{130}{177}$
 $= 0.7344$
 $\theta = 42.7^\circ$
 $\theta = 43^\circ$
 \therefore the wire makes an angle of 43° with the wall.

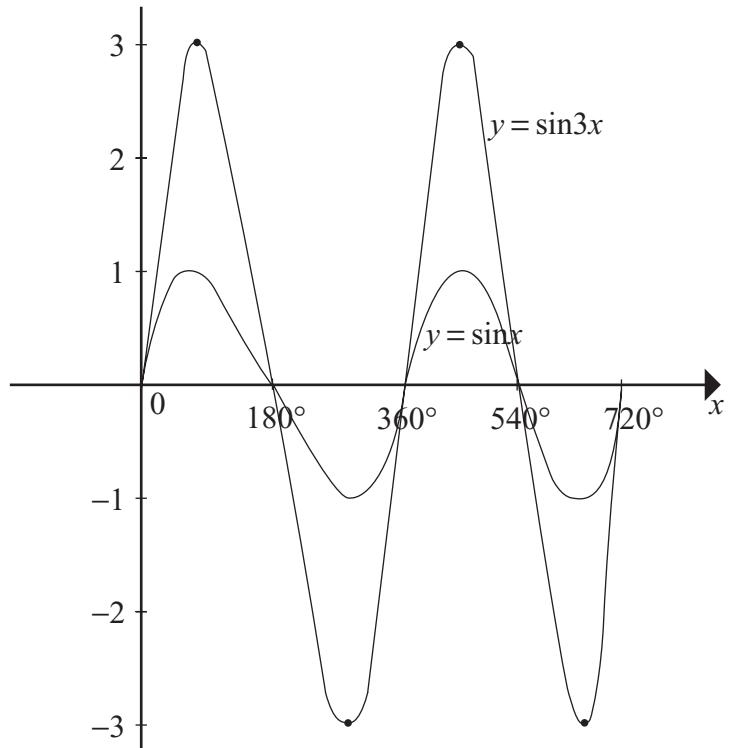


Exercise 2.7

Q1.

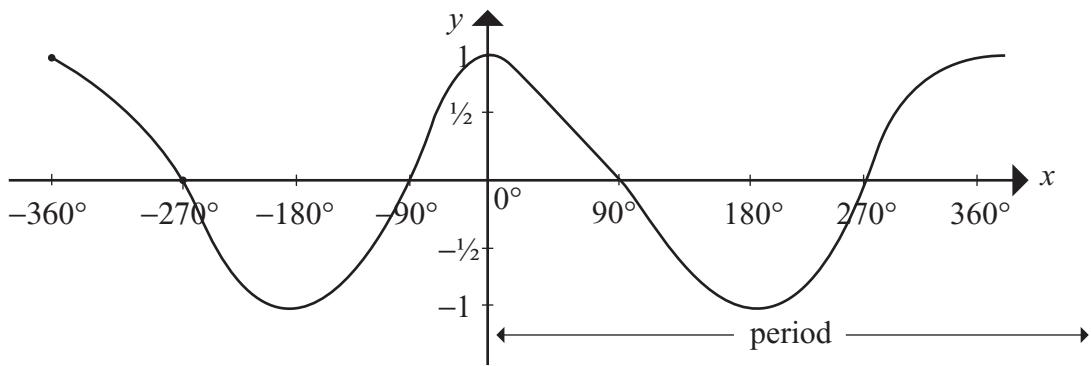
$x =$	0°	45°	90°	135°	180°	225°	270°	315°	360°	450°	540°	630°	720°
$\sin x$	0	0.7	1	0.7	0	-0.7	-1	-0.7	0	1	0	-1	0
$3 \sin x$	0	2.1	3	2.1	0	-2.1	-3	-2.1	0	3	0	-3	0

- (i) Period = 2π (360°)
- (ii) Range = $[-1, 1]$
- (iii) Period = 2π (360°)
- (iv) Range = $[-3, 3]$



Q2.

$x =$	-360°	-315°	-270°	-225°	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°	225°	270°	315°	360°
$\cos x =$	1	0.7	0	-0.7	-1	-0.7	0	0.7	1	0.7	0	-0.7	-1	-0.7	0	0.7	1

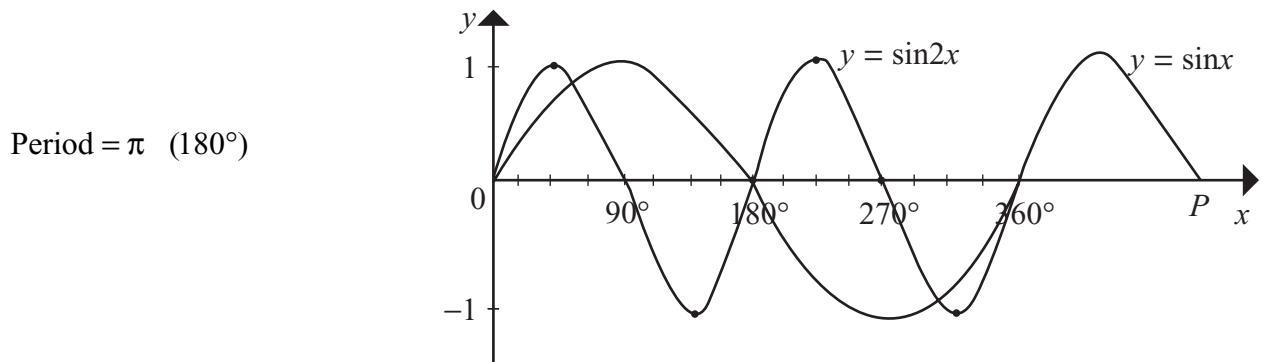


- (i) Period = 2π (360°)
- (ii) Range = $[-1, 1]$

Q3. (i) $P = (540^\circ, 0)$

(ii)

$x =$	0	$22\frac{1}{2}^\circ$	45°	$67\frac{1}{2}^\circ$	90°	$112\frac{1}{2}^\circ$	135°	$157\frac{1}{2}^\circ$	180°
$2x =$	0	45°	90°	135°	180°	225°	270°	315°	360°
$\sin 2x$	0	0.7	1	0.7	0	-0.7	-1	-0.7	0



Q4. (i) Period = 2π (360°) Range = $[-3, 3]$

(ii) Period = π (180°) Range = $[-2, 2]$

(iii) Period = $\frac{2\pi}{3}$ (120°) Range = $[-4, 4]$

Q5. Period = π (180°) Range = $[-3, 3]$

$$y = 3 \cos 2x$$

Q6. (i) Period = π (180°) Range = $[-1, 1]$

$$y = \cos 2x$$

(ii) Period = π (180°) Range = $[-2, 2]$

$$y = 2 \sin 2x$$

(iii) Period = $\frac{\pi}{2}$ (90°) Range = $\left[-\frac{1}{2}, \frac{1}{2}\right]$

$$y = \frac{1}{2} \sin 4x$$

(iv) Period = π (180°) Range = $[-4, 4]$

$$y = 4 \cos 2x$$

Q7. (i) 1

(ii) 0

(iii) 1

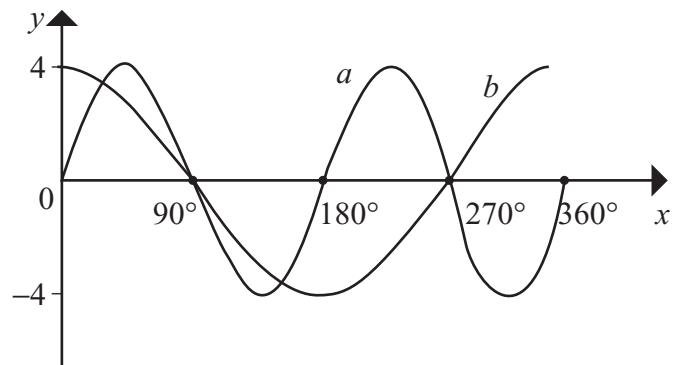
(iv) -1

(v) -1

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

Q8. a is $4 \sin 2x$

b is $4 \cos x$



Q9. $y = 3 \cos 3x$

(i) 0 and $\frac{2\pi}{3}$

(ii) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

(iii) $\frac{\pi}{3}$ and π

Q10. $y = 2 \sin 3x$

Q11. (i) $y = 2 \cos 4x$

(ii) $y = \tan x$

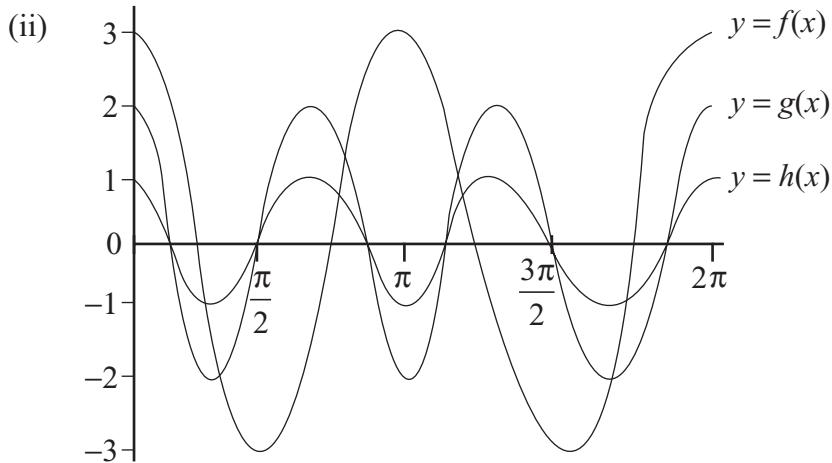
(iii) $y = 3 \sin 2x$

(iv) $y = 5 \cos 2x$

Q12. (i) $f(x) = 3 \cos 2x$

$g(x) = 2 \cos 3x$

$h(x) = \cos 3x$



Exercise 2.8

Q1. $x = 30^\circ$ and 150°

Q2. $x = 30^\circ$ and 330°

Q3. $\theta = \frac{\pi}{4}$ and $\frac{5\pi}{4}$

Q4. $2\theta = 2n\pi + \frac{\pi}{6}$ or $2\theta = 2n\pi + \frac{5\pi}{6}$

$$\Rightarrow \theta = n\pi + \frac{\pi}{12} \quad \Rightarrow \theta = n\pi + \frac{5\pi}{12}$$

Q5. $3\theta = 30^\circ + n(360^\circ)$ or $3\theta = 330^\circ + n(360^\circ)$

$$\Rightarrow \theta = 10^\circ + n(120^\circ) \quad \Rightarrow \theta = 110^\circ + n(120^\circ)$$

Q6. $3\theta = \frac{4\pi}{3} + 2n\pi$ or $3\theta = \frac{5\pi}{3} + 2n\pi$

$$\Rightarrow \theta = \frac{4\pi}{9} + \frac{2n\pi}{3} \quad \Rightarrow \theta = \frac{5\pi}{9} + \frac{2n\pi}{3}$$

Q7. $4\theta = \frac{\pi}{3} + 2n\pi$ or $4\theta = \frac{5\pi}{3} + 2n\pi$

$$\Rightarrow \theta = \frac{\pi}{12} + \frac{n\pi}{2} \quad \Rightarrow \theta = \frac{5\pi}{12} + \frac{n\pi}{2}$$

Q8. $x = \frac{\pi}{6} + n\pi$

Q9. $3x = 225^\circ + n(360^\circ)$ or $3x = 315^\circ + n(360^\circ)$

$$\Rightarrow x = 75^\circ + n(120^\circ) \quad \Rightarrow x = 105^\circ + n(120^\circ)$$

$$\Rightarrow x = 75^\circ, 195^\circ, 315^\circ \quad \Rightarrow x = 105^\circ, 225^\circ, 345^\circ$$

Q10. $2\theta = 150^\circ + n(360^\circ)$ or $2\theta = 210^\circ + n(360^\circ)$

$$\Rightarrow \theta = 75^\circ + n(180^\circ) \quad \Rightarrow \theta = 105^\circ + n(180^\circ)$$

$$\Rightarrow \theta = 75^\circ, 255^\circ \quad \Rightarrow \theta = 105^\circ, 285^\circ$$

Q11. $2\theta = \frac{\pi}{3} + n\pi$

$$\Rightarrow \theta = \frac{\pi}{6} + \frac{n\pi}{2}$$

$$\text{Q12.} \quad 4\theta = \frac{\pi}{6} + 2n\pi \quad \text{or} \quad 4\theta = \frac{11\pi}{6} + 2n\pi$$

$$\Rightarrow \theta = \frac{\pi}{24} + \frac{n\pi}{2} \quad \Rightarrow \quad \theta = \frac{11\pi}{24} + \frac{n\pi}{2}$$

$$\text{Q13.} \quad 3\theta = 120^\circ + n(360^\circ) \quad \text{or} \quad 3\theta = 240^\circ + n(360^\circ)$$

$$\Rightarrow \theta = 40^\circ + n(120^\circ) \quad \Rightarrow \quad \theta = 80^\circ + n(120^\circ)$$

$$\Rightarrow \theta = 40^\circ, 160^\circ, 280^\circ \quad \Rightarrow \quad \theta = 80^\circ, 200^\circ, 320^\circ$$

$$\text{Q14.} \quad 3\theta = 51^\circ + n(360^\circ) \quad \text{or} \quad 3\theta = 129^\circ + n(360^\circ)$$

$$\Rightarrow \theta = 17^\circ + n(120^\circ) \quad \Rightarrow \quad \theta = 43^\circ + n(120^\circ)$$

$$\Rightarrow \theta = 17^\circ, 137^\circ, 257^\circ \quad \Rightarrow \quad \theta = 43^\circ, 163^\circ, 283^\circ$$

Test Yourself 2

A Questions

Q1.

$$\begin{aligned} \text{Area} &= \frac{1}{2}(8)(9)\sin 40^\circ \\ &= 36(0.6428) \\ &= 23.14 \\ &= 23.1 \text{ cm}^2 \end{aligned}$$

Q2. $\theta = 150^\circ, 330^\circ$

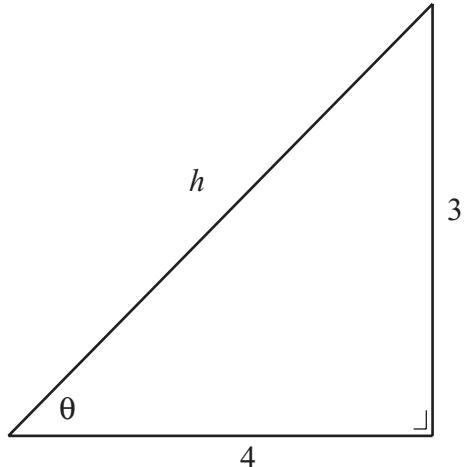
Q3. (i)

$$\begin{aligned} \text{Area} &= \frac{1}{2}(20)^2\theta = 240 \\ \Rightarrow 200\theta &= 240 \\ \Rightarrow \theta &= \frac{240}{200} = \frac{6}{5} \text{ radians} \end{aligned}$$

(ii) Arc length $= (20)\left(\frac{6}{5}\right) = 24 \text{ cm}$

Q4.

$$\begin{aligned} \tan \theta &= \frac{3}{4} \\ h^2 &= 3^2 + 4^2 \\ &= 9 + 16 = 25 \\ \Rightarrow h &= \sqrt{25} = 5 \quad \Rightarrow \sin \theta = \frac{3}{5} \\ \text{Hence, area} &= \frac{1}{2}(8)(7)\sin \theta \\ &= (28)\left(\frac{3}{5}\right) \\ &= 16\frac{4}{5} \text{ cm}^2 \end{aligned}$$



Q5. (i) Period $= 180^\circ$; Range $= [-2, 2]$

(ii) $a = 2$, Period $= \frac{360^\circ}{b} = 180^\circ$

$$\begin{aligned} \Rightarrow 180b &= 360 \\ \Rightarrow b &= 2 \end{aligned}$$

Q6. Right-angled $\Delta \Rightarrow x^2 + x^2 = (2R)^2$

$$\Rightarrow 2x^2 = 4R^2$$

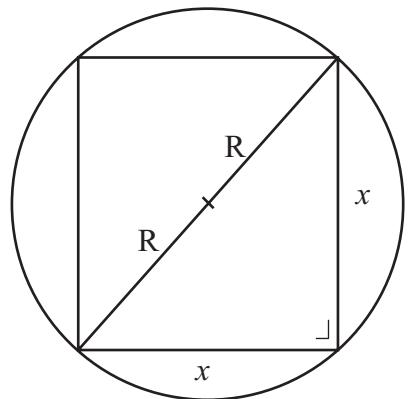
$$\Rightarrow x^2 = 2R^2$$

$$\text{Area Circle} = \pi R^2 = \pi$$

$$\Rightarrow R^2 = 1$$

$$\Rightarrow R = 1$$

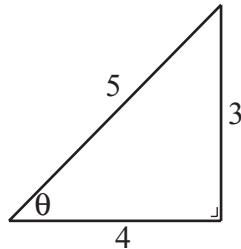
$$\text{Area Square} = x^2 = 2(1)^2 = 2 \text{ sq. units}$$



Q7. $\sin < 0$ and $\cos > 0$

\Rightarrow 4th Quadrant

$$\Rightarrow \tan \theta = -\frac{3}{4}$$



Q8. Area $\Delta PQR = \frac{1}{2}(10)(8)\sin \angle QPR = 20$

$$\Rightarrow 40 \sin \angle QPR = 20$$

$$\Rightarrow \sin \angle QPR = \frac{20}{40} = \frac{1}{2}$$

$$\Rightarrow |\angle QPR| = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ \text{ or } 150^\circ$$

Q9. $4 \sin \theta = 3$

$$\Rightarrow \sin \theta = \frac{3}{4} = 0.75$$

$$\Rightarrow \theta = \sin^{-1}(0.75)$$

$$\Rightarrow \theta = 48.59^\circ \text{ or } 131.41^\circ$$

$$\Rightarrow \theta = 49^\circ \text{ or } 131^\circ$$

Q10. Area $\Delta = \frac{1}{2}(8)(7)\sin \theta = 14\sqrt{3}$

$$\Rightarrow 28 \sin \theta = 14\sqrt{3}$$

$$\Rightarrow \sin \theta = \frac{14\sqrt{3}}{28} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$$

$$\Rightarrow \cos 60^\circ = \frac{1}{2}$$

B Questions

$$\text{Q1. (i)} \quad (17)^2 = (8)^2 + (12)^2 - 2(8)(12)\cos A$$

$$\Rightarrow 289 = 64 + 144 - 192 \cos A$$

$$\Rightarrow 192 \cos A = -81$$

$$\Rightarrow \cos A = -\frac{81}{192} = -0.421875$$

$$\Rightarrow A = \cos^{-1}(-0.421875)$$
$$= 114.95^\circ = 115^\circ$$

$$\text{(ii)} \quad \text{Area} = \frac{1}{2}(8)(12)\sin 115^\circ$$

$$= 48(0.9063)$$

$$= 43.502 = 43.5 \text{ cm}^2$$

$$\text{Q2. (i)} \quad 2\theta = 2n\pi + \frac{5\pi}{6} \quad \text{or} \quad 2n\pi + \frac{7\pi}{6}$$

$$\Rightarrow \theta = n\pi + \frac{5\pi}{12} \quad \text{or} \quad n\pi + \frac{7\pi}{12}$$

$$\text{(ii)} \quad \text{Area} = \frac{1}{2}(4)^2\theta = 12$$

$$\Rightarrow 8\theta = 12$$

$$\Rightarrow \theta = \frac{12}{8} = 1\frac{1}{2} \text{ radians}$$

$$\text{Q3. (i)} \quad (20)^2 = (18)^2 + (23)^2 - 2(18)(23)\cos A$$

$$\Rightarrow 400 = 324 + 529 - (828)\cos A$$

$$\Rightarrow 828 \cos A = 853 - 400 = 453$$

$$\Rightarrow \cos A = \frac{453}{828} = 0.5471$$

$$\Rightarrow A = \cos^{-1}(0.5471) = 56.83 = 57^\circ$$

$$\text{(ii)} \quad \sin 57^\circ = \frac{\text{height}}{47}$$

$$\Rightarrow \text{height} = 47(\sin 57^\circ)$$

$$= 47(0.83867)$$

$$= 39.417 = 39 \text{ cm}$$

Q4. $|OB|^2 = (1600)^2 + (1600)^2$
 $= 2560000 + 2560000$
 $= 5120000$

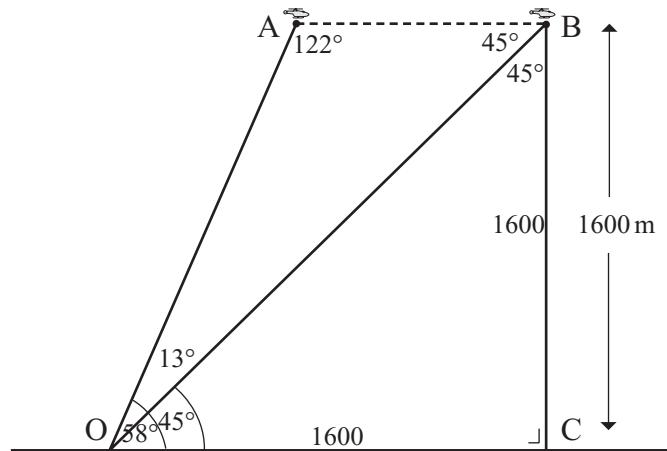
$$|OB| = \sqrt{5120000} = 2262.74$$

Hence, $\frac{|AB|}{\sin 13^\circ} = \frac{2262.74}{\sin 122^\circ}$
 $\Rightarrow |AB| = \frac{(2262.74) \sin 13^\circ}{\sin 122^\circ}$
 $= \frac{509}{0.848}$
 $= 600.236 \text{ m}$

$$\Rightarrow \text{Speed} = 600.236 \text{ m per minute}$$

 $\Rightarrow 600.236 \text{ m} \times 60$
 $= 36014.16 \text{ m}$
 $= 36.01416 \text{ km} = 36 \text{ km}$

$$\Rightarrow \text{Speed} = 36 \text{ km/h}$$



Q5. (i) $(5\sqrt{13})^2 = (10)^2 + (a\sqrt{3})^2 - 2(10)(a\sqrt{3})\cos 150^\circ$

$$\Rightarrow 325 = 100 + 3a^2 - 20\sqrt{3}a\left(-\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow 225 = 3a^2 + 30a$$

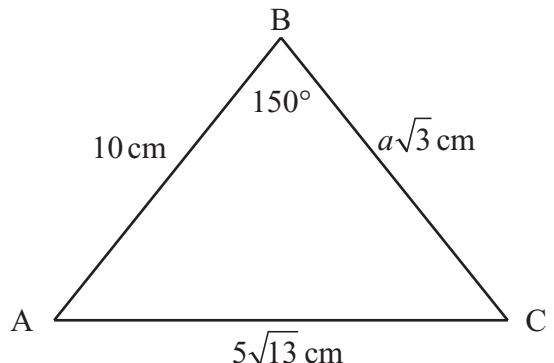
$$\Rightarrow 3a^2 + 30a - 225 = 0$$

$$\Rightarrow a^2 + 10a - 75 = 0$$

$$\Rightarrow (a+15)(a-5) = 0$$

$$\Rightarrow a = -15 \text{ or } a = 5$$

Answer : $a = 5$



(ii) Area $\Delta ABC = \frac{1}{2}(10)(5\sqrt{3})\sin 150^\circ$

$$= 25\sqrt{3}\left(\frac{1}{2}\right)$$

$$= \frac{25\sqrt{3}}{2} \text{ cm}^2$$

Q6. $\cos 33^\circ = \frac{|AC|}{9}$

$$\Rightarrow |AC| = 9 \cos 33^\circ$$

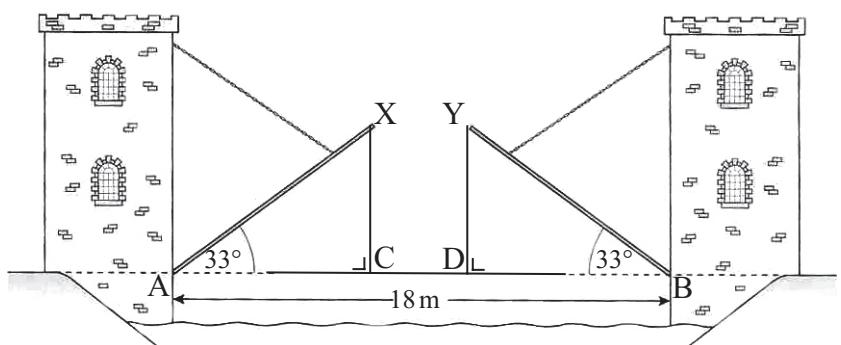
 $= 7.548$

Hence, $|DB| = 7.548$

$$\Rightarrow |CD| = 18 - 2(7.548)$$

 $= 2.904 = 2.9 \text{ m}$

Hence, $|XY| = 2.9 \text{ m}$



Q7. (i) $[-4, 4]$

- (ii) π
- (iii) -4
- (iv) $f(x) = 2 \sin 2x$
 $g(x) = 4 \cos x$

(v) $P = \left(\frac{5\pi}{4}, 2 \right)$

Q8. $\sin 63^\circ = \frac{|OP|}{3}$

$$\Rightarrow |OP| = 3 \sin 63^\circ = 3(0.891) = 2.673$$

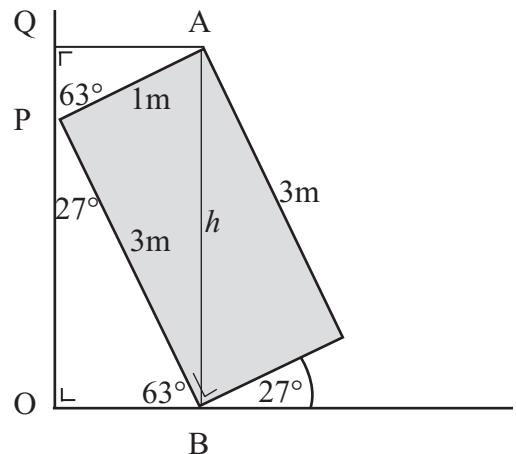
$$\cos 63^\circ = \frac{|QP|}{1}$$

$$\Rightarrow |QP| = 0.454$$

$$\text{Hence, } |OQ| = 2.673 + 0.454 = 3.127 = 3.13 \text{ m}$$

$$\text{Since } |OQ| = |AB|,$$

$$\text{hence, } |AB| = 3.13 \text{ m}$$



Q9. (i) (a) $\tan 23^\circ = \frac{4}{|OA|}$

$$\Rightarrow |OA| = \frac{4}{\tan 23^\circ} = 9.423 = 9.4 \text{ m}$$

(b) $\cos \angle OAD = \frac{|DA|}{|OA|} = \frac{7}{9.4} = 0.74468$

$$\Rightarrow |\angle OAD| = \cos^{-1}(0.74468) = 41.86^\circ = 42^\circ$$

(ii) $|OD|^2 + (7)^2 = (9.4)^2$

$$\Rightarrow |OD|^2 + 49 = 88.36$$

$$\Rightarrow |OD|^2 = 88.36 - 49 = 39.36$$

$$\Rightarrow |OD| = \sqrt{39.36} = 6.274 = 6.3$$

$$\text{Hence, } \tan \angle COD = \frac{4}{6.3} = 0.6349$$

$$\Rightarrow |\angle COD| = \tan^{-1}(0.6349) = 32.41^\circ = 32.4^\circ$$

No; eye moves 32.4° .

$$\text{Q10. (i)} \quad \cos 3\theta = \frac{\sqrt{3}}{2} \Rightarrow 3\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow 3\theta = \frac{\pi}{6} \quad (\text{reference angle})$$

Cosine is positive in the 1st and 4th quadrants

$$\Rightarrow 3\theta = \frac{\pi}{6} \text{ or } \frac{11\pi}{6}$$

$$\text{Solutions : } 3\theta = 2n\pi + \frac{\pi}{6} \text{ or } 3\theta = 2n\pi + \frac{11\pi}{6}$$

$$\Rightarrow \theta = \frac{2n\pi}{3} + \frac{\pi}{18} \text{ or } \theta = \frac{2n\pi}{3} + \frac{11\pi}{18}$$

$$\text{(ii) Yellow Shaded Area} = \frac{1}{2}(6)^2\theta - \frac{1}{2}(4)^2\theta \\ = 18\theta - 8\theta = 10\theta$$

$$\text{Green Shaded Area} = \frac{1}{2}(4)^2(2\pi - \theta) \\ = 8(2\pi - \theta) \\ = 16\pi - 8\theta$$

$$\text{Hence, } 10\theta = 16\pi - 8\theta$$

$$\Rightarrow 18\theta = 16\pi$$

$$\Rightarrow \theta = \frac{16\pi}{18} = \frac{8\pi}{9} \text{ radians}$$

C Questions

$$\text{Q1. (i)} \quad \tan 18^\circ = \frac{|CF|}{20} \Rightarrow |CF| = 20 \tan 18^\circ \\ = 6.49 = 6.5 \text{m}$$

$$\text{(ii)} \quad |DF|^2 = (20)^2 + (6.5)^2 \\ = 400 + 42.25 = 442.25 \\ |DF| = \sqrt{442.25} = 21.02 = 21.0 \text{ m}$$

$$\text{(iii)} \quad \tan \angle CAD = \frac{20}{10} = 2 \\ \Rightarrow |\angle CAD| = \tan^{-1}(2) = 63.43 = 63.4^\circ$$

$$\text{(iv)} \quad |AF|^2 = (10)^2 + (21.0)^2 \\ = 100 + 441 = 541 \\ \Rightarrow |AF| = \sqrt{541} = 23.25 = 23.3 \text{ m}$$

$$\text{(v)} \quad \sin \angle FAC = \frac{|FC|}{|AF|} = \frac{6.5}{23.3} = 0.279 \\ \Rightarrow |\angle FAC| = \sin^{-1}(0.279) = 16.2005 = 16.2^\circ$$

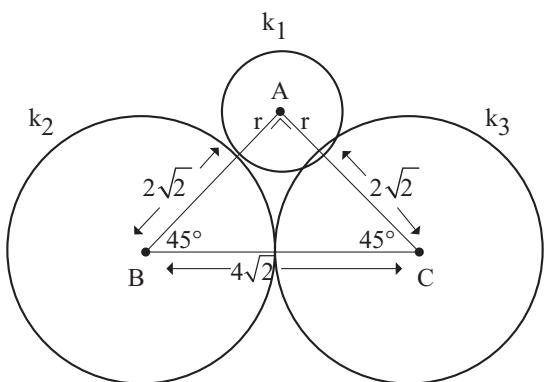
$$\text{Q2. (i)} \quad \tan \theta = \frac{4}{|BP|} \Rightarrow |BP| = \frac{4}{\tan \theta}$$

Hence, $|QC| = \frac{4}{\tan \theta}$

$$|PQ| = k - \left(\frac{4}{\tan \theta} + \frac{4}{\tan \theta} \right) = k - \frac{8}{\tan \theta}$$

$$\text{(ii)} \quad |PQ| = 12 - \frac{8}{\tan \theta} = 12 - 4\sqrt{3} \\ \Rightarrow \frac{8}{\tan \theta} = 4\sqrt{3} \\ \Rightarrow 4\sqrt{3} \tan \theta = 8 \\ \Rightarrow \tan \theta = \frac{8}{4\sqrt{3}} = 1.1547 \\ \Rightarrow \theta = \tan^{-1}(1.1547) = 49.1^\circ = 49^\circ$$

$$\text{Q3. (i)} \quad \sin 45^\circ = \frac{r + 2\sqrt{2}}{4\sqrt{2}} \\ \Rightarrow \frac{1}{\sqrt{2}} = \frac{r + 2\sqrt{2}}{4\sqrt{2}} \\ \Rightarrow r + 2\sqrt{2} = 4 \\ \Rightarrow r = 4 - 2\sqrt{2}$$



$$\text{(ii) Area sector } k_2 = \frac{1}{2}(2\sqrt{2})^2 \left(\frac{\pi}{4}\right)$$

$$= \frac{1}{2}(8)\left(\frac{\pi}{4}\right) = \pi$$

$$\text{Area sector } k_3 = \pi$$

$$\text{Area sector } k_1 = \frac{1}{2}(4-2\sqrt{2})^2 \left(\frac{\pi}{2}\right) = \frac{1}{2}(16-16\sqrt{2}+8)\left(\frac{\pi}{2}\right) = 6\pi - 4\sqrt{2}\pi$$

$$\text{Area } \Delta ABC = \frac{1}{2}(4)(4)\sin 45^\circ = \frac{1}{2}16\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 8$$

$$\Rightarrow \text{Shaded area} = 8 - (\pi + \pi + 6\pi - 4\sqrt{2}\pi)$$

$$= 8 - 8\pi + 4\sqrt{2}\pi$$

Q4. $\tan 35^\circ = \frac{10}{|\text{SO}|} \Rightarrow |\text{SO}| = \frac{10}{\tan 35^\circ} = 14.28$

$$\tan 50^\circ = \frac{10}{|\text{KO}|} \Rightarrow |\text{KO}| = \frac{10}{\tan 50^\circ} = 8.39$$

$$\Rightarrow |\text{SK}|^2 = (14.28)^2 + (8.39)^2 - 2(14.28)(8.39)\cos 60^\circ$$

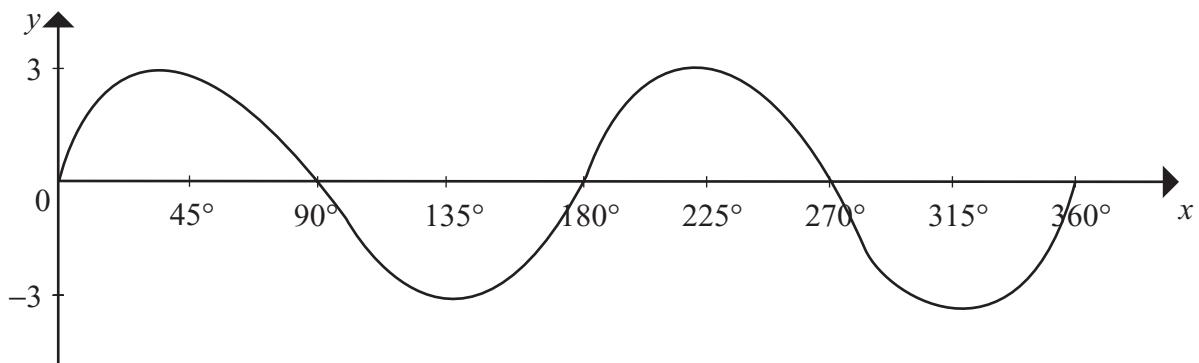
$$= 203.9184 + 70.3921 - (239.6184)(0.5)$$

$$= 154.5013$$

$$\Rightarrow |\text{SK}| = \sqrt{154.5013} = 12.4 = 12\text{m}$$

Q5. (i)

$x =$	0°	45°	90°	135°	180°	225°	270°	315°	360°
$2x =$	0°	90°	180°	270°	360°	450°	540°	630°	720°
$y = \sin 2x$	0	1	0	-1	0	1	0	-1	0
$y = 3 \sin 2x$	0	3	0	-3	0	3	0	-3	0



$$\text{Period} = \pi$$

$$\text{Range} = [-3, 3]$$

$$\text{(ii) (a)} \quad l = r\theta = 5(0.8) + 3(0.8) = 6.4$$

$$\Rightarrow \text{Perimeter} = 6.4 + 2 + 2 = 10.4 \text{ cm}$$

$$\text{(b)} \quad \text{Perimeter} = 5\theta + 3\theta + 2 + 2 = 14$$

$$\Rightarrow 8\theta = 14 - 4 = 10$$

$$\Rightarrow \theta = \frac{10}{8} = 1\frac{1}{4} \text{ radians}$$

Q6. (i) Standard Proof

$$\begin{aligned}
 \text{(ii)} \quad \cos A &= \frac{b^2 + c^2 - a^2}{2bc} = \frac{(a+1)^2 + (a+2)^2 - a^2}{2(a+1)(a+2)} \\
 &= \frac{a^2 + 2a + 1 + a^2 + 4a + 4 - a^2}{2(a+1)(a+2)} \\
 &= \frac{a^2 + 6a + 5}{2(a+1)(a+2)} = \frac{(a+1)(a+5)}{2(a+1)(a+2)} = \frac{a+5}{2a+4}
 \end{aligned}$$

$$\text{Q7. (i)} \quad \tan 25^\circ = \frac{h}{|\text{AO}|} \Rightarrow |\text{AO}| = \frac{h}{\tan 25^\circ}$$

$$\text{(ii)} \quad \tan 33^\circ = \frac{h}{|\text{BO}|} \Rightarrow |\text{BO}| = \frac{h}{\tan 33^\circ}$$

$$\begin{aligned}
 \text{(iii)} \quad (60)^2 &= \left(\frac{h}{\tan 25^\circ} \right)^2 + \frac{h}{\tan 33^\circ} \\
 \Rightarrow 3600 &= \frac{h^2}{0.2174} + \frac{h^2}{0.4217} \\
 \Rightarrow 3600(0.2174)(0.4217) &= h^2(0.4217) + h^2(0.2174) \\
 \Rightarrow h^2(0.6391) &= 330.04 \\
 \Rightarrow h^2 &= \frac{330.04}{0.6391} = 516.41 \\
 \Rightarrow h &= \sqrt{516.41} = 22.72 = 22.7 \text{ m}
 \end{aligned}$$

$$\text{Q8.} \quad \tan 2\theta = \frac{h}{|\text{SA}|} \Rightarrow |\text{SA}| = \frac{h}{\tan 2\theta}$$

$$\tan \theta = \frac{h}{|\text{SD}|} \Rightarrow |\text{SD}| = \frac{h}{\tan \theta}$$

$$\text{Hence, } |\text{SD}| = 5|\text{SA}|$$

$$\Rightarrow \frac{h}{\tan \theta} = 5 \cdot \frac{h}{\tan 2\theta}$$

$$\Rightarrow \tan 2\theta = 5 \tan \theta$$

$$\Rightarrow \frac{2 \tan \theta}{1 - \tan^2 \theta} = 5 \tan \theta$$

$$\Rightarrow 2 \tan \theta = 5 \tan \theta - 5 \tan^3 \theta$$

$$\Rightarrow 5 \tan^3 \theta - 3 \tan \theta = 0$$

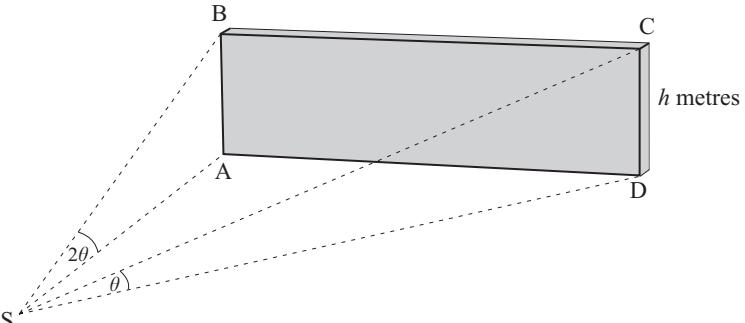
$$\Rightarrow \tan \theta (5 \tan^2 \theta - 3) = 0$$

$$\Rightarrow \tan \theta = 0 \text{ or } \tan^2 \theta = 0.6$$

$$\Rightarrow \theta = \tan^{-1} 0 = 0 \text{ (not valid). Hence, } \tan \theta = \sqrt{0.6} = 0.7746$$

$$\Rightarrow \theta = \tan^{-1} 0.7746$$

$$\Rightarrow \theta = 37.76^\circ = 38^\circ$$



$$\text{Q9. (i)} \quad \cos 3x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow 3x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ \text{ (reference angle)}$$

Cosine is positive in the 1st and 4th quadrants.

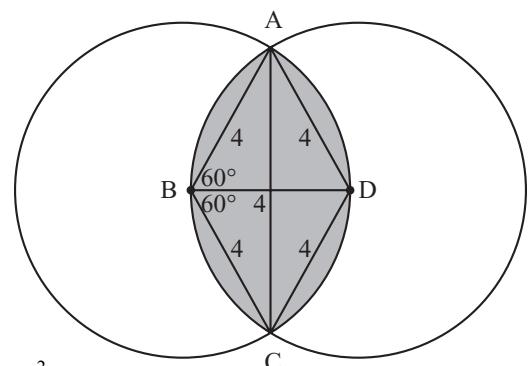
$$\Rightarrow 3x = 30^\circ \text{ or } 330^\circ$$

$$\text{All solutions : } 3x = 30^\circ + (360^\circ)n \text{ or } 3x = 330^\circ + (360^\circ)n$$

$$\Rightarrow x = 10^\circ + (120^\circ)n \text{ or } x = 110^\circ + (120^\circ)n$$

$$\Rightarrow x = 10^\circ, 130^\circ, 250^\circ \text{ or } x = 110^\circ, 230^\circ, 350^\circ$$

$$\Rightarrow x = 10^\circ, 110^\circ, 130^\circ, 230^\circ, 250^\circ, 350^\circ.$$



$$\text{(ii)} \quad \text{Area sector ABC} = \frac{1}{2}(4)^2 \frac{2\pi}{3} = \frac{16\pi}{3}$$

$$\text{Area } \triangle ABC = \frac{1}{2}(4)(4)\sin 120^\circ = 4\sqrt{3}$$

$$\Rightarrow \text{Area segment ADC} = \frac{16\pi}{3} - 4\sqrt{3}$$

$$\Rightarrow \text{Total shaded area} = 2\left(\frac{16\pi}{3} - 4\sqrt{3}\right) = \left(\frac{32\pi}{3} - 8\sqrt{3}\right) \text{ cm}^2$$

$$\text{Q10. (i)} \quad a = (0, 4), b = (0, -4), c = \left(\frac{\pi}{6}, 0\right), d = \left(\frac{\pi}{2}, 0\right)$$

$$e = \left(\frac{5\pi}{6}, 0\right), f = \left(\frac{4\pi}{3}, 0\right)$$

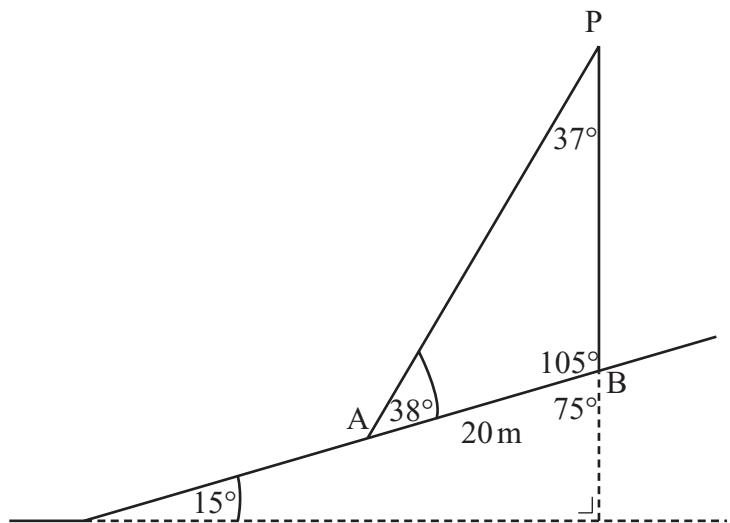
$$\text{(ii)} \quad \frac{|PB|}{\sin 38^\circ} = \frac{20}{\sin 37^\circ}$$

$$\Rightarrow |PB| = \frac{20 \sin 38^\circ}{\sin 37^\circ}$$

$$= \frac{12.313}{0.6018}$$

$$= 20.46$$

$$= 20.5 \text{ m}$$



Chapter 3 Geometry 1

Exercise 3.1

Q1. $a = 46^\circ, b = 180^\circ - 46^\circ = 134^\circ$

$$c = 80^\circ, d = 180^\circ - 80^\circ = 100^\circ$$

$$e = 180^\circ - 115^\circ = 65^\circ$$

Q2. $a = 71^\circ + 40^\circ = 111^\circ, b = 32^\circ + 42^\circ = 74^\circ$

$$\frac{180^\circ - 44^\circ}{2} = \frac{136^\circ}{2} = 68^\circ \Rightarrow c = 68^\circ + 44^\circ = 112^\circ$$

$$\frac{180^\circ - 50^\circ}{2} = \frac{130^\circ}{2} = 65^\circ = d, e = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$$

$$f + f = 105^\circ \Rightarrow 2f = 105^\circ \Rightarrow f = 52\frac{1}{2}^\circ$$

$$g + g = 180^\circ - 60^\circ \Rightarrow 2g = 120^\circ \Rightarrow g = 60^\circ$$

$$h + h = 60^\circ \Rightarrow 2h = 60^\circ \Rightarrow h = 30^\circ$$

Q3. $a + a = 124^\circ \Rightarrow 2a = 124^\circ \Rightarrow a = 62^\circ$

$$b + 35^\circ + 35^\circ = 180^\circ \Rightarrow b = 180^\circ - 70^\circ = 110^\circ$$

$$c + 55^\circ = 110^\circ \Rightarrow c = 110^\circ - 55^\circ = 55^\circ$$

Exterior angle of \triangle = sum of 2 interior opposite angles $\Rightarrow 30^\circ + 43^\circ = 73^\circ$.

$$\text{Hence, } d + 73^\circ + 73^\circ = 180^\circ \Rightarrow d = 180^\circ - 146^\circ = 34^\circ$$

Q4. (i) $x = 55^\circ, y = 180^\circ - (55^\circ + 80^\circ) = 45^\circ$

(ii) $x + 32^\circ + 32^\circ = 180^\circ \Rightarrow x = 180^\circ - 64^\circ = 116^\circ$

$$180^\circ - 116^\circ = 64^\circ \Rightarrow y + 64^\circ + 64^\circ = 180^\circ \\ \Rightarrow y = 180^\circ - 128^\circ = 52^\circ$$

(iii) $x + 50^\circ + 50^\circ = 180^\circ \Rightarrow x = 180^\circ - 100^\circ = 80^\circ$

$$y + 50^\circ = 80^\circ \Rightarrow y = 80^\circ - 50^\circ = 30^\circ$$

Q5. (i) Horizontal line = $y \Rightarrow y^2 + (3)^2 = (7)^2$

$$\Rightarrow y^2 + 9 = 49$$

$$\Rightarrow y^2 = 49 - 9 = 40$$

$$\Rightarrow y = \sqrt{40}$$

$$\text{Hence, } x^2 = (6)^2 + (\sqrt{40})^2$$

$$= 36 + 40 = 76$$

$$\Rightarrow x = \sqrt{76} = 2\sqrt{19}$$

$$\begin{aligned}
 \text{(ii) Vertical line } &= y \Rightarrow y^2 + (3)^2 = (5)^2 \\
 &\Rightarrow y^2 + 9 = 25 \\
 &\Rightarrow y^2 = 25 - 9 = 16 \\
 &\Rightarrow y = \sqrt{16} = 4
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence, } &x^2 = (4)^2 + (4)^2 \\
 &\Rightarrow x^2 = 16 + 16 = 32 \\
 &\Rightarrow x = \sqrt{32} = 4\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) Vertical line } &= y \Rightarrow y^2 + (6)^2 = (8)^2 \\
 &\Rightarrow y^2 + 36 = 64 \\
 &\Rightarrow y^2 = 64 - 36 = 28 \\
 &\Rightarrow y = \sqrt{28}
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence, } &x^2 = (\sqrt{28})^2 + (4)^2 \\
 &= 28 + 16 \\
 &= 44 \\
 &\Rightarrow x = \sqrt{44} = 2\sqrt{11}
 \end{aligned}$$

$$\begin{aligned}
 \text{Q6. } &x^2 + (\sqrt{44})^2 = (12)^2 \\
 &\Rightarrow x^2 + 44 = 144 \\
 &\Rightarrow x^2 = 144 - 44 = 100 \\
 &\Rightarrow x = \sqrt{100} = 10 \\
 \text{Hence, } &y^2 + (6)^2 = (10)^2 \\
 &\Rightarrow y^2 + 36 = 100 \\
 &\Rightarrow y^2 = 100 - 36 = 64 \\
 &\Rightarrow y = \sqrt{64} = 8
 \end{aligned}$$

$$\begin{aligned}
 \text{Q7. } |AC| &= |AD| \Rightarrow |\angle ACD| = |\angle ADC| \\
 &\Rightarrow 180^\circ - |\angle ACD| = 180^\circ - |\angle ADC| \\
 &\Rightarrow |\angle ACB| = |\angle ADE| \\
 |BD| &= |CE| \Rightarrow |BC| + |CD| = |CD| + |DE| \\
 &\Rightarrow |BC| = |DE|
 \end{aligned}$$

In Δ s ABC, ADE

$$\begin{aligned}
 |AC| &= |AD| \\
 |\angle ACB| &= |\angle ADE| \\
 |BC| &= |DE|
 \end{aligned}$$

\Rightarrow Δ s are congruent by S.A.S.

Q8. (i) alternate angles

(ii) In Δs AMD, MBP

$$|\angle DAM| = |\angle MBP|$$

$$|AM| = |MB|$$

$$|\angle AMD| = |\angle BMP|$$

$\Rightarrow \Delta s$ are congruent by A.S.A.

(iii) Δs AMD, MBP are congruent $\Rightarrow |AD| = |PB|$

$$ABCD \text{ is a parallelogram} \Rightarrow |AD| = |BC|$$

$$\Rightarrow |PB| = |BC|$$

$\Rightarrow B$ is the midpoint of $[CP]$

Q9. (i) Lengths of sides may be different

(ii) In Δs PTQ, STR

$$|\angle TPQ| = |\angle TSR|$$

$$|PQ| = |RS|$$

$$|\angle PQT| = |\angle TRS|$$

$\Rightarrow \Delta s$ are congruent by A.S.A.

Q10. (i) $90^\circ + |\angle CBG| = 90^\circ + |\angle CBE|$

$$\Rightarrow |\angle ABG| = |\angle CBE|$$

(ii) In Δs ABG, CBE

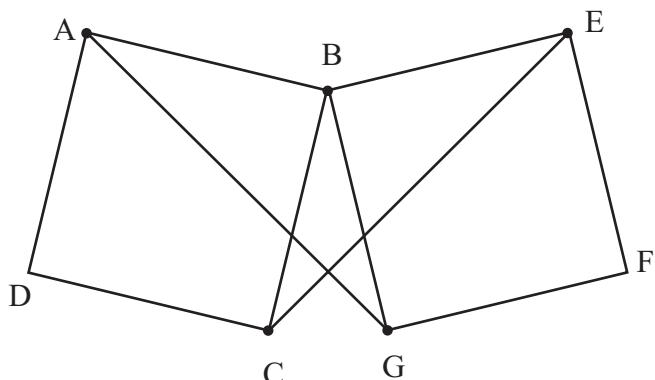
$$|AB| = |BE|$$

$$|\angle ABG| = |\angle CBE|$$

$$|BG| = |BC|$$

$\Rightarrow \Delta s$ are congruent by S.A.S.

$$\Rightarrow |AG| = |CE|$$



Q11. $|ED|^2 = (4)^2 + (8)^2$

$$= 16 + 64$$

$$= 80$$

$$\Rightarrow |ED| = \sqrt{80}$$

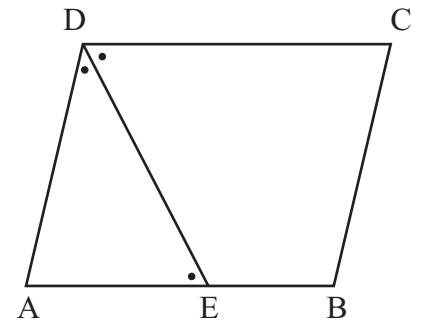
$$|HD|^2 = |HE|^2 + |ED|^2 = (5)^2 + (\sqrt{80})^2$$

$$= 25 + 80$$

$$= 105$$

$$\Rightarrow |HD| = \sqrt{105}$$

Q12. $|\angle ADE| = |\angle EDC|$
 $|\angle DEA| = |\angle EDC|$
 $\Rightarrow |\angle ADE| = |\angle DEA|$
 $\Rightarrow |AD| = |AE|$
 ABCD is a parallelogram $\Rightarrow |AD| = |BC|$
 $\Rightarrow |AE| = |BC|$



Exercise 3.2

Q1. (i) Area $\triangle ABC = \frac{1}{2}(9)(8) = 36 \text{ cm}^2$

(ii) Area $\triangle ABC = \frac{1}{2}(12)(h) = 36$
 $\Rightarrow 6h = 36$
 $\Rightarrow h = \frac{36}{6} = 6 \text{ cm}$

Q2. (i) Area $\triangle = \frac{1}{2}(6)(8) = 24$
 Area $\triangle = \frac{1}{2}(10)(h) = 24$
 $\Rightarrow 5h = 24$
 $\Rightarrow h = \frac{24}{5} = 4.8 \text{ cm}$

(ii) Area $\triangle = \frac{1}{2}(12)(14) = 84$
 Area $\triangle = \frac{1}{2}(16)(h) = 84$
 $\Rightarrow 8h = 84$
 $\Rightarrow h = \frac{84}{8} = 10.5 \text{ cm}$

(iii) Area $\triangle = \frac{1}{2}(24)(18) = 216$
 Area $\triangle = \frac{1}{2}(20)(h) = 216$
 $\Rightarrow 10h = 216$
 $\Rightarrow h = \frac{216}{10} = 21.6 \text{ cm}$

Q3. (i) Area parallelogram = $(12)(8) = 96 \text{ cm}^2$

(ii) Area parallelogram = $(14)(9) = 126 \text{ cm}^2$

(iii) Area parallelogram = $(13)(11) = 143 \text{ cm}^2$

Q4. Area ABCD = $(22)(14) = 308 \text{ cm}^2$

Area ABCD = $|BC|.(18) = 308$

$$\Rightarrow |BC| = \frac{308}{18} = 17\frac{1}{9} \text{ cm}$$

Q5. (i) Largest is $\angle BAC$; Smallest is $\angle ACB$

(ii) $|AC| > 5 \text{ cm}$ and $< 15 \text{ cm}$

Q6. (i) Area ABCD = 80

$$\Rightarrow \text{Area triangle ADC} = 40$$

$$\Rightarrow \frac{1}{2}|AC|.|DE| = 40$$

$$\Rightarrow \frac{1}{2}(16)|DE| = 40$$

$$\Rightarrow 8|DE| = 40$$

$$\Rightarrow |DE| = \frac{40}{8} = 5 \text{ cm}$$

(ii) Area $\triangle ADC = \text{Area } \triangle ABC$

$$\Rightarrow \frac{1}{2}|AC|.|DE| = \frac{1}{2}|AC|.|BF|$$

$$\Rightarrow |DE| = |BF|$$

(iii) Area ABCD = $|AB|.h = 80$

$$\Rightarrow (10).h = 80$$

$$\Rightarrow h = \frac{80}{10} = 8 \text{ cm}$$

Q7. (i) Area ABCD = 2 Area $\triangle DCB = 2(15) = 30 \text{ cm}^2$

(ii) Area ADBE = $2(15) = 30 \text{ cm}^2$

(iii) Area ADCE = $3(15) = 45 \text{ cm}^2$

(iv) Area ABCD = $|DC| \cdot h = 30$
 $\Rightarrow 7.5h = 30$
 $\Rightarrow h = \frac{30}{7.5} = 4 \text{ cm}$

Q8. (i) [AF] bisects area ABFE.

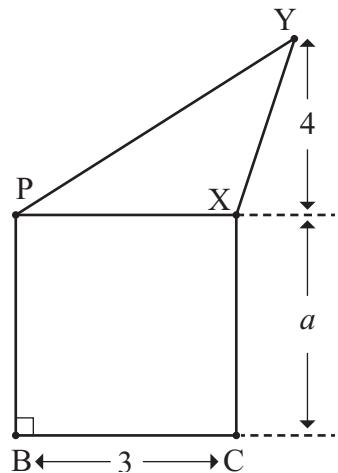
Since area $\triangle AFE = 30 \text{ sq.units} \Rightarrow$ Area $\triangle AFB = 30 \text{ sq.units}$

(ii) Area ABCDE = $30 + 30 + 30 + 60 = 150 \text{ sq.units}$

Q9. (i) Area $\triangle PXY = \frac{1}{2}|PX| \cdot h = \frac{1}{2}(3)(4) = 6$

Area Rectangle BPXC = $(3)(a) = 3a$

\Rightarrow Total area = $3a + 6 = 3(a + 2)$



Shaded area = Area $\triangle DEG -$ Area $\triangle EFG$

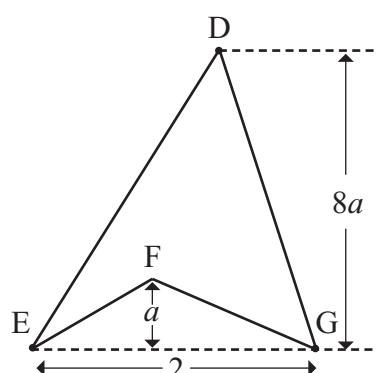
$$= \frac{1}{2}(2)(8a) - \frac{1}{2}(2)(a)$$

$$= 8a - a = 7a$$

(ii) Hence, $7a = 3a + 6$

$$\Rightarrow 4a = 6$$

$$\Rightarrow a = \frac{6}{4} = 1.5$$



Q10. (i) Area ABCD + Area \triangle ADE

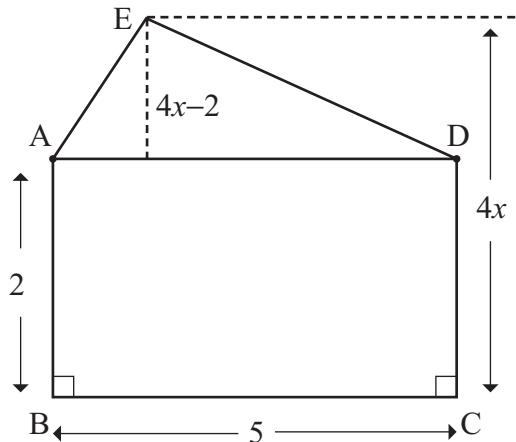
$$\begin{aligned} &= (5)(2) + \frac{1}{2}(5)(4x - 2) \\ &= 10 + 10x - 5 \\ &= 10x + 5 = 5(2x + 1) \end{aligned}$$

Area EFGH + Area HGQP

$$\begin{aligned} &= (3)(x) + (3)(3x) \\ &= 3x + 9x \\ &= 12x \end{aligned}$$

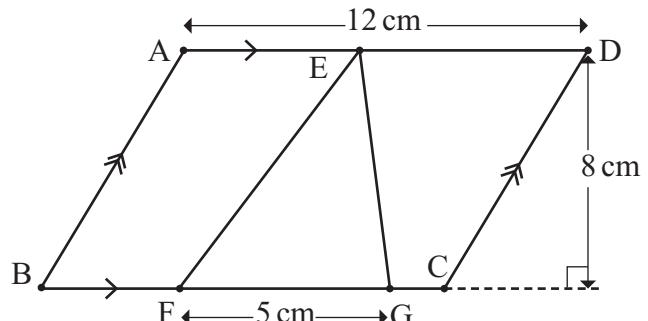
(ii) Hence, $12x = 10x + 5$

$$\begin{aligned} &\Rightarrow 2x = 5 \\ &\Rightarrow x = \frac{5}{2} = 2.5 \end{aligned}$$



Q11. Shaded area = Area ABCD - Area \triangle EFG

$$\begin{aligned} &= (12)(8) - \frac{1}{2}(5)(8) \\ &= 96 - 20 \\ &= 76 \text{ cm}^2 \end{aligned}$$



Q12. (i) AFED is a parallelogram

and diagonals [AE] and [DF] bisect each other.

In \triangle s AXD, FXE;

$$|AX| = |XE|$$

$$|\angle AXD| = |\angle FXE|$$

$$|AX| = |XF|$$

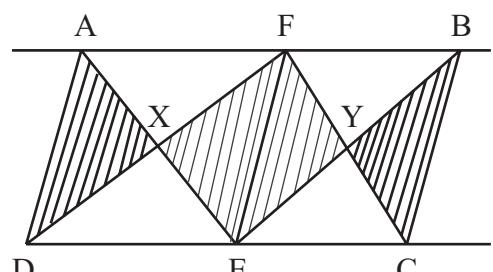
\Rightarrow \triangle s are congruent by S.A.S.

\Rightarrow \triangle s AXD, FXE are equal in area.

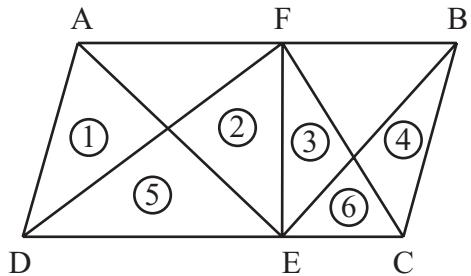
Similarly, \triangle s FYE, BYC are equal in area.

Hence, Area \triangle AXD + Area \triangle BYC = Area \triangle FXE + Area \triangle FYE

i.e., Blue shaded Area = Yellow shaded Area



- (ii) $AB \parallel DC \Rightarrow \text{Area } \triangle ADE = \text{Area } \triangle FDE$
Hence, Area Triangles $\textcircled{1} + \textcircled{5} = \text{Area } \triangle \textcircled{2} + \textcircled{5}$
 $\Rightarrow \text{Area } \triangle \textcircled{1} = \text{Area } \triangle \textcircled{2}$
Similarly, Area $\triangle FEC = \text{Area } \triangle BEC$
Hence, Area $\triangle \textcircled{3} + \textcircled{6} = \text{Area } \triangle \textcircled{4} + \textcircled{6}$
 $\Rightarrow \text{Area } \triangle \textcircled{3} = \text{Area } \triangle \textcircled{4}$
Hence, Area $\triangle \textcircled{1} + \textcircled{4} = \text{Area } \triangle \textcircled{2} + \textcircled{3}$
ie, Blue shaded Area = Yellow shaded Area



Q13. (i) $\text{Area } \triangle AFD = \frac{1}{2} \text{Area } ABCD$

$$\text{Area } \triangle DEC = \frac{1}{2} \text{Area } ABCD$$

$$\Rightarrow \text{Area } \triangle AFD = \frac{1}{2} \text{Area } \triangle DEC$$

ie., Area $\triangle \textcircled{1} + \textcircled{2} + \text{shaded green} = \text{Area } \triangle \textcircled{3} + \textcircled{4} + \text{shaded green}$

$$\Rightarrow \text{Area } \triangle \textcircled{1} + \textcircled{2} = \text{Area } \triangle \textcircled{3} + \textcircled{4}$$

(ii) $\text{Area } \triangle AFD = \frac{1}{2} \text{Area } ABCD$

$$\text{Hence, remainder} = \frac{1}{2} \text{Area } ABCD$$

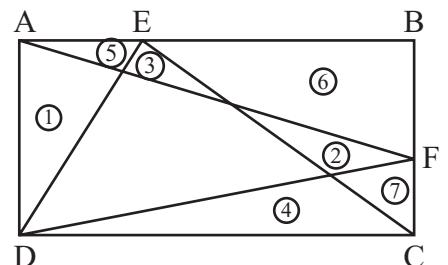
$$\text{ie, Area } \triangle \text{ABF} + \triangle FCD = \frac{1}{2} \text{Area } ABCD$$

Hence, Area $\triangle AFD = \text{Area } \triangle ABF + \text{Area } \triangle FCD$

$$\Rightarrow \text{Area } \triangle \textcircled{1} + \textcircled{2} + \text{shaded green} = \text{Area } \triangle \textcircled{5} + \textcircled{3} + \textcircled{6} + \textcircled{4} + \textcircled{7}$$

$$\Rightarrow \text{shaded green} = \text{Area } \triangle \textcircled{5} + \textcircled{6} + \textcircled{7}$$

ie, Region shaded in green = Region shaded in blue



Exercise 3.3

Q1. (i) $\frac{6}{3} = \frac{5}{x} \Rightarrow 6x = 15$
 $\Rightarrow x = \frac{15}{6} = 2.5$

(ii) $\frac{6}{4} = \frac{x}{6} \Rightarrow 4x = 36$
 $\Rightarrow x = \frac{36}{4} = 9$

(iii) $\frac{12}{5} = \frac{14}{x} \Rightarrow 12x = 70$
 $\Rightarrow x = \frac{70}{12} = 5\frac{5}{6}$

$$\text{Q2. } \frac{5}{7} = \frac{6}{x} \Rightarrow 5x = 42 \quad \frac{5}{2} = \frac{7}{y}$$

$$\Rightarrow x = \frac{42}{5} = 8.4 \quad \Rightarrow 5y = 14$$

$$\Rightarrow y = 2.8$$

$$\frac{a}{4} = \frac{6}{4} \Rightarrow 4a = 24$$

$$\Rightarrow a = \frac{24}{4} = 6$$

$$\frac{7}{10} = \frac{b}{12} \Rightarrow 10b = 84$$

$$\Rightarrow b = \frac{84}{10} = 8.4$$

$$\text{Q3. } \frac{|RS|}{|RX|} = \frac{|RT|}{|RY|} \Rightarrow \frac{12}{5} = \frac{|RT|}{6}$$

$$\Rightarrow 5|RT| = 72$$

$$\Rightarrow |RT| = \frac{72}{5} = 14.4$$

$$\text{Q4. } \frac{|AX|}{|AB|} = \frac{|AY|}{|AC|} \Rightarrow \frac{3}{5} = \frac{|AY|}{8}$$

$$\Rightarrow 5|AY| = 24$$

$$\Rightarrow |AY| = \frac{24}{5} = 4.8$$

$$\text{Q5. } \frac{|BC|}{|PQ|} = \frac{|AC|}{|AQ|} \Rightarrow \frac{|BC|}{3} = \frac{7}{5}$$

$$\Rightarrow 5|BC| = 21$$

$$\Rightarrow |BC| = \frac{21}{5} = 4.2$$

$$\frac{|BP|}{|PA|} = \frac{|CQ|}{|QA|} \Rightarrow \frac{|BP|}{4} = \frac{2}{5}$$

$$\Rightarrow 5|BP| = 8$$

$$\Rightarrow |BP| = \frac{8}{5} = 1.6$$

$$\text{Q6. (i)} \quad \frac{|AY|}{|YC|} = \frac{|AX|}{|XB|} \Rightarrow \frac{|AY|}{10} = \frac{2}{1} \\ \Rightarrow |AY| = 20 \text{ cm}$$

$$\text{(ii)} \quad \frac{|XY|}{|BC|} = \frac{|AX|}{|AB|} \Rightarrow \frac{|XY|}{|BC|} = \frac{2}{3} \\ \Rightarrow |XY| : |BC| = 2 : 3$$

$$\text{(iii)} \quad \frac{|XY|}{|BC|} = \frac{|AX|}{|AB|} \Rightarrow \frac{|XY|}{30} = \frac{2}{3} \\ \Rightarrow 3|XY| = 60 \\ \Rightarrow |XY| = \frac{60}{3} = 20 \text{ cm}$$

$$\text{Q7. (i)} \quad \frac{|DE|}{|EF|} = \frac{|GH|}{|HI|} \Rightarrow \frac{1}{1} = \frac{8}{|HI|} \\ \Rightarrow |HI| = 8 \text{ cm}$$

$$\text{(ii)} \quad \frac{|DE|}{|EF|} = \frac{|GJ|}{|JK|} \Rightarrow \frac{1}{1} = \frac{|GJ|}{7} \\ \Rightarrow |GJ| = 7 \text{ cm}$$

$$\text{Q8.} \quad \frac{4}{x} = \frac{|AB|}{3} \Rightarrow x \cdot |AB| = 12 \\ \Rightarrow |AB| = \frac{12}{x}$$

- Q9. (i)** Δ s ABC, DEF are equiangular
because • $|\angle BAC| = |\angle EDF|$
• $|\angle ABC| = |\angle DEF|$
• $|\angle ACB| = |\angle DFE|.$

Hence, Δ s ABC, DEF are similar.

(ii) [DF]

$$\begin{aligned} \text{(iii)} \quad \frac{3.5}{8} &= \frac{6}{x} \Rightarrow (3.5)x = 48 \\ &\Rightarrow x = \frac{48}{3.5} = \frac{96}{7} \\ \frac{3.5}{8} &= \frac{4}{y} \Rightarrow (3.5)y = 32 \\ &\Rightarrow y = \frac{32}{3.5} = \frac{64}{7} \end{aligned}$$

Q10. $\frac{x}{6} = \frac{6}{8} \Rightarrow 8x = 36$

$$\Rightarrow x = \frac{36}{8} = 4.5$$

$$\frac{3}{y} = \frac{6}{8} \Rightarrow 6y = 24$$

$$\Rightarrow y = \frac{24}{6} = 4$$

Q11. (i) [XY] corresponds to [AB] because $|\angle YZX| = |\angle BCA|$.

$$\begin{aligned} \text{(ii)} \quad \frac{x}{6} &= \frac{15}{10} \Rightarrow 10x = 90 \\ &\Rightarrow x = \frac{90}{10} = 9 \end{aligned}$$

$$\begin{aligned} \frac{y}{9} &= \frac{15}{10} \Rightarrow 10y = 135 \\ &\Rightarrow y = \frac{135}{10} = 13.5 \end{aligned}$$

Q12. (i) $\frac{y}{6} = \frac{15}{9} \Rightarrow 9y = 90$

$$\Rightarrow y = \frac{90}{9} = 10$$

$$\frac{x}{10} = \frac{15}{9} \Rightarrow 9x = 150$$

$$\Rightarrow x = \frac{150}{9} = 16\frac{2}{3}$$

(ii) $\frac{x}{12} = \frac{4}{6}$

$$\Rightarrow 6x = 48 \Rightarrow x = 8$$

$$\frac{y}{x} = \frac{4}{6}$$

$$\Rightarrow \frac{y}{8} = \frac{4}{6} \dots (x = 8)$$

$$\Rightarrow 6y = 32$$

$$\Rightarrow y = 5\frac{2}{6} = 5\frac{1}{3}$$

$$\text{Q13. (i)} \quad \frac{|CD|}{6} = \frac{12}{8} \Rightarrow 8|CD| = 72 \\ \Rightarrow |CD| = \frac{72}{8} = 9$$

$$\text{(ii)} \quad \frac{|AD|}{12} = \frac{12}{8} \Rightarrow 8|AD| = 144 \\ \Rightarrow |AD| = \frac{144}{8} = 18$$

Q14. (i) $\Delta s DAB, DBC$ are equiangular because

- $|\angle DAB| = |\angle DBC|$
- $|\angle ABD| = |\angle CDB|$

Hence, $\Delta s DAB, DBC$ are similar.

$$\text{(ii)} \quad \frac{4}{|DB|} = \frac{|DB|}{9} \Rightarrow |DB|^2 = 36 \\ \Rightarrow |DB| = \sqrt{36} = 6$$

Q15. $\Delta s ABD, ACD$ are equiangular

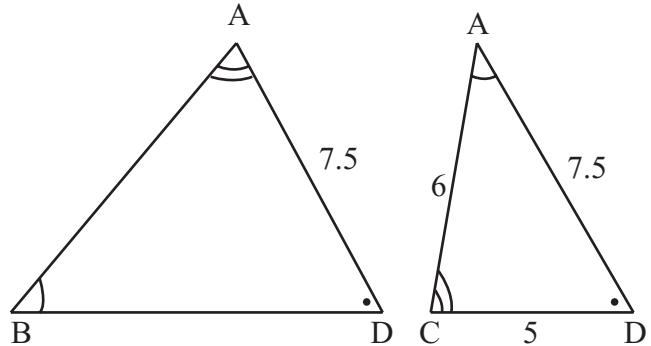
because (i) $|\angle ADB| = |\angle ADC|$
(ii) $|\angle ABD| = |\angle CAD|$

Hence, $|\angle BAD| = |\angle ACD|$

$\Rightarrow \Delta s ABD, ACD$ are similar.

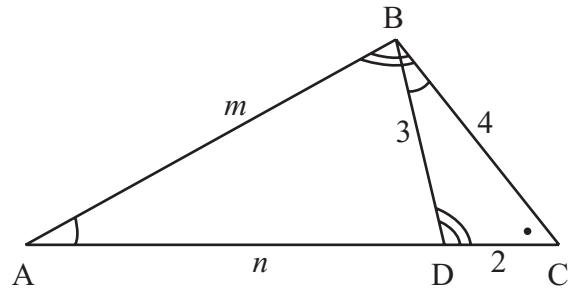
$$\frac{|BD|}{|AD|} = \frac{|AD|}{|CD|} \Rightarrow \frac{|BD|}{7.5} = \frac{7.5}{5} \\ \Rightarrow 5|BD| = 56.25 \\ \Rightarrow |BD| = \frac{56.25}{5} = 11.25$$

$$\frac{|AB|}{|AC|} = \frac{|AD|}{|CD|} \Rightarrow \frac{|AB|}{6} = \frac{7.5}{5} \\ \Rightarrow 5|AB| = 45 \\ \Rightarrow |AB| = \frac{45}{5} = 9$$



Q16. (i) $\triangle ABC$ and $\triangle DBC$

$$\begin{aligned} \text{(ii)} \quad & \frac{|AB|}{|BD|} = \frac{|BC|}{|DC|} \Rightarrow \frac{m}{3} = \frac{4}{2} \\ & \Rightarrow 2m = 12 \\ & \Rightarrow m = 6 \\ & \frac{|AC|}{|BC|} = \frac{|BC|}{|DC|} \Rightarrow \frac{n+2}{4} = \frac{4}{2} \\ & \Rightarrow 2n + 4 = 16 \\ & \Rightarrow 2n = 12 \\ & \Rightarrow n = 6 \end{aligned}$$



Q17. $\frac{3}{6} = \frac{4}{3+x}$

$$\Rightarrow 9 + 3x = 24$$

$$\Rightarrow 3x = 15$$

$$\Rightarrow x = 5$$

Q18. (i) $\triangle WXZ$

$$\begin{aligned} \text{(ii)} \quad & \frac{|XZ|}{|WX|} = \frac{|WX|}{|XY|} \Rightarrow \frac{v+3}{5} = \frac{5}{3} \\ & \Rightarrow 3v + 9 = 25 \\ & \Rightarrow 3v = 16 \\ & \Rightarrow v = \frac{16}{3} = 5\frac{1}{3} \end{aligned}$$

$$w^2 = \left(5\frac{1}{3}\right)^2 + (4)^2$$

$$= \frac{256}{9} + 16 = \frac{400}{9}$$

$$\Rightarrow w = \sqrt{\frac{400}{9}} = \frac{20}{3} = 6\frac{2}{3}$$

Q19.

$$\frac{x}{1} = \frac{1}{x+1}$$

$$\Rightarrow x^2 + x = 1$$

$$\Rightarrow x^2 + x - 1 = 0$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

$$= \frac{-1 + 2.236}{2}$$

$$= 0.618$$

Hence, ratio = $\frac{1}{0.618+1} = \frac{1}{1.618}$ or 1:1.618

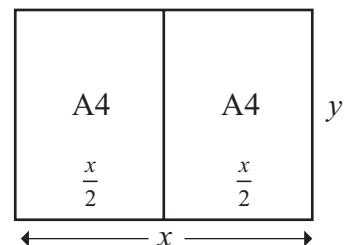
Q20.

$$\frac{\frac{x}{2}}{y} = \frac{y}{x}$$

$$\Rightarrow y^2 = \frac{x^2}{2}$$

$$\Rightarrow x^2 = 2y^2$$

$$\Rightarrow x = \sqrt{2y^2} = \sqrt{2}y \Rightarrow \frac{x}{y} = \frac{\sqrt{2}}{1}$$



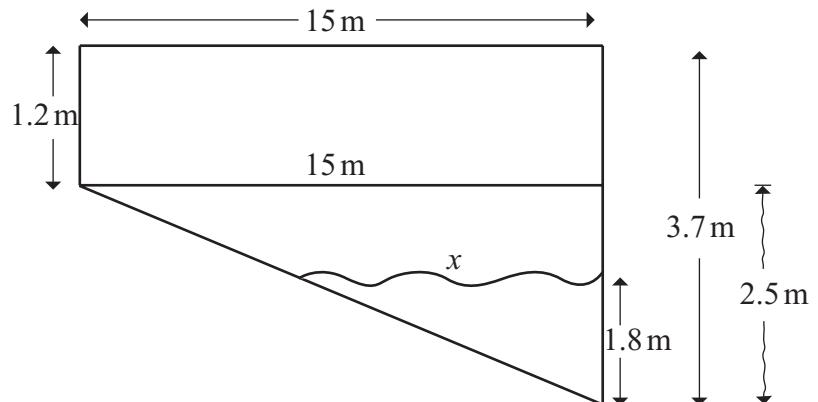
Q21.

$$\frac{15}{2.5} = \frac{x}{1.8}$$

$$\Rightarrow 2.5x = 27$$

$$\Rightarrow x = \frac{27}{2.5}$$

$$\Rightarrow x = 10.8 \text{ m}$$



Exercise 3.4

Q1. (i) $\angle a = 2(48^\circ) = 96^\circ$
 $\angle b = 2(44^\circ) = 88^\circ$ and $\angle c = 180^\circ - 44^\circ = 136^\circ$
 $\angle d = 2(42^\circ) = 84^\circ$ and $\angle e = \frac{1}{2}(180^\circ - 84^\circ) = 48^\circ$

Q2. $\angle a = 52^\circ, \angle b = 44^\circ, \angle c = 45^\circ, \angle d = 20^\circ$

Q3. $\angle a = 47^\circ$, $\angle b = 2(47^\circ) = 94^\circ$, $\angle c = \frac{1}{2}(180^\circ - 94^\circ) = 43^\circ$

Q4. $\angle f = 40^\circ$, $\angle g = 180^\circ - (85^\circ + 40^\circ) = 55^\circ$, $\angle h = \angle g = 55^\circ$

Q5. $\angle a = 42^\circ$, $\angle b = 90^\circ - 42^\circ = 48^\circ$

$$50^\circ + 50^\circ = 100^\circ \Rightarrow 180^\circ - 100^\circ = 80^\circ$$

$$\Rightarrow \angle c = \frac{1}{2}(80^\circ) = 40^\circ$$

$$\angle d = \frac{1}{2}(180^\circ - 70^\circ) = 55^\circ$$

$$\angle e = 90^\circ - \angle d = 90^\circ - 55^\circ = 35^\circ$$

Q6. $\angle a = 180^\circ - 85^\circ = 95^\circ$

$$\angle b = 180^\circ - 105^\circ = 75^\circ$$

$$180^\circ - 86^\circ = 94^\circ \Rightarrow 2\angle c = 180^\circ - 94^\circ = 86^\circ$$

$$\Rightarrow \angle c = \frac{1}{2}(86^\circ) = 43^\circ$$

$$\angle d = 180^\circ - (32^\circ + 32^\circ) = 116^\circ$$

$$\angle e = 180^\circ - 116^\circ = 64^\circ$$

Q7. $360^\circ - (108^\circ + 120^\circ) = 132^\circ$

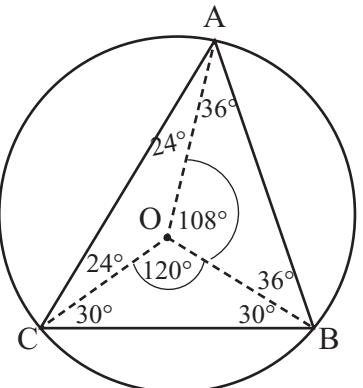
$$180^\circ - 132^\circ = 48^\circ$$

$$\Rightarrow 48^\circ \div 2 = 24^\circ$$

$$\Rightarrow \angle A = 24^\circ + 36^\circ = 60^\circ$$

$$\angle B = 36^\circ + 30^\circ = 66^\circ$$

$$\angle C = 24^\circ + 30^\circ = 54^\circ$$



Q8. (i) $|\angle OST| = 90^\circ$

(ii) $|\angle OSP| = 90^\circ - 40^\circ = 50^\circ$

(iii) $|\angle OPS| = |\angle OSP| = 50^\circ$

(iv) $|\angle SOP| = 180^\circ - (50^\circ + 50^\circ) = 80^\circ$

Q9. $\angle a = 55^\circ$
 $\angle b = 90^\circ, \angle c = 180^\circ - (90^\circ + 41^\circ) = 49^\circ$
 $\angle d = 90^\circ - 41^\circ = 49^\circ$
 $\angle e = 90^\circ - 23^\circ = 67^\circ$

Q10. (i) $|\angle BAC| = 62^\circ$

(ii) $|\angle ABD| = 44^\circ$

Q11. (i) In Δs AOP, BOP

$$\begin{aligned} |OA| &= |OB| \\ |\angle OAP| &= |\angle OBP| \\ |OP| &= |OP| \\ \Rightarrow \Delta s \text{ are congruent by R.H.S.} \end{aligned}$$

(ii) Since Δs AOP, BOP are congruent

$$\Rightarrow |PA| = |PB|$$

(iii) $|\angle APB| + |\angle AOB| = 360^\circ - (90^\circ + 90^\circ)$
 $= 180^\circ$

Q12. $|\angle AOT| = 180^\circ - 40^\circ = 140^\circ$

$$\begin{aligned} \Rightarrow |\angle ATO| + |\angle OAT| &= 2|\angle ATO| = 40^\circ \\ &= |\angle ATO| = \frac{40^\circ}{2} = 20^\circ \end{aligned}$$

Q13. (i) $|\angle ABC| = 180^\circ - 34^\circ = 146^\circ$

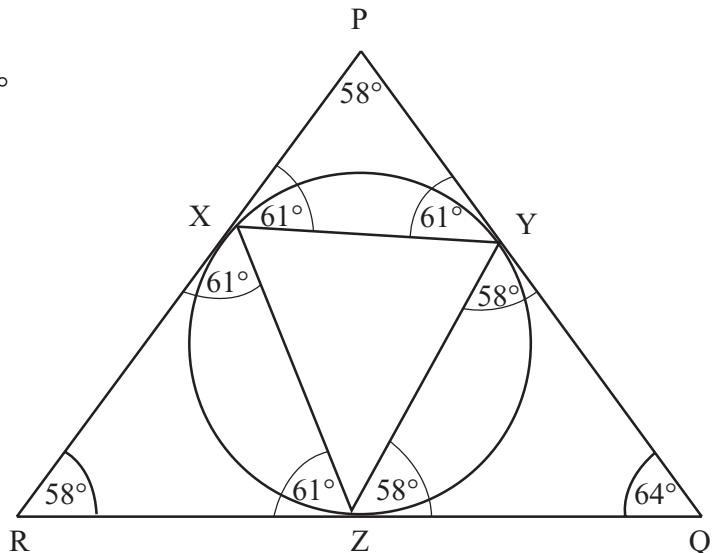
(ii) $|\angle BAC| + |\angle BCA| = 180^\circ - 146^\circ$
 $\Rightarrow 2|\angle BAC| = 34^\circ \quad \dots |\angle BAC| = |\angle BCA| \text{ as } \triangle ABC \text{ is isosceles}$
 $\Rightarrow |\angle BAC| = \frac{34^\circ}{2} = 17^\circ$

Q14. $\angle x + \angle x + \angle y + \angle y = 180^\circ$

$$\begin{aligned} \Rightarrow 2(\angle x + \angle y) &= 180^\circ \\ \Rightarrow \angle x + \angle y &= \frac{180^\circ}{2} = 90^\circ \end{aligned}$$

Q15. (i) Δs XRZ, YQZ, PYX

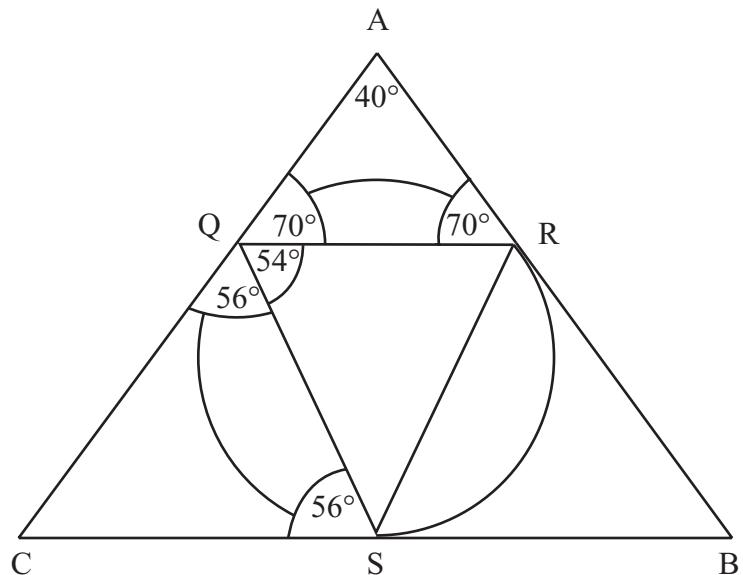
$$\begin{aligned} \text{(ii)} \quad & |\angle RPQ| = 180^\circ - (58^\circ + 64^\circ) = 58^\circ \\ & \Rightarrow |\angle PXY| + |\angle PYX| = 180^\circ - 58^\circ = 122^\circ \\ & \Rightarrow 2|\angle PXY| = 122^\circ \\ & \Rightarrow |\angle PXY| = 61^\circ \end{aligned}$$



$$\begin{aligned} \text{(iii)} \quad & |\angle XZY| = 180^\circ - (61^\circ + 58^\circ) = 61^\circ \\ & |\angle ZYX| = 180^\circ - (61^\circ + 58^\circ) = 61^\circ \\ & |\angle ZXZ| = 180^\circ - (61^\circ + 61^\circ) = 58^\circ \end{aligned}$$

Q16. $|\angle QAR| = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$

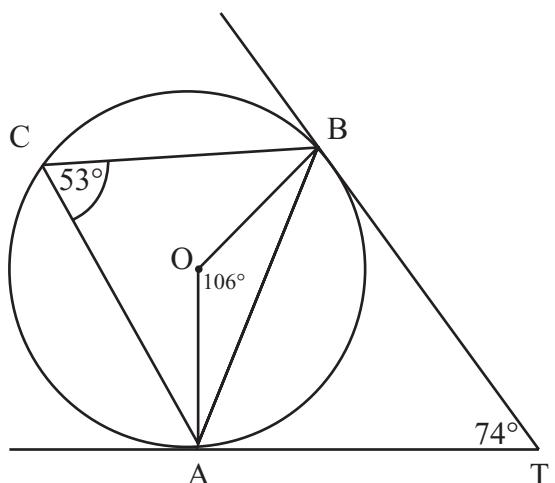
$$\begin{aligned} |\angle CQS| &= 180^\circ - (70^\circ + 54^\circ) = 56^\circ \\ |\angle ACB| &= 180^\circ - (56^\circ + 56^\circ) = 68^\circ \end{aligned}$$



Q17. (i) $|\angle AOB| = 2(53^\circ) = 106^\circ$

$$\text{(ii)} \quad |\angle BTA| = 180^\circ - 106^\circ = 74^\circ$$

$$\begin{aligned} \text{(iii)} \quad & |\angle ABT| + |\angle BAT| = 180^\circ - 74^\circ \\ & \Rightarrow 2|\angle ABT| = 106^\circ \\ & \Rightarrow |\angle ABT| = \frac{106^\circ}{2} = 53^\circ \end{aligned}$$

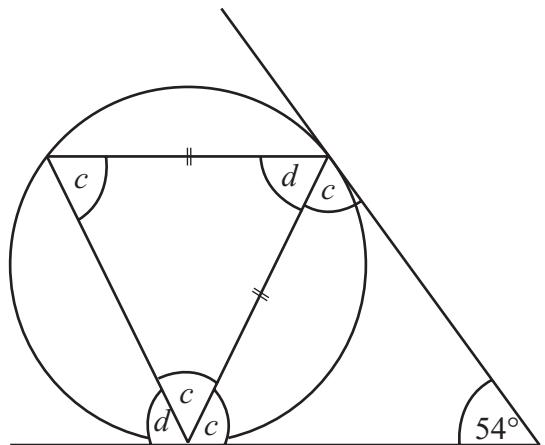


Q18. (i) $|\angle DBA| + |\angle BDA| = 180^\circ - 84^\circ$
 $\Rightarrow 2|\angle DBA| = 96^\circ$
 $\Rightarrow |\angle DBA| = \frac{96^\circ}{2} = 48^\circ$

(ii) $|\angle BCA| = |\angle BDA| = 48^\circ$

Q20. Using circle theorem in Q19:

$$\begin{aligned}\angle a &= 78^\circ, \quad \angle b = 39^\circ \\ \angle x &= 50^\circ \\ \angle y &= 180^\circ - (62^\circ + 50^\circ) = 68^\circ \\ \angle c + \angle c &= 180^\circ - 54^\circ \\ \Rightarrow 2\angle c &= 126^\circ \\ \Rightarrow \angle c &= 63^\circ \\ \angle c + \angle c + \angle d &= 180^\circ \\ \Rightarrow \angle d &= 180^\circ - (63^\circ + 63^\circ) = 54^\circ\end{aligned}$$

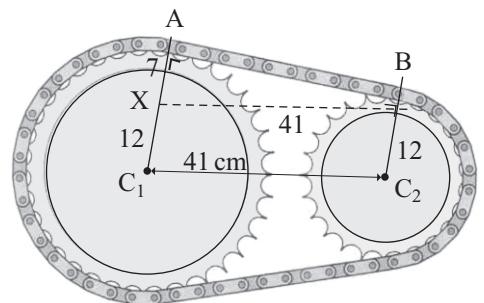


Q21. In Δs ABE, ECD;
 $|\angle BAE| = |\angle EDC|$
 $|\angle ABE| = |\angle ECD|$
 $|\angle AEB| = |\angle CED|$
 $\Rightarrow \Delta s$ are equiangular
 $\Rightarrow \Delta s$ are similar

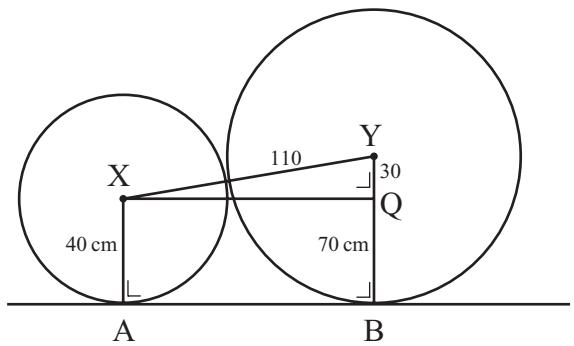
Q22. (i) $|\angle BAE| = |\angle ECD|$
 $|\angle ABE| = |\angle ECD|$
 $\Rightarrow |\angle BAE| = |\angle ABE|$
 $\Rightarrow \triangle AEB$ is isosceles

(ii) (b) is not always true.

Q23. $x^2 + (7)^2 = (41)^2$
 $\Rightarrow x^2 + 49 = 1681$
 $\Rightarrow x^2 = 1681 - 49$
 $\Rightarrow x = \sqrt{1632} = 4\sqrt{102} \quad (= 40.4 \text{ cm})$

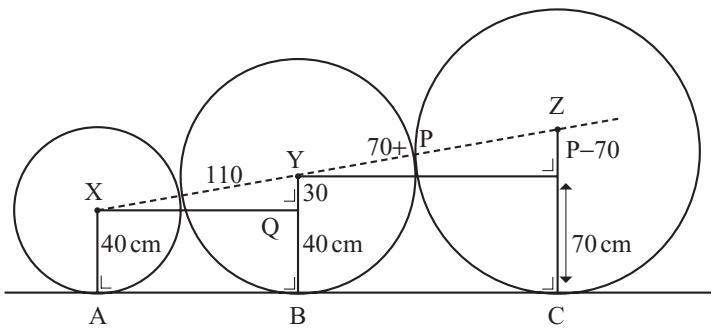


Q24. (i) $|XQ|^2 + (30)^2 = (110)^2$
 $\Rightarrow |XQ|^2 + 900 = 12100$
 $\Rightarrow |XQ|^2 = 12100 - 900$
 $\Rightarrow \quad = 11200$
 $\Rightarrow |XQ| = \sqrt{11200}$
 $\quad = 40\sqrt{7} \quad (= 105.8 \text{ cm})$
 $\Rightarrow |AB| = (40\sqrt{7}) \text{ cm}$



(ii) Triangles XYQ and YZT are similar

$$\begin{aligned}\Rightarrow \frac{110}{30} &= \frac{70+P}{P-70} \\ \Rightarrow 110P - 7700 &= 2100 + 30P \\ \Rightarrow 110P - 30P &= 2100 + 7700 \\ \Rightarrow 80P &= 9800 \\ \Rightarrow P &= \frac{9800}{80} = 122.5 \text{ cm} \\ \Rightarrow |ZC| &= 52.5 \text{ cm}\end{aligned}$$



Chapter 4 Co-ordinate Geometry: The Circle

Exercise 4.1

Q1. (i) $x^2 + y^2 = (2)^2 \Rightarrow x^2 + y^2 = 4$

(ii) $x^2 + y^2 = (5)^2 \Rightarrow x^2 + y^2 = 25$

(iii) $x^2 + y^2 = (\sqrt{2})^2 \Rightarrow x^2 + y^2 = 2$

(iv) $x^2 + y^2 = (3\sqrt{2})^2 \Rightarrow x^2 + y^2 = 18$

(v) $x^2 + y^2 = \left(\frac{3}{4}\right)^2 \Rightarrow x^2 + y^2 = \frac{9}{16}$
 $\Rightarrow 16x^2 + 16y^2 = 9$

(vi) $x^2 + y^2 = \left(2\frac{1}{2}\right)^2 \Rightarrow x^2 + y^2 = \frac{25}{4}$
 $\Rightarrow 4x^2 + 4y^2 = 25$

Q2. $(0,0), (3,4) \Rightarrow \text{radius} = \sqrt{(3-0)^2 + (4-0)^2}$
 $= \sqrt{9+16}$
 $= \sqrt{25} = 5$

Hence, $x^2 + y^2 = (5)^2 \Rightarrow x^2 + y^2 = 25$

Q3. $(0,0), (-4,1) \Rightarrow \text{radius} = \sqrt{(-4-0)^2 + (1-0)^2}$
 $= \sqrt{16+1}$
 $= \sqrt{17}$

Hence, $x^2 + y^2 = (\sqrt{17})^2 \Rightarrow x^2 + y^2 = 17$

Q4. (i) $(-4,-3), (4,3) \Rightarrow \text{midpoint} = \left(\frac{-4+4}{2}, \frac{-3+3}{2}\right)$
 $= (0,0)$ [the midpoint is the centre]

(ii) $(0,0)(4,3) \Rightarrow \text{radius} = \sqrt{(4-0)^2 + (3-0)^2}$
 $= \sqrt{16+9}$
 $= \sqrt{25} = 5$

(iii) $x^2 + y^2 = (5)^2 \Rightarrow x^2 + y^2 = 25$

Q5. $(4, -1) (-4, 1) \Rightarrow \text{midpoint} = \left(\frac{4-4}{2}, \frac{-1+1}{2} \right) = (0, 0)$

$$(0, 0) (4, -1) \Rightarrow \text{radius} = \sqrt{(4-0)^2 + (-1-0)^2} = \sqrt{16+1} = \sqrt{17}$$

Hence, $x^2 + y^2 = (\sqrt{17})^2 \Rightarrow x^2 + y^2 = 17$ is the equation of the circle.

Q6. (i) $r^2 = 9 \Rightarrow r = \sqrt{9} = 3$

(ii) $r^2 = 1 \Rightarrow r = \sqrt{1} = 1$

(iii) $r^2 = 27 \Rightarrow r = \sqrt{27} = 3\sqrt{3}$

(iv) $4x^2 + 4y^2 = 25$

$$\Rightarrow x^2 + y^2 = \frac{25}{4}$$

$$\Rightarrow r^2 = \frac{25}{4} \Rightarrow r = \sqrt{\frac{25}{4}} = \frac{5}{2}$$

(v) $9x^2 + 9y^2 = 4$

$$\Rightarrow x^2 + y^2 = \frac{4}{9}$$

$$\Rightarrow r^2 = \frac{4}{9} \Rightarrow r = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

(vi) $16x^2 + 16y^2 = 49$

$$\Rightarrow x^2 + y^2 = \frac{49}{16}$$

$$\Rightarrow r^2 = \frac{49}{16} \Rightarrow r = \sqrt{\frac{49}{16}} = \frac{7}{4}$$

Q7. (i) Radius = perpendicular distance from $(0, 0)$ to $2x + y - 5 = 0$.

$$\Rightarrow \text{radius} = \frac{|2(0) + 1(0) - 5|}{\sqrt{(2)^2 + (1)^2}} = \frac{5}{\sqrt{5}} = \frac{5}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \sqrt{5}$$

(ii) Equation of circle : $x^2 + y^2 = (\sqrt{5})^2 \Rightarrow x^2 + y^2 = 5$

Q8. Point $(0, 0)$; tangent : $4x - 3y - 25 = 0$

$$\Rightarrow \text{radius} = \frac{|4(0) - 3(0) - 25|}{\sqrt{(4)^2 + (-3)^2}} = \frac{25}{\sqrt{25}} = \frac{25}{5} = 5$$

Hence, equation of circle : $x^2 + y^2 = 5^2 \Rightarrow x^2 + y^2 = 25$

Q9. Centre = $(0, 0)$; tangent : $3x - y + 10 = 0$

$$\Rightarrow \text{radius} = \frac{|3(0) - 1(0) + 10|}{\sqrt{(3)^2 + (-1)^2}} = \frac{10}{\sqrt{10}} = \sqrt{10}$$

Hence, equation of circle : $x^2 + y^2 = (\sqrt{10})^2$

$$\Rightarrow x^2 + y^2 = 10$$

Q10. Centre $(0, 0)$, radius = $2\sqrt{5}$

\Rightarrow Equation of circle : $x^2 + y^2 = (2\sqrt{5})^2$

$$\Rightarrow x^2 + y^2 = 20$$

centre $(0, 0)$; line t : $x - 2y + 10 = 0$

$$\begin{aligned}\Rightarrow \text{perpendicular distance} &= \frac{|1(0) - 2(0) + 10|}{\sqrt{(1)^2 + (-2)^2}} \\ &= \frac{10}{\sqrt{5}} = \frac{10\sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5} = \text{radius}\end{aligned}$$

Hence, t is a tangent.

Exercise 4.2

Q1. (i) Centre $(3, 1)$, radius = 2

$$\text{Equation of circle} : (x - 3)^2 + (y - 1)^2 = (2)^2 = 4$$

(ii) Centre $(1, -4)$, radius = $\sqrt{8}$

$$\text{Equation of circle} : (x - 1)^2 + (y + 4)^2 = (\sqrt{8})^2 = 8$$

(iii) Centre $(4, 0)$, radius = $2\sqrt{3}$

$$\text{Equation of circle} : (x - 4)^2 + (y - 0)^2 = (2\sqrt{3})^2 = 12$$

(iv) Centre $(0, -5)$, radius = $3\sqrt{2}$

$$\text{Equation of circle} : (x - 0)^2 + (y + 5)^2 = (3\sqrt{2})^2 = 18$$

Q2. Centre $(2, 2)$, point $(5, 1)$ \Rightarrow radius = $\sqrt{(5-2)^2 + (1-2)^2}$

$$\begin{aligned}&= \sqrt{3^2 + (-1)^2} \\ &= \sqrt{9+1} = \sqrt{10}\end{aligned}$$

$$\text{Equation of circle} : (x - 2)^2 + (y - 2)^2 = (\sqrt{10})^2 = 10$$

Q3. (i) $(3, 5), (-1, 1) \Rightarrow$ midpoint = $\left(\frac{3-1}{2}, \frac{5+1}{2}\right) = (1, 3)$

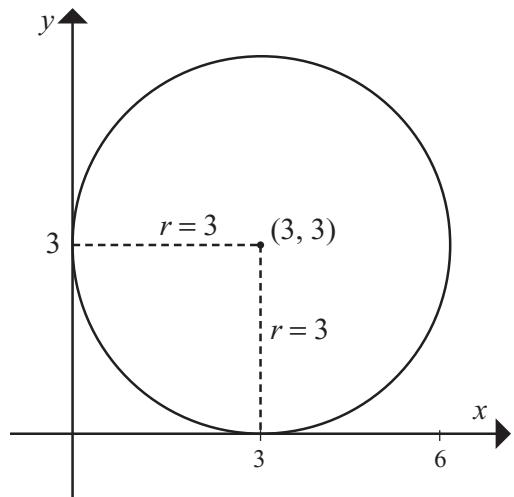
$$\begin{aligned}\text{(ii) Centre } (1, 3), \text{ point } (3, 5) \Rightarrow \text{radius} &= \sqrt{(3-1)^2 + (5-3)^2} \\ &= \sqrt{2^2 + 2^2} = \sqrt{8}\end{aligned}$$

$$\text{Equation of circle} : (x - 1)^2 + (y - 3)^2 = (\sqrt{8})^2 = 8$$

- Q4.** (i) Centre = $(3, 2)$, radius = $\sqrt{16} = 4$
(ii) Centre = $(-2, 6)$, radius = $\sqrt{8} = 2\sqrt{2}$
(iii) Centre = $(3, 0)$, radius = $\sqrt{5}$
(iv) Centre = $(0, -2)$, radius = $\sqrt{10}$

Q5. Radius = $\sqrt{18} = 3\sqrt{2}$
 \Rightarrow diameter = $2(3\sqrt{2}) = 6\sqrt{2}$
Hence, centre = $(-2, 5)$, radius = $6\sqrt{2}$
 \Rightarrow Equation of circle : $(x + 2)^2 + (y - 5)^2 = (6\sqrt{2})^2 = 72$

Q6. Centre = $(3, 3)$, radius = 3
Equation of circle : $(x - 3)^2 + (y - 3)^2 = (3)^2$
 $\Rightarrow (x - 3)^2 + (y - 3)^2 = 9$



Q7. (i) $2g = -4 \Rightarrow g = -2$
 $2f = 8 \Rightarrow f = 4$
 \Rightarrow centre = $(-g, -f) = (2, -4)$
and radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(-2)^2 + (4)^2 + 5}$
 $= \sqrt{4+16+5} = \sqrt{25} = 5$

(ii) $2g = -2 \Rightarrow g = -1$
 $2f = -6 \Rightarrow f = -3$
 \Rightarrow centre = $(-g, -f) = (1, 3)$
and radius = $\sqrt{(-1)^2 + (-3)^2 + 15}$
 $= \sqrt{1+9+15} = \sqrt{25} = 5$

(iii) $2g = -8 \Rightarrow g = -4$
 $2f = 0 \Rightarrow f = 0$
 \Rightarrow centre = $(4, 0)$
and radius = $\sqrt{(-4)^2 + (0)^2 + 8} = \sqrt{16+8} = \sqrt{24} = 2\sqrt{6}$

$$\begin{aligned}
 \text{(iv)} \quad 2g = 5 &\Rightarrow g = \frac{5}{2} = 2\frac{1}{2} \\
 2f = -6 &\Rightarrow f = -3 \\
 \Rightarrow \text{ centre} &= \left(-2\frac{1}{2}, 3 \right) \\
 \text{and radius} &= \sqrt{\left(2\frac{1}{2} \right)^2 + (-3)^2 + 5} = \sqrt{20\frac{1}{4}} = \frac{9}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad x^2 + y^2 - 2x + \frac{3}{2}y &= 0 \\
 \Rightarrow 2g = -2 &\Rightarrow g = -1 \\
 \text{and } 2f = \frac{3}{2} &\Rightarrow f = \frac{3}{4} \\
 \Rightarrow \text{ centre} &= \left(1, -\frac{3}{4} \right) \\
 \text{and radius} &= \sqrt{(-1)^2 + \left(\frac{3}{4} \right)^2} = \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad x^2 + y^2 - 7y + \frac{33}{4} &= 0 \\
 \Rightarrow 2g = 0 &\Rightarrow g = 0 \\
 \text{and } 2f = -7 &\Rightarrow f = -\frac{7}{2} = -3\frac{1}{2} \\
 \Rightarrow \text{ centre} &= \left(0, 3\frac{1}{2} \right) \\
 \text{and radius} &= \sqrt{(0)^2 + \left(-3\frac{1}{2} \right)^2 - \frac{33}{4}} = \sqrt{\frac{49}{4} - \frac{33}{4}} = \sqrt{4} = 2
 \end{aligned}$$

Q8. Circle : $x^2 + y^2 - 4x + 2y - 20 = 0$
 Point $(5, -5) \Rightarrow (5)^2 + (-5)^2 - 4(5) + 2(-5) - 20$
 $= 25 + 25 - 20 - 10 - 20 = 50 - 50 = 0$, true

Q9. Circle : $x^2 + y^2 + 2x - 4y - 20 = 0$
 Point $(3, 6) \Rightarrow (3)^2 + (6)^2 + 2(3) - 4(6) - 20$
 $= 9 + 36 + 6 - 24 - 20$
 $= 51 - 44 = 7 > 0 \Rightarrow \text{outside circle}$

Q10. Circle : $x^2 + y^2 - 2x + 4y - 15 = 0$
 Point $(3, 1) \Rightarrow (3)^2 + (1)^2 - 2(3) + 4(1) - 15$
 $= 9 + 1 - 6 + 4 - 15$
 $= 14 - 21 = -7 < 0 \Rightarrow \text{inside circle}$

Q11. Circle : $x^2 + y^2 - 6x + 4y + 4 = 0$

$$\begin{aligned} \text{Point } (1,1) &\Rightarrow (1)^2 + (1)^2 - 6(1) + 4(1) + 4 \\ &= 1 + 1 - 6 + 4 + 4 \\ &= 10 - 6 = 4 > 0 \Rightarrow \text{outside circle} \end{aligned}$$

Q12. $2g = -8 \Rightarrow g = -4$

$$2f = 10 \Rightarrow f = 5$$

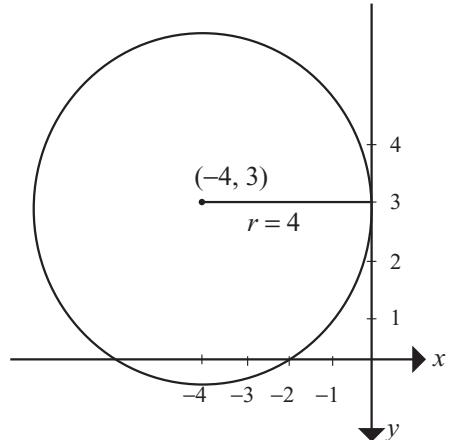
$$\begin{aligned} \text{radius} = 7 &\Rightarrow \sqrt{g^2 + f^2 - c} = \sqrt{(-4)^2 + (5)^2 - k} = 7 \\ &\Rightarrow 16 + 25 - k = 49 \\ &\Rightarrow -k = 49 - 41 = 8 \\ &\Rightarrow k = -8 \end{aligned}$$

Q13. (i) Sketch circle k

(ii) Radius = 4

(iii) Centre = $(-4, 3)$

$$\text{Equation of circle : } (x + 4)^2 + (y - 3)^2 = (4)^2 = 16$$



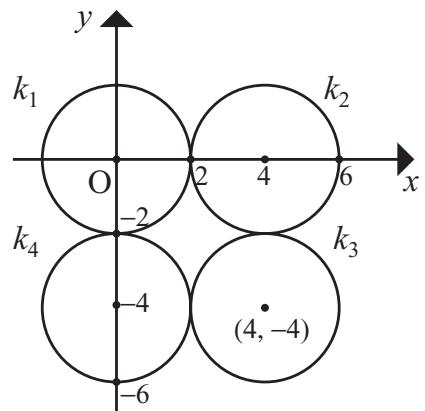
Q14. (i) $r^2 = 4 \Rightarrow r = 2$

(ii) Centre $k_3 = (4, -4)$

(iii) k_3 has centre $(4, -4)$, radius = 2

$$\Rightarrow \text{Equation } k_3 : (x - 4)^2 + (y + 4)^2 = (2)^2 = 4$$

(iv) k_4 its centre is $(0, -4)$.

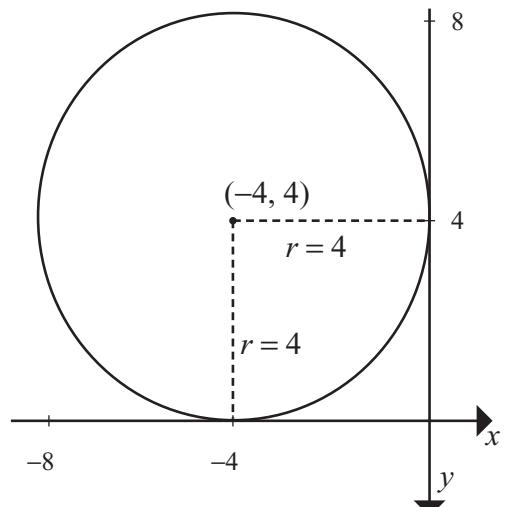


Q15. Centre = $(-4, 4)$

Radius = 4

$$\text{Equation of circle : } (x + 4)^2 + (y - 4)^2 = (4)^2$$

$$\Rightarrow (x + 4)^2 + (y - 4)^2 = 16$$



Q16. (i) $(x-2)^2 + (y-6)^2 = 100$

centre = (2, 6) = C

(ii) C = (2, 6), P = (10, 0) slope CP = $\frac{0-6}{10-2} = \frac{-6}{8} = \frac{-3}{4}$

line $4x - 3y - 40 = 0 \Rightarrow \text{slope} = \frac{-a}{b} = \frac{-4}{-3} = \frac{4}{3}$

product of slopes $w_1 \cdot w_2 = \left(\frac{-3}{4}\right)\left(\frac{4}{3}\right) = \frac{-12}{12} = -1$

Hence, CP is perpendicular to the given line.

Q17. Centre $\Rightarrow y = x \cap y = -x + 4$

$$\Rightarrow x = -x + 4$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = 2 \Rightarrow y = 2$$

Centre = (2, 2) and diameter = 2 \Rightarrow radius = 1

Equation of circle : $(x-2)^2 + (y-2)^2 = (1)^2 = 1$

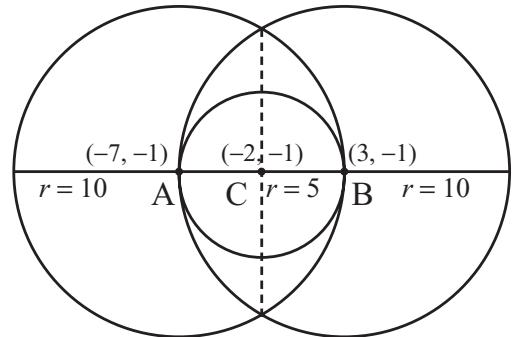
Q18. (i) $(x+2)^2 + (y+1)^2 = 25$

centre C = (-2, -1), radius = 5

centre B = (3, -1), radius = 10

centre A = (-7, -1), radius = 10

(ii) $(x+7)^2 + (y+1)^2 = (10)^2 = 100$



Exercise 4.3

Q1. Circle : $x^2 + y^2 = 10$

$$\Rightarrow \text{centre} = (0, 0), \text{ radius} = \sqrt{10}$$

$$l : 3x + y + 10 = 0$$

$$\text{perpendicular distance} = \frac{|3(0) + 1(0) + 10|}{\sqrt{(3)^2 + (1)^2}} = \frac{10}{\sqrt{10}}$$

$$= \frac{10\sqrt{10}}{\sqrt{10}\sqrt{10}} = \sqrt{10} = \text{radius}$$

Hence, l is a tangent to the circle.

Q2. $(x-3)^2 + (y+4)^2 = 50$

\Rightarrow centre = (3, -4), radius = $\sqrt{50} = 5\sqrt{2}$

line: $x - y + 3 = 0$

$$\begin{aligned}\text{perpendicular distance} &= \frac{|1(3) - 1(-4) + 3|}{\sqrt{(1)^2 + (-1)^2}} \\ &= \frac{10}{\sqrt{2}} = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2} = \text{radius}\end{aligned}$$

Hence, the line is a tangent to the circle.

Q3. $(x+2)^2 + (y-1)^2 = 16$

\Rightarrow centre = (-2, 1), radius = $\sqrt{16} = 4$

line: $3x - 4y - 12 = 0$

$$\begin{aligned}\text{perpendicular distance} &= \frac{|3(-2) - 4(1) - 12|}{\sqrt{(3)^2 + (-4)^2}} \\ &= \frac{|-6 - 4 - 12|}{\sqrt{25}} \\ &= \frac{22}{5} = 4\frac{2}{5} \neq 4\end{aligned}$$

Hence, the line is not a tangent to the circle.

Q4. Centre = (-1, 2), tangent: $2x - 3y - 5 = 0$

$$\text{Radius} = \frac{|2(-1) - 3(2) - 5|}{\sqrt{(2)^2 + (-3)^2}} = \frac{|-2 - 6 - 5|}{\sqrt{13}} = \frac{13}{\sqrt{13}} = \sqrt{13}$$

Equation of circle: $(x+1)^2 + (y-2)^2 = (\sqrt{13})^2 = 13$

Q5. Centre = (2, 1), tangent: $x - y + 5 = 0$

$$\begin{aligned}\text{Radius} &= \frac{|1(2) - 1(1) + 5|}{\sqrt{(1)^2 + (-1)^2}} = \frac{|2 - 1 + 5|}{\sqrt{2}} = \frac{6}{\sqrt{2}} \\ &= \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}\end{aligned}$$

Equation of circle: $(x-2)^2 + (y-1)^2 = (3\sqrt{2})^2 = 18$

Q6. $x^2 + y^2 - 2x - 2y + 1 = 0$

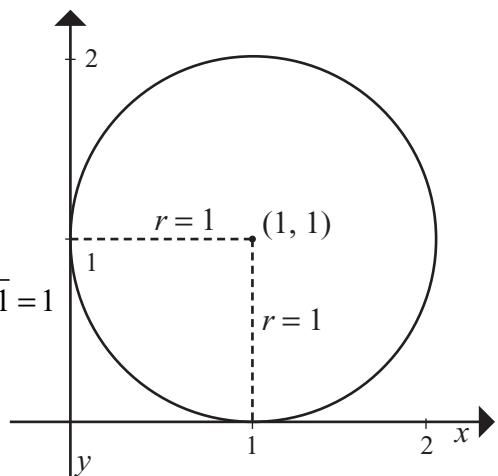
(i) $2g = -2 \Rightarrow g = -1$

$2f = -2 \Rightarrow f = -1$

\Rightarrow centre = (1, 1)

(ii) Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(-1)^2 + (-1)^2 - 1} = \sqrt{1+1-1} = \sqrt{1} = 1$

(iii) Sketch of circle



(iv) $|g| = |-1| = 1 = \text{radius}$
 $|f| = |-1| = 1 = \text{radius}$

Q7. Centre $= (2, 2) = (-g, -f)$
Radius $= |g| = |f| = |2| = 2$
Equation of circle: $(x - 2)^2 + (y - 2)^2 = (2)^2 = 4$

Q8. Centre $= (2, 3) = (-g, -f)$
Touches y -axis \Rightarrow radius $= |g| = |2| = 2$
Equation of circle: $(x - 2)^2 + (y - 3)^2 = (2)^2 = 4$

Q9. $x^2 + y^2 - 4x + 6y - 12 = 0$
(i) $2g = -4 \Rightarrow g = -2$
 $2f = 6 \Rightarrow f = 3$
centre $= (-g, -f) = (2, -3)$
radius $= \sqrt{g^2 + f^2 - c} = \sqrt{(-2)^2 + (3)^2 + 12}$
 $= \sqrt{4 + 9 + 12} = \sqrt{25} = 5$

(ii) Tangent: $3x + 4y - k = 0$
Radius $= \frac{|3(2) + 4(-3) - k|}{\sqrt{(3)^2 + (4)^2}} = 5$
 $\Rightarrow \frac{|6 - 12 - k|}{\sqrt{25}} = 5$
 $\Rightarrow |6 - 12 - k| = 25$
 $\Rightarrow -6 - k = 25 \quad \text{or} \quad 6 + k = 25$
 $\Rightarrow -k = 31 \quad \text{or} \quad k = 25 - 6$
 $\Rightarrow k = -31 \quad \text{or} \quad k = 19$

Q10. (i) Midpoint [AB] $= \left[\frac{0+1}{2}, \frac{-2+2}{2} \right] = \left(\frac{1}{2}, 0 \right)$
Slope AB $= \frac{2+2}{1-0} = \frac{4}{1} = 4$
Perpendicular slope $= -\frac{1}{4}$
 \Rightarrow Equation: $y - 0 = -\frac{1}{4} \left(x - \frac{1}{2} \right)$
 $\Rightarrow 4y = -1 \left(x - \frac{1}{2} \right)$
 $\Rightarrow 4y = -x + \frac{1}{2}$
 $\Rightarrow 8y = -2x + 1$
 $\Rightarrow 2x + 8y - 1 = 0$

$$(ii) \text{ Midpoint } [BC] = \left[\frac{0+4}{2}, \frac{-2-3}{2} \right] = \left(2, -2 \frac{1}{2} \right)$$

$$\text{Slope BC} = \frac{-3+2}{4-0} = -\frac{1}{4}$$

Perpendicular slope = 4

$$\Rightarrow \text{ Equation : } y + 2 \frac{1}{2} = 4(x - 2)$$

$$\Rightarrow y + 2 \frac{1}{2} = 4x - 8$$

$$\Rightarrow 2y + 5 = 8x - 16$$

$$\Rightarrow 8x - 2y - 21 = 0$$

$$(iii) \quad 8x - 2y = 21 \quad (\times 4)$$

$$\frac{2x + 8y = 1}{\underline{32x - 8y = 84}} \quad (\times 1)$$

$$\Rightarrow 32x - 8y = 84$$

$$\frac{2x + 8y = 1}{\underline{34x = 85}} \quad (\text{adding})$$

$$\Rightarrow x = \frac{85}{34} = \frac{5}{2}$$

$$\Rightarrow 8\left(\frac{5}{2}\right) - 2y = 21$$

$$\Rightarrow 20 - 2y = 21$$

$$\Rightarrow -2y = 1$$

$$\Rightarrow y = -\frac{1}{2} \Rightarrow \text{point} = \left(\frac{5}{2}, -\frac{1}{2} \right) = \text{centre}$$

$$(iv) \quad (1, 2), \left(\frac{5}{2}, -\frac{1}{2} \right) \Rightarrow \text{radius} = \sqrt{\left(\frac{5}{2} - 1 \right)^2 + \left(-\frac{1}{2} - 2 \right)^2}$$

$$= \sqrt{\frac{9}{4} + \frac{25}{4}} = \sqrt{\frac{17}{2}}$$

$$(v) \quad \text{Equation of circle : } \left(x - \frac{5}{2} \right)^2 + \left(y + \frac{1}{2} \right)^2 = \left(\sqrt{\frac{17}{2}} \right)^2 = \frac{17}{2}$$

Q11. General equation : $x^2 + y^2 + 2gx + 2fy + c = 0$

point $(0,0) \Rightarrow (0)^2 + (0)^2 + 2g(0) + 2f(0) + c = 0$
 $\Rightarrow 0 + 0 + 0 + 0 + c = 0 \Rightarrow c = 0$

point $(2,0) \Rightarrow (2)^2 + (0)^2 + 2g(2) + 2f(0) + c = 0$
 $\Rightarrow 4 + 0 + 4g + 0 + c = 0$
 $\Rightarrow 4g + c = -4$

point $(3,-1) \Rightarrow (3)^2 + (-1)^2 + 2g(3) + 2f(-1) + c = 0$
 $\Rightarrow 9 + 1 + 6g - 2f + c = 0$
 $\Rightarrow 6g - 2f + c = -10$

$c = 0 \Rightarrow 4g + 0 = -4$
 $4g = -4 \Rightarrow g = -1$

Hence, $6(-1) - 2f + 0 = -10$
 $-2f = -4$
 $f = 2$

$\Rightarrow x^2 + y^2 + 2(-1)x + 2(2)y + 0 = 0$
 $\Rightarrow x^2 + y^2 - 2x + 4y = 0$

Q12. General equation : $x^2 + y^2 + 2gx + 2fy + c = 0$

point $(0,0) \Rightarrow (0)^2 + (0)^2 + 2g(0) + 2f(0) + c = 0$
 $\Rightarrow 0 + 0 + 0 + 0 + c = 0 \Rightarrow c = 0$

point $(-2,4) \Rightarrow (-2)^2 + (4)^2 + 2g(-2) + 2f(4) + c = 0$
 $\Rightarrow 4 + 16 - 4g + 8f + c = 0$
 $\Rightarrow -4g + 8f + c = -20$

$c = 0 \Rightarrow -4g + 8f = -20$
 $\Rightarrow -g + 2f = -5$

point $(-1,7) \Rightarrow (-1)^2 + (7)^2 + 2g(-1) + 2f(7) + c = 0$
 $\Rightarrow 1 + 49 - 2g + 14f + c = 0$
 $\Rightarrow -2g + 14f + c = -50$

$c = 0 \Rightarrow -2g + 14f = -50$
 $\Rightarrow -g + 7f = -25$

$-g + 2f = -5$
 $\underline{-g + 7f = -25} \quad (\text{subtracting})$
 $-5f = +20$
 $\Rightarrow f = -4$

$\Rightarrow -g + 2(-4) = -5$
 $\Rightarrow -g - 8 = -5$
 $\Rightarrow -g = 3 \Rightarrow g = -3$

$\Rightarrow x^2 + y^2 + 2(-3)x + 2(-4)y + 0 = 0$
 $\Rightarrow x^2 + y^2 - 6x - 8y = 0$

Q13. Centre $(-g, -f)$ on line $x + 2y - 6 = 0$

$$\Rightarrow -g - 2f - 6 = 0 \Rightarrow -g - 2f = 6$$

$$\text{point } (3, 5) \Rightarrow (3)^2 + (5)^2 + 2g(3) + 2f(5) + c = 0$$

$$\Rightarrow 9 + 25 + 6g + 10f + c = 0$$

$$\Rightarrow 6g + 10f + c = -34$$

$$\text{point } (-1, 3) \Rightarrow (-1)^2 + (3)^2 + 2g(-1) + 2f(3) + c = 0$$

$$\Rightarrow 1 + 9 - 2g + 6f + c = 0$$

$$\Rightarrow -2g + 6f + c = -10$$

$$\underline{6g + 10f + c = -34} \quad (\text{subtracting})$$

$$-8g - 4f = 24$$

$$\Rightarrow -2g - f = 6$$

$$\underline{-2g - 4f = 12}$$

$$3f = -6$$

$$\Rightarrow f = -2$$

$$\Rightarrow -g - 2(-2) = 6$$

$$\Rightarrow -g + 4 = 6$$

$$\Rightarrow -g = 2 \Rightarrow g = -2$$

$$\text{Hence, } 6(-2) + 10(-2) + c = -34$$

$$\Rightarrow -12 - 20 + c = -34$$

$$\Rightarrow c = -2$$

$$\Rightarrow x^2 + y^2 + 2(-2)x + 2(-2)y - 2 = 0$$

$$\Rightarrow x^2 + y^2 - 4x - 4y - 2 = 0$$

Q14. Centre $(-g, -f)$ is on x -axis $\Rightarrow -f = 0$

$$\Rightarrow f = 0$$

$$\text{Point } (4, 5) \Rightarrow (4)^2 + (5)^2 + 2g(4) + 2f(5) + c = 0$$

$$\Rightarrow 16 + 25 + 8g + 10f + c = 0$$

$$\Rightarrow 8g + 10f + c = -41$$

$$f = 0 \Rightarrow 8g + 10(0) + c = -41 \Rightarrow 8g + c = -41$$

$$\text{Point } (-2, 3) \Rightarrow (-2)^2 + (3)^2 + 2g(-2) + 2f(3) + c = 0$$

$$\Rightarrow 4 + 9 - 4g + 6f + c = 0$$

$$\Rightarrow -4g + 6f + c = -13$$

$$f = 0 \Rightarrow -4g + 6(0) + c = -13 \Rightarrow -4g + c = -13$$

$$8g + c = -41$$

$$\begin{array}{r} -4g + c = -13 \\ \hline \end{array} \quad (\text{subtracting})$$

$$12g = -28$$

$$\Rightarrow g = \frac{-28}{12} = -\frac{7}{3}$$

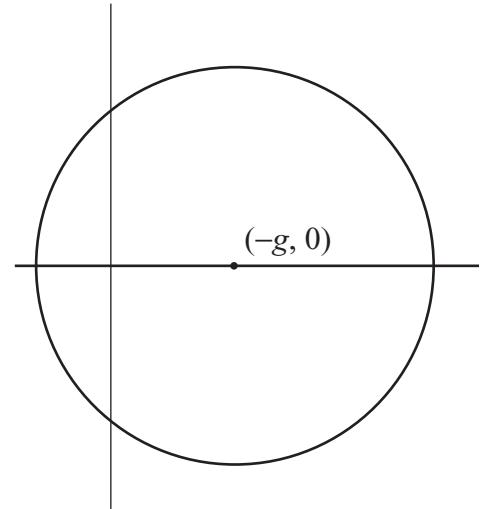
$$\Rightarrow -4\left(-\frac{7}{3}\right) + c = -13$$

$$\Rightarrow \frac{28}{3} + c = -13$$

$$\Rightarrow c = -13 - \frac{28}{3} = -\frac{67}{3}$$

$$\text{Hence, } x^2 + y^2 + 2\left(-\frac{7}{3}\right)x + 2(0)y - \frac{67}{3} = 0$$

$$\Rightarrow 3x^2 + 3y^2 - 14x - 67 = 0$$



Q15. (i) $x^2 + y^2 - 4x - 6y + k = 0$

$$2g = -4 \Rightarrow g = -2$$

$$2f = -6 \Rightarrow f = -3$$

$$\Rightarrow \text{centre } (-g, -f) = (2, 3)$$

$$\text{circle touches } x\text{-axis} \Rightarrow r = |-3| = 3$$

$$\text{(ii)} \quad \text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{(-2)^2 + (-3)^2 - k} = 3$$

$$\Rightarrow 4 + 9 - k = 9$$

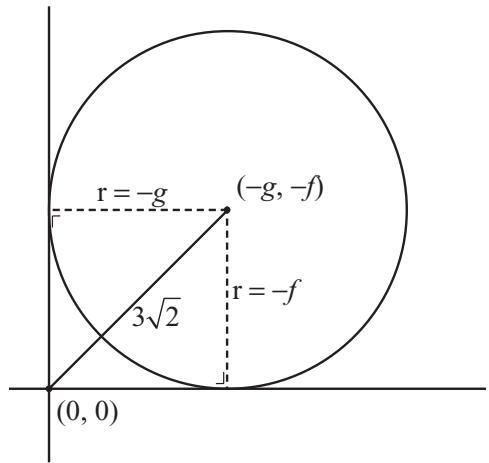
$$\Rightarrow -k = -4 \Rightarrow k = 4$$

and point T = (2, 0).

Q16. (i) Centre = $(-g, -f)$
 $\Rightarrow r = -g$ and $r = -f$
 $\Rightarrow -g = -f \Rightarrow g = f$
Centre in the first quadrant $\Rightarrow g, f < 0$

(ii) $(0,0)(-g, -f) \Rightarrow \text{distance} = 3\sqrt{2}$
 $\Rightarrow \sqrt{(-g-0)^2 + (-f-0)^2} = 3\sqrt{2}$
 $\Rightarrow g^2 + f^2 = (3\sqrt{2})^2 = 18$
 $g = f \Rightarrow g^2 + g^2 = 18$
 $\Rightarrow 2g^2 = 18$
 $\Rightarrow g^2 = 9$
 $\Rightarrow g = -3 \text{ or } g = 3 \text{ (Not valid)}$
 $\Rightarrow -g = 3, -f = 3$
centre $(3,3)$ radius = 3

Equation of circle : $(x-3)^2 + (y-3)^2 = 3^2$
 $\Rightarrow x^2 - 6x + 9 + y^2 - 6y + 9 = 9$
 $\Rightarrow x^2 + y^2 - 6x - 6y + 9 = 0$



Q17. (i) The perpendicular to a tangent at the point of contact passes through the centre of the circle.

(ii) Tangent : $3x + 2y - 12 = 0$
 $\Rightarrow \text{slope} = -\frac{a}{b} = -\frac{3}{2} \Rightarrow \text{perpendicular slope} = \frac{2}{3}$
point B = $(4,0)$
 $\Rightarrow \text{equation} : y - 0 = \frac{2}{3}(x - 4)$
 $\Rightarrow 3y = 2x - 8$
 $\Rightarrow 2x - 3y - 8 = 0$

(iii) $A = (3, -5), B = (4, 0) \Rightarrow \text{midpoint} = \left(\frac{3+4}{2}, \frac{-5+0}{2} \right) = \left(\frac{7}{2}, \frac{-5}{2} \right)$
slope AB = $\frac{0+5}{4-3} = \frac{5}{1} \Rightarrow \text{perpendicular slope} = -\frac{1}{5}$
 $\Rightarrow \text{equation} : y + \frac{5}{2} = -\left(x - \frac{7}{2}\right)$
 $\Rightarrow 5y + \frac{25}{2} = -x + \frac{7}{2}$
 $\Rightarrow x + 5y + 9 = 0$

$$(iv) \quad 2x - 3y = 8$$

$$\underline{x + 5y = -9} \quad \textcircled{2}$$

$$2x - 3y = 8$$

$$\underline{2x + 10y = -18} \text{ (subtracting)}$$

$$-13y = 26 \Rightarrow y = -2$$

$$\Rightarrow 2x - 3(-2) = 8$$

$$\Rightarrow 2x + 6 = 8$$

$$\Rightarrow 2x = 2 \Rightarrow x = 1$$

$$\Rightarrow \text{centre} = (1, -2)$$

$$\text{point A} = (3, -5)$$

$$\Rightarrow \text{radius} = \sqrt{(3-1)^2 + (-5+2)^2}$$

$$= \sqrt{2^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$$

$$(v) \quad \text{Equation of circle: } (x-1)^2 + (y+2)^2 = (\sqrt{13})^2 = 13$$

$$\Rightarrow x^2 - 2x + 1 + y^2 + 4y + 4 = 13$$

$$\Rightarrow x^2 + y^2 - 2x + 4y - 8 = 0$$

Q18. (7,2)(7,10) Equation of chord : $x = 7$

Midpoint = (7, 6)

\Rightarrow Equation of perpendicular bisector : $y = 6$

$$\Rightarrow -f = 6 \Rightarrow f = -6$$

Radius = distance between $(-g, -f)$ and $(-1, -f)$

Radius = $-1 + g$

$$\Rightarrow \sqrt{g^2 + f^2 - c} = -1 + g$$

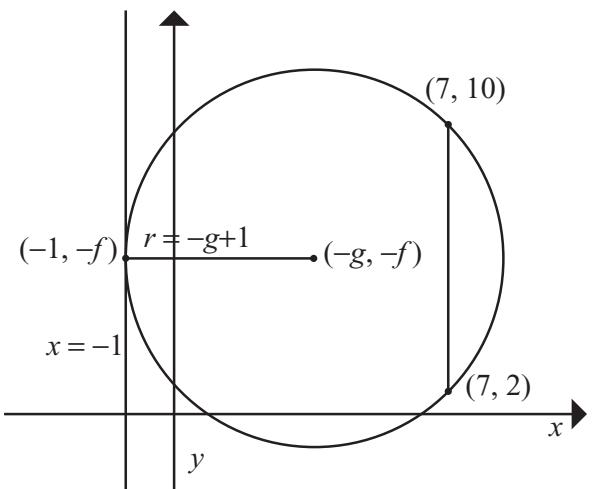
$$\Rightarrow g^2 + f^2 - c = (-1 + g)^2 = 1 - 2g + g^2$$

$$\Rightarrow f^2 - c = 1 - 2g$$

$$f = -6 \Rightarrow (-6)^2 - c = 1 - 2g$$

$$\Rightarrow 36 - c = 1 - 2g$$

$$\Rightarrow 2g - c = -35$$



$$\text{Point } (7, 2) \Rightarrow (7)^2 + (2)^2 + 2g(7) + 2f(2) + c = 0$$

$$\Rightarrow 14g + 4f + c = -53$$

$$f = -6 \Rightarrow 14g + 4(-6) + c = -53$$

$$14g + c = -53 + 24 = -29$$

$$2g - c = -35$$

$$\underline{14g + c = -29} \quad (\text{adding})$$

$$16g = -64$$

$$\Rightarrow g = -4$$

$$14(-4) + c = -29$$

$$\Rightarrow -56 + c = -29$$

$$\Rightarrow c = -29 + 56 = 27$$

Hence, equation of circle : $x^2 + y^2 + 2(-4)x + 2(-6)y + 27 = 0$

$$\Rightarrow x^2 + y^2 - 8x - 12y + 27 = 0$$

Q19. Radius = $\sqrt{20} \Rightarrow \sqrt{g^2 + f^2 - c} = \sqrt{20}$
 $\Rightarrow g^2 + f^2 - c = 20$

Point $(-1, 3) \Rightarrow (-1)^2 + (3)^2 + 2g(-1) + 2f(3) + c = 0$
 $\Rightarrow 1 + 9 - 2g + 6f + c = 0$
 $\Rightarrow -2g + 6f + c = -10$
and $\frac{g^2 + f^2 - c = 20 \text{ (adding)}}$
 $\Rightarrow g^2 - 2g + f^2 + 6f = 10$

Centre $(-g, -f)$ on line $x + y = 0$

$$\begin{aligned}\Rightarrow -g - f &= 0 \\ \Rightarrow g &= -f\end{aligned}$$

Hence, $(-f)^2 - 2(-f) + f^2 + 6f = 10$
 $\Rightarrow f^2 + 2f + f^2 + 6f - 10 = 0$
 $\Rightarrow 2f^2 + 8f - 10 = 0$
 $\Rightarrow f^2 + 4f - 5 = 0$
 $\Rightarrow (f + 5)(f - 1) = 0$
 $\Rightarrow f = -5 \text{ or } f = 1$
 $\Rightarrow g = -(-5) = 5 \text{ or } g = -(1) = -1$
 $\Rightarrow (5)^2 + (-5)^2 - c = 20 \text{ or } (-1)^2 + (1)^2 - c = 20$
 $\Rightarrow 25 + 25 - c = 20 \Rightarrow 1 + 1 - c = 20$
 $\Rightarrow c = 30 \Rightarrow c = -18$

$C_1 : x^2 + y^2 + 10x - 10y + 30 = 0$

$C_2 : x^2 + y^2 - 2x + 2y - 18 = 0$

Q20. (i) $(x - 5)^2 + (y - 3)^2 = 4$
centre = $(5, 3)$

$$r^2 = 4 \Rightarrow r = 2$$

Line is parallel to x -axis and goes through $(5, 1)$

$$\Rightarrow \text{equation: } y = 1$$

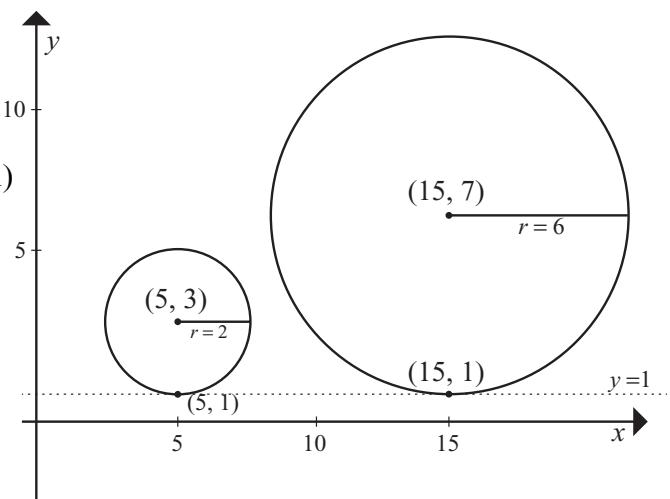
(ii) Rear wheel radius = $3(2) = 6$
10 units from $(5, 1) = (15, 1)$

\Rightarrow centre of rear wheel's rim = $(15, 7)$

$$\Rightarrow \text{equation: } (x - 15)^2 + (y - 7)^2 = (6)^2 = 36$$

(iii) 2 units to the left \Rightarrow centre = $(13, 7)$
and radius = 6

$$\Rightarrow \text{equation: } (x - 13)^2 + (y - 7)^2 = (6)^2 = 36$$



Exercise 4.4

Q1. Circle : $x^2 + y^2 = 8 \Rightarrow$ centre = (0,0), point P(2,2)

$$\Rightarrow \text{slope of the radius [op]} = \frac{2-0}{2-0} = \frac{2}{2} = 1$$

$$\Rightarrow \text{perpendicular slope} = -1$$

$$\text{Equation of tangent : } y - 2 = -1(x - 2)$$

$$\Rightarrow y - 2 = -x + 2$$

$$\Rightarrow x + y = 4$$

Q2. Circle : $x^2 + y^2 = 10 \Rightarrow$ centre = (0,0), point (-3,1)

$$\Rightarrow \text{slope of radius} = \frac{1-0}{-3-0} = -\frac{1}{3}$$

$$\Rightarrow \text{perpendicular slope} = 3$$

$$\text{Equation of tangent : } y - 1 = 3(x + 3)$$

$$\Rightarrow y - 1 = 3x + 9$$

$$\Rightarrow 3x - y + 10 = 0$$

Q3. Circle : $x^2 + y^2 = 17 \Rightarrow$ centre = (0,0), point (4,-1)

$$\Rightarrow \text{slope of radius} = \frac{-1-0}{4-0} = -\frac{1}{4}$$

$$\Rightarrow \text{perpendicular slope} = 4$$

$$\text{Equation of tangent : } y + 1 = 4(x - 4)$$

$$\Rightarrow y + 1 = 4x - 16$$

$$\Rightarrow 4x - y - 17 = 0$$

Q4. Circle : $(x-1)^2 + (y+2)^2 = 20$

(i) Point P(3,2) $\Rightarrow (3-1)^2 + (2+2)^2 = 20$

$$\Rightarrow (2)^2 + (4)^2 = 20$$

$$\Rightarrow 4 + 16 = 20$$

$$\Rightarrow 20 = 20 \Rightarrow \text{True}$$

(ii) Centre = (1,-2)

(iii) C(1,-2) P(3,2) \Rightarrow slope CP = $\frac{2+2}{3-1} = \frac{4}{2} = 2$

$$\Rightarrow \text{perpendicular slope} = -\frac{1}{2}$$

$$\text{Equation of tangent : } y - 2 = -\frac{1}{2}(x - 3)$$

$$\Rightarrow 2y - 4 = -x + 3$$

$$\Rightarrow x + 2y - 7 = 0$$

Q5. Circle : $(x+4)^2 + (y-3)^2 = 17$

centre = $(-4, 3)$

point $(0, 2)$

$$\text{slope radius} = \frac{3-2}{-4-0} = -\frac{1}{4}$$

perpendicular slope = 4

$$\text{Equation of tangent : } y-2=4(x-0)$$

$$\Rightarrow y-2=4x$$

$$\Rightarrow 4x-y+2=0$$

Q6. Circle : $x^2 + y^2 - 4x + 10y - 8 = 0$

$$2g = -4 \Rightarrow g = -2$$

$$2f = 10 \Rightarrow f = 5$$

centre = $(-g, -f) = (2, -5)$

$$\begin{aligned}\text{radius} &= \sqrt{g^2 + f^2 - c} = \sqrt{(-2)^2 + (5)^2 + 8} \\ &= \sqrt{4 + 25 + 8} = \sqrt{37}\end{aligned}$$

point $(3, 1)$

$$\text{slope radius} = \frac{-5-1}{2-3} = \frac{-6}{-1} = 6$$

$$\text{perpendicular slope} = -\frac{1}{6}$$

$$\text{Equation of tangent : } y-1 = -\frac{1}{6}(x-3)$$

$$\Rightarrow 6y-6 = -x+3$$

$$\Rightarrow x+6y-9=0$$

Q7. Point $(0, 0)$ line : $3x - 4y - 25 = 0$

$$\begin{aligned}\text{Perpendicular distance} &= \frac{|3(0) - 4(0) - 25|}{\sqrt{(3)^2 + (-4)^2}} = \frac{|-25|}{\sqrt{25}} \\ &= \frac{25}{5} = 5\end{aligned}$$

Circle $x^2 + y^2 = 25$

$$\Rightarrow \text{centre} = (0, 0) \quad \text{radius} = \sqrt{25} = 5$$

Perpendicular distance from $(0, 0)$ to line = 5

Hence, line $3x - 4y - 25 = 0$ is a tangent to the circle.

Q8. Circle: $x^2 + y^2 - 6x - 4y + 8 = 0$

$$2g = -6 \Rightarrow g = -3$$

$$2f = -4 \Rightarrow f = -2$$

$$\Rightarrow \text{centre} = (-g, -f) = (3, 2)$$

$$\text{radius} = \sqrt{g^2 + f^2 - c} = \sqrt{(-3)^2 + (-2)^2 - 8} = \sqrt{5}$$

$$\text{line: } x + 2y - 12 = 0$$

$$\text{Perpendicular distance} = \frac{|1(3) + 2(2) - 12|}{\sqrt{(1)^2 + (2)^2}} = \frac{|-5|}{\sqrt{5}} = \sqrt{5}$$

Hence, line $x + 2y - 12 = 0$ is a tangent to the circle because the perpendicular distance from the centre of the circle to the line = the radius length.

Q9. Circle: $x^2 + y^2 - 6x - 2y - 15 = 0$

$$2g = -6 \Rightarrow g = -3$$

$$2f = -2 \Rightarrow f = -1$$

$$\Rightarrow \text{centre} = (-g, -f) = (3, 1)$$

$$\text{radius} = \sqrt{g^2 + f^2 - c} = \sqrt{(-3)^2 + (-1)^2 + 15} = \sqrt{25} = 5$$

$$\text{Tangent: } 3x + 4y + c = 0 \Rightarrow \text{perpendicular distance} = \frac{|3(3) + 4(1) + c|}{\sqrt{(3)^2 + (4)^2}} = 5$$

$$\Rightarrow \frac{|13 + c|}{\sqrt{25}} = \frac{|13 + c|}{5} = 5$$

$$\Rightarrow |13 + c| = 25 \Rightarrow 13 + c = -25 \Rightarrow c = -38$$

OR $13 + c = 25 \Rightarrow c = 12$

Q10. circle : $x^2 + y^2 + 4x - 4y - 5 = 0$

$$\Rightarrow 2g = 4 \Rightarrow g = 2$$

$$\text{and } 2f = -4 \Rightarrow f = -2$$

$$\Rightarrow \text{centre} = (-g, -f) = (-2, 2)$$

$$\text{and radius} = \sqrt{g^2 + f^2 - c} = \sqrt{(2)^2 + (-2)^2 + 5} = \sqrt{4 + 4 + 5} = \sqrt{13}$$

$$\text{tangent : } 2x - ky - 3 = 0$$

$$\Rightarrow \text{perpendicular distance} = \frac{|2(-2) - k(2) - 3|}{\sqrt{(2)^2 + (-k)^2}} = \sqrt{13}$$

$$\Rightarrow \frac{|-4 - 2k - 3|}{\sqrt{4 + k^2}} = \sqrt{13}$$

$$\Rightarrow |-2k - 7| = \sqrt{13} \sqrt{4 + k^2}$$

$$\Rightarrow |-2k - 7| = \sqrt{52 + 13k^2}$$

$$\Rightarrow (-2k - 7)^2 = 52 + 13k^2$$

$$\Rightarrow 4k^2 + 28k + 49 - 52 - 13k^2 = 0$$

$$\Rightarrow -9k^2 + 28k - 3 = 0$$

$$\Rightarrow 9k^2 - 28k + 3 = 0$$

$$\Rightarrow (k - 3)(9k - 1) = 0$$

$$\Rightarrow k = 3, k = \frac{1}{9}$$

Q11. $x^2 + y^2 = 13 \Rightarrow$ centre = $(0, 0)$, radius = $\sqrt{13}$

point $(-2, 3) \Rightarrow$ slope radius = $\frac{3-0}{-2-0} = -\frac{3}{2}$

\Rightarrow perpendicular slope = $\frac{2}{3}$

\Rightarrow Equation of tangent: $y - 3 = \frac{2}{3}(x + 2)$

$$\Rightarrow 3y - 9 = 2x + 4$$

$$\Rightarrow 2x - 3y + 13 = 0$$

Circle: $x^2 + y^2 - 10x + 2y - 26 = 0$

\Rightarrow centre = $(5, -1)$ and radius = $\sqrt{(-5)^2 + (1)^2 + 26} = \sqrt{52}$
 $= 2\sqrt{13}$

Line: $2x - 3y + 13 = 0$

$$\begin{aligned}\Rightarrow \text{perpendicular distance} &= \frac{|2(5) - 3(-1) + 13|}{\sqrt{(2)^2 + (-3)^2}} \\ &= \frac{|10 + 3 + 13|}{\sqrt{13}} \\ &= \frac{26}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} \\ &= \frac{26\sqrt{13}}{13} \\ &= 2\sqrt{13}\end{aligned}$$

Hence, $2x - 3y + 13 = 0$ is a tangent to both circles.

Q12. Centre $(2, -1)$, tangent: $3x + y = 0$

$$\begin{aligned}\text{perpendicular distance} &= \frac{|3(2) + 1(-1)|}{\sqrt{(3)^2 + (1)^2}} = \frac{5}{\sqrt{10}} = \frac{5\sqrt{10}}{\sqrt{10}\sqrt{10}} \\ &= \frac{5\sqrt{10}}{10} = \frac{\sqrt{10}}{2} = \text{radius}\end{aligned}$$

Equation of circle: $(x - 2)^2 + (y + 1)^2 = \left(\frac{\sqrt{10}}{2}\right)^2 = \frac{10}{4} = \frac{5}{2}$

Q13. Point = $(0, 0)$, slope = m

Equation of line: $y - 0 = m(x - 0)$

$$\Rightarrow y = mx \Rightarrow mx - y = 0$$

Circle: $x^2 + y^2 - 4x - 2y + 4 = 0$

$$2g = -4 \Rightarrow g = -2$$

$$2f = -2 \Rightarrow f = -1$$

$$\Rightarrow \text{centre} = (-g, -f) = (2, 1)$$

$$\begin{aligned}\text{and radius} &= \sqrt{g^2 + f^2 - c} = \sqrt{(-2)^2 + (-1)^2 - 4} = \sqrt{4+1-4} \\ &= \sqrt{1} = 1\end{aligned}$$

Hence, perpendicular distance from centre $(2, 1)$ to

the tangent $mx - y = 0$ equals radius = 1.

$$\Rightarrow \frac{|m(2) - 1(1)|}{\sqrt{(m)^2 + (-1)^2}} = 1$$

$$\Rightarrow 2m - 1 = 1\sqrt{m^2 + 1}$$

$$\Rightarrow (2m - 1)^2 = m^2 + 1$$

$$\Rightarrow 4m^2 - 4m + 1 = m^2 + 1$$

$$\Rightarrow 3m^2 - 4m = 0$$

$$\Rightarrow m(3m - 4) = 0$$

$$\Rightarrow m = 0 \quad \text{or} \quad 3m = 4$$

$$\Rightarrow m = \frac{4}{3}$$

Hence, $m = 0 \Rightarrow \text{tangent: } (0)x - y = 0$

$$\Rightarrow y = 0$$

$$\text{and } m = \frac{4}{3} \Rightarrow \text{tangent: } \frac{4}{3}(x) - y = 0$$

$$\Rightarrow 4x - 3y = 0$$

Q14. Point $(3, 5)$, slope $= m$

$$\Rightarrow \text{Equation of line : } y - 5 = m(x - 3)$$

$$\Rightarrow y - 5 = mx - 3m$$

$$\Rightarrow mx - y - 3m + 5 = 0$$

$$\text{Circle : } x^2 + y^2 + 2x - 4y - 4 = 0$$

$$\Rightarrow \text{centre} = (-1, 2), \text{ radius} = \sqrt{(1)^2 + (-2)^2 + 4} = \sqrt{9} = 3$$

$$\text{Perpendicular distance} = \frac{|m(-1) - 1(2) - 3m + 5|}{\sqrt{(m)^2 + (-1)^2}} = 3$$

$$\Rightarrow \frac{|-m - 2 - 3m + 5|}{\sqrt{m^2 + 1}} = 3$$

$$\Rightarrow |-4m + 3| = 3\sqrt{m^2 + 1}$$

$$\Rightarrow (-4m + 3)^2 = (3\sqrt{m^2 + 1})^2$$

$$\Rightarrow 16m^2 - 24m + 9 = 9(m^2 + 1) = 9m^2 + 9$$

$$\Rightarrow 7m^2 - 24m = 0$$

$$\Rightarrow m(7m - 24) = 0$$

$$\Rightarrow m = 0, 7m = 24$$

$$\Rightarrow m = \frac{24}{7}$$

$$m = 0 \Rightarrow \text{tangent : } (0)x - y - 3(0) + 5 = 0$$

$$\Rightarrow y - 5 = 0$$

$$m = \frac{24}{7} \Rightarrow \text{tangent : } \left(\frac{24}{7}\right)x - y - 3\left(\frac{24}{7}\right) + 5 = 0$$

$$\Rightarrow 24x - 7y - 72 + 35 = 0$$

$$\Rightarrow 24x - 7y - 37 = 0$$

Q15. Line parallel to $3x + 4y - 6 = 0$

$$\text{is } 3x + 4y + c = 0$$

$$\text{Circle : } x^2 + y^2 - 2x - 2y - 7 = 0$$

$$\Rightarrow \text{centre} = (1, 1), \text{ radius} = \sqrt{(-1)^2 + (-1)^2 + 7} = \sqrt{9} = 3$$

$$\text{Perpendicular distance : } \frac{|3(1) + 4(1) + c|}{\sqrt{(3)^2 + (4)^2}} = 3$$

$$\Rightarrow \frac{|7 + c|}{\sqrt{25}} = \frac{|7 + c|}{5} = 3$$

$$\Rightarrow |7 + c| = 15$$

$$\Rightarrow 7 + c = 15 \text{ or } 7 + c = -15$$

$$\Rightarrow c = 8 \text{ or } c = -22$$

$$\text{Hence, tangents are } 3x + 4y + 8 = 0$$

$$\text{and } 3x + 4y - 22 = 0.$$

Q16. (i) Centre = (3, 5), tangent: $2x - y + 4 = 0$

$$\begin{aligned}\text{Radius} &= \text{perpendicular distance} = \frac{|2(3) - 1(5) + 4|}{\sqrt{(2)^2 + (-1)^2}} \\ &= \frac{|6 - 5 + 4|}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}\end{aligned}$$

(ii) Equation of circle: $(x - 3)^2 + (y - 5)^2 = (\sqrt{5})^2 = 5$

(iii) Point (1, 4), centre = (3, 5)

$$\text{slope radius} = \frac{5 - 4}{3 - 1} = \frac{1}{2}$$

$$\Rightarrow \text{perpendicular slope} = -2$$

$$\text{Equation of tangent: } y - 4 = -2(x - 1)$$

$$\Rightarrow y - 4 = -2x + 2$$

$$\Rightarrow 2x + y - 6 = 0$$

Q17. (i) Circle: $x^2 + y^2 - 10kx + 6y + 60 = 0$

$$2g = -10k \Rightarrow g = -5k$$

$$2f = 6 \Rightarrow f = 3$$

$$\Rightarrow \text{Centre} = (-g, -f) = (5k, -3)$$

$$(ii) \text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{(-5k)^2 + (3)^2 - 60} = 7$$

$$\Rightarrow 25k^2 + 9 - 60 = 49$$

$$\Rightarrow 25k^2 = 100$$

$$\Rightarrow k^2 = 4$$

$$\Rightarrow k = 2 \text{ or } k = -2 \text{ (Not valid)}$$

(iii) Tangent: $3x + 4y + d = 0$, centre = (10, -3)

$$\text{Perpendicular distance} = \frac{|3(10) + 4(-3) + d|}{\sqrt{(3)^2 + (4)^2}} = 7$$

$$\Rightarrow \frac{|30 - 12 + d|}{\sqrt{25}} = \frac{|18 + d|}{5} = 7$$

$$\Rightarrow |18 + d| = 35$$

$$\Rightarrow 18 + d = 35 \text{ or } 18 + d = -35$$

$$\Rightarrow d = 17 \text{ or } d = -53$$

Q18. Circle: $x^2 + y^2 + 4x - 2y - 4 = 0$

$$(i) 2g = 4 \Rightarrow g = 2$$

$$2f = -2 \Rightarrow f = -1$$

$$\text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{(2)^2 + (-1)^2 + 4} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

(ii) The perpendicular to a tangent at the point of contact passes through the centre of the circle.

$$(iii) \quad C(-2,1), P(3,1) \Rightarrow |CP| = \sqrt{(3+2)^2 + (1-1)^2} \\ = \sqrt{25+0} = 5$$

$$\text{Radius} = |CT| = 3$$

$$\begin{aligned} \text{Pythagoras' theorem} &\Rightarrow |PT|^2 + |CT|^2 = |CP|^2 \\ &\Rightarrow |PT|^2 + (3)^2 = (5)^2 \\ &\Rightarrow |PT|^2 + 9 = 25 \\ &\Rightarrow |PT|^2 = 25 - 9 = 16 \\ &\Rightarrow |PT| = \sqrt{16} = 4 \end{aligned}$$

Q19. Circle: $x^2 + y^2 - 14x - 2y + 34 = 0$

$$2g = -14 \Rightarrow g = -7$$

$$2f = -2 \Rightarrow f = -1$$

$$\Rightarrow \text{Centre} = (-g, -f) = (7, 1) \text{ and radius} = \sqrt{(-7)^2 + (-1)^2 - 34} \\ = \sqrt{16} = 4$$

$$\text{Point } (2, 5) \Rightarrow \text{hypotenuse} = \sqrt{(7-2)^2 + (1-5)^2} = \sqrt{(5)^2 + (-4)^2} = \sqrt{41}$$

$$\text{Radius} = 4$$

$$\Rightarrow \text{Length of tangent} = \sqrt{(\text{hypotenuse})^2 - (\text{radius})^2} \\ = \sqrt{41-16} \\ = \sqrt{25} \\ = 5$$

Q20. Centre = (4, 2), radius = $\sqrt{(-4)^2 + (-2)^2 - 10}$ \\ $= \sqrt{16+4-10} = \sqrt{10}$

$$\text{Point } (0, 0) \Rightarrow \text{hypotenuse} = \sqrt{(4-0)^2 + (2-0)^2} \\ = \sqrt{4^2 + 2^2} = \sqrt{20}$$

$$\Rightarrow \text{Length of tangent} = \sqrt{(\text{hypotenuse})^2 - (\text{radius})^2} \\ = \sqrt{20-10} = \sqrt{10}$$

Q21. Centre = (2, 5), radius = $\sqrt{16} = 4$

$$\text{Point } (7, 8) \Rightarrow \text{hypotenuse} = \sqrt{(7-2)^2 + (8-5)^2} \\ = \sqrt{5^2 + 3^2} = \sqrt{34}$$

$$\Rightarrow \text{Length of tangent} = \sqrt{(\text{hypotenuse})^2 - (\text{radius})^2} \\ = \sqrt{34-16} = \sqrt{18} = 3\sqrt{2}$$

Q22. Centre = $(2, 3)$, radius = $\sqrt{(-2)^2 + (-3)^2 - c}$
 $= \sqrt{4 + 9 - c} = \sqrt{13 - c}$

Point $(1, 1)$ \Rightarrow hypotenuse = $\sqrt{(2-1)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$
Length of tangent = 2

$$\Rightarrow (\text{radius})^2 + (2)^2 = (\sqrt{5})^2$$

$$\Rightarrow (\text{radius})^2 + 4 = 5$$

$$\Rightarrow (\text{radius})^2 = 5 - 4 = 1$$

$$\Rightarrow \text{radius} = 1$$

Hence, $\sqrt{13 - c} = 1$

$$\Rightarrow 13 - c = (1)^2 = 1$$

$$\Rightarrow c = 12$$

Exercise 4.5

Q1. Line: $3x - y + 5 = 0 \Rightarrow y = 3x + 5$

Circle: $x^2 + y^2 = 5 \Rightarrow x^2 + (3x + 5)^2 = 5$
 $\Rightarrow x^2 + 9x^2 + 30x + 25 - 5 = 0$
 $\Rightarrow 10x^2 + 30x + 20 = 0$
 $\Rightarrow x^2 + 3x + 2 = 0$
 $\Rightarrow (x+1)(x+2) = 0$
 $\Rightarrow x = -1, \quad x = -2$
 $\Rightarrow y = 3(-1) + 5 \quad y = 3(-2) + 5$
 $= 2 \quad = -1$

Points = $(-1, 2)$ and $(-2, -1)$

Q2. Circle: $x^2 + y^2 = 10 \Rightarrow \text{centre} = (0, 0)$, radius = $\sqrt{10}$

Line: $x - 3y - 10 = 0$

$$\text{Perpendicular distance} = \frac{|1(0) - 3(0) - 10|}{\sqrt{(1)^2 + (-3)^2}} = \frac{|-10|}{\sqrt{10}} = \sqrt{10} = \text{radius}$$

Hence, the line is a tangent to the circle.

Line: $x = 3y + 10 \Rightarrow \text{Circle: } (3y + 10)^2 + y^2 = 10$

$$\Rightarrow 9y^2 + 60y + 100 + y^2 - 10 = 0$$

$$\Rightarrow 10y^2 + 60y + 90 = 0$$

$$\Rightarrow y^2 + 6y + 9 = 0$$

$$\Rightarrow (y+3)(y+3) = 0$$

$$\Rightarrow y = -3$$

$$\Rightarrow x = 3(-3) + 10$$

$$x = -9 + 10 = 1$$

$$\Rightarrow \text{Point of contact} = (1, -3)$$

Q3. Line : $2x - y - 5 = 0 \cap$ circle : $x^2 + y^2 = 5$

$$\begin{aligned} \Rightarrow y &= 2x - 5 \Rightarrow x^2 + (2x - 5)^2 = 5 \\ &\Rightarrow x^2 + 4x^2 - 20x + 25 - 5 = 0 \\ &\Rightarrow 5x^2 - 20x + 20 = 0 \\ &\Rightarrow x^2 - 4x + 4 = 0 \\ &\Rightarrow (x - 2)(x - 2) = 0 \\ &\Rightarrow x = 2 \\ &\Rightarrow y = 2(2) - 5 = -1 \end{aligned}$$

\Rightarrow Point of intersection = (2, -1)

Q4. (i) $x + y = 6 \cap x^2 + y^2 + 2x - 4y - 20 = 0$

$$\begin{aligned} \Rightarrow y &= -x + 6 \Rightarrow x^2 + (-x + 6)^2 + 2x - 4(-x + 6) - 20 = 0 \\ &\Rightarrow x^2 + x^2 - 12x + 36 + 2x + 4x - 24 - 20 = 0 \\ &\Rightarrow 2x^2 - 6x - 8 = 0 \\ &\Rightarrow x^2 - 3x - 4 = 0 \\ &\Rightarrow (x - 4)(x + 1) = 0 \\ &\Rightarrow x = 4 \quad \text{or} \quad x = -1 \\ &\Rightarrow y = -4 + 6 = 2 \quad \text{or} \quad y = -(-1) + 6 = 7 \end{aligned}$$

\Rightarrow Points = (4, 2) and (-1, 7)

(ii) $2x + y - 2 = 0 \cap x^2 + y^2 - 10x - 4y - 11 = 0$

$$\begin{aligned} \Rightarrow y &= -2x + 2 \Rightarrow x^2 + (-2x + 2)^2 - 10x - 4(-2x + 2) - 11 = 0 \\ &\Rightarrow x^2 + 4x^2 - 8x + 4 - 10x + 8x - 8 - 11 = 0 \\ &\Rightarrow 5x^2 - 10x - 15 = 0 \\ &\Rightarrow x^2 - 2x - 3 = 0 \\ &\Rightarrow (x + 1)(x - 3) = 0 \\ &\Rightarrow x = -1 \quad \text{or} \quad x = 3 \\ &\Rightarrow y = -2(-1) + 2 \quad \text{or} \quad y = -2(3) + 2 \\ &\qquad\qquad\qquad = 4 \qquad\qquad\qquad = -4 \end{aligned}$$

\Rightarrow Points = (-1, 4) and (3, -4)

(iii) $3x - y - 5 = 0 \cap x^2 + y^2 - 2x + 4y - 5 = 0$

$$\begin{aligned} \Rightarrow y &= 3x - 5 \Rightarrow x^2 + (3x - 5)^2 - 2x + 4(3x - 5) - 5 = 0 \\ &\Rightarrow x^2 + 9x^2 - 30x + 25 - 2x + 12x - 20 - 5 = 0 \\ &\Rightarrow 10x^2 - 20x = 0 \\ &\Rightarrow x^2 - 2x = 0 \\ &\Rightarrow x(x - 2) = 0 \\ &\Rightarrow x = 0 \quad \text{or} \quad x = 2 \\ &\Rightarrow y = 3(0) - 5 \quad \text{or} \quad y = 3(2) - 5 \\ &\qquad\qquad\qquad = -5 \qquad\qquad\qquad = 1 \end{aligned}$$

Points = (0, -5) and (2, 1)

Q5. $x - 2y + 12 = 0 \cap x^2 + y^2 - x - 31 = 0$

$$\Rightarrow x = 2y - 12 \Rightarrow (2y - 12)^2 + y^2 - (2y - 12) - 31 = 0$$

$$\Rightarrow 4y^2 - 48y + 144 + y^2 - 2y + 12 - 31 = 0$$

$$\Rightarrow 5y^2 - 50y + 125 = 0$$

$$\Rightarrow y^2 - 10y + 25 = 0$$

$$\Rightarrow (y - 5)(y - 5) = 0$$

$$\Rightarrow y = 5$$

$$\Rightarrow x = 2(5) - 12 = -2$$

$$\Rightarrow \text{Point of contact} = (-2, 5)$$

Q6. (i) $x - 2y - 1 = 0 \cap x^2 + y^2 + 2x - 8y - 8 = 0$

$$\Rightarrow x = 2y + 1 \Rightarrow (2y + 1)^2 + y^2 + 2(2y + 1) - 8y - 8 = 0$$

$$\Rightarrow 4y^2 + 4y + 1 + y^2 + 4y + 2 - 8y - 8 = 0$$

$$\Rightarrow 5y^2 - 5 = 0$$

$$\Rightarrow y^2 - 1 = 0$$

$$\Rightarrow (y - 1)(y + 1) = 0$$

$$\Rightarrow y = 1 \quad \text{or} \quad y = -1$$

$$\Rightarrow x = 2(1) + 1 = 3 \quad \text{or} \quad x = 2(-1) + 1 = -1$$

$$\Rightarrow L = (3, 1) \text{ and } M = (-1, -1)$$

(ii) $L = (3, 1) \quad M = (-1, -1)$
 Midpoint = $\left(\frac{3-1}{2}, \frac{1-1}{2}\right) = (1, 0)$

(iii) Centre = $(1, 0) \quad L = (3, 1)$
 $\Rightarrow \text{Radius} = \sqrt{(3-1)^2 = (1-0)^2} = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$
 $\Rightarrow \text{Equation of circle: } (x-1)^2 + (y-0)^2 = (\sqrt{5})^2$
 $\Rightarrow (x-1)^2 + y^2 = 5$

Q7. Circle: $x^2 + y^2 - 4x - 6y - 12 = 0$
 $x\text{-axis} \Rightarrow y = 0 \Rightarrow x^2 + (0)^2 - 4x - 6(0) - 12 = 0$
 $\Rightarrow x^2 - 4x - 12 = 0$
 $\Rightarrow (x-6)(x+2) = 0$
 $\Rightarrow x = 6 \text{ or } x = -2$

Points = $(6, 0)$ and $(-2, 0)$
 Hence, length of intercept = $6 - (-2) = 8$ units.

Q8. Circle: $x^2 + y^2 - 4x + 6y - 7 = 0$

$$\begin{aligned}y\text{-axis } &\Rightarrow x=0 \Rightarrow (0)^2 + y^2 - 4(0) + 6y - 7 = 0 \\&\Rightarrow y^2 + 6y - 7 = 0 \\&\Rightarrow (y+7)(y-1) = 0 \\&\Rightarrow y = -7 \text{ or } y = 1\end{aligned}$$

Points = (0, -7) and (0, 1)

Hence, length of chord = $1 - (-7) = 8$ units

Q9. Circle: $x^2 + y^2 - 4x + 11y - 12 = 0$

$$\begin{aligned}x\text{-axis } &\Rightarrow y=0 \Rightarrow x^2 + (0)^2 - 4x + 11(0) - 12 = 0 \\&\Rightarrow x^2 - 4x - 12 = 0 \\&\Rightarrow (x-6)(x+2) = 0 \\&\Rightarrow x = 6 \text{ or } x = -2\end{aligned}$$

\Rightarrow positive x-axis = (6, 0) $\Rightarrow a = 6$

$$\begin{aligned}y\text{-axis } &\Rightarrow x=0 \Rightarrow (0)^2 + y^2 - 4(0) + 11y - 12 = 0 \\&\Rightarrow y^2 + 11y - 12 = 0 \\&\Rightarrow (y-1)(y+12) = 0 \\&\Rightarrow y = 1 \text{ or } y = -12\end{aligned}$$

\Rightarrow positive y-axis = (0, 1) $\Rightarrow b = 1$

Q10. Circle: $x^2 + y^2 - 4x - 8y - 5 = 0$

$$\begin{aligned}x\text{-axis } &\Rightarrow y=0 \Rightarrow x^2 + (0)^2 - 4x - 8(0) - 5 = 0 \\&\Rightarrow x^2 - 4x - 5 = 0 \\&\Rightarrow (x+1)(x-5) = 0 \\&\Rightarrow x = -1 \text{ or } x = 5\end{aligned}$$

Points = (-1, 0) and (5, 0)

Length of intercept = $5 - (-1) = 6$ units

Q11. Circle $s_1 : x^2 + y^2 - 3x + 5y - 4 = 0$

Circle $s_2 : x^2 + y^2 - x + 4y - 7 = 0$

Common chord is $s_1 - s_2 = 0$

$$\Rightarrow x^2 + y^2 - 3x + 5y - 4 - (x^2 + y^2 - x + 4y - 7) = 0$$

$$\Rightarrow x^2 + y^2 - 3x + 5y - 4 - x^2 - y^2 + x - 4y + 7 = 0$$

$$\Rightarrow -2x + y + 3 = 0$$

$$\Rightarrow 2x - y - 3 = 0$$

Chord: $y = 2x - 3 \cap$ circle: $x^2 + y^2 - 3x + 5y - 4 = 0$

$$\Rightarrow x^2 + (2x - 3)^2 - 3x + 5(2x - 3) - 4 = 0$$

$$\Rightarrow x^2 + 4x^2 - 12x + 9 - 3x + 10x - 15 - 4 = 0$$

$$\Rightarrow 5x^2 - 5x - 10 = 0$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x - 2)(x + 1) = 0$$

$$\Rightarrow x = 2 \quad \text{or} \quad x = -1$$

$$\Rightarrow y = 2(2) - 3 = 1 \quad \text{or} \quad y = 2(-1) - 3 = -5$$

Points = (2, 1) and (-1, -5)

Q12. Common tangent: $s_1 - s_2 = 0$

$$\Rightarrow x^2 + y^2 + 14x - 10y - 26 - (x^2 + y^2 - 4x + 14y + 28) = 0$$

$$\Rightarrow x^2 + y^2 + 14x - 10y - 26 - x^2 - y^2 + 4x - 14y - 28 = 0$$

$$\Rightarrow 18x - 24y - 54 = 0$$

$$\Rightarrow 3x - 4y - 9 = 0$$

$$\Rightarrow 3x = 4y + 9 \quad \cap \quad x^2 + y^2 + 14x - 10y - 26 = 0$$

$$\Rightarrow x = \frac{4y + 9}{3} \quad \Rightarrow \quad \left(\frac{4y + 9}{3}\right)^2 + y^2 + 14\left(\frac{4y + 9}{3}\right) - 10y - 26 = 0$$

$$\Rightarrow \frac{16y^2 + 72y + 81}{9} + y^2 + \frac{56y + 126}{3} - 10y - 26 = 0$$

$$\Rightarrow 16y^2 + 72y + 81 + 9y^2 + 168y + 378 - 90y - 234 = 0$$

$$\Rightarrow 25y^2 + 150y + 225 = 0$$

$$\Rightarrow y^2 + 6y + 9 = 0$$

$$\Rightarrow (y + 3)(y + 3) = 0$$

$$\Rightarrow y = -3$$

$$\Rightarrow x = \frac{4(-3) + 9}{3} = \frac{-12 + 9}{3} = -1$$

$$\Rightarrow \text{Point of intersection} = (-1, -3)$$

Q13. Common chord : $s_1 - s_2 = 0$

$$\begin{aligned}
 &\Rightarrow x^2 + y^2 + 4x - 2y - 5 - (x^2 + y^2 + 14x - 12y + 65) = 0 \\
 &\Rightarrow x^2 + y^2 + 4x - 2y - 5 - x^2 - y^2 - 14x + 12y - 65 = 0 \\
 &\Rightarrow -10x + 10y - 70 = 0 \\
 &\Rightarrow x - y + 7 = 0 \\
 &\Rightarrow y = x + 7 \quad \cap \quad x^2 + y^2 + 4x - 2y - 5 = 0 \\
 &\quad \Rightarrow x^2 + (x+7)^2 + 4x - 2(x+7) - 5 = 0 \\
 &\quad \Rightarrow x^2 + x^2 + 14x + 49 + 4x - 2x - 14 - 5 = 0 \\
 &\quad \Rightarrow 2x^2 + 16x + 30 = 0 \\
 &\quad \Rightarrow x^2 + 8x + 15 = 0 \\
 &\quad \Rightarrow (x+3)(x+5) = 0 \\
 &\quad \Rightarrow x = -3 \quad \text{or} \quad x = -5 \\
 &\quad \Rightarrow y = -3 + 7 = 4 \quad \text{or} \quad y = -5 + 7 = 2 \\
 &\Rightarrow \text{Points of intersection} = (-3, 4) \text{ and } (-5, 2)
 \end{aligned}$$

Q14. Circle : $x^2 + y^2 + 2x - 8y + 4 = 0$

(i) Centre = $(-1, 4)$ = co-ordinates of the Polar star

$$\begin{aligned}
 \text{(ii)} \quad &y = 1 \quad \cap \quad x^2 + y^2 + 2x - 8y + 4 = 0 \\
 &\Rightarrow x^2 + (1)^2 + 2x - 8(1) + 4 = 0 \\
 &\Rightarrow x^2 + 2x - 3 = 0 \\
 &\Rightarrow (x+3)(x-1) = 0 \\
 &\Rightarrow x = -3 \text{ or } x = 1
 \end{aligned}$$

\Rightarrow rising : $(-3, 1)$; setting $(1, 1)$

Exercise 4.6

Q1. $s_1 : x^2 + y^2 - 2x - 15 = 0 \Rightarrow \text{centre} = (1, 0)$

$$\text{and radius} = \sqrt{(-1)^2 + (0)^2 + 15} = \sqrt{16} = 4$$

$s_2 : x^2 + y^2 - 14x - 16y + 77 = 0 \Rightarrow \text{centre} = (7, 8)$

$$\begin{aligned}
 \text{and radius} &= \sqrt{(-7)^2 + (-8)^2 - 77} \\
 &= \sqrt{49 + 64 - 77} = \sqrt{36} = 6
 \end{aligned}$$

$$\begin{aligned}
 \text{Distance between centres} &= \sqrt{(7-1)^2 + (8-0)^2} \\
 &= \sqrt{(6)^2 + (8)^2} = \sqrt{100} = 10
 \end{aligned}$$

Sum of 2 radii = $4 + 6 = 10$

Hence, the circles touch externally.

Q2. $s_1 : x^2 + y^2 + 4x - 6y + 12 = 0 \Rightarrow \text{centre} = (-2, 3)$
 and radius = $\sqrt{(2)^2 + (-3)^2 - 12} = \sqrt{1} = 1$

$$s_2 : x^2 + y^2 - 12x + 6y - 76 = 0 \Rightarrow \text{centre} = (6, -3)$$

$$\text{and radius} = \sqrt{(-6)^2 + (3)^2 + 76}$$

$$= \sqrt{36 + 9 + 76} = \sqrt{121} = 11$$

$$\text{Distance between centres} = \sqrt{(6+2)^2 + (-3-3)^2}$$

$$= \sqrt{(8)^2 + (-6)^2} = \sqrt{100} = 10$$

Difference between the 2 radii = $11 - 1 = 10$

Hence, the circles touch internally.

Q3. $s_1 : x^2 + y^2 - 4x - 2y - 20 = 0 \Rightarrow \text{centre} = (2, 1)$
 and radius = $\sqrt{(-2)^2 + (-1)^2 + 20}$
 $= \sqrt{4+1+20} = \sqrt{25} = 5$

$$s_2 : x^2 + y^2 - 16x - 18y + 120 = 0 \Rightarrow \text{centre} = (8, 9)$$

$$\text{and radius} = \sqrt{(-8)^2 + (-9)^2 - 120}$$

$$= \sqrt{64+81-120} = \sqrt{25} = 5$$

$$\text{Distance between centres} = \sqrt{(8-2)^2 + (9-1)^2}$$

$$= \sqrt{(6)^2 + (8)^2} = \sqrt{100} = 10$$

Sum of the 2 radii = $5 + 5 = 10$

Hence, the circles touch externally.

Q4. $s_1 : x^2 + y^2 - 16y + 32 = 0 \Rightarrow \text{centre} = (0, 8)$
 and radius = $\sqrt{(0)^2 + (-8)^2 - 32}$
 $= \sqrt{64-32} = \sqrt{32} = 4\sqrt{2}$

$$s_2 : x^2 + y^2 - 18x + 2y + 32 = 0 \Rightarrow \text{centre} = (9, -1)$$

$$\text{and radius} = \sqrt{(-9)^2 + (1)^2 - 32}$$

$$= \sqrt{81+1-32} = \sqrt{50} = 5\sqrt{2}$$

$$\text{Distance between centres} = \sqrt{(9-0)^2 + (-1-8)^2}$$

$$= \sqrt{(9)^2 + (-9)^2} = \sqrt{162} = 9\sqrt{2}$$

Sum of the 2 radii = $4\sqrt{2} + 5\sqrt{2} = 9\sqrt{2}$

Hence, the circles touch externally.

Q5. (i) $s_1 : x^2 + y^2 - 4x - 6y + 5 = 0 \Rightarrow \text{centre} = (2, 3)$

$$\begin{aligned}\text{and radius} &= \sqrt{(-2)^2 + (-3)^2 - 5} \\ &= \sqrt{4 + 9 - 5} = \sqrt{8} = 2\sqrt{2}\end{aligned}$$

$$s_2 : x^2 + y^2 - 6x - 8y + 23 = 0 \Rightarrow \text{centre} = (3, 4)$$

$$\begin{aligned}\text{and radius} &= \sqrt{(-3)^2 + (-4)^2 - 23} \\ &= \sqrt{9 + 16 - 23} = \sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Distance between centres} &= \sqrt{(3-2)^2 + (4-3)^2} \\ &= \sqrt{(1)^2 + (1)^2} = \sqrt{2}\end{aligned}$$

$$\text{Difference between the radii} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$

Hence, the circles touch internally.

(ii) Common tangent : $s_1 - s_2 = 0$

$$\begin{aligned}\Rightarrow x^2 + y^2 - 4x - 6y + 5 - (x^2 + y^2 - 6x - 8y + 23) &= 0 \\ \Rightarrow x^2 + y^2 - 4x - 6y + 5 - x^2 - y^2 + 6x + 8y - 23 &= 0 \\ \Rightarrow 2x + 2y - 18 &= 0 \\ \Rightarrow x + y - 9 &= 0\end{aligned}$$

(iii) $y = -x + 9 \cap x^2 + y^2 - 4x - 6y + 5 = 0$

$$\begin{aligned}\Rightarrow x^2 + (-x + 9)^2 - 4x - 6(-x + 9) + 5 &= 0 \\ \Rightarrow x^2 + x^2 - 18x + 81 - 4x + 6x - 54 + 5 &= 0 \\ \Rightarrow 2x^2 - 16x + 32 &= 0 \\ \Rightarrow x^2 - 8x + 16 &= 0 \\ \Rightarrow (x-4)(x-4) &= 0 \\ \Rightarrow x &= 4 \\ \Rightarrow y &= -4 + 9 = 5\end{aligned}$$

Point of contact = (4, 5)

Q6. (i) $r^2 = (3)^2 + (4)^2 = 9 + 16 = 25$

$$\Rightarrow r = \sqrt{25} = 5$$

(ii) Centre = (3, 0), radius = 5

$$\text{Equation of circle : } (x-3)^2 + (y-0)^2 = (5)^2 = 25$$

$$\Rightarrow (x-3)^2 + y^2 = 25$$

Q7. (i) Sketch circle : centre = (2, 3), radius = 4

$$(ii) \text{ Equation of circle : } (x-2)^2 + (y-3)^2 = 4^2 = 16$$

$$\Rightarrow x^2 - 4x + 4 + y^2 - 6y + 9 = 16$$

$$\Rightarrow x^2 + y^2 - 4x - 6y - 3 = 0$$

$$y\text{-axis} \Rightarrow x = 0$$

$$\Rightarrow (0)^2 + y^2 - 4(0) - 6y - 3 = 0$$

$$\Rightarrow y^2 - 6y - 3 = 0$$

$$\Rightarrow y = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36+12}}{2} = \frac{6 \pm \sqrt{48}}{2} = \frac{6 \pm 4\sqrt{3}}{2} = 3 \pm 2\sqrt{3}$$

Points on y -axis = $(0, 3 - 2\sqrt{3})$ and $(0, 3 + 2\sqrt{3})$

$$\text{Distance between points} = \sqrt{(0-0)^2 + [3+2\sqrt{3} - (3-2\sqrt{3})]^2}$$

$$= \sqrt{0 + (3+2\sqrt{3} - 3+2\sqrt{3})^2}$$

$$= \sqrt{(4\sqrt{3})^2} = 4\sqrt{3}$$

$$(iii) \text{ } x\text{-axis} \Rightarrow y = 0$$

$$\Rightarrow x^2 + (0)^2 - 4x - 6(0) - 3 = 0$$

$$\Rightarrow x^2 - 4x - 3 = 0$$

$$\Rightarrow x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

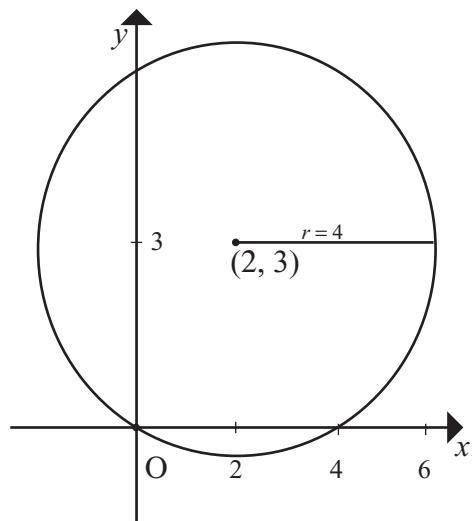
$$= \frac{4 \pm \sqrt{16+12}}{2}$$

$$= \frac{4 \pm \sqrt{28}}{2} = \frac{4 \pm 2\sqrt{7}}{2} = 2 \pm \sqrt{7}$$

Points on x -axis = $(2 - \sqrt{7}, 0)$ and $(2 + \sqrt{7}, 0)$

$$\Rightarrow \text{Length of intercept} = (2 + \sqrt{7}) - (2 - \sqrt{7})$$

$$= 2 + \sqrt{7} - 2 + \sqrt{7} = 2\sqrt{7}$$



$$Q8. (i) \quad x^2 + (3)^2 = 5^2$$

$$\Rightarrow x^2 + 9 = 25$$

$$\Rightarrow x^2 = 25 - 9 = 16$$

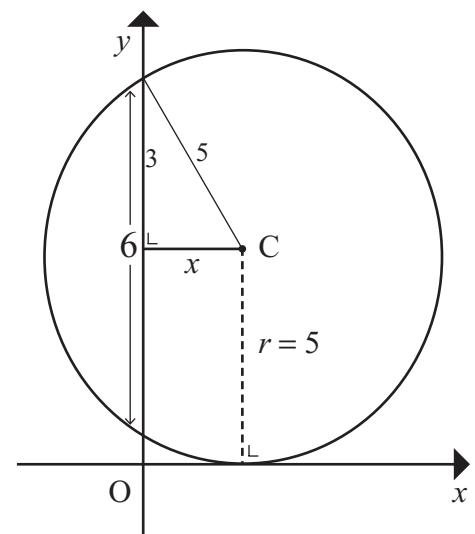
$$\Rightarrow x = \sqrt{16} = 4$$

$$\Rightarrow \text{centre} = (4, 5)$$

$$(ii) \text{ Equation of circle : } (x-4)^2 + (y-5)^2 = 5^2$$

$$\Rightarrow x^2 - 8x + 16 + y^2 - 10y + 25 = 25$$

$$\Rightarrow x^2 + y^2 - 8x - 10y + 16 = 0$$

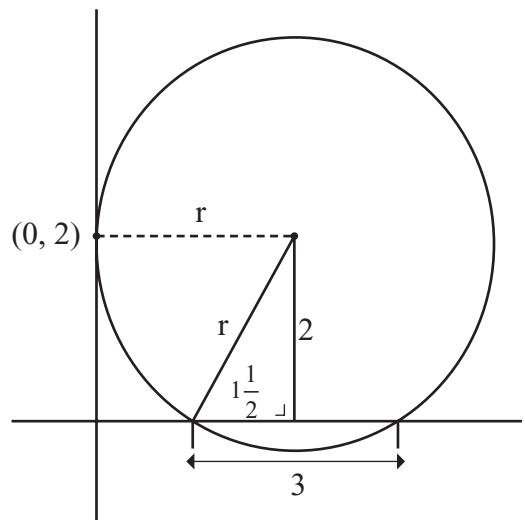


Q9. $r^2 = \left(1\frac{1}{2}\right)^2 + (2)^2$
 $= 6\frac{1}{4}$

$\Rightarrow r = \sqrt{6\frac{1}{4}} = \frac{5}{2}$

$\Rightarrow \text{Centre} = \left(\frac{5}{2}, 2\right)$

$\text{Equation of circle: } \left(x - \frac{5}{2}\right)^2 + (y - 2)^2 = \left(\frac{5}{2}\right)^2$



Q10. (i) A(-1, -4) B(2, 0)

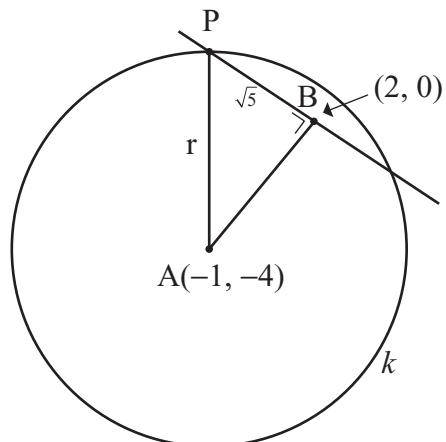
$\Rightarrow |AB| = \sqrt{(2+1)^2 + (0+4)^2}$
 $= \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

$\text{radius} = |AP|$

$\Rightarrow |AP|^2 = (\sqrt{5})^2 + (5)^2$
 $= 5 + 25 = 30$

$\Rightarrow \text{radius} = |AP| = \sqrt{30}$

(ii) Equation of circle: $(x+1)^2 + (y+4)^2 = (\sqrt{30})^2 = 30$



Q11. (i) $x^2 + (4)^2 = (5)^2$

$\Rightarrow x^2 + 16 = 25$

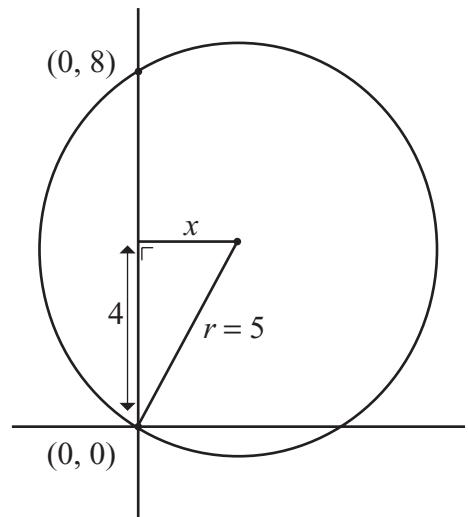
$\Rightarrow x^2 = 25 - 16 = 9$

$\Rightarrow x = \sqrt{9} = 3$

$\Rightarrow \text{centre} = (3, 4)$

(ii) Equation of circle: $(x-3)^2 + (y-4)^2 = (5)^2$

$\Rightarrow (x-3)^2 + (y-4)^2 = 25$



Q12. $s_1 : x^2 + y^2 - 6x + 4y - 12 = 0$
 \Rightarrow centre = (3, -2) and radius = $\sqrt{(-3)^2 + (2)^2 + 12} = \sqrt{9 + 4 + 12} = \sqrt{25} = 5$

$s_2 : x^2 + y^2 + 12x - 20y + k = 0$
 \Rightarrow centre = (-6, 10) and radius = $\sqrt{(6)^2 + (-10)^2 - k} = \sqrt{136 - k}$

Distance between centres = $\sqrt{(-6 - 3)^2 + (10 + 2)^2} = \sqrt{81 + 144} = \sqrt{225} = 15$

Circles touch externally $\Rightarrow 5 + \sqrt{136 - k} = 15$
 $\Rightarrow \sqrt{136 - k} = 15 - 5 = 10$
 $\Rightarrow (\sqrt{136 - k})^2 = (10)^2$
 $\Rightarrow 136 - k = 100$
 $\Rightarrow k = 36$

Exercise 4.7

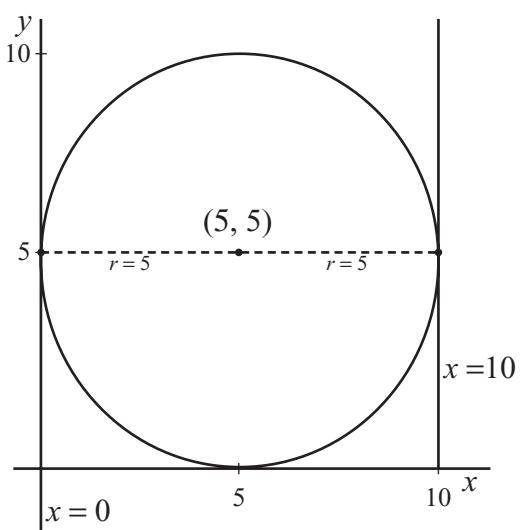
Q1. Centre = $(-g, -f) = (3, -4)$
Circle touching the x -axis $\Rightarrow r = |-f| = |-4| = 4$
Equation of circle: $(x - 3)^2 + (y + 4)^2 = (4)^2 = 16$

Q2. Centre = $(-g, -f) = (-3, 2)$
Circle touching the y -axis $\Rightarrow r = |-g| = |-3| = 3$
Equation of circle k : $(x + 3)^2 + (y - 2)^2 = (3)^2 = 9$

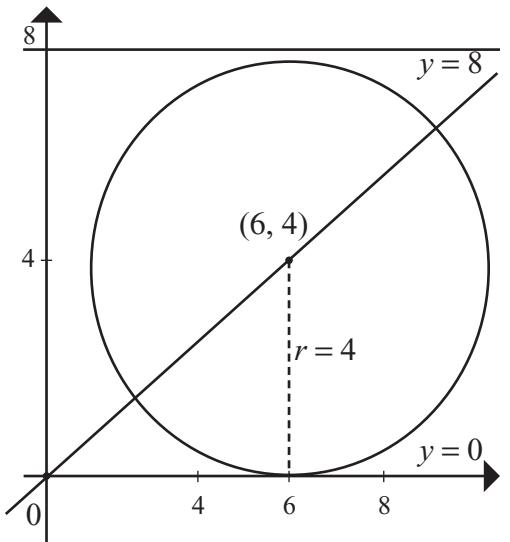
Q3. Centre = $(5, y) = (-g, -f)$
(i) Circle touching the x -axis and y -axis $\Rightarrow r = |-g| = |-f| \Rightarrow |5| = |y| \Rightarrow y = 5$

(ii) Centre = (5, 5), radius = 5
Equation of circle: $(x - 5)^2 + (y - 5)^2 = (5)^2 = 25$

(iii) y -axis is a tangent $\Rightarrow x = 0$
2nd tangent parallel to the y -axis is: $x = 10$



- Q4.** (i) Sketch of the circle
(ii) Diameter = 8 \Rightarrow radius = 4
(iii) Circle touching the x -axis
 $\Rightarrow r = |-f| = 4$
 $\Rightarrow -f = 4$
Centre $(-g, -f)$ lies on the line: $2x - 3y = 0$
 $\Rightarrow -2g - 3(4) = 0$
 $\Rightarrow -2g = 12$
 $\Rightarrow -g = 6$
 \Rightarrow Centre $= (-g, -f) = (6, 4)$
(iv) Equation of circle: $(x - 6)^2 + (y - 4)^2 = (4)^2 = 16$



- Q5.** (i) Lines $x = 0$ and $x = 8$ are tangents to the circle
 \Rightarrow diameter = 8 \Rightarrow radius = 4
(ii) Circle touching the y -axis
 $\Rightarrow r = |-g| = 4 \Rightarrow x = 4$
Centre $(4, y)$ lies on the line: $2x - y - 3 = 0$
 $\Rightarrow 2(4) - y - 3 = 0$
 $\Rightarrow 8 - y - 3 = 0$
 $\Rightarrow y = 5$
 \Rightarrow Centre $= (4, 5)$
(iii) Equation of circle: $(x - 4)^2 + (y - 5)^2 = (4)^2 = 16$

Q6. x -axis is a tangent \Rightarrow radius $= |-f|$
 $\Rightarrow \sqrt{g^2 + f^2 - c} = |-f|$
 $\Rightarrow g^2 + f^2 - c = f^2$
 $\Rightarrow g^2 - c = 0 \Rightarrow g^2 = c$
 x -axis is a tangent at $(4, 0) \Rightarrow -g = 4 \Rightarrow g = -4$
 $\Rightarrow c = (4)^2 = 16$

Point $(1, 3) \Rightarrow (1)^2 + (3)^2 + 2g(1) + 2f(3) + c = 0$
 $\Rightarrow 2g + 6f + c = -10$
 $\Rightarrow 2(-4) + 6f + 16 = -10$
 $\Rightarrow -8 + 6f + 16 = -10$
 $\Rightarrow 6f = -18$
 $\Rightarrow f = -3$
 \Rightarrow Equation of circle: $x^2 + y^2 + 2(-4)x + 2(-3)y + 16 = 0$
 $\Rightarrow x^2 + y^2 - 8x - 6y + 16 = 0$

Q7. y -axis is a tangent \Rightarrow radius $= |-g|$

$$\Rightarrow \sqrt{g^2 + f^2 - c} = |-g|$$

$$\Rightarrow g^2 + f^2 - c = g^2$$

$$\Rightarrow f^2 - c = 0 \Rightarrow f^2 = c$$

y -axis is a tangent at $(0, -3) \Rightarrow -f = -3 \Rightarrow f = 3$

$$\Rightarrow c = (3)^2 = 9$$

Point $(4, 1) \Rightarrow (4)^2 + (1)^2 + 2g(4) + 2f(1) + c = 0$

$$\Rightarrow 16 + 1 + 8g + 2f + c = 0$$

$$\Rightarrow 8g + 2f + c = -17$$

$$\Rightarrow 8g + 2(3) + 9 = -17$$

$$\Rightarrow 8g = -32$$

$$\Rightarrow g = -4$$

\Rightarrow Equation of circle: $x^2 + y^2 + 2(-4)x + 2(3)y + 9 = 0$

$$\Rightarrow x^2 + y^2 - 8x + 6y + 9 = 0$$

Q8. x -axis is a tangent

$$\Rightarrow \text{radius} = |-f|$$

$$\Rightarrow \sqrt{g^2 + f^2 - c} = |-f|$$

$$\Rightarrow g^2 + f^2 - c = f^2$$

$$\Rightarrow g^2 = c$$

$y = 0$ and $y = 10$ are tangents to the circle

\Rightarrow diameter $= 10 \Rightarrow$ radius $= 5$

\Rightarrow centre $(-g, -f)$ lies on line $y = 5$

$\Rightarrow -f = 5 \Rightarrow f = -5$

Point $(1, 5) \Rightarrow (1)^2 + (5)^2 + 2g(1) + 2f(5) + c = 0$

$$\Rightarrow 2g + 10f + c = -26$$

$$f = -5 \Rightarrow 2g + 10(-5) + c = -26$$

$$\Rightarrow 2g + c = 24$$

$$g^2 = c \Rightarrow 2g + g^2 - 24 = 0$$

$$\Rightarrow g^2 + 2g - 24 = 0$$

$$\Rightarrow (g+6)(g-4) = 0$$

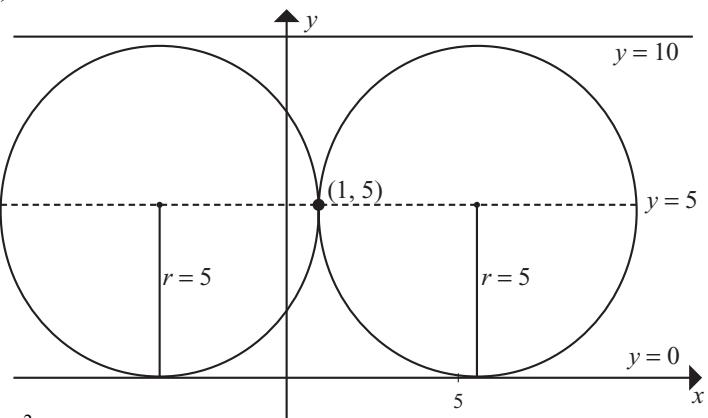
$$\Rightarrow g = -6 \text{ or } g = 4$$

Centre $= (6, 5)$, radius $= 5$

Equation of circle: $(x - 6)^2 + (y - 5)^2 = (5)^2 = 25$

OR centre $= (-4, 5)$, radius $= 5$

Equation of circle: $(x + 4)^2 + (y - 5)^2 = (5)^2 = 25$



Test Yourself 4

A Questions

Q1. (i) Point $(1, 2)$, centre $(-1, 5)$ \Rightarrow length of radius $= \sqrt{(-1-1)^2 + (5-2)^2}$
 $= \sqrt{4+9} = \sqrt{13}$

(ii) Equation of circle: $(x+1)^2 + (y-5)^2 = (\sqrt{13})^2 = 13$

Q2. Circle: $x^2 + y^2 - 2x - 4y - 9 = 0$
 \Rightarrow Centre $= (1, 2)$ and radius $= \sqrt{(-1)^2 + (-2)^2 + 9}$
 $= \sqrt{1+4+9} = \sqrt{14}$

New circle: centre $= (0, 0)$, radius $= \sqrt{14}$

Equation of circle: $x^2 + y^2 = (\sqrt{14})^2 = 14$

Q3. Centre $= (2, 3)$ \Rightarrow circle touches x -axis at $(2, 0)$
 \Rightarrow radius $= 3$
Equation of circle: $(x-2)^2 + (y-3)^2 = (3)^2 = 9$

Q4. $x^2 + y^2 = 25 \Rightarrow$ centre $= (0, 0)$ and radius $= \sqrt{25} = 5$
Line: $3x - 4y + 25 = 0$
 \Rightarrow Perpendicular distance $= \frac{|3(0) - 4(0) + 25|}{\sqrt{(3)^2 + (-4)^2}}$
 $= \frac{25}{\sqrt{25}} = \frac{25}{5} = 5 = \text{radius}$

Hence, the line is a tangent to the circle.

Q5. A $(-1, -3)$, B $(3, 1)$ \Rightarrow centre = midpoint $= \left(\frac{-1+3}{2}, \frac{-3+1}{2} \right)$
 $= (1, -1)$

Radius $= \sqrt{(-1-1)^2 + (-3+1)^2}$
 $= \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Equation of circle: $(x-1)^2 + (y+1)^2 = (2\sqrt{2})^2 = 8$

Q6. Circle: $(x-5)^2 + y^2 = 36$
 x -axis $\Rightarrow y=0 \Rightarrow (x-5)^2 + (0)^2 = 36$
 $\Rightarrow x^2 - 10x + 25 - 36 = 0$
 $\Rightarrow x^2 - 10x - 11 = 0$
 $\Rightarrow (x-11)(x+1) = 0$
 $\Rightarrow x = 11 \text{ or } x = -1$
 $\Rightarrow P = (11, 0); Q = (-1, 0)$

Q7. Circle : $x^2 + y^2 - 2x + 4y + 4 = 0$

$$\Rightarrow \text{centre} = (1, -2)$$

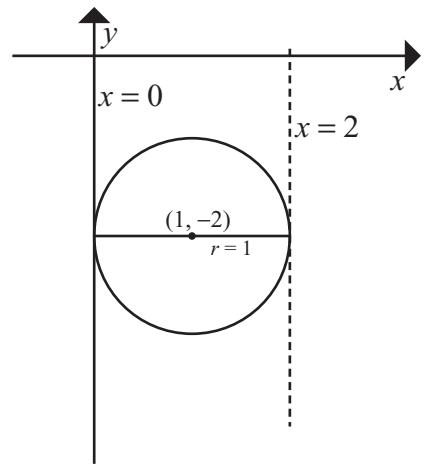
$$\text{and radius} = \sqrt{(-1)^2 + (2)^2 - 4}$$

$$= \sqrt{1+4-4}$$

$$= \sqrt{1} = 1$$

Line $x = k$ is parallel to the y -axis

$$\Rightarrow k = 0, 2$$



Q8. Centres (8, 5) and (2, -3).

$$\text{Distance between centres} = \sqrt{(2-8)^2 + (-3-5)^2}$$

$$= \sqrt{(-6)^2 + (-8)^2} = \sqrt{36+64} = \sqrt{100} = 10$$

Circles touch externally \Rightarrow sum of the two radii = 10

$$\Rightarrow \text{radius of } k_1 + \text{radius of } k_2 = 10$$

$$\Rightarrow 6 + \text{radius of } k_2 = 10$$

$$\Rightarrow \text{radius of } k_2 = 10 - 6 = 4$$

Q9. Circle : $(x-5)^2 + (y+2)^2 = 30$

$$\text{Point } (0,0) \Rightarrow (0-5)^2 + (0+2)^2$$

$$= 25 + 4 = 29 < 30$$

\Rightarrow point is inside the circle.

Q10. A = (1, 6) B = (11, 10)

$$\text{Centre} = \text{midpoint} = \left(\frac{1+11}{2}, \frac{6+10}{2} \right) = (6, 8)$$

$$\text{Radius} = \sqrt{(6-1)^2 + (8-6)^2} = \sqrt{(5)^2 + (2)^2} = \sqrt{25+4} = \sqrt{29}$$

$$\text{Equation of circle : } (x-6)^2 + (y-8)^2 = (\sqrt{29})^2 = 29$$

B Questions

Q1. Circle : $x^2 + y^2 - 6x - 2y - 3 = 0$

$$\text{centre} = (3,1) \text{ and radius} = \sqrt{(-3)^2 + (-1)^2 + 3} \\ = \sqrt{9+1+3} = \sqrt{13}$$

$$\text{Centre } (3,1), \text{ point } (5,4) \Rightarrow \text{slope radius} = \frac{4-1}{5-3} = \frac{3}{2}$$

$$\Rightarrow \text{perpendicular slope} = \frac{-2}{3}$$

$$\Rightarrow \text{equation of tangent: } y - 4 = \frac{-2}{3}(x - 5)$$

$$\Rightarrow 3y - 12 = -2x + 10$$

$$\Rightarrow 2x + 3y - 22 = 0$$

Q2. (i) Centre = $(5, -1)$, line $l: 3x - 4y + 11 = 0$

$$\text{Perpendicular distance} = \frac{|3(5) - 4(-1) + 11|}{\sqrt{(3)^2 + (-4)^2}} \\ = \frac{|15 + 4 + 11|}{\sqrt{25}} = \frac{30}{5} = 6 = \text{radius}$$

(ii) Centre = $(5, -1)$, line : $x + py + 1 = 0$, radius = 6

$$\text{Perpendicular distance} = \frac{|1(5) + p(-1) + 1|}{\sqrt{(1)^2 + (p)^2}} = 6 \\ \Rightarrow \frac{|6 - p|}{\sqrt{1 + p^2}} = 6 \\ \Rightarrow |6 - p| = 6\sqrt{1 + p^2} \\ \Rightarrow (6 - p)^2 = 36(1 + p^2) \\ \Rightarrow 36 - 12p + p^2 = 36 + 36p^2 \\ \Rightarrow 35p^2 + 12p = 0 \\ \Rightarrow p(35p + 12) = 0 \\ \Rightarrow p = 0 \quad \text{or} \quad 35p = -12 \\ \Rightarrow p = \frac{-12}{35}$$

Q3. Circle : $x^2 + y^2 + 4x - 3y - 12 = 0$

$$\Rightarrow \text{Centre} = \left(-2, \frac{3}{2}\right) \text{ and radius} = \sqrt{(2)^2 + \left(\frac{-3}{2}\right)^2 + 12}$$

$$= \sqrt{4 + \frac{9}{4} + 12} = \sqrt{18\frac{1}{4}}$$

Tangent : $8x + 3y + k = 0$

$$\Rightarrow \text{Perpendicular distance} = \frac{|8(-2) + 3\left(\frac{3}{2}\right) + k|}{\sqrt{(8)^2 + (3)^2}} = \sqrt{18\frac{1}{4}}$$

$$\Rightarrow \frac{|-16 + \frac{9}{2} + k|}{\sqrt{64+9}} = \sqrt{\frac{73}{4}}$$

$$\Rightarrow \left|-11\frac{1}{2} + k\right| = \sqrt{\frac{73}{4}} \cdot \sqrt{73}$$

$$\Rightarrow \left|-11\frac{1}{2} + k\right| = \frac{73}{2}$$

$$\Rightarrow -11\frac{1}{2} + k = -\frac{73}{2} \quad \text{or} \quad -11\frac{1}{2} + k = \frac{73}{2}$$

$$\Rightarrow k = -\frac{73}{2} + \frac{23}{2} = -25 \quad \text{or} \quad k = \frac{73}{2} + \frac{23}{2} = 48$$

Q4. (i) Circle k : $x^2 + y^2 + px - 2y + 5 = 0$

$$\text{Point A}(5, 2) \Rightarrow (5)^2 + (2)^2 + p(5) - 2(2) + 5 = 0$$

$$\Rightarrow 25 + 4 + 5p - 4 + 5 = 0$$

$$\Rightarrow 5p + 30 = 0$$

$$\Rightarrow 5p = -30 \Rightarrow p = -6$$

(ii) Line : $x - y - 1 = 0$ meets circle : $x^2 + y^2 - 6x - 2y + 5 = 0$

$$\Rightarrow x = y + 1 \Rightarrow (y + 1)^2 + y^2 - 6(y + 1) - 2y + 5 = 0$$

$$\Rightarrow y^2 + 2y + 1 + y^2 - 6y - 6 - 2y + 5 = 0$$

$$\Rightarrow 2y^2 - 6y = 0$$

$$\Rightarrow y^2 - 3y = 0$$

$$\Rightarrow y(y - 3) = 0$$

$$\Rightarrow y = 0 \quad \text{or} \quad y = 3$$

$$\Rightarrow x = 0 + 1 = 1 \quad \text{or} \quad x = 3 + 1 = 4$$

\Rightarrow Points of intersection = (1, 0) and (4, 3)

Q5. (i) $c_1 : x^2 + y^2 + 2x - 2y - 23 = 0$

Centre = $(-1, 1)$

$$\text{Radius} = \sqrt{(1)^2 + (-1)^2 + 23} = \sqrt{25} = 5$$

$$c_2 : x^2 + y^2 - 14x - 2y + 41 = 0$$

Centre = $(7, 1)$

$$\text{Radius} = \sqrt{(-7)^2 + (-1)^2 - 41} = \sqrt{9} = 3$$

$$\begin{aligned}\text{Distance between centres} &= \sqrt{(7+1)^2 + (1-1)^2} \\ &= \sqrt{8^2 + 0^2} = \sqrt{64} = 8\end{aligned}$$

Sum of the two radii = $5 + 3 = 8$

Hence, the circles touch externally.

(ii) $P = (-6, 1), Q = (10, 1)$

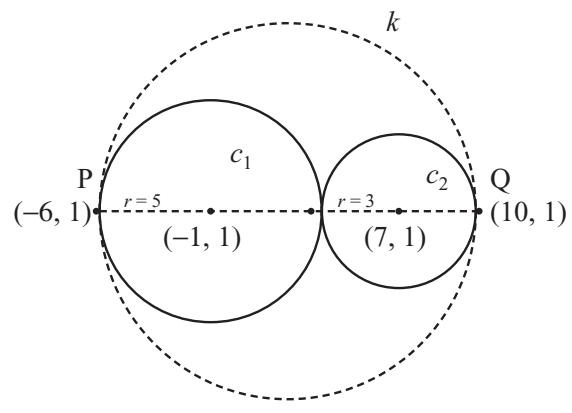
$$\text{Centre of circle } k = \text{midpoint } [PQ] = \left(\frac{-6+10}{2}, \frac{1+1}{2} \right) = (2, 1)$$

Diameter $c_1 = 10$, diameter $c_2 = 6$

Diameter of circle $k = 10 + 6 = 16$

$$\Rightarrow \text{Radius of circle } k = \frac{16}{2} = 8$$

$$\text{Equation of circle } k : (x-2)^2 + (y-1)^2 = (8)^2 = 64$$



Q6. Circle : $x^2 + y^2 - 10y + 20 = 0$

(i) Centre C = $(0, 5)$

(ii) Radius = $\sqrt{(0)^2 + (-5)^2 - 20} = \sqrt{25 - 20} = \sqrt{5}$

(iii) $y = 2x \cap x^2 + y^2 - 10y + 20 = 0$

$$\Rightarrow x^2 + (2x)^2 - 10(2x) + 20 = 0$$

$$\Rightarrow x^2 + 4x^2 - 20x + 20 = 0$$

$$\Rightarrow 5x^2 - 20x + 20 = 0$$

$$\Rightarrow x^2 - 4x + 4 = 0$$

$$\Rightarrow (x-2)(x-2) = 0$$

$$\Rightarrow x = 2 \Rightarrow y = 2(2) = 4$$

One point of contact = $(2, 4)$

\Rightarrow Line is a tangent to the circle.

Q7. $x^2 + y^2 = 4 \Rightarrow$ centre = $(0, 0)$ and radius = $\sqrt{4} = 2$

$$x^2 + y^2 - 8x - 6y + 16 = 0$$

$$\Rightarrow \text{Centre} = (4, 3) \text{ and radius} = \sqrt{(-4)^2 + (-3)^2 - 16} \\ = \sqrt{16 + 9 - 16} = \sqrt{9} = 3$$

$$\text{Distance between centres} = \sqrt{(4-0)^2 + (3-0)^2} \\ = \sqrt{16+9} = \sqrt{25} = 5$$

$$\text{Sum of the two radii} = 2 + 3 = 5$$

Hence, the circles touch externally.

$$\text{Common tangent: } s_1 - s_2 = 0$$

$$\Rightarrow x^2 + y^2 - 4 - (x^2 + y^2 - 8x - 6y + 16) = 0$$

$$\Rightarrow x^2 + y^2 - 4 - x^2 - y^2 + 8x + 6y - 16 = 0$$

$$\Rightarrow 8x + 6y - 20 = 0$$

$$\Rightarrow 4x + 3y - 10 = 0$$

Q8. Circle: $x^2 + y^2 + 2gx + 2fy + 7 = 0$

(i) Point $(2, 5) \Rightarrow (2)^2 + (5)^2 + 2g(2) + 2f(5) + 7 = 0$

$$\Rightarrow 4 + 25 + 4g + 10f + 7 = 0$$

$$\Rightarrow 4g + 10f = -36$$

$$\Rightarrow 2g + 5f = -18$$

Point $(-2, 1) \Rightarrow (-2)^2 + (1)^2 + 2g(-2) + 2f(1) + 7 = 0$

$$\Rightarrow 4 + 1 - 4g + 2f + 7 = 0$$

$$\Rightarrow -4g + 2f = -12$$

$$\Rightarrow 2g - f = 6$$

(ii) $2g + 5f = -18$

$$\begin{array}{r} 2g - f = 6 \\ \hline 6f = -24 \end{array}$$

$$f = -4 \Rightarrow 2g - (-4) = 6$$

$$2g = 2 \Rightarrow g = 1$$

(iii) Circle: $x^2 + y^2 + 2(1)x + 2(-4)y + 7 = 0$

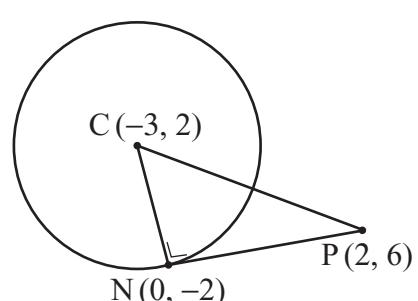
$$\Rightarrow x^2 + y^2 + 2x - 8y + 7 = 0$$

$$\text{Centre} = (-1, 4) \text{ and radius} = \sqrt{(1)^2 + (-4)^2 - 7} = \sqrt{10}$$

Q9. Circle: $(x + 3)^2 + (y - 2)^2 = 25$

(i) Centre C = $(-3, 2)$

(ii) Radius = $\sqrt{25} = 5$



$$\begin{aligned}
 \text{(iii) Point } N(0, -2) &\Rightarrow (0+3)^2 + (-2-2)^2 = 25 \\
 &\Rightarrow (3)^2 + (-4)^2 = 9+16 = 25 = 25 \\
 &\Rightarrow \text{Point } N(0, -2) \text{ lies on the circle.}
 \end{aligned}$$

$$|CP| = \sqrt{(2+3)^2 + (6-2)^2} = \sqrt{25+16} = \sqrt{41}$$

$$\begin{aligned}
 \text{Radius} &= |CN| = 5 \\
 \Rightarrow |NP|^2 + |CN|^2 &= |CP|^2 \\
 \Rightarrow |NP|^2 + (5)^2 &= (\sqrt{41})^2 \\
 \Rightarrow |NP|^2 + 25 &= 41 \\
 \Rightarrow |NP|^2 &= 41 - 25 = 16 \\
 \Rightarrow |NP|^2 &= \sqrt{16} = 4
 \end{aligned}$$

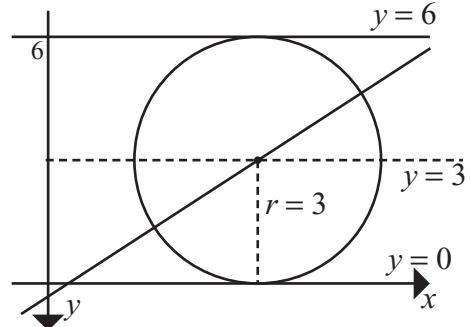
Q10. Centre $(-g, -f)$ on line $x - 2y - 1 = 0$

$$\begin{aligned}
 \Rightarrow -g - 2(-f) - 1 &= 0 \\
 \Rightarrow -g + 2f - 1 &= 0
 \end{aligned}$$

Circle touching the x -axis

$$\begin{aligned}
 \Rightarrow g^2 &= c \\
 y = 0 \text{ and } y = 6 \text{ are parallel tangents} &\Rightarrow \text{equation of diameter is } y = 3 \\
 \Rightarrow -f = 3 &\Rightarrow f = -3 \\
 \Rightarrow -g + 2(-3) - 1 &= 0 \\
 \Rightarrow -g - 6 - 1 &= 0 \\
 \Rightarrow -g = 7 &\Rightarrow g = -7 \\
 \Rightarrow c = g^2 &= (-7)^2 = 49
 \end{aligned}$$

$$\begin{aligned}
 \text{Equation of circle: } x^2 + y^2 + 2(-7)x + 2(-3)y + 49 &= 0 \\
 \Rightarrow x^2 + y^2 - 14x - 6y + 49 &= 0
 \end{aligned}$$



C Questions

Q1. Circle: $x^2 + y^2 - 8x - 7y + 12 = 0$

$$\begin{aligned} \text{y-axis } \Rightarrow x = 0 &\Rightarrow (0)^2 + y^2 - 8(0) - 7y + 12 = 0 \\ &\Rightarrow y^2 - 7y + 12 = 0 \\ &\Rightarrow (y-3)(y-4) = 0 \\ &\Rightarrow y = 3 \text{ or } y = 4 \end{aligned}$$

Points = (0, 3) and (0, 4)

$$\Rightarrow \text{length of the intercept} = 4 - 3 = 1$$

$$\begin{aligned} \text{Circle has centre} &= \left(4, 3\frac{1}{2}\right) \text{ and radius} = \sqrt{\left(-4\right)^2 + \left(-3\frac{1}{2}\right)^2 - 12} \\ &= \sqrt{16 + 12\frac{1}{4} - 12} = \sqrt{16\frac{1}{4}} \end{aligned}$$

$$\begin{aligned} \text{Point } (9, 2), \text{ centre} &= \left(4, 3\frac{1}{2}\right) \Rightarrow \text{hypotenuse} = \sqrt{\left(4-9\right)^2 + \left(3\frac{1}{2}-2\right)^2} \\ &= \sqrt{25 + 2\frac{1}{4}} = \sqrt{27\frac{1}{4}} \end{aligned}$$

$$\begin{aligned} \Rightarrow \text{Length of tangent} &= \sqrt{\left(\sqrt{27\frac{1}{4}}\right)^2 - \left(16\frac{1}{4}\right)^2} \\ &= \sqrt{27\frac{1}{4} - 16\frac{1}{4}} = \sqrt{11} \end{aligned}$$

Q2. Line: $3x - 4y + 1 = 0$

$$\text{Equation of parallel line: } 3x - 4y + c = 0$$

$$\text{Circle: } x^2 + y^2 - 8x + 2y - 8 = 0$$

$$\begin{aligned} \text{Centre} &= (4, -1) \text{ and radius} = \sqrt{\left(-4\right)^2 + \left(1\right)^2 + 8} \\ &= \sqrt{16 + 1 + 8} = \sqrt{25} = 5 \end{aligned}$$

$$\begin{aligned} \text{Perpendicular distance} &= \frac{|3(4) - 4(-1) + c|}{\sqrt{(3)^2 + (-4)^2}} = 5 \\ &\Rightarrow \frac{|12 + 4 + c|}{\sqrt{25}} = 5 \\ &\Rightarrow \frac{|16 + c|}{5} = 5 \\ &\Rightarrow |16 + c| = 25 \end{aligned}$$

$$\Rightarrow 16 + c = 25 \quad \text{or} \quad 16 + c = -25$$

$$\Rightarrow c = 25 - 16 = 9 \quad \text{or} \quad c = -25 - 16 = -41$$

$$\text{Tangents: } 3x - 4y + 9 = 0 \quad \text{or} \quad 3x - 4y - 41 = 0$$

Q3. (i) $C = (8, 1)$; $D = (2, 1)$

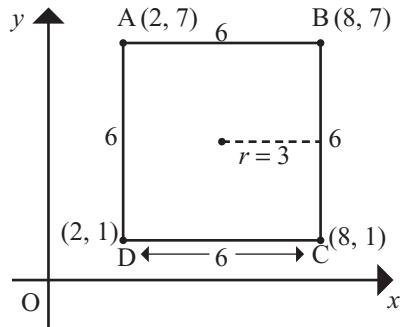
(ii) $A = (2, 7)$, $C = (8, 1)$

$$\text{centre} = \text{midpoint} = \left(\frac{2+8}{2}, \frac{7+1}{2} \right) \\ = (5, 4)$$

(iii) Diameter $= |AB| = 6$

$$\text{Radius} = \frac{6}{2} = 3$$

$$\text{Equation of circle: } (x-5)^2 + (y-4)^2 = 3^2 = 9$$



Q4. x -axis is a tangent at $(3, 0)$

$$\Rightarrow \text{centre} = (3, k)$$

chord $= 8 \Rightarrow$ length of chord

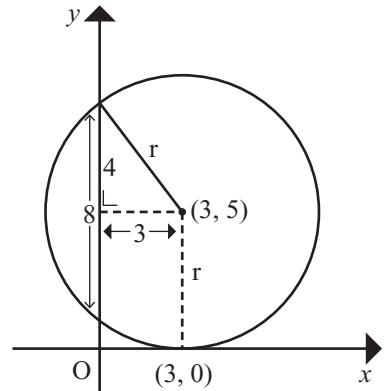
$$\text{bisected} = \frac{8}{2} = 4$$

$$\text{Hence, } r^2 = (3)^2 + (4)^2 = 25$$

$$\Rightarrow r = \sqrt{25} = 5$$

$$\Rightarrow \text{centre} = (3, 5)$$

$$\text{Equation of circle: } (x-3)^2 + (y-5)^2 = 5^2 = 25$$



Q5. Circle: $(x-20)^2 + (y-18)^2 = 16$

$$\Rightarrow \text{centre } A = (20, 18), \text{ radius} = \sqrt{16} = 4$$

$$|AD|^2 + (12)^2 = (20)^2$$

$$\Rightarrow |AD|^2 + 144 = 400$$

$$\Rightarrow |AD|^2 = 400 - 144 = 256$$

$$\Rightarrow |AD| = \sqrt{256} = 16$$

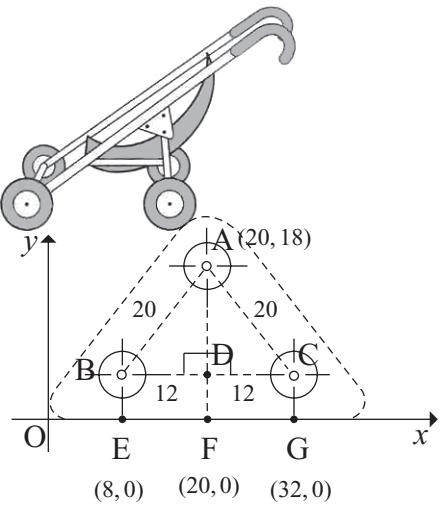
$$|AF| = 18 \Rightarrow |DF| = 18 - 16 = 2$$

$$\text{centre } B = (8, 2), \text{ radius} = 4$$

$$\text{equation of circle: } (x-8)^2 + (y-2)^2 = 4^2 = 16$$

$$\text{centre } C = (32, 2), \text{ radius} = 4$$

$$\text{equation of circle: } (x-32)^2 + (y-2)^2 = 4^2 = 16$$



Q6. End-points of chord = (0, 0) and (4, 2)

$$\text{midpoint} = \left(\frac{0+4}{2}, \frac{0+2}{2} \right) = (2, 1)$$

$$\text{slope of chord} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$$

$$\text{perpendicular slope} = -2$$

$$\text{equation of diameter: } y-1 = -2(x-2)$$

$$\Rightarrow y-1 = -2x+4$$

$$\Rightarrow 2x+y = 5$$

$$\therefore \text{second diameter: } \frac{x+y=1}{x=4}$$

$$\Rightarrow 4+y=1$$

$$\Rightarrow y = -3$$

centre = (4, -3), point (0, 0)

$$\text{radius} = \sqrt{(4-0)^2 + (-3-0)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5$$

$$\text{equation of circle: } (x-4)^2 + (y+3)^2 = (5)^2 = 25$$

Q7. (i) Sketch the circle

$$\text{(ii) Point } (3,3) \Rightarrow (3)^2 + (3)^2 + 2g(3) + 2f(3) + c = 0 \\ \Rightarrow 6g + 6f + c = -18$$

$$\text{Point } (4,1) \Rightarrow (4)^2 + (1)^2 + 2g(4) + 2f(1) + c = 0 \\ \Rightarrow 8g + 2f + c = -17 \\ \underline{6g + 6f + c = -18} \\ \Rightarrow 2g - 4f = 1$$

$$\text{tangent: } 3x - y - 6 = 0 \Rightarrow \text{slope} = -\frac{a}{b} = -\frac{3}{-1} = 3$$

$$\text{perpendicular slope} = -\frac{1}{3}, \text{ point } (3,3)$$

$$\Rightarrow \text{equation of diameter: } y - 3 = -\frac{1}{3}(x - 3)$$

$$\Rightarrow 3y - 9 = -x + 3$$

$$\Rightarrow x + 3y = 12$$

$$\text{centre } (-g, -f) \Rightarrow -g - 3f = 12$$

$$\Rightarrow -2g - 6f = 24$$

$$-2g - 6f = 24$$

$$\underline{2g - 4f = 1} \text{ (adding)} \\ \Rightarrow -10f = 25$$

$$\Rightarrow f = \frac{-25}{10} = \frac{-25}{2}$$

$$\Rightarrow 2g - 4\left(-\frac{5}{2}\right) = 1$$

$$\Rightarrow 2g + 10 = 1$$

$$\Rightarrow 2g = -9$$

$$\Rightarrow g = \frac{-9}{2}$$

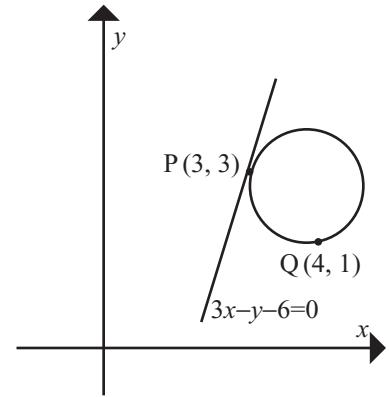
$$\Rightarrow 6\left(\frac{-9}{2}\right) + 6\left(\frac{-5}{2}\right) + c = -18$$

$$\Rightarrow -27 - 15 + c = -18$$

$$\Rightarrow c = -18 + 42 = 24$$

$$\text{Equation of circle: } x^2 + y^2 + 2\left(\frac{-9}{2}\right)x + 2\left(\frac{-5}{2}\right)y + 24 = 0$$

$$\Rightarrow x^2 + y^2 - 9x - 5y + 24 = 0$$



Q8. Circle touches the y -axis $\Rightarrow f^2 = c$
 $P(1, 0), Q(7, 0) \Rightarrow \text{midpoint} = \left(\frac{1+7}{2}, 0\right)$

$$\Rightarrow D = (4, 0)$$

$$\text{centre} = (-g, -f) = (4, -f)$$

$$\Rightarrow -g = 4 \Rightarrow g = -4$$

$$\text{points } (1, 0), C(4, -f)$$

$$\text{radius} = \sqrt{(4-1)^2 + (-f-0)^2} = 4$$

$$\Rightarrow \sqrt{9 + f^2} = 4$$

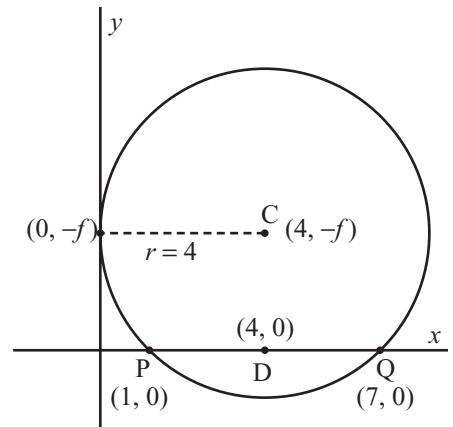
$$\Rightarrow 9 + f^2 = 16$$

$$\Rightarrow f^2 = 16 - 9 = 7 \Rightarrow f = -\sqrt{7} \text{ or } f = \sqrt{7} \text{ (not valid)}$$

$$\Rightarrow c = f^2 = (-\sqrt{7})^2 = 7$$

$$\Rightarrow \text{centre} = (4, \sqrt{7}) \text{ and radius} = 4$$

$$\text{Equation of circle: } (x-4)^2 + (y-\sqrt{7})^2 = 4^2 = 16$$



Q9. (i) Circle touches the y -axis $\Rightarrow r = |-g|$

$$\Rightarrow \sqrt{g^2 + f^2 - c} = |-g|$$

$$\Rightarrow g^2 + f^2 - c = g^2$$

$$\Rightarrow f^2 - c = 0$$

$$\Rightarrow f^2 = c$$

$$\begin{aligned}
 \text{(ii) Point } (-3,6) &\Rightarrow (-3)^2 + (6)^2 + 2g(-3) + 2f(6) + c = 0 \\
 &\Rightarrow 9 + 36 - 6g + 12f + c = 0 \\
 &\Rightarrow -6g + 12f + c = -45
 \end{aligned}$$

$$\begin{aligned}
 \text{Point } (-6,3) &\Rightarrow (-6)^2 + (3)^2 + 2g(-6) + 2f(3) + c = 0 \\
 &\Rightarrow 36 + 9 - 12g + 6f + c = 0 \\
 &\Rightarrow -12g + 6f + c = -45 \\
 &\Rightarrow \underline{-6g + 12f + c = -45} \quad (\text{subtracting}) \\
 &\Rightarrow -6g - 6f = 0 \\
 &\Rightarrow g + f = 0 \\
 &\Rightarrow g = -f
 \end{aligned}$$

$$\text{Hence, } -6(-f) + 12f + f^2 = -45$$

$$\begin{aligned}
 &\Rightarrow 6f + 12f + f^2 + 45 = 0 \\
 &\Rightarrow f^2 + 18f + 45 = 0 \\
 &\Rightarrow (f+3)(f+15) = 0 \\
 &\Rightarrow f = -3 \text{ or } f = -15 \\
 &\Rightarrow g = 3 \text{ or } g = 15 \\
 &\Rightarrow c = (-3)^2 = 9 \text{ or } c = (-15)^2 = 225
 \end{aligned}$$

\Rightarrow Equation of circles:

$$\begin{aligned}
 &x^2 + y^2 + 2(3)x + 2(-3)y + 9 = 0 \\
 &\Rightarrow x^2 + y^2 + 6x - 6y + 9 = 0 \\
 \text{and } &x^2 + y^2 + 2(15)x + 2(-15)y + 225 = 0 \\
 &\Rightarrow x^2 + y^2 + 30x - 30y + 225 = 0
 \end{aligned}$$

$$\text{Q10. Circle: } x^2 + y^2 - 2ax - 2by + b^2 = 0$$

$$\begin{aligned}
 \text{(i) } 2g &= -2a \Rightarrow g = -a \\
 2f &= -2b \Rightarrow f = -b \\
 \Rightarrow \text{centre } (-g, -f) &= (a, b) \\
 \text{and radius } &= \sqrt{(-a)^2 + (-b)^2 - b^2} \\
 &= \sqrt{a^2 + b^2 - b^2} = \sqrt{a^2} = a
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) Circle touches the } y\text{-axis} &\Rightarrow f^2 = c \\
 &\Rightarrow (-b)^2 = b^2 \\
 &\Rightarrow b^2 = b^2 \text{ true}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) Point } (1,2) &\Rightarrow (1)^2 + (2)^2 - 2a(1) - 2b(2) + b^2 = 0 \\
 &\Rightarrow 1 + 4 - 2a - 4b + b^2 = 0 \\
 &\Rightarrow -2a - 4b + b^2 = -5
 \end{aligned}$$

$$\begin{aligned}
 \text{Point } (2,3) &\Rightarrow (2)^2 + (3)^2 - 2a(2) - 2b(3) + b^2 = 0 \\
 &\Rightarrow 4 + 9 - 4a - 6b + b^2 = 0 \\
 &\Rightarrow -4a - 6b + b^2 = -13 \\
 &\Rightarrow \underline{-2a - 4b + b^2 = -5} \quad (\text{subtracting}) \\
 &\Rightarrow -2a - 2b = -8 \\
 &\Rightarrow a + b = 4 \\
 &\Rightarrow a = 4 - b \\
 \\
 &\Rightarrow -2(4 - b) - 4b + b^2 = -5 \\
 &\Rightarrow -8 + 2b - 4b + b^2 = -5 \\
 &\Rightarrow b^2 - 2b - 3 = 0 \\
 &\Rightarrow (b+1)(b-3) = 0 \\
 &\Rightarrow b = -1 \text{ or } b = 3 \\
 &\Rightarrow a = 4 - (-1) = 5 \text{ or } a = 4 - 3 = 1
 \end{aligned}$$

centre = (5, -1), radius = 5

$$\text{Circle: } \Rightarrow (x-5)^2 + (y+1)^2 = (5)^2 = 25$$

and centre = (1, 3), radius = 1

$$\text{Circle: } (x-1)^2 + (y-3)^2 = (1)^2 = 1$$

$$\begin{aligned}
 \text{(iv) Distance between centres} &= \sqrt{(1-5)^2 + (3+1)^2} \\
 &= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}
 \end{aligned}$$

Chapter 5 Trigonometry 2

Exercise 5.1

Q1. Prove : $\cos A \tan A = \sin A$

$$\begin{aligned}\text{Proof : } \cos A \tan A &= \cos A \cdot \frac{\sin A}{\cos A} \\ &= \sin A\end{aligned}$$

Q2. Prove : $\sin \theta \sec \theta = \tan \theta$

$$\begin{aligned}\text{Proof : } \sin \theta \sec \theta &= \sin \theta \cdot \frac{1}{\cos \theta} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta\end{aligned}$$

Q3. Prove : $\sin \theta \tan \theta + \cos \theta = \sec \theta$

$$\begin{aligned}\text{Proof : } \sin \theta \tan \theta + \cos \theta &= \sin \theta \cdot \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{1} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} \\ &= \frac{1}{\cos \theta} \\ &= \sec \theta\end{aligned}$$

Q4. Prove : $\frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}} = \tan \theta$

$$\begin{aligned}\text{Proof : } \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}} &= \frac{\sin \theta}{\sqrt{\cos^2 \theta}} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta\end{aligned}$$

Q5. Prove : $\sec A - \sin A \tan A = \cos A$

$$\begin{aligned}\text{Proof : } \sec A - \sin A \tan A &= \frac{1}{\cos A} - \sin A \cdot \frac{\sin A}{\cos A} \\ &= \frac{1 - \sin^2 A}{\cos A} \\ &= \frac{\cos^2 A}{\cos A} \\ &= \cos A\end{aligned}$$

Q6. Prove : $1 - \tan^2 \theta \cos^2 \theta = \cos^2 \theta$

$$\begin{aligned}\text{Proof : } 1 - \tan^2 \theta \cos^2 \theta &= 1 - \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \cos^2 \theta \\ &= 1 - \sin^2 \theta \\ &= \cos^2 \theta\end{aligned}$$

Q7. Prove : $\frac{(1+\cos\theta)(1-\cos\theta)}{\cos^2\theta} = \tan^2\theta$

$$\begin{aligned}\text{Proof : } \frac{(1+\cos\theta)(1-\cos\theta)}{\cos^2\theta} &= \frac{1-\cos\theta + \cos\theta - \cos^2\theta}{\cos^2\theta} \\ &= \frac{1-\cos^2\theta}{\cos^2\theta} \\ &= \frac{\sin^2\theta}{\cos^2\theta} \\ &= \tan^2\theta\end{aligned}$$

Q8. Prove : $\sec^2 A - \tan^2 A = 1$

$$\begin{aligned}\text{Proof : } \sec^2 A - \tan^2 A &= \frac{1}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A} \\ &= \frac{1 - \sin^2 A}{\cos^2 A} \\ &= \frac{\cos^2 A}{\cos^2 A} \\ &= 1\end{aligned}$$

Q9. Prove : $\frac{\sqrt{1-\cos^2\theta}}{\tan\theta} = \cos\theta$

$$\begin{aligned}\text{Proof : } \frac{\sqrt{1-\cos^2\theta}}{\tan\theta} &= \frac{\sqrt{\sin^2\theta}}{\frac{\sin\theta}{\cos\theta}} \\ &= \sin\theta \cdot \frac{\cos\theta}{\sin\theta} \\ &= \cos\theta\end{aligned}$$

Q10. Prove : $(1 + \tan^2 \theta) \cos^2 \theta = 1$

$$\begin{aligned}\text{Proof : } (1 + \tan^2 \theta) \cdot \cos^2 \theta &= \sec^2 \theta \cdot \cos^2 \theta \\ &= \frac{1}{\cos^2 \theta} \cdot \cos^2 \theta \\ &= 1\end{aligned}$$

Q11. Prove : $(\cos \theta + \sin \theta)^2 + (\cos \theta - \sin \theta)^2 = 2$

Proof :
$$\begin{aligned} & (\cos \theta + \sin \theta)^2 + (\cos \theta - \sin \theta)^2 \\ &= \cos^2 \theta + 2 \sin \theta \cos \theta + \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta + \sin^2 \theta \\ &= 2(\cos^2 \theta + \sin^2 \theta) \\ &= 2 \cdot 1 = 2 \end{aligned}$$

Q12. Prove : $(1 + \tan^2 A)(1 - \sin^2 A) = 1$

Proof :
$$\begin{aligned} & (1 + \tan^2 A)(1 - \sin^2 A) = \sec^2 A \cdot \cos^2 A \\ &= \frac{1}{\cos^2 A} \cdot \cos^2 A \\ &= 1 \end{aligned}$$

Q13. Prove : $(\sin \theta + \cos \theta)^2 - 2 \sin \theta \cos \theta = 1$

Proof :
$$\begin{aligned} & (\sin \theta + \cos \theta)^2 - 2 \sin \theta \cos \theta \\ &= \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta - 2 \sin \theta \cos \theta \\ &= \sin^2 \theta + \cos^2 \theta \\ &= 1 \end{aligned}$$

Q14. Prove : $\frac{1 - \cos^2 A}{\sin A \cos A} = \tan A$

Proof :
$$\begin{aligned} \frac{1 - \cos^2 A}{\sin A \cos A} &= \frac{\sin^2 A}{\sin A \cos A} \\ &= \frac{\sin A}{\cos A} \\ &= \tan A \end{aligned}$$

Q15. Prove : $\frac{1}{1 - \sin A} + \frac{1}{1 + \sin A} = 2 \sec^2 A$

Proof :
$$\begin{aligned} \frac{1}{1 - \sin A} + \frac{1}{1 + \sin A} &= \frac{1(1 + \sin A) + 1(1 - \sin A)}{(1 - \sin A)(1 + \sin A)} \\ &= \frac{1 + \sin A + 1 - \sin A}{1 + \sin A - \sin A - \sin^2 A} \\ &= \frac{2}{1 - \sin^2 A} \\ &= \frac{2}{\cos^2 A} \\ &= 2 \sec^2 A \end{aligned}$$

Q16. Prove : $(1 - \sin^2 A) \tan^2 A + \cos^2 A = 1$

$$\begin{aligned}\text{Proof : } (1 - \sin^2 A) \tan^2 A + \cos^2 A &= \cos^2 A \cdot \frac{\sin^2 A}{\cos^2 A} + \cos^2 A \\ &= \sin^2 A + \cos^2 A = 1\end{aligned}$$

Q17. Prove : $\operatorname{cosec}^2 \theta (\tan^2 \theta - \sin^2 \theta) = \tan^2 \theta$

$$\begin{aligned}\text{Proof : } \operatorname{cosec}^2 \theta (\tan^2 \theta - \sin^2 \theta) &= \frac{1}{\sin^2 \theta} \left(\frac{\sin^2 \theta}{\cos^2 \theta} - \frac{\sin^2 \theta}{1} \right) \\ &= \frac{1}{\sin^2 \theta} \left(\frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta} \right) \\ &= \frac{1}{\sin^2 \theta} \cdot \sin^2 \theta \left(\frac{1 - \cos^2 \theta}{\cos^2 \theta} \right) = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta\end{aligned}$$

Q18. Prove : $(1 - \sin A)(\sec A + \tan A) = \cos A$

$$\begin{aligned}\text{Proof : } (1 - \sin A)(\sec A + \tan A) &= (1 - \sin A) \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) \\ &= (1 - \sin A) \cdot \frac{(1 + \sin A)}{\cos A} \\ &= \frac{1 + \sin A - \sin A - \sin^2 A}{\cos A} \\ &= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A\end{aligned}$$

Q19. Prove : $b \cos C + c \cos B = a$

$$\text{Proof : cosine rule: } \cos B = \frac{a^2 + c^2 - b^2}{2ac}, \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\begin{aligned}\text{Thus, } b \cos C + c \cos B &= b \cdot \frac{(a^2 + b^2 - c^2)}{2ab} + c \cdot \frac{(a^2 + c^2 - b^2)}{2ac} \\ &= \frac{a^2 + b^2 - c^2}{2a} + \frac{a^2 + c^2 - b^2}{2a} \\ &= \frac{a^2 + b^2 - c^2 + a^2 + c^2 - b^2}{2a} = \frac{2a^2}{2a} = a\end{aligned}$$

Q20. Prove : $bc \cos A + ca \cos B = c^2$

Proof : cosine rule : $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

Hence, $bc \cos A + ca \cos B = \frac{bc(b^2 + c^2 - a^2)}{2bc} + \frac{ca(a^2 + c^2 - b^2)}{2ac}$
 $= \frac{b^2 + c^2 - a^2}{2} + \frac{a^2 + c^2 - b^2}{2}$
 $= \frac{b^2 + c^2 - a^2 + a^2 + c^2 - b^2}{2}$
 $= \frac{2c^2}{2} = c^2$

Q21. Prove : $c = b \cos A + a \cos B$

Proof : cosine rule : $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

Hence, $b \cos A + a \cos B = \frac{b(b^2 + c^2 - a^2)}{2bc} + \frac{a(a^2 + c^2 - b^2)}{2ac}$
 $= \frac{b^2 + c^2 - a^2}{2c} + \frac{a^2 + c^2 - b^2}{2c}$
 $= \frac{b^2 + c^2 - a^2 + a^2 + c^2 - b^2}{2c}$
 $= \frac{2c^2}{2c} = c$

Q22. Prove : $a \cos B - b \cos A = \frac{a^2 - b^2}{c}$

Proof : cosine rule : $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

Hence, $a \cos B - b \cos A = \frac{a(a^2 + c^2 - b^2)}{2ac} - \frac{b(b^2 + c^2 - a^2)}{2bc}$
 $= \frac{a^2 + c^2 - b^2}{2c} - \frac{(b^2 + c^2 - a^2)}{2c}$
 $= \frac{a^2 + c^2 - b^2 - b^2 - c^2 + a^2}{2c}$
 $= \frac{2(a^2 - b^2)}{2c} = \frac{a^2 - b^2}{c}$

Q23. Prove : $ab \cos C - ac \cos B = b^2 - c^2$

Proof : cosine rule : $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Hence,
$$\begin{aligned} ab \cos C - ac \cos B &= \frac{ab(a^2 + b^2 - c^2)}{2ab} - \frac{ac(a^2 + c^2 - b^2)}{2ac} \\ &= \frac{a^2 + b^2 - c^2}{2} - \frac{(a^2 + c^2 - b^2)}{2} \\ &= \frac{a^2 + b^2 - c^2 - a^2 - c^2 + b^2}{2} \\ &= \frac{2(b^2 - c^2)}{2} = b^2 - c^2 \end{aligned}$$

Q24. Prove : $c \cos B - b \cos C = \frac{c^2 - b^2}{a}$

Proof : cosine rule: $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Hence,
$$\begin{aligned} c \cos B - b \cos C &= \frac{c(a^2 + c^2 - b^2)}{2ac} - \frac{b(a^2 + b^2 - c^2)}{2ab} \\ &= \frac{a^2 + c^2 - b^2}{2a} - \frac{(a^2 + b^2 - c^2)}{2a} \\ &= \frac{a^2 + c^2 - b^2 - a^2 - b^2 + c^2}{2a} \\ &= \frac{2(c^2 - b^2)}{2a} = \frac{c^2 - b^2}{a} \end{aligned}$$

Q25. Prove : $\frac{\sin A - \sin B}{\sin B} = \frac{a - b}{b}$

Proof : sine rule : $\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow b \sin A = a \sin B$
 $\Rightarrow \sin A = \frac{a \sin B}{b}$

Hence,
$$\begin{aligned} \frac{\sin A - \sin B}{\sin B} &= \frac{\frac{a \sin B}{b} - \frac{\sin B}{1}}{\sin B} \\ &= \frac{\frac{a \sin B - b \sin B}{b}}{\sin B} \\ &= \frac{\sin B \left[\frac{a - b}{b} \right]}{\sin B} \\ &= \frac{a - b}{b} \end{aligned}$$

Q26. Prove : $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$

Proof : cosine rule : $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$; $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Hence,
$$\begin{aligned}\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} &= \frac{\frac{b^2 + c^2 - a^2}{2bc}}{a} + \frac{\frac{a^2 + c^2 - b^2}{2ac}}{b} + \frac{\frac{a^2 + b^2 - c^2}{2ab}}{c} \\ &= \frac{b^2 + c^2 - a^2}{2abc} + \frac{a^2 + c^2 - b^2}{2abc} + \frac{a^2 + b^2 - c^2}{2abc} \\ &= \frac{b^2 + c^2 - a^2 + a^2 + c^2 - b^2 + a^2 + b^2 - c^2}{2abc} \\ &= \frac{a^2 + b^2 + c^2}{2abc}\end{aligned}$$

Exercise 5.2

Q1. (i) $\cos 15^\circ = \cos (60^\circ - 45^\circ)$
 $= \cos 60^\circ \cdot \cos 45^\circ + \sin 60^\circ \cdot \sin 45^\circ$
 $= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$
 $= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) $\sin 75^\circ = \sin (45^\circ + 30^\circ)$
 $= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$
 $= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$
 $= \frac{\sqrt{3}+1}{2\sqrt{2}}$

(iii) $\cos 105^\circ = \cos (60^\circ + 45^\circ)$
 $= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$
 $= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$
 $= \frac{1-\sqrt{3}}{2\sqrt{2}}$

$$\begin{aligned}
 \text{Q2. (i)} \quad \tan 15^\circ &= \tan(45^\circ - 30^\circ) \\
 &= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} \\
 &= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} \\
 &= \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \sin 135^\circ &= \sin(90^\circ + 45^\circ) \\
 &= \sin 90^\circ \cos 45^\circ + \cos 90^\circ \sin 45^\circ \\
 &= 1 \cdot \frac{1}{\sqrt{2}} + 0 \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \tan 75^\circ &= \tan(45^\circ + 30^\circ) \\
 &= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} \\
 &= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} \\
 &= \frac{\frac{\sqrt{3} + 1}{\sqrt{3}}}{\frac{\sqrt{3} - 1}{\sqrt{3}}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}
 \end{aligned}$$

$$\begin{aligned}
 \text{Q3.} \quad \text{First Triangle : } \sin A &= \frac{3}{5}, \quad \cos A = \frac{4}{5}, \quad \tan A = \frac{3}{4} \\
 \text{Second Triangle : } \sin B &= \frac{5}{13}, \quad \cos B = \frac{12}{13}, \quad \tan B = \frac{5}{12}
 \end{aligned}$$

$$\begin{aligned}
 \text{(i)} \quad \cos(A + B) &= \cos A \cos B - \sin A \sin B \\
 &= \frac{4}{5} \cdot \frac{12}{13} - \frac{3}{5} \cdot \frac{5}{13} = \frac{48}{65} - \frac{15}{65} = \frac{33}{65}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\
 &= \frac{\frac{3}{4} - \frac{5}{12}}{1 + \frac{3}{4} \cdot \frac{5}{12}} = \frac{\frac{1}{3}}{\frac{21}{16}} = \frac{16}{63}
 \end{aligned}$$

Q4. (i) $\sin 45^\circ \cos 15^\circ + \cos 45^\circ \sin 15^\circ$

$$= \sin(45^\circ + 15^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

(ii) $\cos 40^\circ \cos 50^\circ - \sin 40^\circ \sin 50^\circ$

$$= \cos(40^\circ + 50^\circ) = \cos 90^\circ = 0$$

(iii) $\cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ$

$$= \cos(80^\circ - 20^\circ) = \cos 60^\circ = \frac{1}{2}$$

(iv) $\frac{\tan 25^\circ + \tan 20^\circ}{1 - \tan 25^\circ \tan 20^\circ} = \tan(25^\circ + 20^\circ) = \tan 45^\circ = 1$

Q5. (i) $\frac{\tan 2A + \tan A}{1 - \tan 2A \tan A} = \tan(2A + A) = \tan 3A$

(ii) $\sin 2\theta \cos \theta + \cos 2\theta \sin \theta$

$$= \sin(2\theta + \theta) = \sin 3\theta$$

Q6. (i) $\sin(90^\circ - A) = \sin 90^\circ \cos A - \cos 90^\circ \sin A$

$$= 1 \cdot \cos A - 0 \cdot \sin A = \cos A$$

(ii) $\cos(90^\circ + A) = \cos 90^\circ \cos A - \sin 90^\circ \sin A$

$$= 0 \cdot \cos A - 1 \cdot \sin A = -\sin A$$

Q7. $\tan(A - B) = 2 \quad \text{and} \quad \tan B = \frac{1}{4}$

$$\Rightarrow \frac{\tan A - \tan B}{1 + \tan A \tan B} = 2$$

$$\Rightarrow \frac{\tan A - \frac{1}{4}}{1 + \tan A \cdot \frac{1}{4}} = \frac{2}{1}$$

$$\Rightarrow \tan A - \frac{1}{4} = 2 + \tan A \cdot \frac{1}{2}$$

$$\Rightarrow 4 \tan A - 1 = 8 + 2 \tan A$$

$$\Rightarrow 2 \tan A = 9$$

$$\Rightarrow \tan A = \frac{9}{2} = 4\frac{1}{2}$$

Q8. $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$$\Rightarrow (A+B) = \tan^{-1}(1) = \frac{\pi}{4} \text{ (or } 45^\circ\text{)}$$

Q9. $\tan(A+B) = 1$ and $\tan A = \frac{1}{3}$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\Rightarrow \frac{\frac{1}{3} + \tan B}{1 - \frac{1}{3} \cdot \tan B} = \frac{1}{1}$$

$$\Rightarrow \frac{1}{3} + \tan B = 1 - \frac{1}{3} \tan B$$

$$\Rightarrow 1 + 3 \tan B = 3 - \tan B$$

$$\Rightarrow 4 \tan B = 2$$

$$\Rightarrow \tan B = \frac{2}{4} = \frac{1}{2}$$

Q10. If $\sin x = \frac{1}{2}$ \Rightarrow Pythagoras: $p^2 + 1^2 = 2^2$

$$\Rightarrow p^2 = 4 - 1 = 3$$

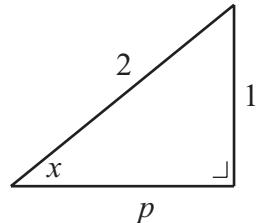
$$\Rightarrow p = \sqrt{3}$$

Hence, $\cos x = \frac{\sqrt{3}}{2}$

$$\sin\left(x + \frac{\pi}{4}\right) = \sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}}$$



$$Q11. \tan 15^\circ = \tan(45^\circ - 30^\circ)$$

$$\begin{aligned} &= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} \\ &= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \end{aligned}$$

$$\begin{aligned} \text{Hence, } \frac{\sqrt{3}-1}{\sqrt{3}+1} &= \frac{\sqrt{3}-1}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} \\ &= \frac{\sqrt{3}(\sqrt{3}-1) - 1(\sqrt{3}-1)}{\sqrt{3}(\sqrt{3}-1) + 1(\sqrt{3}-1)} \\ &= \frac{3 - \sqrt{3} - \sqrt{3} + 1}{3 - \sqrt{3} + \sqrt{3} - 1} \\ &= \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{Hence, } \tan^2 15^\circ &= (\tan 15^\circ)(\tan 15^\circ) \\ &= (2 - \sqrt{3})(2 - \sqrt{3}) \\ &= 2(2 - \sqrt{3}) - \sqrt{3}(2 - \sqrt{3}) \\ &= 4 - 2\sqrt{3} - 2\sqrt{3} + 3 = 7 - 4\sqrt{3} \end{aligned}$$

$$Q12. \text{ Prove : } \tan\left(\frac{\pi}{4} + A\right) = \frac{\cos A + \sin A}{\cos A - \sin A}$$

$$\begin{aligned} \text{Proof : } \tan\left(\frac{\pi}{4} + A\right) &= \frac{\tan \frac{\pi}{4} + \tan A}{1 - \tan \frac{\pi}{4} \tan A} \\ &= \frac{1 + \tan A}{1 - 1 \cdot \tan A} \\ &= \frac{1 + \frac{\sin A}{\cos A}}{1 - \frac{\sin A}{\cos A}} \\ &= \frac{\frac{\cos A + \sin A}{\cos A}}{\frac{\cos A - \sin A}{\cos A}} \\ &= \frac{\cos A + \sin A}{\cos A - \sin A} \end{aligned}$$

$$\begin{aligned} Q13. \cos(A+B)\cos B + \sin(A+B)\sin B \\ = \cos[(A+B)-B] = \cos(A+B-B) = \cos A \end{aligned}$$

Q14. First Triangle : $\tan A = \frac{2}{h}$

Second Triangle : $\tan B = \frac{3}{h}$

If $A + B = 45^\circ$

$$\Rightarrow \tan(A + B) = \tan 45^\circ$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\Rightarrow \frac{\frac{2}{h} + \frac{3}{h}}{1 - \frac{2}{h} \cdot \frac{3}{h}} = \frac{\frac{2+3}{h}}{\frac{h^2 - 6}{h^2}}$$

$$\Rightarrow \frac{5}{h} \cdot \frac{h^2}{h^2 - 6} = 1$$

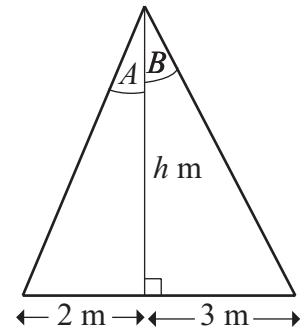
$$\Rightarrow \frac{5h}{h^2 - 6} = \frac{1}{1}$$

$$\Rightarrow h^2 - 6 = 5h$$

$$\Rightarrow h^2 - 5h - 6 = 0$$

$$\Rightarrow (h-6)(h+1) = 0$$

$$\Rightarrow h = 6 \text{ or } h = -1 \text{ (not valid)}$$



Q15. $\sin A = \sin(A + 30^\circ)$

$$\Rightarrow \sin A = \sin A \cos 30^\circ + \cos A \sin 30^\circ$$

$$\Rightarrow \sin A = \sin A \cdot \frac{\sqrt{3}}{2} + \cos A \cdot \frac{1}{2}$$

$$\Rightarrow 2\sin A = \sqrt{3}\sin A + \cos A \cdot 1$$

$$\Rightarrow \sin A(2 - \sqrt{3}) = \cos A \cdot 1$$

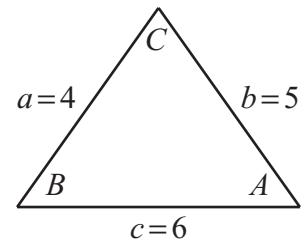
$$\Rightarrow \frac{\sin A}{\cos A} = \frac{1}{2 - \sqrt{3}}$$

$$\Rightarrow \tan A = \frac{1}{2 - \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{4 - 3} = \frac{2 + \sqrt{3}}{1} = 2 + \sqrt{3}$$

$$\begin{aligned} \text{Q16. (i) Cos rule : } \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \\ &= \frac{(5)^2 + (6)^2 - (4)^2}{2(5)(6)} \\ &= \frac{45}{60} = \frac{3}{4} \end{aligned}$$

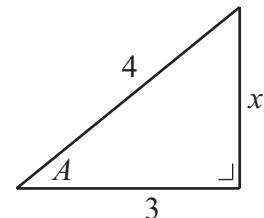
$$\begin{aligned} \cos C &= \frac{a^2 + b^2 - c^2}{2ab} \\ &= \frac{(4)^2 + (5)^2 - (6)^2}{2(4)(5)} \\ &= \frac{5}{40} = \frac{1}{8} \end{aligned}$$

$$\text{Hence, } \cos A + \cos C = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$$

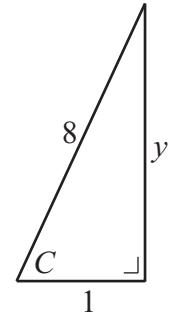


$$\begin{aligned} \text{(ii) } \cos A &= \frac{3}{4} \Rightarrow \text{ Pythagoras : } x^2 + (3)^2 = (4)^2 \\ &\Rightarrow x^2 + 9 = 16 \\ &\Rightarrow x^2 = 16 - 9 = 7 \\ &\Rightarrow x = \sqrt{7} \end{aligned}$$

$$\text{Hence, } \sin A = \frac{\sqrt{7}}{4}$$



$$\begin{aligned} \cos C &= \frac{1}{8} \Rightarrow \text{ Pythagoras : } y^2 + (1)^2 = (8)^2 \\ &\Rightarrow y^2 + 1 = 64 \\ &\Rightarrow y^2 = 64 - 1 = 63 \\ &\Rightarrow y = \sqrt{63} = 3\sqrt{7} \end{aligned}$$



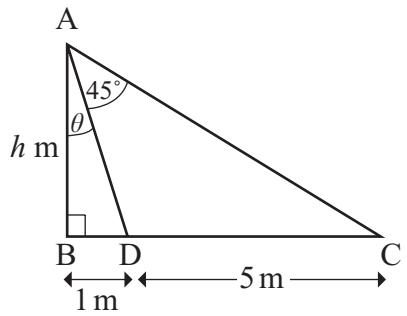
$$\text{Hence, } \sin C = \frac{3\sqrt{7}}{8}$$

$$\cos(A+C) = \cos A \cos C - \sin A \sin C$$

$$\begin{aligned} &= \frac{3}{4} \cdot \frac{1}{8} - \frac{\sqrt{7}}{4} \cdot \frac{3\sqrt{7}}{8} \\ &= \frac{3}{32} - \frac{3.7}{32} \\ &= \frac{3-21}{32} = \frac{-18}{32} = \frac{-9}{16} \end{aligned}$$

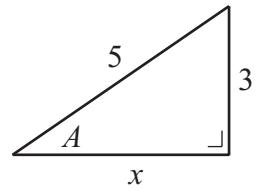
Q17. Triangle ABD $\Rightarrow \tan \theta = \frac{1}{h}$

$$\begin{aligned} \text{Triangle ABC} &\Rightarrow \tan(\theta + 45^\circ) = \frac{6}{h} \\ &\Rightarrow \frac{\tan \theta + \tan 45^\circ}{1 - \tan \theta \cdot \tan 45^\circ} = \frac{6}{h} \\ &\Rightarrow \frac{\frac{1}{h} + 1}{1 - \frac{1}{h} \cdot 1} = \frac{6}{h} \\ &\Rightarrow h\left(\frac{1}{h} + 1\right) = 6\left(1 - \frac{1}{h}\right) \\ &\Rightarrow 1 + h = 6 - \frac{6}{h} \\ &\Rightarrow h + h^2 = 6h - 6 \\ &\Rightarrow h^2 - 5h + 6 = 0 \\ &\Rightarrow (h-2)(h-3) = 0 \\ &\Rightarrow h = 2 \text{ m or } 3 \text{ m} \end{aligned}$$



Exercise 5.3

Q1. $\sin A = \frac{3}{5} \Rightarrow \text{Pythagoras: } x^2 + (3)^2 = (5)^2$
 $\Rightarrow x^2 + 9 = 25$
 $\Rightarrow x^2 = 25 - 9 = 16$
 $\Rightarrow x = \sqrt{16} = 4$



Hence, $\cos A = \frac{4}{5}$ and $\tan A = \frac{3}{4}$

(i) $\sin 2A = 2 \sin A \cos A = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$

(ii) $\cos 2A = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

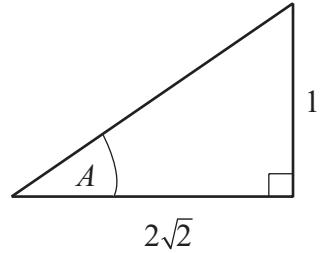
(iii) $\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{\frac{24}{25}}{\frac{7}{25}} = \frac{24}{7}$

Q2. $\tan A = \frac{1}{2}$

$$(i) \quad \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} = \frac{2\left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2}\right)^2} = \frac{1}{1 - \frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$(ii) \quad \sin 2A = \frac{2 \tan A}{1 + \tan^2 A} = \frac{2\left(\frac{1}{2}\right)}{1 + \left(\frac{1}{2}\right)^2} = \frac{1}{1 + \frac{1}{4}} = \frac{1}{1\frac{1}{4}} = \frac{4}{5}$$

Q3. Using the diagram, $\tan A = \frac{1}{2\sqrt{2}}$.



$$\begin{aligned} \Rightarrow \cos 2A &= \frac{1 - \tan^2 A}{1 + \tan^2 A} \\ &= \frac{1 - \left(\frac{1}{2\sqrt{2}}\right)^2}{1 + \left(\frac{1}{2\sqrt{2}}\right)^2} = \frac{1 - \frac{1}{8}}{1 + \frac{1}{8}} = \frac{\frac{7}{8}}{\frac{9}{8}} = \frac{7}{9} \end{aligned}$$

Q4. $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$$\cos 2A = \frac{3}{8}$$

$$\Rightarrow 1 - 2\sin^2 A = \frac{3}{8}$$

$$\Rightarrow -2\sin^2 A = \frac{3}{8} - 1 = \frac{-5}{8}$$

$$\Rightarrow \sin^2 A = \frac{5}{16}$$

$$\Rightarrow \sin A = \sqrt{\frac{5}{16}} = \frac{\sqrt{5}}{4} \quad \text{where } 0^\circ < A < 90^\circ$$

$$\cos 2A = \frac{3}{8}$$

$$\Rightarrow 2\cos^2 A - 1 = \frac{3}{8}$$

$$\Rightarrow 2\cos^2 A = 1 + \frac{3}{8} = \frac{11}{8}$$

$$\Rightarrow \cos^2 A = \frac{11}{16}$$

$$\Rightarrow \cos A = \sqrt{\frac{11}{16}} = \frac{\sqrt{11}}{4} \quad \text{where } 0^\circ < A < 90^\circ$$

Q5. Given $\sin 2A = 2 \sin A \cos A$ and $\cos 2A = \cos^2 A - \sin^2 A$

$$(i) \quad 2 \sin 15^\circ \cos 15^\circ = \sin 2(15^\circ) = \sin 30^\circ = \frac{1}{2}$$

$$(ii) \quad 2 \sin 75^\circ \cos 75^\circ = \sin 2(75^\circ)$$

$$= \sin 150^\circ = \frac{1}{2}$$

$$(iii) \quad \cos^2 22\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$$

$$= \cos 2\left(22\frac{1}{2}^\circ\right)$$

$$= \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\begin{aligned} Q6. \quad \frac{2 \tan 22\frac{1}{2}^\circ}{1 - \tan^2 22\frac{1}{2}^\circ} &= \tan 2\left(22\frac{1}{2}^\circ\right) \\ &= \tan 45^\circ = 1 \end{aligned}$$

$$Q7. \quad \cos 3A = \cos(2A + A)$$

$$\begin{aligned} &= \cos 2A \cos A - \sin 2A \sin A \\ &= (2 \cos^2 A - 1) \cos A - 2 \sin A \cos A \sin A \\ &= 2 \cos^3 A - \cos A - 2 \cos A \sin^2 A \\ &= 2 \cos^3 A - \cos A - 2 \cos A (1 - \cos^2 A) \\ &= 2 \cos^3 A - \cos A - 2 \cos A + 2 \cos^3 A \\ &= 4 \cos^3 A - 3 \cos A \end{aligned}$$

$$Q8. (i) \quad \text{Prove : } (\sin A + \cos A)^2 = 1 + \sin 2A$$

$$\begin{aligned} \text{Proof : } (\sin A + \cos A)^2 &= \sin^2 A + 2 \sin A \cos A + \cos^2 A \\ &= \cos^2 A + \sin^2 A + 2 \sin A \cos A \\ &= 1 + \sin 2A \end{aligned}$$

$$(ii) \quad \text{Prove : } \frac{\cos 2A}{\cos A + \sin A} = \cos A - \sin A$$

$$\begin{aligned} \text{Proof : } \frac{\cos 2A}{\cos A + \sin A} &= \frac{\cos^2 A - \sin^2 A}{\cos A + \sin A} \\ &= \frac{(\cos A - \sin A)(\cos A + \sin A)}{\cos A + \sin A} \\ &= \cos A - \sin A \end{aligned}$$

Q9. Show that $1 - (\cos x - \sin x)^2 = \sin 2x$

$$\begin{aligned}\text{Proof: } 1 - (\cos x - \sin x)^2 &= 1 - [\cos^2 x - 2 \sin x \cos x + \sin^2 x] \\ &= 1 - [\cos^2 x + \sin^2 x - \sin 2x] \\ &= 1 - [1 - \sin 2x] \\ &= 1 - 1 + \sin 2x = \sin 2x\end{aligned}$$

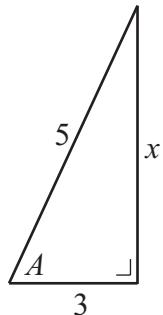
Q10. $\tan A = \frac{1}{2}$

$$\begin{aligned}\Rightarrow \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A} = \frac{2\left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2}\right)^2} \\ &= \frac{1}{1 - \frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}\end{aligned}$$

Q11. $\cos A = \frac{3}{5} \Rightarrow$ Pythagoras: $x^2 + (3)^2 = (5)^2$

$$\begin{aligned}\Rightarrow x^2 + 9 &= 25 \\ \Rightarrow x^2 &= 25 - 9 = 16 \\ \Rightarrow x &= \sqrt{16} = 4\end{aligned}$$

Hence, $\sin A = \frac{4}{5}$



(i) $\sin 2A = 2 \sin A \cos A = 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{24}{25}$

(ii) $\cos 2A = \cos^2 A - \sin^2 A = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$

$$\begin{aligned}&= \frac{9}{25} - \frac{16}{25} \\ &= -\frac{7}{25}\end{aligned}$$

Q12. Prove that $\frac{1-\cos 2A}{\sin 2A} = \tan A$

Proof :
$$\begin{aligned}\frac{1-\cos 2A}{\sin 2A} &= \frac{1 - \frac{1-\tan^2 A}{1+\tan^2 A}}{\frac{2\tan A}{1+\tan^2 A}} \\ &= \frac{\frac{1+\tan^2 A - (1-\tan^2 A)}{1+\tan^2 A}}{\frac{2\tan A}{1+\tan^2 A}} \\ &= \frac{\frac{1+\tan^2 A - 1 + \tan^2 A}{1+\tan^2 A}}{2\tan A} \\ &= \frac{2\tan^2 A}{2\tan A} = \tan A\end{aligned}$$

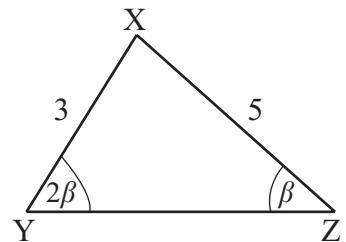
Q13. Show that $\frac{2\tan A}{1+\tan^2 A} = \sin 2A$

Proof :
$$\begin{aligned}\frac{2\tan A}{1+\tan^2 A} &= \frac{\frac{2\sin A}{\cos A}}{\frac{\sec^2 A}{\cos^2 A}} = \frac{\frac{2\sin A}{\cos A}}{\frac{1}{\cos^2 A}} \\ &= 2 \frac{\sin A}{\cos A} \cdot \frac{\cos^2 A}{1} \\ &= 2\sin A \cdot \cos A \\ &= \sin 2A\end{aligned}$$

Q14. $\tan 2\theta = \frac{4}{3}$

$$\begin{aligned}\Rightarrow \quad \frac{2\tan\theta}{1-\tan^2\theta} &= \frac{4}{3} \\ \Rightarrow \quad 6\tan\theta &= 4 - 4\tan^2\theta \\ \Rightarrow \quad 4\tan^2\theta + 6\tan\theta - 4 &= 0 \\ \Rightarrow \quad 2\tan^2\theta + 3\tan\theta - 2 &= 0 \\ \Rightarrow \quad (2\tan\theta - 1)(\tan\theta + 2) &= 0 \\ \Rightarrow \quad \tan\theta &= \frac{1}{2} \text{ or } -2\end{aligned}$$

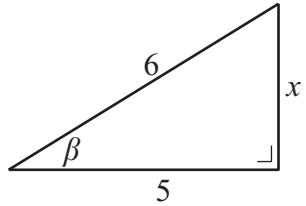
Q15. (i) sine rule : $\frac{5}{\sin 2\beta} = \frac{3}{\sin \beta}$
 $\Rightarrow 3\sin 2\beta = 5\sin \beta$
 $\Rightarrow \sin 2\beta = \frac{5\sin \beta}{3} = \frac{5}{3}\sin \beta$



$$\begin{aligned}
 \text{(ii)} \quad \sin 2\beta &= 2 \sin \beta \cos \beta = \frac{5 \sin \beta}{3} \\
 \Rightarrow 6 \sin \beta \cos \beta &= 5 \sin \beta \\
 \Rightarrow \cos \beta &= \frac{5}{6}
 \end{aligned}$$

$$\begin{aligned}
 \text{Pythagoras: } x^2 + (5)^2 &= (6)^2 \\
 \Rightarrow x^2 + 25 &= 36 \\
 \Rightarrow x^2 &= 36 - 25 = 11 \\
 \Rightarrow x &= \sqrt{11}
 \end{aligned}$$

$$\text{Hence, } \tan \beta = \frac{\sqrt{11}}{5}$$



$$\text{Q16. (i)} \quad \tan(A+B) = -1 \quad \text{and} \quad \tan A = \frac{4}{3}$$

$$\begin{aligned}
 \Rightarrow \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} &= -1 \\
 \Rightarrow \frac{\frac{4}{3} + \tan B}{1 - \frac{4}{3} \cdot \tan B} &= \frac{-1}{1} \\
 \Rightarrow \frac{4}{3} + \tan B &= -1 + \frac{4}{3} \tan B \\
 \Rightarrow 4 + 3 \tan B &= -3 + 4 \tan B \\
 \Rightarrow -\tan B &= -7 \\
 \Rightarrow \tan B &= 7
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \sin 2B &= \frac{2 \tan B}{1 + \tan^2 B} \\
 &= \frac{2(7)}{1 + (7)^2} = \frac{14}{50} = \frac{7}{25}
 \end{aligned}$$

$$\text{Q17. (i)} \quad \text{Show that} \quad \frac{\sin 2A}{1 + \cos 2A} = \tan A$$

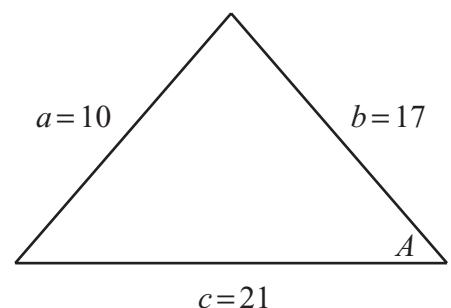
$$\begin{aligned}
 \text{Proof:} \quad \frac{\sin 2A}{1 + \cos 2A} &= \frac{\frac{2 \tan A}{1 + \tan^2 A}}{1 + \frac{1 - \tan^2 A}{1 + \tan^2 A}} \\
 &= \frac{\frac{2 \tan A}{1 + \tan^2 A}}{\frac{1 + \tan^2 A + 1 - \tan^2 A}{1 + \tan^2 A}} \\
 &= \frac{2 \tan A}{2} = \tan A
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \text{Let } A = 22\frac{1}{2}^\circ &\Rightarrow \frac{\sin 2\left(22\frac{1}{2}^\circ\right)}{1 + \cos 2\left(22\frac{1}{2}^\circ\right)^2} = \tan 22\frac{1}{2}^\circ \\
 &\Rightarrow \frac{\sin 45^\circ}{1 + \cos 45^\circ} = \frac{\frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}} = \tan 22\frac{1}{2}^\circ \\
 &= \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{2}+1}{\sqrt{2}}} = \tan 22\frac{1}{2}^\circ \\
 &= \frac{1}{\sqrt{2}+1} = \tan 22\frac{1}{2}^\circ \\
 &\Rightarrow \tan 22\frac{1}{2}^\circ = \frac{1}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \frac{\sqrt{2}-1}{1} = \sqrt{2}-1
 \end{aligned}$$

$$\begin{aligned}
 \text{Q18. (i)} \quad \cos 2A &= 1 - 2 \sin^2 A \\
 \Rightarrow \cos 4A &= \cos 2(2A) = 1 - 2 \sin^2 2A \\
 \text{(ii)} \quad \cos 2A &= 2 \cos^2 A - 1 \\
 \Rightarrow \cos 4A &= \cos 2(2A) = 2 \cos^2 2A - 1
 \end{aligned}$$

$$\begin{aligned}
 \text{Show that } \frac{1-\cos 4A}{1+\cos 4A} &= \tan^2 2A \\
 \text{Proof: } \frac{1-\cos 4A}{1+\cos 4A} &= \frac{1-(1-2 \sin^2 2A)}{1+(2 \cos^2 2A-1)} \\
 &= \frac{1-1+2 \sin^2 2A}{1+2 \cos^2 2A-1} \\
 &= \frac{2 \sin^2 2A}{2 \cos^2 2A} = \tan^2 2A
 \end{aligned}$$

$$\begin{aligned}
 \text{Q19. (i)} \quad \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \\
 &= \frac{(17)^2 + (21)^2 - (10)^2}{2(17)(21)} \\
 &= \frac{289 + 441 - 100}{714} = \frac{630}{714} = \frac{15}{17}
 \end{aligned}$$



$$\begin{aligned}
 \text{(ii)} \quad & \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A} \\
 \Rightarrow & \cos A = \frac{1 - \tan^2 \frac{A}{2}}{1 + \tan^2 \frac{A}{2}} \\
 \Rightarrow & \frac{15}{17} = \frac{1 - \tan^2 \frac{A}{2}}{1 + \tan^2 \frac{A}{2}} \\
 \Rightarrow & 15 + 15 \tan^2 \frac{A}{2} = 17 - 17 \tan^2 \frac{A}{2} \\
 \Rightarrow & 32 \tan^2 \frac{A}{2} = 2 \\
 \Rightarrow & \tan^2 \frac{A}{2} = \frac{1}{16} \\
 \Rightarrow & \tan \frac{A}{2} = \sqrt{\frac{1}{16}} = \frac{1}{4} \quad \text{if } A \text{ is acute}
 \end{aligned}$$

Q20. (i) In ΔSRP

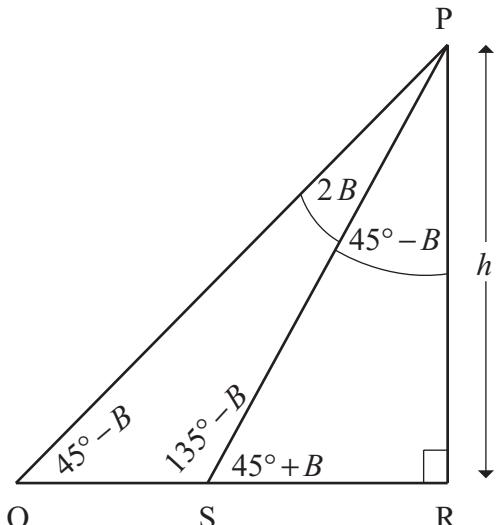
$$\begin{aligned}
 \frac{\tan(45^\circ - B)}{1} &= \frac{|SR|}{h} \\
 \Rightarrow |SR| &= h \tan(45^\circ - B)
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \tan(45^\circ - B) = \frac{\tan 45^\circ - \tan B}{1 + \tan 45^\circ \tan B} \\
 & = \frac{1 - \tan B}{1 + 1 \cdot \tan B} = \frac{1 - \tan B}{1 + \tan B} \\
 \Rightarrow |SR| &= h \left(\frac{1 - \tan B}{1 + \tan B} \right)
 \end{aligned}$$

$$\begin{aligned}
 \text{In } \Delta PQR : \tan(45^\circ - B) &= \frac{h}{|QR|} \\
 \Rightarrow |QR| \tan(45^\circ - B) &= h \\
 \Rightarrow |QR| &= \frac{h}{\tan(45^\circ - B)} \\
 \Rightarrow |QR| &= \frac{h}{\frac{1 - \tan B}{1 + \tan B}} = \frac{h(1 + \tan B)}{1 - \tan B}
 \end{aligned}$$

$$\text{Hence, } |QS| = |QR| - |SR|$$

$$\begin{aligned}
 &= \frac{h(1 + \tan B)}{1 - \tan B} - \frac{h(1 - \tan B)}{1 + \tan B} \\
 &= \frac{h(1 + \tan B)(1 + \tan B) - h(1 - \tan B)(1 - \tan B)}{(1 - \tan B)(1 + \tan B)} \\
 &= \frac{h[1 + 2 \tan B + \tan^2 B - (1 - 2 \tan B + \tan^2 B)]}{1 - \tan^2 B} \\
 &= \frac{h[1 + 2 \tan B + \tan^2 B - 1 + 2 \tan B - \tan^2 B]}{1 - \tan^2 B} \\
 &= \frac{h \cdot 2(2 \tan B)}{1 - \tan^2 B} = 2h \tan 2B
 \end{aligned}$$



Exercise 5.4

$$\text{Q1. (i)} \quad \sin 5x + \sin 3x = 2 \sin \frac{5x+3x}{2} \cos \frac{5x-3x}{2} \\ = 2 \sin 4x \cos x$$

$$\text{(ii)} \quad \sin 4x - \sin 2x = 2 \cos \frac{4x+2x}{2} \sin \frac{4x-2x}{2} \\ = 2 \cos 3x \sin x$$

$$\text{(iii)} \quad \cos 3x + \cos x = 2 \cos \frac{3x+x}{2} \cos \frac{3x-x}{2} \\ = 2 \cos 2x \cos x$$

$$\text{(iv)} \quad \cos 7\theta - \cos 5\theta = -2 \sin \frac{7\theta+5\theta}{2} \sin \frac{7\theta-5\theta}{2} \\ = -2 \sin 6\theta \sin \theta$$

$$\text{(v)} \quad \cos 3\theta - \cos \theta = -2 \sin \frac{3\theta+\theta}{2} \sin \frac{3\theta-\theta}{2} \\ = -2 \sin 2\theta \sin \theta$$

$$\text{(vi)} \quad \sin 3\theta - \sin 7\theta = 2 \cos \frac{3\theta+7\theta}{2} \sin \frac{3\theta-7\theta}{2} \\ = 2 \cos 5\theta \sin(-2\theta) \\ = -2 \cos 5\theta \sin 2\theta$$

$$\text{Q2. (i)} \quad \cos 80^\circ + \cos 40^\circ = 2 \cos \frac{80^\circ+40^\circ}{2} \cos \frac{80^\circ-40^\circ}{2} \\ = 2 \cos 60^\circ \cos 20^\circ \\ = 2 \cdot \frac{1}{2} \cos 20^\circ = \cos 20^\circ$$

$$\text{(ii)} \quad \sin 125^\circ - \sin 55^\circ = 2 \cos \frac{125^\circ+55^\circ}{2} \sin \frac{125^\circ-55^\circ}{2} \\ = 2 \cos 90^\circ \sin 35^\circ \\ = 2(0) \sin 35^\circ = 0$$

$$\text{(iii)} \quad \cos 75^\circ - \cos 15^\circ = -2 \sin \frac{75^\circ+15^\circ}{2} \sin \frac{75^\circ-15^\circ}{2} \\ = -2 \sin 45^\circ \sin 30^\circ \\ = -2 \left(\frac{1}{\sqrt{2}} \right) \left(\frac{1}{2} \right) = -\frac{1}{\sqrt{2}}$$

$$\text{Q3. (i)} \quad \sin 75^\circ - \sin 15^\circ = 2 \cos \frac{75^\circ+15^\circ}{2} \sin \frac{75^\circ-15^\circ}{2} \\ = 2 \cos 45^\circ \sin 30^\circ \\ = 2 \left(\frac{1}{\sqrt{2}} \right) \left(\frac{1}{2} \right) = \frac{1}{\sqrt{2}}$$

$$\begin{aligned}
 \text{(ii)} \quad \sin 10^\circ + \sin 80^\circ &= 2 \sin \frac{10^\circ + 80^\circ}{2} \cos \frac{10^\circ - 80^\circ}{2} \\
 &= 2 \sin 45^\circ \cos (-35^\circ) \\
 &= 2 \cdot \frac{1}{\sqrt{2}} \cos 35^\circ \\
 &= \sqrt{2} \cos 35^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{Q4. (i)} \quad \cos(x+45^\circ) + \cos(x-45^\circ) &= 2 \cos \frac{x+45^\circ + x-45^\circ}{2} \cos \frac{x+45^\circ - (x-45^\circ)}{2} \\
 &= 2 \cos \frac{2x}{2} \cos \frac{x+45^\circ - x+45^\circ}{2} \\
 &= 2 \cos x \cos 45^\circ \\
 &= 2 \cos x \cdot \frac{1}{\sqrt{2}} = \sqrt{2} \cos x
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \cos(x+60^\circ) - \cos(x-60^\circ) &= -2 \sin \frac{x+60^\circ + x-60^\circ}{2} \cdot \sin \frac{x+60^\circ - (x-60^\circ)}{2} \\
 &= -2 \sin \frac{2x}{2} \cdot \sin \frac{x+60^\circ - x+60^\circ}{2} \\
 &= -2 \sin x \cdot \sin 60^\circ \\
 &= -2 \sin x \cdot \frac{\sqrt{3}}{2} = -\sqrt{3} \sin x
 \end{aligned}$$

$$\begin{aligned}
 \text{Q5. (i)} \quad 2 \sin 3A \cos 2A &= \sin(3A+2A) + \sin(3A-2A) \\
 &= \sin 5A + \sin A \\
 \text{(ii)} \quad 2 \cos 4x \sin x &= \sin(4x+x) - \sin(4x-x) \\
 &= \sin 5x - \sin 3x \\
 \text{(iii)} \quad 2 \cos 5A \cos 2A &= \cos(5A+2A) + \cos(5A-2A) \\
 &= \cos 7A + \cos 3A \\
 \text{(iv)} \quad -2 \sin 6A \sin 2A &= -[\cos(6A-2A) - \cos(6A+2A)] \\
 &= -[\cos 4A - \cos 8A] \\
 &= \cos 8A - \cos 4A
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad \sin 2A \sin A &= \frac{1}{2} [\cos(2A-A) - \cos(2A+A)] \\
 &= \frac{1}{2} [\cos A - \cos 3A] \\
 &= -\frac{1}{2} [\cos 3A - \cos A]
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad \sin x \cos 5x &= \frac{1}{2} [\sin(x+5x) + \sin(x-5x)] \\
 &= \frac{1}{2} [\sin 6x + \sin(-4x)] \\
 &= \frac{1}{2} [\sin 6x - \sin 4x]
 \end{aligned}$$

$$\begin{aligned}
 \text{Q6. (i)} \quad 2 \sin 75^\circ \cos 45^\circ &= \sin(75^\circ + 45^\circ) + \sin(75^\circ - 45^\circ) \\
 &= \sin 120^\circ + \sin 30^\circ \\
 &= \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1}{2}(\sqrt{3} + 1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad 10 \sin 67\frac{1}{2}^\circ \sin 22\frac{1}{2}^\circ &= 5 \left[2 \sin 67\frac{1}{2}^\circ \sin 22\frac{1}{2}^\circ \right] \\
 &= 5 \left[\cos \left(67\frac{1}{2}^\circ - 22\frac{1}{2}^\circ \right) - \cos \left(67\frac{1}{2}^\circ + 22\frac{1}{2}^\circ \right) \right] \\
 &= 5 [\cos 45^\circ - \cos 90^\circ] \\
 &= 5 \left[\frac{1}{\sqrt{2}} - 0 \right] \\
 &= 5 \cdot \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}
 \end{aligned}$$

$$\text{Q7. (i)} \quad \text{Show : } 2 \cos(A+45^\circ) \sin(A-45^\circ) = \sin 2A - 1$$

$$\begin{aligned}
 \text{Proof : } 2 \cos(A+45^\circ) \sin(A-45^\circ) &= \sin[A+45^\circ + (A-45^\circ)] - \sin[A+45^\circ - (A-45^\circ)] \\
 &= \sin[A+45^\circ + A-45^\circ] - \sin[A+45^\circ - A+45^\circ] \\
 &= \sin 2A - \sin 90^\circ \\
 &= \sin 2A - 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \text{Show that } \frac{\cos 50^\circ - \cos 70^\circ}{\sin 70^\circ - \sin 50^\circ} &= \sqrt{3} \\
 \text{Proof : } \frac{\cos 50^\circ - \cos 70^\circ}{\sin 70^\circ - \sin 50^\circ} &= \frac{-2 \sin \frac{50^\circ + 70^\circ}{2} \sin \frac{50^\circ - 70^\circ}{2}}{2 \cos \frac{70^\circ + 50^\circ}{2} \sin \frac{70^\circ - 50^\circ}{2}} \\
 &= \frac{-2 \sin 60^\circ \cdot \sin(-10^\circ)}{2 \cos 60^\circ \cdot \sin 10^\circ} \\
 &= \frac{2 \sin 60^\circ \sin 10^\circ}{2 \cos 60^\circ \sin 10^\circ} \\
 &= \tan 60^\circ = \sqrt{3}
 \end{aligned}$$

Q8. Show that $\frac{\sin(\theta+15^\circ) + \sin(\theta-15^\circ)}{\cos(\theta+15^\circ) + \cos(\theta-15^\circ)} = \tan \theta$

Proof:
$$\frac{\sin(\theta+15^\circ) + \sin(\theta-15^\circ)}{\cos(\theta+15^\circ) + \cos(\theta-15^\circ)}$$

$$\begin{aligned} &= \frac{2\sin\frac{\theta+15^\circ+\theta-15^\circ}{2}\cos\frac{\theta+15^\circ-(\theta-15^\circ)}{2}}{2\cos\frac{\theta+15^\circ+\theta-15^\circ}{2}\cos\frac{\theta+15^\circ-(\theta-15^\circ)}{2}} \\ &= \frac{2\sin\frac{2\theta}{2}\cos\frac{\theta+15^\circ-\theta+15^\circ}{2}}{2\cos\frac{2\theta}{2}\cos\frac{\theta+15^\circ-\theta+15^\circ}{2}} \\ &= \frac{\sin\theta\cos15^\circ}{\cos\theta\cos15^\circ} = \tan\theta \end{aligned}$$

Q9. Show that $\frac{\sin 4A + \sin 2A}{2\sin 3A} = \cos A$

Proof:
$$\begin{aligned} \frac{\sin 4A + \sin 2A}{2\sin 3A} &= \frac{2\sin\frac{4A+2A}{2}\cos\frac{4A-2A}{2}}{2\sin 3A} \\ &= \frac{2\sin 3A \cos A}{2\sin 3A} = \cos A \end{aligned}$$

Q10. Show $\frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A} = \tan A$

Proof:
$$\begin{aligned} \frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A} &= \frac{2\cos\frac{5A+3A}{2}\sin\frac{5A-3A}{2}}{2\cos\frac{5A+3A}{2}\cos\frac{5A-3A}{2}} \\ &= \frac{\cos 4A \cdot \sin A}{\cos 4A \cdot \cos A} = \tan A \end{aligned}$$

Q11. Show that $2\sin(135^\circ + A)\sin(45^\circ + A) = \cos 2A$

Proof:
$$\begin{aligned} 2\sin(135^\circ + A)\sin(45^\circ + A) &= \cos[135 + A - (45 + A)] - \cos[135 + A + (45 + A)] \\ &= \cos[135 + A - 45 - A] - \cos[135 + A + 45 + A] \\ &= \cos 90^\circ - \cos(180 + 2A) \\ &= 0 - [\cos 180 \cos 2A - \sin 180 \sin 2A] \\ &= -[(-1) \cdot \cos 2A - 0 \cdot \sin 2A] = \cos 2A \end{aligned}$$

Q12. Given that $\tan 3\theta = 2$,

$$\begin{aligned} \text{evaluate } \frac{\sin \theta + \sin 3\theta + \sin 5\theta}{\cos \theta + \cos 3\theta + \cos 5\theta} &= \frac{(\sin 5\theta + \sin \theta) + \sin 3\theta}{(\cos 5\theta + \cos \theta) + \cos 3\theta} \\ &= \frac{2 \sin \frac{5\theta + \theta}{2} \cos \frac{5\theta - \theta}{2} + \sin 3\theta}{2 \cos \frac{5\theta + \theta}{2} \cos \frac{5\theta - \theta}{2} + \cos 3\theta} \\ &= \frac{2 \sin 3\theta \cos 2\theta + \sin 3\theta}{2 \cos 3\theta \cos 2\theta + \cos 3\theta} \\ &= \frac{\sin 3\theta (2 \cos 2\theta + 1)}{\cos 3\theta (2 \cos 2\theta + 1)} \\ &= \tan 3\theta = 2 \end{aligned}$$

Exercise 5.5

Q1. (i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$

(ii) $\cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$

(iii) $\tan^{-1}(1) = 45^\circ$

(iv) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$

(v) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -60^\circ$

(vi) $\tan^{-1}(-1) = -45^\circ$

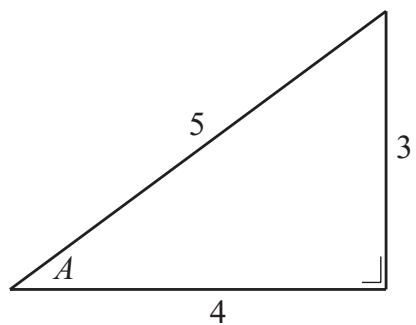
(vii) $\cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ$

(viii) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -30^\circ$

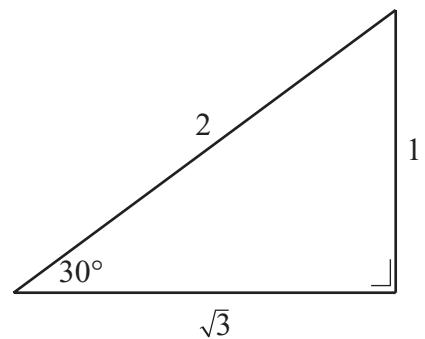
Q2. (i) $\sin^{-1}\left(\frac{3}{5}\right) = A$

and $\tan^{-1}\left(\frac{3}{4}\right) = A$

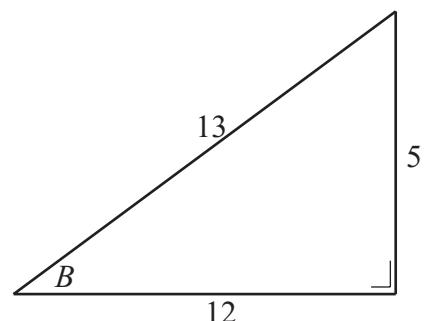
$\Rightarrow \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$



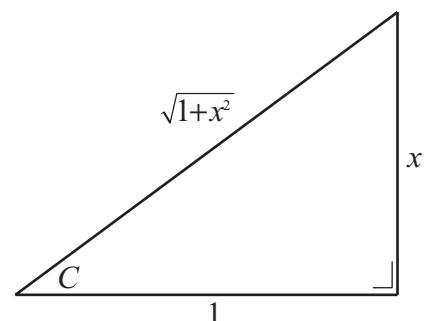
$$\begin{aligned}
 \text{(ii)} \quad & \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ \\
 \text{and} \quad & \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ \\
 \Rightarrow \quad & \sin^{-1}\left(\frac{1}{2}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)
 \end{aligned}$$



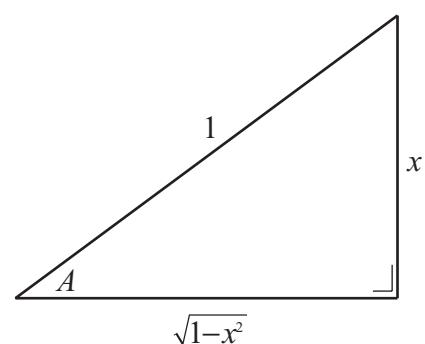
$$\begin{aligned}
 \text{(iii)} \quad & \sin^{-1}\left(\frac{5}{13}\right) = B \\
 \text{and} \quad & \tan^{-1}\left(\frac{5}{12}\right) = B \\
 \Rightarrow \quad & \sin^{-1}\left(\frac{5}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)
 \end{aligned}$$



$$\begin{aligned}
 \text{(iv)} \quad & \tan^{-1}(x) = C \\
 & \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = C \\
 \Rightarrow \quad & \tan^{-1}(x) = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)
 \end{aligned}$$



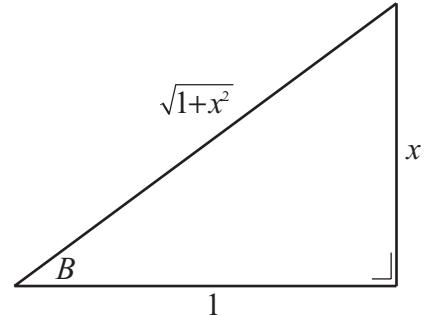
$$\begin{aligned}
 \text{Q3. (i)} \quad & \sin^{-1}\left(\frac{x}{1}\right) = A \\
 \Rightarrow \quad & \sin(\sin^{-1} x) = \sin A = \frac{x}{1} = x
 \end{aligned}$$



$$\begin{aligned}
 & x^2 + (\sqrt{1-x^2})^2 = 1^2 \\
 \Rightarrow \quad & x^2 + 1 - x^2 = 1 \\
 \Rightarrow \quad & 1 = 1
 \end{aligned}$$

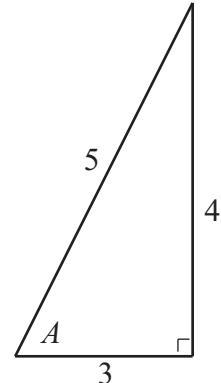
$$\begin{aligned}
 \text{(ii)} \quad & \sin^{-1}\left(\frac{x}{1}\right) = A \\
 \Rightarrow & \cos(\sin^{-1} x) \\
 = & \cos A = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & \tan^{-1}\left(\frac{x}{1}\right) = B \\
 \Rightarrow & \sin\left[\tan^{-1}\left(\frac{x}{1}\right)\right] \\
 = & \sin B = \frac{x}{\sqrt{1+x^2}}
 \end{aligned}$$

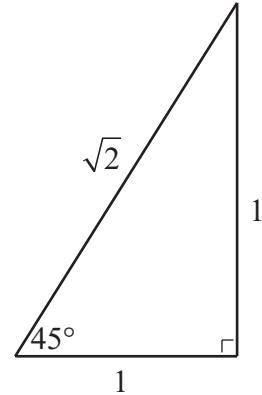


$$\begin{aligned}
 x^2 + 1^2 &= (\sqrt{1+x^2})^2 \\
 \Rightarrow x^2 + 1 &= 1 + x^2
 \end{aligned}$$

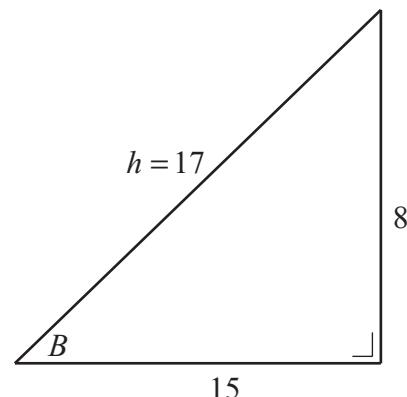
$$\begin{aligned}
 \text{Q4. (i)} \quad & \cos^{-1}\left(\frac{3}{5}\right) = A \\
 \Rightarrow & \sin\left[\cos^{-1}\left(\frac{3}{5}\right)\right] \\
 = & \sin A = \frac{4}{5}
 \end{aligned}$$



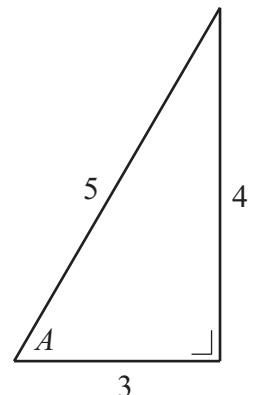
$$\begin{aligned}
 \text{(ii)} \quad & \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ \\
 \Rightarrow & \cos(\tan^{-1} 1) \\
 = & \cos 45^\circ = \frac{1}{\sqrt{2}}
 \end{aligned}$$



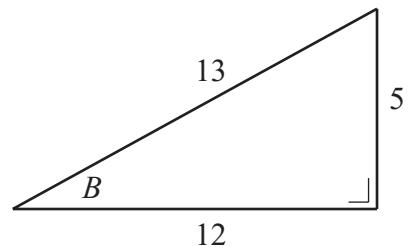
$$\begin{aligned}
 \text{(iii)} \quad & \tan^{-1}\left(\frac{8}{15}\right) = B \\
 & \sin\left[\tan^{-1}\left(\frac{8}{15}\right)\right] \\
 & = \sin B = \frac{8}{17}
 \end{aligned}$$



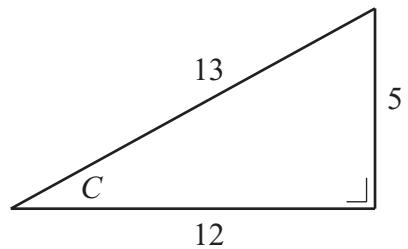
$$\begin{aligned}
 \text{Q5. (i)} \quad & \cos^{-1}\left(\frac{3}{5}\right) = A \\
 \Rightarrow \quad & \sin\left[2\cos^{-1}\left(\frac{3}{5}\right)\right] \\
 & = \sin 2A \\
 & = 2\sin A \cos A = 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{24}{25}
 \end{aligned}$$



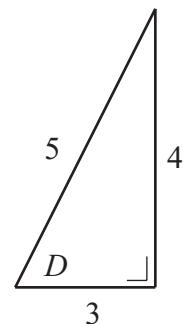
$$\begin{aligned}
 \text{(ii)} \quad & \sin^{-1}\left(\frac{5}{13}\right) = B \\
 & \cos\left[2\sin^{-1}\left(\frac{5}{13}\right)\right] = \cos 2B \\
 & = \cos^2 B - \sin^2 B \\
 & = \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \frac{119}{169}
 \end{aligned}$$



Q6. (i) $\sin^{-1}\left(\frac{5}{13}\right) = C \Rightarrow \sin C = \frac{5}{13}$
 and $\cos C = \frac{12}{13}$

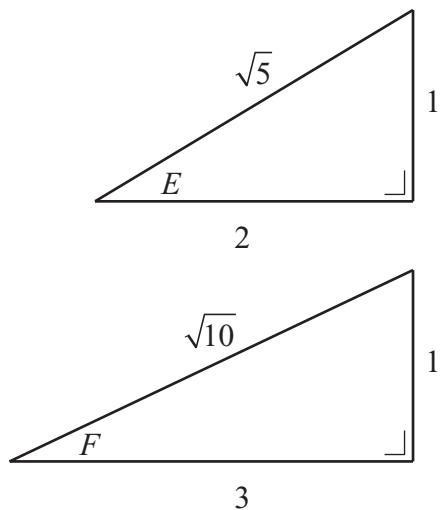


$$\begin{aligned} \sin^{-1}\left(\frac{4}{5}\right) &= D \Rightarrow \sin D = \frac{4}{5} \\ \text{and } \cos D &= \frac{3}{5} \\ \sin\left[\sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right] \\ &= \sin(C + D) \\ &= \sin C \cos D + \cos C \sin D \\ &= \frac{5}{13} \cdot \frac{3}{5} + \frac{12}{13} \cdot \frac{4}{5} = \frac{15}{65} + \frac{48}{65} = \frac{63}{65} \end{aligned}$$

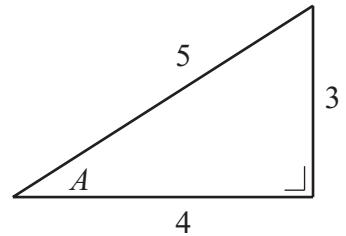


(ii) $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = E \Rightarrow \sin E = \frac{1}{\sqrt{5}}$
 and $\cos E = \frac{2}{\sqrt{5}}$

$$\begin{aligned} \sin^{-1}\left(\frac{1}{\sqrt{10}}\right) &= F \Rightarrow \sin F = \frac{1}{\sqrt{10}} \\ \text{and } \cos F &= \frac{3}{\sqrt{10}} \\ \sin\left[\sin^{-1}\frac{1}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{10}}\right] \\ &= \sin(E + F) = \sin E \cos F + \cos E \sin F \\ &= \frac{1}{\sqrt{5}} \cdot \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{10}} = \frac{3}{\sqrt{50}} + \frac{2}{\sqrt{50}} = \frac{5}{\sqrt{50}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \end{aligned}$$

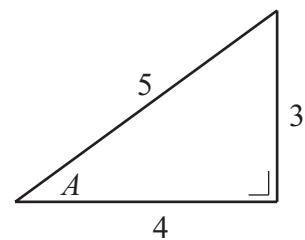


Q7. $\sin^{-1}\left(\frac{3}{5}\right) = A \Rightarrow \sin A = \frac{3}{5}$
 and $\tan A = \frac{3}{4}$

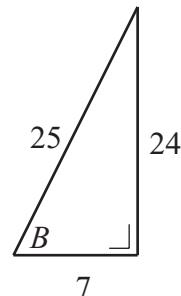


$$\begin{aligned} \sin^{-1}\left(\frac{5}{13}\right) &= B \Rightarrow \sin B = \frac{5}{13} \\ \text{and } \tan B &= \frac{5}{12} \\ \tan\left[\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right)\right] &= \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ &= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{\frac{7}{6}}{1 - \frac{5}{16}} = \frac{\frac{7}{6}}{\frac{11}{16}} = \frac{56}{33} \end{aligned}$$

Q8. $\tan^{-1}\left(\frac{3}{4}\right) = A \Rightarrow \sin A = \frac{3}{5}$
 and $\cos A = \frac{4}{5}$



$$\begin{aligned} \cos^{-1}\left(\frac{7}{25}\right) &= B \Rightarrow \sin B = \frac{24}{25} \\ \text{and } \cos B &= \frac{7}{25} \\ \sin\left[2\tan^{-1}\left(\frac{3}{4}\right)\right] &= \sin 2A = 2\sin A \cos A \\ &= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25} \end{aligned}$$



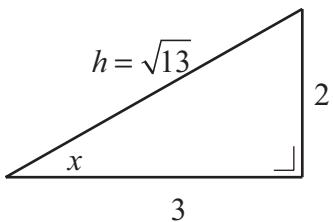
Hence, $\sin\left[2\tan^{-1}\left(\frac{3}{4}\right)\right] = \sin\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$

Test Yourself 5

A Questions

Q1. $\sin 2x = 2 \sin x \cos x$

$$= 2 \cdot \frac{2}{\sqrt{13}} \cdot \frac{3}{\sqrt{13}} = \frac{12}{13}$$



$$h^2 = 2^2 + 3^2 = 13$$

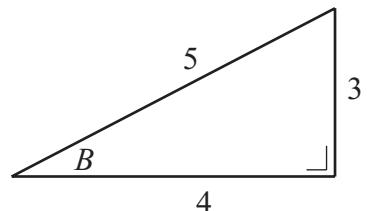
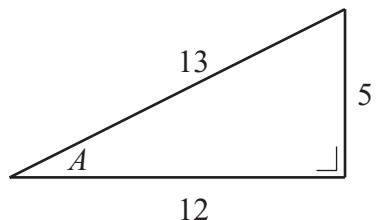
$$\Rightarrow h = \sqrt{13}$$

Q2. $\tan A = \frac{5}{12} \Rightarrow \sin A = \frac{5}{13}$

and $\cos A = \frac{12}{13}$

$\tan B = \frac{3}{4} \Rightarrow \sin B = \frac{3}{5}$

and $\cos B = \frac{4}{5}$



$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{12}{13} \cdot \frac{4}{5} + \frac{5}{13} \cdot \frac{3}{5} = \frac{48+15}{65} = \frac{63}{65}$$

Q3. Show that $(\cos A + \sin A)^2 = 1 + \sin 2A$

Proof: $(\cos A + \sin A)^2 = \cos^2 A + 2 \sin A \cos A + \sin^2 A$
 $= 1 + \sin 2A$

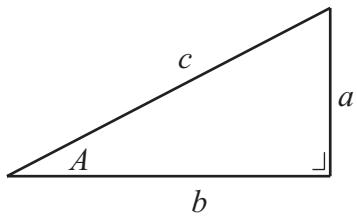
Q4. $\cos x = \frac{4}{5} \Rightarrow \tan x = \frac{3}{4}$

Hence, $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} = \frac{2 \left(\frac{3}{4}\right)}{1 - \left(\frac{3}{4}\right)^2}$

$$= \frac{\frac{6}{4}}{1 - \frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{24}{7}$$

Q5. Prove : $\sin^2 A + \cos^2 A = 1$

$$\begin{aligned}\text{Proof: } \sin^2 A + \cos^2 A &= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 \\ &= \frac{a^2}{c^2} + \frac{b^2}{c^2} \\ &= \frac{a^2 + b^2}{c^2} \quad \text{as } a^2 + b^2 = c^2 \text{ (Pythagoras' theorem)} \\ &= \frac{c^2}{c^2} = 1\end{aligned}$$

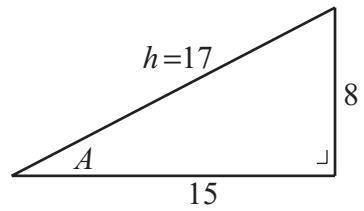


Q6. $\tan A = \frac{8}{15} \Rightarrow \text{Pythagoras: } h^2 = 8^2 + 15^2$

$$= 64 + 225$$

$$= 289$$

$$\Rightarrow h = \sqrt{289} = 17$$



(i) $\cos A = \frac{15}{17}$

(ii) $\sin 2A = 2 \sin A \cos A = 2 \left(\frac{8}{17}\right) \left(\frac{15}{17}\right) = \frac{240}{289}$

Q7. (i) $\sin 75^\circ \cos 15^\circ - \cos 75^\circ \sin 15^\circ$

$$= \sin(75^\circ - 15^\circ)$$

$$= \sin 60^\circ = \frac{\sqrt{3}}{2}$$

(ii) $\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1$

$$\text{Hence, } 2 + 2 \cos 2x = 2 + 2(2 \cos^2 x - 1)$$

$$= 2 + 4 \cos^2 x - 2 = 4 \cos^2 x$$

$$\text{Q8. } \tan 75^\circ = \tan(45^\circ + 30^\circ)$$

$$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{3 + \sqrt{3} + \sqrt{3} + 1}{3 + \sqrt{3} - \sqrt{3} - 1}$$

$$= \frac{4 + 2\sqrt{3}}{2}$$

$$= 2 + \sqrt{3} = a + b\sqrt{3}$$

Hence, $a = 2, b = 1$.

$$\text{Q9. (i) Show that } \tan \theta \sin \theta + \cos \theta = \sec \theta$$

$$\begin{aligned}\text{Proof: } \tan \theta \sin \theta + \cos \theta &= \frac{\sin \theta}{\cos \theta} \cdot \sin \theta + \frac{\cos \theta}{1} \\ &= \frac{\sin \theta \cdot \sin \theta + \cos \theta \cdot \cos \theta}{\cos \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} = \frac{1}{\cos \theta} = \sec \theta\end{aligned}$$

$$\text{(ii)} \quad \cos \theta = \frac{5}{13} \quad x^2 + (5)^2 = (13)^2$$

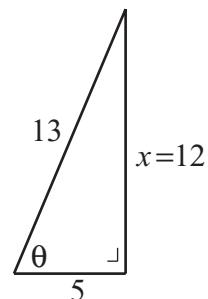
$$\Rightarrow x^2 + 25 = 169$$

$$\Rightarrow \sin \theta = \frac{12}{13} \quad \Rightarrow x^2 = 169 - 25 = 144$$

$$\Rightarrow x = \sqrt{144} = 12$$

$$\text{Hence, } \sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \left(\frac{12}{13} \right) \cdot \left(\frac{5}{13} \right) = \frac{120}{169}$$



$$\begin{aligned}
 Q10. \text{ (i)} \quad \sin 75^\circ - \sin 15^\circ &= 2 \cos \frac{75^\circ + 15^\circ}{2} \cdot \sin \frac{75^\circ - 15^\circ}{2} \\
 &= 2 \cos 45^\circ \cdot \sin 30^\circ \\
 &= 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{k}} \Rightarrow k = 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad A &= \sin^{-1} \frac{1}{2} = 30^\circ \\
 \Rightarrow \tan 2A &= \tan 2(30^\circ) = \tan 60^\circ = \sqrt{3}
 \end{aligned}$$

B Questions

$$\begin{aligned}
 Q1. \text{ (i)} \quad \cos 2A &= \cos^2 A - \sin^2 A \\
 &= \cos^2 A - (1 - \cos^2 A) \\
 &= \cos^2 A - 1 + \cos^2 A \\
 &= 2 \cos^2 A - 1 \\
 \Rightarrow \cos 2A + 1 &= 2 \cos^2 A \\
 \Rightarrow \cos^2 A &= \frac{1}{2}(1 + \cos 2A)
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \sin 40^\circ \cos 20^\circ + \cos 40^\circ \sin 20^\circ &= \sin(40^\circ + 20^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}
 \end{aligned}$$

$$\begin{aligned}
 Q2. \text{ (i)} \quad \text{Given } \sin \theta &= \frac{4}{5} \\
 \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\
 &= (1 - \sin^2 \theta) - \sin^2 \theta \\
 &= 1 - 2 \sin^2 \theta \\
 &= 1 - 2 \left(\frac{4}{5}\right)^2 = 1 - \frac{32}{25} = -\frac{7}{25}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \text{Show that } 2 \cos^2 A - \cos 2A - 1 &= 0 \\
 \text{Proof: } 2 \cos^2 A - \cos 2A - 1 &= 2 \cos^2 A - (\cos^2 A - \sin^2 A) - 1 \\
 &= 2 \cos^2 A - \cos^2 A + \sin^2 A - 1 \\
 &= (\cos^2 A + \sin^2 A) - 1 = 1 - 1 = 0
 \end{aligned}$$

Q3. (i) $2\sin 4\theta \cos 2\theta = \sin(4\theta + 2\theta) + \sin(4\theta - 2\theta)$
 $= \sin 6\theta + \sin 2\theta$

(ii) $(\cos x + \sin x)^2 + (\cos x - \sin x)^2$
 $= \cos^2 x + 2\sin x \cos x + \sin^2 x + \cos^2 x - 2\sin x \cos x + \sin^2 x$
 $= 2(\cos^2 x + \sin^2 x)$
 $= 2(1) = 2$

Q4. (i) Prove: $\cos(45^\circ + \theta) - \cos(45^\circ - \theta) = -\sqrt{2} \sin \theta$

Proof: $\cos(45^\circ + \theta) - \cos(45^\circ - \theta)$
 $= -2 \sin \frac{45^\circ + \theta + 45^\circ - \theta}{2} \cdot \sin \frac{45^\circ + \theta - (45^\circ - \theta)}{2}$
 $= -2 \sin \frac{90^\circ}{2} \cdot \sin \frac{45^\circ + \theta - 45^\circ + \theta}{2}$
 $= -2 \sin 45^\circ \cdot \sin \theta$
 $= -2 \cdot \frac{1}{\sqrt{2}} \sin \theta = -\sqrt{2} \sin \theta$

(ii) Prove that $\frac{1}{\cos \theta} - \cos \theta = \tan \theta \sin \theta$

Proof: $\frac{1}{\cos \theta} - \cos \theta = \frac{1 - \cos \theta \cdot \cos \theta}{\cos \theta}$
 $= \frac{1 - \cos^2 \theta}{\cos \theta}$
 $= \frac{\sin^2 \theta}{\cos \theta}$
 $= \frac{\sin \theta}{\cos \theta} \cdot \sin \theta = \tan \theta \cdot \sin \theta$

Q5. (i) $\cos^2 15^\circ - \sin^2 15^\circ$

$$= \cos 2(15^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

(ii) Prove that $\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} = 2$

Proof: $\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} = \frac{\sin 3\theta \cos \theta - \cos 3\theta \sin \theta}{\sin \theta \cos \theta}$
 $= \frac{\sin(3\theta - \theta)}{\sin \theta \cos \theta}$
 $= \frac{\sin 2\theta}{\sin \theta \cos \theta}$
 $= \frac{2 \sin \theta \cdot \cos \theta}{\sin \theta \cdot \cos \theta} = 2$

Q6. (i) Show that $\tan 15^\circ = 2 - \sqrt{3}$

Proof: $\tan 15^\circ = \tan(45^\circ - 30^\circ)$

$$\begin{aligned} &= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} \\ &= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} \\ &= \frac{\sqrt{3}-1}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} \\ &= \frac{3-\sqrt{3}-\sqrt{3}+1}{3-\sqrt{3}+\sqrt{3}-1} = \frac{4-2\sqrt{3}}{2} = 2 - \sqrt{3} \end{aligned}$$

(ii) Prove that $\frac{\cos 5\theta - \cos 3\theta}{\sin 4\theta} = -2 \sin \theta$

Proof: $\frac{\cos 5\theta - \cos 3\theta}{\sin 4\theta}$

$$\begin{aligned} &= \frac{-2 \sin \frac{5\theta + 3\theta}{2} \sin \frac{5\theta - 3\theta}{2}}{\sin 4\theta} \\ &= \frac{-2 \sin 4\theta \sin \theta}{\sin 4\theta} = -2 \sin \theta \end{aligned}$$

Q7. Prove: $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\begin{aligned} \text{Proof: } \tan(A+B) &= \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \\ &= \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}} \\ &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \end{aligned}$$

$$Q8. \quad A + B = \frac{\pi}{4}$$

$$\Rightarrow \tan(A + B) = \tan \frac{\pi}{4}$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\Rightarrow \tan A + \tan B = 1 - \tan A \tan B$$

$$\Rightarrow \tan A + \tan B \tan B = 1 - \tan B$$

$$\Rightarrow \tan A(1 + \tan B) = 1 - \tan B$$

$$\Rightarrow \tan A = \frac{1 - \tan B}{1 + \tan B}$$

$$\text{Hence, } (1 + \tan A)(1 + \tan B)$$

$$= \left(1 + \frac{1 - \tan B}{1 + \tan B}\right)(1 + \tan B)$$

$$= 1 + \tan B + \left(\frac{1 - \tan B}{1 + \tan B}\right)(1 + \tan B)$$

$$= 1 + \tan B + 1 - \tan B = 2$$

$$Q9. \quad \sin 105^\circ - \sin 15^\circ = 2 \cos \frac{105^\circ + 15^\circ}{2} \sin \frac{105^\circ - 15^\circ}{2}$$

$$= 2 \cos 60^\circ \sin 45^\circ$$

$$= 2 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$Q10. \quad \text{Triangle PTS} \Rightarrow \tan 2\theta = \frac{h}{x}$$

$$\text{Triangle PQR} \Rightarrow \tan \theta = \frac{h}{3x} = \frac{1}{3} \cdot \frac{h}{x} = \frac{1}{3} \tan 2\theta$$

$$\Rightarrow 3 \tan \theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\Rightarrow 3 = \frac{2}{1 - \tan^2 \theta}$$

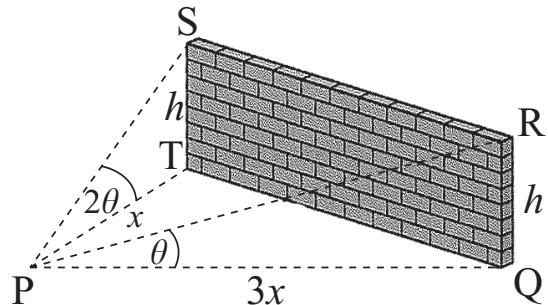
$$\Rightarrow 2 = 3 - 3 \tan^2 \theta$$

$$\Rightarrow 3 \tan^2 \theta = 1$$

$$\Rightarrow \tan^2 \theta = \frac{1}{3}$$

$$\Rightarrow \tan \theta = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} \quad (\theta \text{ is acute})$$

$$\Rightarrow \theta = \frac{\pi}{6}$$



C Questions

Q1. Prove that $\cos 2x = 1 - 2\sin^2 x$

$$\begin{aligned}\text{Proof : } \cos 2x &= \cos(x+x) \\ &= \cos x \cos x - \sin x \sin x \\ &= \cos^2 x - \sin^2 x \\ &= 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x\end{aligned}$$

Prove that $\sin 3x = 3\sin x - 4\sin^3 x$

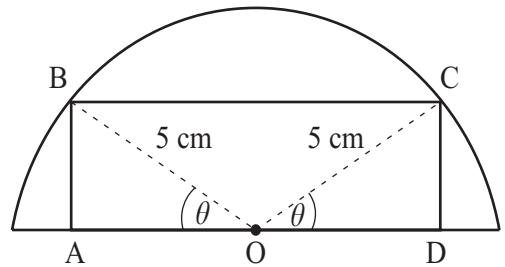
$$\begin{aligned}\text{Proof : } \sin 3x &= \sin(2x+x) \\ &= \sin 2x \cos x + \cos 2x \sin x \\ &= 2\sin x \cos x \cdot \cos x + (1 - 2\sin^2 x) \sin x \\ &= 2\sin x(\cos^2 x) + \sin x - 2\sin^3 x \\ &= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x \\ &= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x \\ &= 3\sin x - 4\sin^3 x\end{aligned}$$

Q2. Show that $(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 2 + 2\cos(A - B)$

$$\begin{aligned}\text{Proof : } (\cos A + \cos B)^2 + (\sin A + \sin B)^2 &= \cos^2 A + 2\cos A \cos B + \cos^2 B + \sin^2 A + 2\sin A \sin B + \sin^2 B \\ &= (\cos^2 A + \sin^2 A) + (\cos^2 B + \sin^2 B) + 2(\cos A \cos B + \sin A \sin B) \\ &= 1 + 1 + 2\cos(A - B) = 2 + 2\cos(A - B)\end{aligned}$$

$$\begin{aligned}\text{Q3. (i) Triangle AOB : } \sin \theta &= \frac{|AB|}{5} \\ &\Rightarrow |AB| = 5\sin \theta = |CD| \\ \text{and } \cos \theta &= \frac{|AO|}{5} \\ &\Rightarrow |AO| = 5\cos \theta = |OD| \\ &\Rightarrow |AD| = 5\cos \theta + 5\cos \theta = 10\cos \theta\end{aligned}$$

$$\begin{aligned}\text{Hence, perimeter } p &= 2(|AD| + |AB|) \\ &= 2(10\cos \theta + 5\sin \theta) \\ &= 20\cos \theta + 10\sin \theta\end{aligned}$$



$$\begin{aligned}\text{(ii) Area rectangle} &= |AB| \cdot |AD| \\ &= 5\sin \theta \cdot 10\cos \theta \\ &= 50\sin \theta \cos \theta \\ &= 25 \cdot 2\sin \theta \cos \theta \\ &\Rightarrow k \sin 2\theta = 25 \sin 2\theta \Rightarrow k = 25\end{aligned}$$

$$Q4. \quad \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\Rightarrow \cos(A-B) = \cos[A+(-B)] = \cos A \cos(-B) - \sin A \sin(-B) \\ = \cos A \cos B + \sin A \sin B$$

$$\text{Hence, } \sin(A+B) = \cos[90^\circ - (A+B)] \\ = \cos[(90^\circ - A) - B] \\ = \cos(90^\circ - A) \cos B + \sin(90^\circ - A) \sin B \\ = \sin A \cos B + \cos A \sin B$$

$$Q5. (i) \quad \text{Show that } \sqrt{2\sin^2 \theta + 6\cos^2 \theta - 2} = 2\cos \theta$$

$$\text{Proof : } \sqrt{2\sin^2 \theta + 6\cos^2 \theta - 2} = \sqrt{2(1 - \cos^2 \theta) + 6\cos^2 \theta - 2} \\ = \sqrt{2 - 2\cos^2 \theta + 6\cos^2 \theta - 2} \\ = \sqrt{4\cos^2 \theta} \\ = 2\cos \theta$$

$$(ii) \quad \text{Equation : } a\sin^2 2x + \cos 2x - b = 0$$

$$x = 0^\circ \Rightarrow a\sin^2 2(0^\circ) + \cos 2(0^\circ) - b = 0 \\ \Rightarrow a(0) + \cos 0^\circ - b = 0 \\ \Rightarrow 0 + 1 - b = 0 \Rightarrow b = 1$$

$$x = 60^\circ \Rightarrow a\sin^2 2(60^\circ) + \cos 2(60^\circ) - b = 0 \\ \Rightarrow a[\sin 120^\circ]^2 + \cos 120^\circ - b = 0 \\ \Rightarrow a\left(\frac{3}{4}\right) - \frac{1}{2} - 1 = 0 \\ \Rightarrow 3a - 2 - 4 = 0 \\ \Rightarrow 3a = 6 \\ \Rightarrow a = 2$$

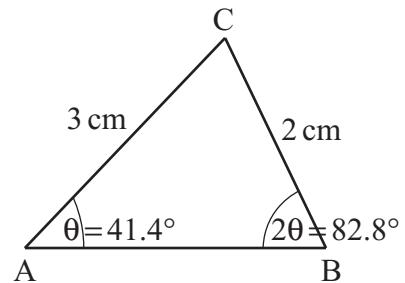
$$Q6. \quad \frac{3}{\sin 2\theta} = \frac{2}{\sin \theta} \Rightarrow 2\sin 2\theta = 3\sin \theta$$

$$\Rightarrow 2 \cdot 2\sin \theta \cos \theta = 3\sin \theta \\ \Rightarrow \cos \theta = \frac{3}{4} = 0.75 \\ \Rightarrow \theta = \cos^{-1}(0.75) = 41.409^\circ = 41.4^\circ$$

$$\theta = 41.4^\circ \Rightarrow 2\theta = 2(41.4^\circ) = 82.8^\circ$$

$$\Rightarrow \text{angle ACB} = 180^\circ - (41.4^\circ + 82.8^\circ) \\ = 180^\circ - 124.2^\circ \\ = 55.8^\circ$$

Since $55.8^\circ > 41.4^\circ \Rightarrow |AB| > 2\text{ cm}$ as bigger side is opposite greater angle.



$$\text{Q7. (i)} \quad \sin 2\theta = 1 \Rightarrow 2\theta = \sin^{-1}(1) = 90^\circ$$

$$\Rightarrow \theta = \frac{90^\circ}{2} = 45^\circ$$

$$(a) \sin \theta = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$(b) \tan \theta = \tan 45^\circ = 1$$

$$\text{(ii) Show that } \frac{\sin 4\theta(1 - \cos 2\theta)}{\cos 2\theta(1 - \cos 4\theta)} = \tan \theta$$

$$\begin{aligned} \text{Proof : } \frac{\sin 4\theta(1 - \cos 2\theta)}{\cos 2\theta(1 - \cos 4\theta)} &= \frac{2 \sin 2\theta \cos 2\theta(1 - \cos 2\theta)}{\cos 2\theta(1 - \cos 4\theta)} \\ &= \frac{2 \sin 2\theta(1 - \cos 2\theta)}{1 - (1 - 2 \sin^2 2\theta)} \\ &= \frac{2 \sin 2\theta(1 - \cos 2\theta)}{1 - 1 + 2 \sin^2 2\theta} \\ &= \frac{2 \sin 2\theta(1 - \cos 2\theta)}{2 \sin^2 2\theta} \\ &= \frac{1 - \cos 2\theta}{\sin 2\theta} \\ &= \frac{1 - \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}}{\frac{2 \tan \theta}{1 + \tan^2 \theta}} \\ &= \frac{\frac{1 + \tan^2 \theta - (1 - \tan^2 \theta)}{1 + \tan^2 \theta}}{\frac{2 \tan \theta}{1 + \tan^2 \theta}} \\ &= \frac{1 + \tan^2 \theta - 1 + \tan^2 \theta}{2 \tan \theta} \\ &= \frac{2 \tan^2 \theta}{2 \tan \theta} = \tan \theta \end{aligned}$$

$$\text{Q8. (i)} \quad \text{Area } \Delta ACB = \text{Area } \Delta DCE$$

$$\Rightarrow \frac{1}{2}x \cdot x \sin 4\theta = \frac{1}{2}x \cdot x \sin 2\theta$$

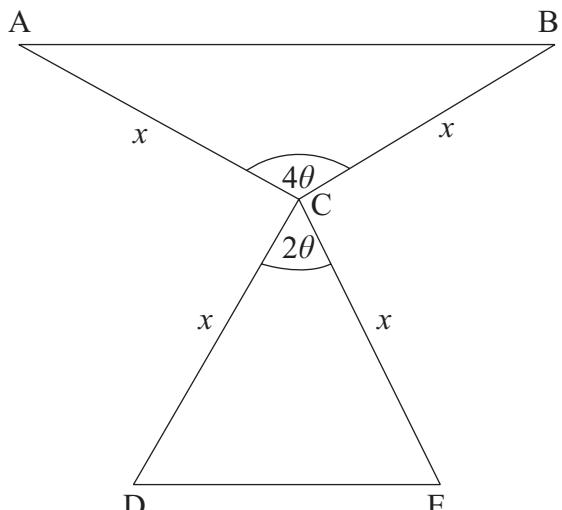
$$\Rightarrow \sin 4\theta = \sin 2\theta$$

$$\Rightarrow 2 \sin 2\theta \cos 2\theta = \sin 2\theta$$

$$\Rightarrow \cos 2\theta = \frac{1}{2}$$

$$\Rightarrow 2\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

$$\Rightarrow \theta = \frac{60^\circ}{2} = 30^\circ$$



$$(ii) \quad \theta = 30^\circ \Rightarrow 2\theta = 60^\circ \text{ and } 4\theta = 120^\circ$$

$$\begin{aligned}\text{Triangle ABC: } |\overline{AB}|^2 &= x^2 + x^2 - 2x \cdot x \cos 60^\circ \\ &= 2x^2 - 2x \cdot x \left(\frac{1}{2}\right) \\ &= 2x^2 - x^2 = x^2\end{aligned}$$

$$\begin{aligned}\text{Triangle DCE: } |\overline{DE}|^2 &= x^2 + x^2 - 2x \cdot x \cos 120^\circ \\ &= 2x^2 - 2x^2 \left(-\frac{1}{2}\right) \\ &= 2x^2 + x^2 = 3x^2\end{aligned}$$

$$\text{Given } |\overline{AB}|^2 + |\overline{DE}|^2 = 24$$

$$\Rightarrow x^2 + 3x^2 = 24$$

$$\Rightarrow 4x^2 = 24$$

$$\Rightarrow x^2 = 6$$

$$\Rightarrow x = \sqrt{6}$$

$$\text{Q9. (i) Area sector ADBC} = \frac{1}{2}(2)^2 2\theta = 4\theta \text{ radians}$$

$$(ii) \quad \text{Area triangle ABC} = \frac{1}{2}(2)(2)\sin 2\theta = 2\sin 2\theta$$

$$\text{Area } \Delta \text{ ABC} = \frac{3}{4} \text{ Area sector ADBC}$$

$$\Rightarrow 2\sin 2\theta = \frac{3}{4}(4\theta)$$

$$\Rightarrow 2\sin 2\theta = 3\theta$$

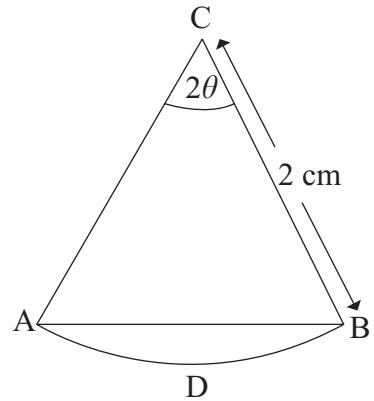
$$\text{Area } \Delta \text{ ABC} = \sqrt{3}$$

$$\Rightarrow 2\sin 2\theta = \sqrt{3}$$

$$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow 2\theta = \sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

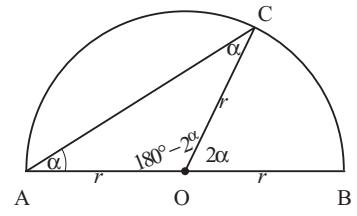
$$\Rightarrow \theta = \frac{\pi}{6} \text{ radians}$$



$$\begin{aligned}
 \text{Q10. (i) Triangle ACO : } & \frac{|AC|}{\sin(180^\circ - 2\alpha)} = \frac{r}{\sin \alpha} \\
 \Rightarrow & \frac{|AC|}{\sin 2\alpha} = \frac{r}{\sin \alpha} \\
 \Rightarrow & |AC| = \frac{r \sin 2\alpha}{\sin \alpha} \\
 &= \frac{r \cdot 2 \sin \alpha \cos \alpha}{\sin \alpha} \\
 &= 2r \cos \alpha
 \end{aligned}$$

(ii) [AC] bisects the area of the semicircular region

$$\begin{aligned}
 \Rightarrow 2[\text{Area } \Delta AOC + \text{Area sector COB}] &= \text{Area semicircle} \\
 \Rightarrow 2 \left[\frac{1}{2} r \cdot r \sin(180^\circ - 2\alpha) + \frac{1}{2} r^2 2\alpha \right] &= \frac{1}{2} \pi r^2 \\
 \Rightarrow 2 \cdot \frac{1}{2} r^2 \sin 2\alpha + 2 \cdot \frac{1}{2} r^2 \cdot 2\alpha &= \frac{\pi}{2} r^2 \\
 \Rightarrow \sin 2\alpha + 2\alpha &= \frac{\pi}{2}
 \end{aligned}$$



Chapter 6 Geometry 2: Enlargements/Constructions

Exercise 6.1

Q1. (i) Scale factor = 2

$$(ii) \quad x = \frac{12}{2} = 6 \text{ cm}$$

$$y = 2(9) = 18 \text{ cm}$$

(iii) Each small square has length of side = 3 cm

$$\Rightarrow \text{Area each small square} = (3)(3) = 9 \text{ cm}^2$$

$$\text{Area object} = 5(9) = 45 \text{ cm}^2$$

$$\text{Area image} = 20(9) = 180 \text{ cm}^2 = 4(45) \text{ cm}^2$$

\Rightarrow Scale factor: $k = 2$

$$\text{Area (image)} = 2^2 = 4 \text{ times area (object)} = k^2$$

Q2. Scale factor = 2

$$(i) \quad |BC| = 4 \Rightarrow |B'C'| = 2(4) = 8$$

$$(ii) \quad |AC| = 6 \Rightarrow |A'C'| = 2(6) = 12$$

$$(iii) \quad |A'B'| = 10 \Rightarrow |AB| = \frac{1}{2}(10) = 5$$

$$\text{Area } \triangle A'B'C' = 30 \text{ sq.units}$$

$$\Rightarrow \text{Area } \triangle ABC = \frac{30}{4} = 7\frac{1}{2} \text{ sq.units}$$

Q3. (i) Scale factor = 2

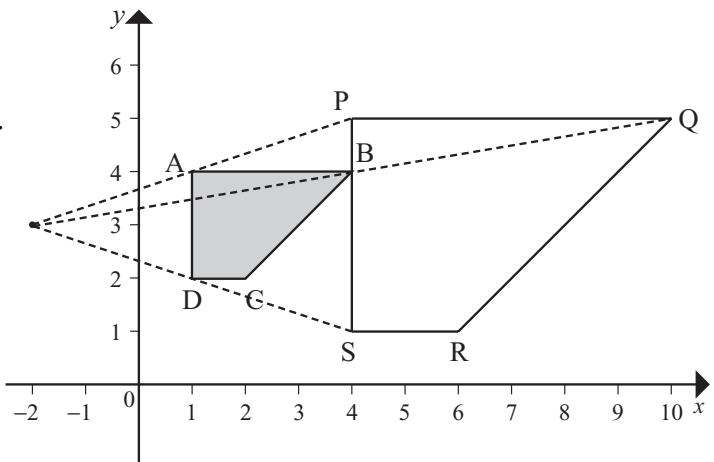
(ii) Centre of enlargement is found by joining 2 sets of corresponding points and continuing the lines until they meet.

$$(iii) (-2, 3)$$

$$(iv) \quad \text{Area } ABCD = 30 \text{ sq.units}$$

$$\Rightarrow \text{Area } PQRS = (2^2)(30)$$

$$= (4)(30) = 120 \text{ sq.units}$$



Q4. Scale factor = $1\frac{1}{2}$

(i) $|AC| = 8 \Rightarrow |AC'| = (8)\left(1\frac{1}{2}\right) = 12$

(ii) $|B'C'| = 9 \Rightarrow |BC| = 9 \div 1\frac{1}{2} = 6$

(iii) $|AB| = x \Rightarrow |AB'| = x + 3$

$$\Rightarrow \frac{|AB'|}{|AB|} = \frac{x+3}{x} = 1\frac{1}{2} \Rightarrow \frac{x+3}{x} = \frac{3}{2}$$

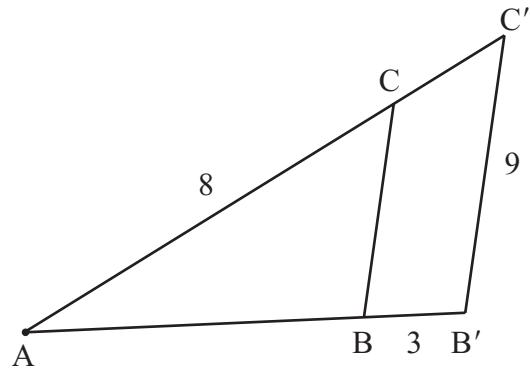
$$\Rightarrow 3x = 2x + 6$$

$$\Rightarrow x = 6 \Rightarrow |AB| = 6$$

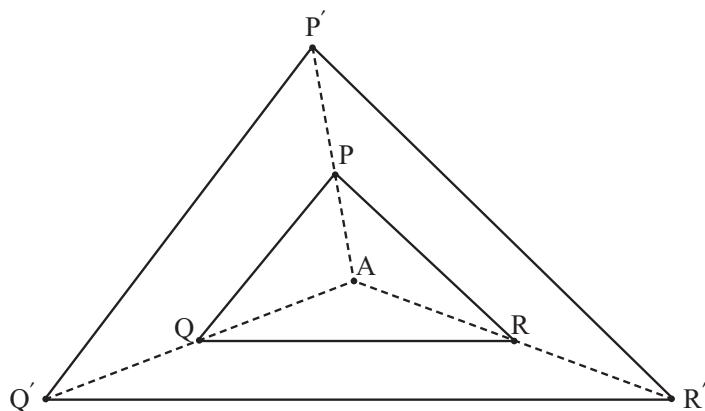
Scale factor $k = 1\frac{1}{2} = \frac{3}{2} \Rightarrow k^2 = \frac{9}{4}$

Area $\triangle ABC = 20$ sq.units

$$\Rightarrow \text{Area } \triangle AB'C' = (20)\left(\frac{9}{4}\right) = 45 \text{ sq.units}$$



Q5.



Q6. (i) Scale factor = $\frac{12.8}{3.2} = 4$

(ii) $|XZ| = 4.1 \text{ cm} \Rightarrow |X'Z'| = (4.1)(4) = 16.4 \text{ cm}$

(iii) $|X'Y'| = 12 \text{ cm} \Rightarrow |XY| = \frac{12}{4} = 3 \text{ cm}$

(iv) $|OZ'| = 4|OZ|$

$\Rightarrow |ZZ'| = 3|OZ|$

$\Rightarrow |OZ| : |ZZ'| = 1 : 3$

(v) $k = 4 \Rightarrow k^2 = 16$

Area $\triangle X'Y'Z' = 64 \text{ cm}^2$

$$\Rightarrow \text{Area } \triangle XYZ = \frac{64}{16} = 4 \text{ cm}^2$$

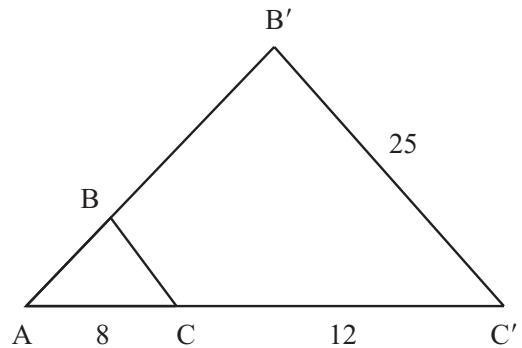
Q7. (i) Scale factor = $\frac{|AC'|}{|AC|} = \frac{20}{8} = 2\frac{1}{2}$

(ii) $|B'C'| = 25 \Rightarrow |BC| = 25 \div 2\frac{1}{2} = 10$

(iii) $|AB'| = 2\frac{1}{2}|AB| \Rightarrow |AB| : |AB'| = 1 : 2\frac{1}{2} = 2 : 5$

(iv) $k = 2\frac{1}{2} \Rightarrow k^2 = \left(2\frac{1}{2}\right)^2 = 6\frac{1}{4}$

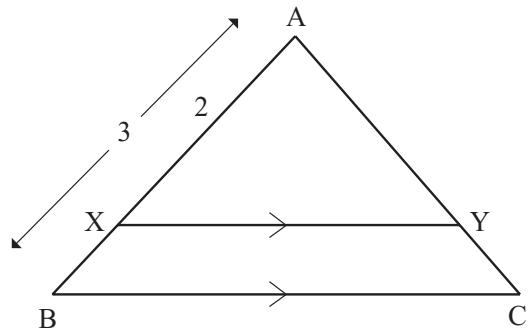
$\text{Area } \triangle ABC = 16 \text{ sq.units} \Rightarrow \text{Area } \triangle AB'C' = (16) \left(6\frac{1}{4}\right) = 100 \text{ sq.units}$



Q8. Scale factor = $\frac{|AB|}{|AX|} = \frac{3}{2} = k \Rightarrow k^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$\text{Area } \triangle AXY = 4 \text{ cm}^2$

$\Rightarrow \text{Area } \triangle ABC = (4) \left(\frac{9}{4}\right) = 9 \text{ cm}^2$



Q9. Scale factor = $k = 2.5 \Rightarrow k^2 = (2.5)^2 = 6.25$

$\text{Area design} = 176 \text{ cm}^2$

$\Rightarrow \text{Area completed mosaic} = (176)(6.25) = 1100 \text{ cm}^2$

Q10. $k^2 = 4 \Rightarrow \text{scale factor } k = \sqrt{4} = 2$

$k^2 = 2 \Rightarrow \text{scale factor } k = \sqrt{2}$

Q11. Scale factor $k = \frac{12}{8} = 1.5 \Rightarrow k^2 = (1.5)^2 = 2.25$

$\text{Area larger shape} = 27 \text{ cm}^2$

$\Rightarrow \text{Area smaller shape A} = \frac{27}{2.25} = 12 \text{ cm}^2$

Q12. Scale factor $k = \frac{2}{3}$

(i) Original height = 156 mm

$\Rightarrow \text{Reduced height} = (156) \left(\frac{2}{3}\right) = 104 \text{ mm}$

(ii) Reduced label height = 28 mm

$\Rightarrow \text{Original label height} = 28 \div \frac{2}{3} = 42 \text{ mm}$

Q13. Scale factor $k = \frac{300 \text{ m}}{15 \text{ cm}} = \frac{30000 \text{ cm}}{15 \text{ cm}} = 2000$

$$\Rightarrow k^2 = (2000)^2 = 4,000,000$$

$$\text{Model Area} = 25.5 \text{ cm}^2$$

$$\Rightarrow \text{Tower Area} = (25.5)(4,000,000) = 102,000,000 \text{ cm}^2 \\ = 10,200 \text{ m}^2$$

Q14. Scale factor $k = 25 \Rightarrow k^2 = (25)^2 = 625$

(i) Plan pond area = 24 cm^2

$$\Rightarrow \text{Real pond area} = (24)(625) = 15000 \text{ cm}^2 \\ = 1.5 \text{ m}^2$$

(ii) Real lawn area = $17 \text{ m}^2 = 170000 \text{ cm}^2$

$$\Rightarrow \text{Plan lawn area} = \frac{170000}{625} = 272 \text{ cm}^2$$

Q15. Scale factor = $k = 2$

(i) Map scale is 1:1000

Enlarged map scale 1:500

(ii) Street length = 6 cm

$$\text{Real-life street length} = (6)(1000) \\ = 6000 \text{ cm} = 60 \text{ m}$$

(iii) Scale factor = $k = \frac{1}{2}$

Map scale is 1:1000

Sean's enlarged map scale is 1:2000

(iv) Distance railway stations = 1 km = 100,000 cm

$$\text{Distance on sean's enlarged map} = \frac{100,000}{2000} = 50 \text{ cm}$$

Q16. (i) Scale factor $k = \frac{6}{2} = 3$

(ii) Scale factor of volume = k^3

Q17. (i) Scale factor $k = \frac{100}{40} = \frac{5}{2} = 2\frac{1}{2}$

(ii) Volume scale factor $= k^3 = \left(\frac{5}{2}\right)^3 = \frac{125}{8}$

(iii) Model volume $= 240 \text{ cm}^3$

$$\text{Final sculpture volume} = (240) \left(\frac{125}{8} \right) = 3750 \text{ cm}^3$$

Q18. Scale factor $k = 2 \Rightarrow \text{Volume} = k^3 = (2)^3 = 8$

$$\begin{aligned} \text{Volume small cylinder} &= 200 \text{ cm}^3 \Rightarrow \text{Volume large cylinder} = (200)(8) \\ &= 1600 \text{ cm}^3 \end{aligned}$$

Q19. Scale factor $k = \frac{18}{15} = 1.2 \Rightarrow k^3 = (1.2)^3 = 1.728$

Smaller bottle volume $= 400 \text{ ml}$

$$\begin{aligned} \Rightarrow \text{Larger bottle volume} &= (400)(1.728) \\ &= 691.2 \text{ ml} \quad = 691 \text{ ml} \end{aligned}$$

Q20. $k^3 = \frac{400}{300} = 1\frac{1}{3}$

$$\Rightarrow k = \sqrt[3]{1\frac{1}{3}} = 1.1006424$$

Small tin height $= 10 \text{ cm}$

$$\begin{aligned} \Rightarrow \text{Large tin height} &= (10)(1.1006424) \\ &= 11.006424 \text{ cm} \quad = 11 \text{ cm} \end{aligned}$$

Q21. $k^2 = \frac{45 \text{ cm}^2}{5 \text{ cm}^2} = 9 \Rightarrow k = \sqrt{9} = 3$

$$\Rightarrow k^3 = (3)^3 = 27$$

Small sphere mass $= 2 \text{ kg}$

$$\Rightarrow \text{Large sphere mass} = (2)(27) = 54 \text{ kg}$$

Q22. Scale factor $= k = \frac{22}{10} = 2.2 \Rightarrow k^3 = (2.2)^3 = 10.648$

Child's rugby ball volume $= 200 \text{ cm}^3$

$$\begin{aligned} \Rightarrow \text{Full-size ball volume} &= (200)(10.648) \\ &= 2129.6 \text{ cm}^3 \end{aligned}$$

Q23. Scale factor $k = \frac{8\text{ m}}{20\text{ cm}} = \frac{800}{20} = 40 \Rightarrow k^3 = (40)^3 = 64000$

Pond volume = $50\text{ m}^3 = 50,000,000\text{ cm}^3$

Model volume = $\frac{50,000,000}{64000} = 781.25\text{ cm}^3$

Q24. $k^3 = \frac{54}{16} = 3.375 \Rightarrow k = \sqrt[3]{3.375} = 1.5$

Small cylinder length = 6 units

\Rightarrow Larger cylinder length $m = (6)(1.5) = 9$ units

Q25. $k^2 = \frac{1.2\text{ m}^2}{400\text{ cm}^2} = \frac{12000}{400} = 30 \Rightarrow k = \sqrt{30} = 5.477$
 $\Rightarrow k^3 = (5.477)^3 = 164.296$

Volume real trough = $0.1\text{ m}^3 = 100,000\text{ cm}^3$

\Rightarrow Volume model = $\frac{100,000}{164.296} = 608.6 = 609\text{ cm}^3$

Q26. $k^3 = 8 \Rightarrow k = \sqrt[3]{8} = 2$

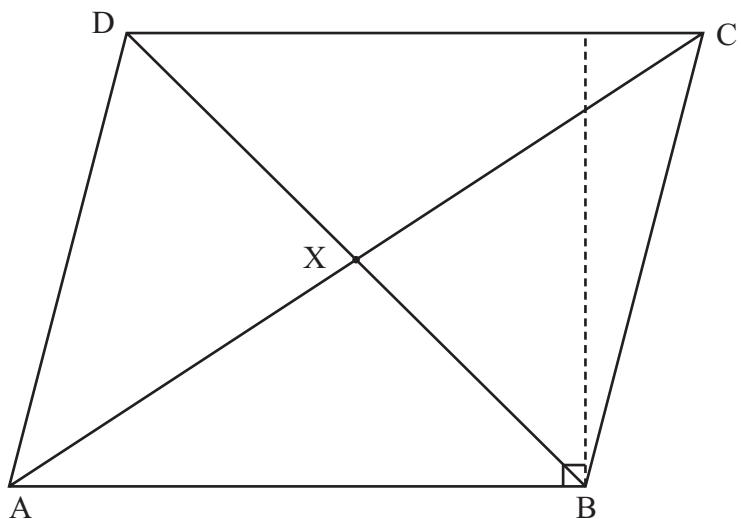
Statue height = 40 cm

\Rightarrow Replica height = $\frac{40}{2} = 20\text{ cm}$

Exercise 6.2

Q1. (i) Construct the parallelogram.

(ii) Yes, the diagonals bisect each other.



(iii) Given : Parallelogram ABCD with diagonals [AC] and [DB] meeting at X.

To prove : $|DX| = |XB|$

Proof : In Δs D XC, A XB;

$$|\angle XDC| = |\angle XBA| \quad (\text{alternate angles})$$

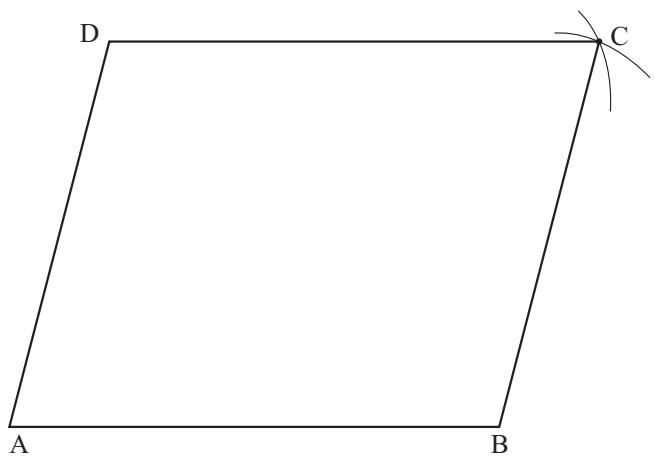
$$|DC| = |AB| \quad (\text{opposite sides of parallelogram})$$

$$|\angle XCD| = |\angle XAB| \quad (\text{alternate angles})$$

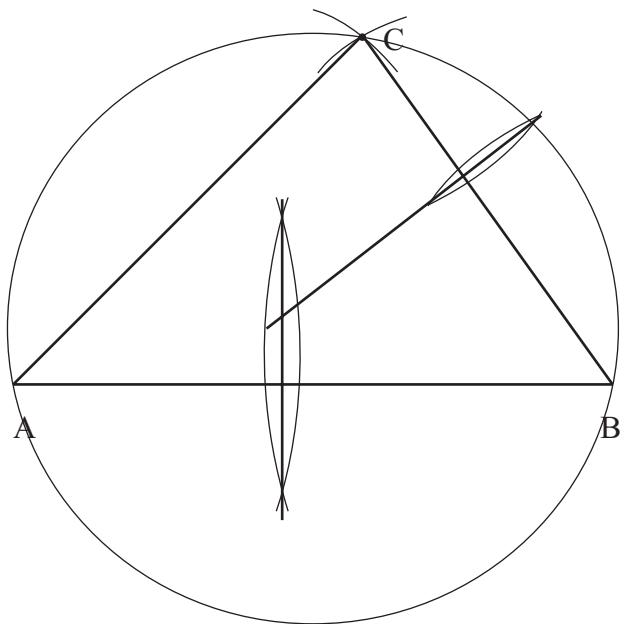
$\Rightarrow \Delta s$ are congruent by A.S.A.

$$\Rightarrow |DX| = |XB|$$

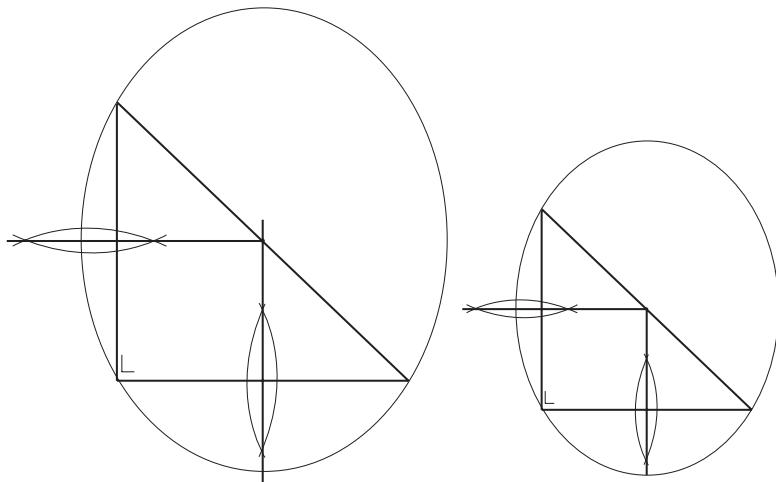
Q2. $|\angle ABC| = 105^\circ$



Q3.



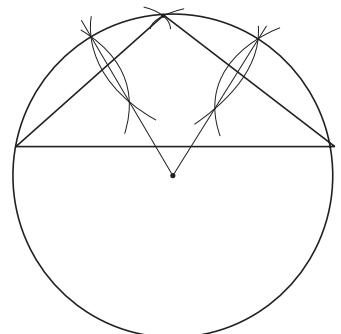
Q4.



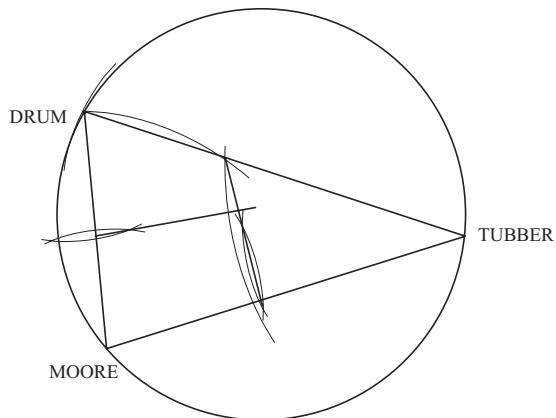
Yes : You get the same result each time.

\Rightarrow Circumcentre is the midpoint of the hypotenuse.

- Q5. In an obtuse-angled triangle, the circumcentre is outside the triangle.

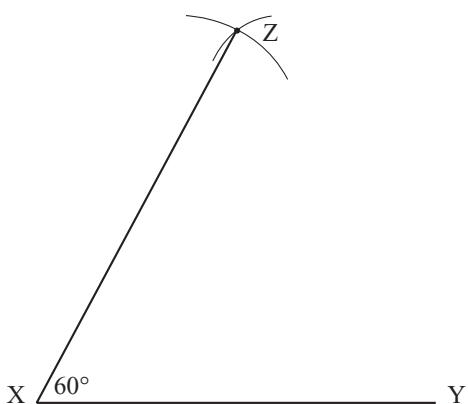


Q6.



School should be built at the circumcentre of the triangle.

Q7.



Draw a line segment [XY].

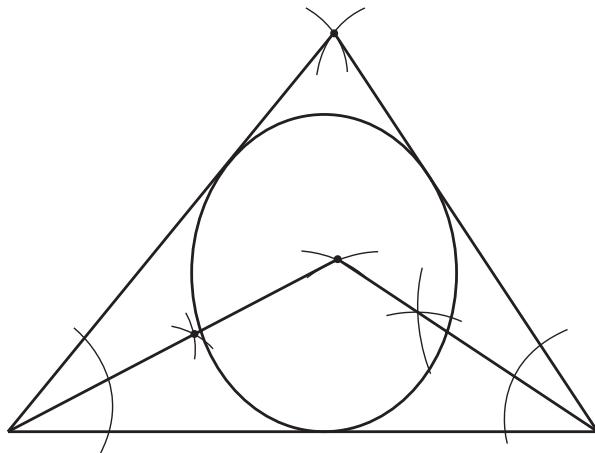
Set the compass to a radius of [XY] with X as centre and radius [XY].

Draw an arc.

Repeat at Y.

The arcs meet at Z. Join XZ $\Rightarrow |\angle ZXY| = 60^\circ$

Q8.



Q9. (i) Area $\triangle BOC = \frac{1}{2}ar$

(ii) Area $\triangle ABC = \text{Area } \triangle BOC + \text{Area } \triangle AOC + \text{Area } \triangle BOA$

$$\begin{aligned} &= \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr \\ &= \frac{1}{2}r(a+b+c) \end{aligned}$$

Q10. $|\angle G| + |\angle E| = |\angle H| + |\angle F| = 90^\circ$

Since $|\angle G| = |\angle H| \Rightarrow |\angle E| = |\angle F|$

In $\Delta s XKZ, XKY$

$$|\angle E| = |\angle F|$$

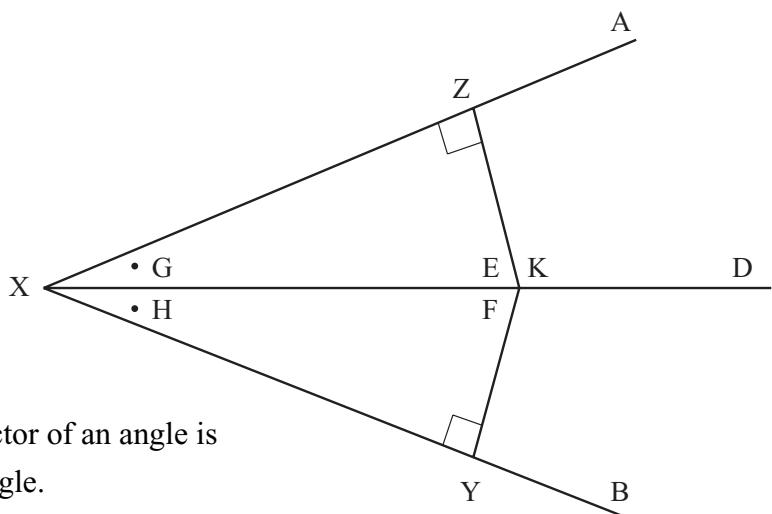
$$|XK| = |XK|$$

$$|\angle G| = |\angle H|$$

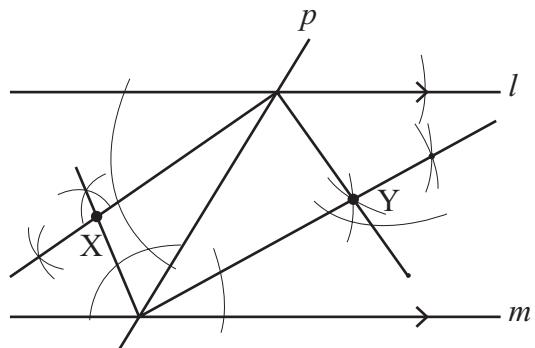
$\Rightarrow \Delta s$ are congruent by A.S.A.

$$\Rightarrow |KZ| = |KY|$$

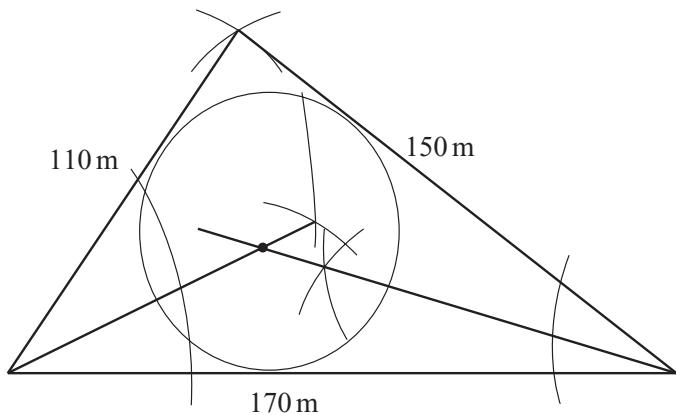
Conclusion: Any point on the bisector of an angle is equidistant from the arms of the angle.



Q11. Construct the bisectors of all 4 angles to locate X and Y.



Q12. (i) Using $20\text{ m} = 1\text{ cm}$, draw a scaled diagram.



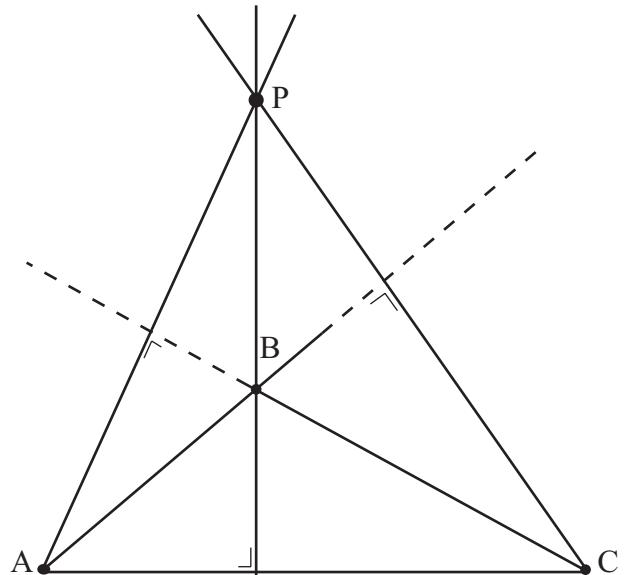
(ii) Best position to pitch a tent is the incentre.

Q13. (i) Radius $= \frac{1}{2}(10) = 5\text{ cm}$

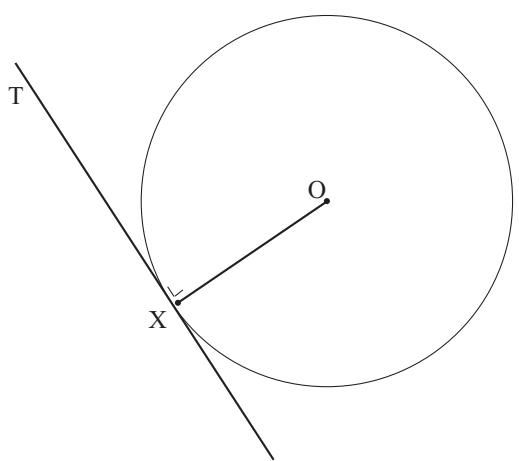
(ii) B is the point of intersection of all 3 altitudes (i.e. the orthocentre) of $\triangle ABC$.

Q14. Orthocentre P is outside the triangle ABC.

Result will hold for all obtuse-angled triangles.

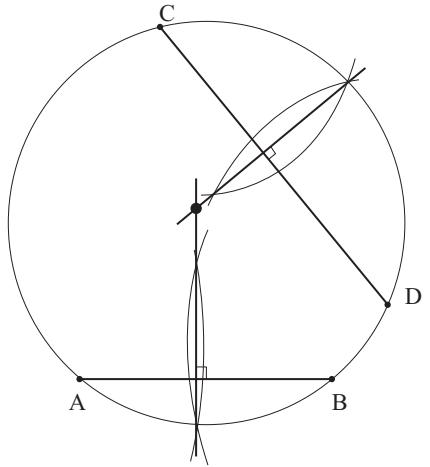


Q15. Tangent XT is perpendicular to radius OX.

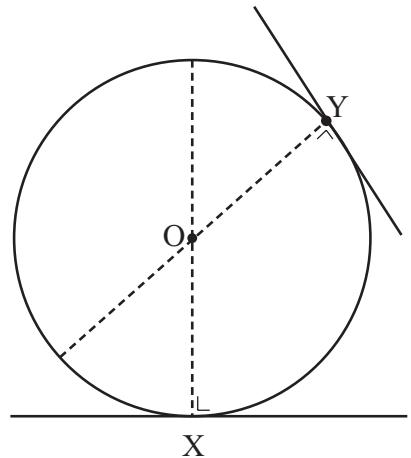


- Q16.** Perpendicular bisector of $[AB]$
is equidistant from the end-points A and B.

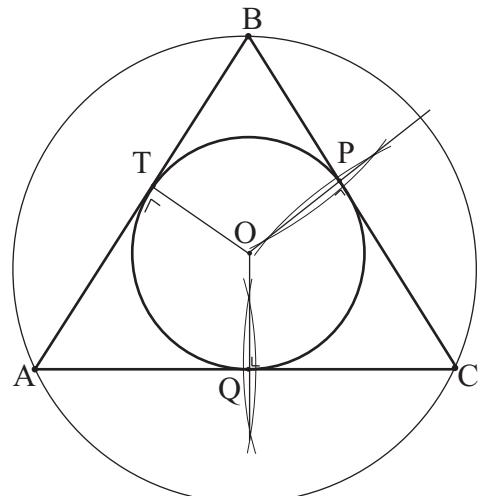
Point of intersection of the perpendicular bisectors of the two chords is the centre of the circle.



- Q17.** Draw tangents at both points X and Y.
The perpendicular lines to both tangents are diameters which meet at point O, the centre of the circle.



- Q18. (i)** Construct the perpendicular bisectors of $[AC]$ and $[BC]$ to meet at the point O, the circumcentre.
Hence, $|OA| = |OB| = |OC|$.

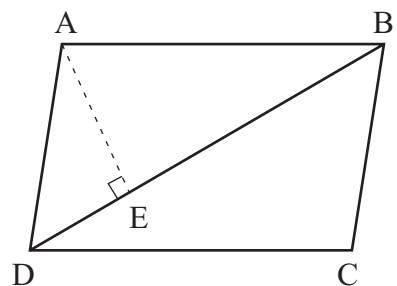


- (ii)** In an equilateral triangle, the bisectors of the angles at A,B,C also meet at the point O.
Hence, $|OP| = |OQ| = |OT|$ = radius of incircle.

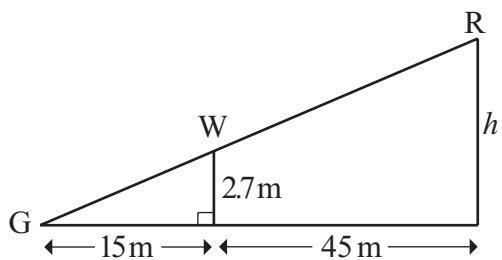
Test Yourself 6

A Questions

Q1. Area parallelogram = 2 Area $\triangle ADB$ = 40
 $= 2 \cdot \frac{1}{2} |\overline{DB}| \cdot |\overline{AE}| = 40$
 $= 15 \cdot |\overline{AE}| = 40$
 $\Rightarrow |\overline{AE}| = \frac{40}{15} = 2\frac{2}{3} \text{ cm}$

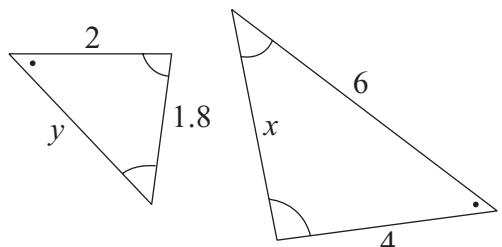


Q2. $\tan \angle G = \frac{2.7}{15} = \frac{h}{60}$
 $\Rightarrow 15h = (60)(2.7) = 162$
 $\Rightarrow h = \frac{162}{15} = 10.8 \text{ m}$



- Q3. (i) Equal angles are marked on the diagram
 \Rightarrow the two triangles are equiangular (i.e. all the same size but not necessarily all 60°).
 \Rightarrow the two triangles are similar. (Not congruent as the corresponding sides differ in length.)

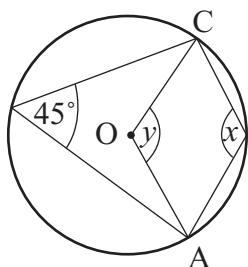
(ii) $\frac{x}{1.8} = \frac{4}{2} = \frac{6}{y}$
 $\frac{x}{1.8} = \frac{4}{2} \quad \text{and} \quad \frac{4}{2} = \frac{6}{y}$
 $\Rightarrow 2x = 7.2 \quad \Rightarrow 4y = 12$
 $\Rightarrow x = \frac{7.2}{2} = 3.6 \quad \Rightarrow y = \frac{12}{4} = 3$



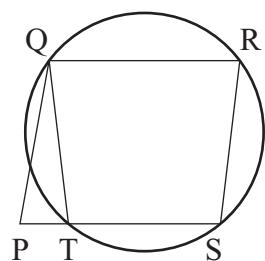
- Q4. (i) Cyclic quadrilateral $\Rightarrow \angle x + 45^\circ = 180^\circ$
 $\Rightarrow \angle x = 180^\circ - 45^\circ = 135^\circ$

$$\angle y = 2(45^\circ) = 90^\circ$$

- (ii) $\angle y = 90^\circ \Rightarrow [\overline{AC}]$ is the diameter of the circumcircle.
Hence, a circle drawn on $[\overline{AC}]$ as diameter must pass through O.

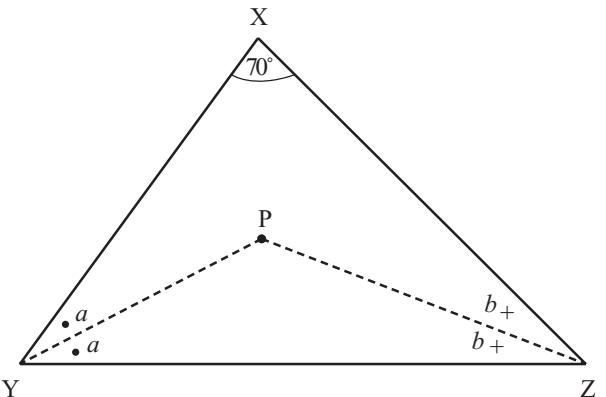


- Q5. (i) False as QT is not parallel to RS
(ii) False as PQRS is not a rectangle
(iii) True as opposite angles of a cyclic quadrilateral add to 180°
(iv) False as PQRS is not a rectangle.



Q6. (i) Point of intersection of the perpendicular bisectors of $[AB]$ and $[BC]$.

$$\begin{aligned}
 \text{(ii)} \quad & 2\angle a + 2\angle b + 70^\circ = 180^\circ \\
 & \Rightarrow 2\angle a + 2\angle b = 180^\circ - 70^\circ = 110^\circ \\
 & \Rightarrow \angle a + \angle b = \frac{110^\circ}{2} = 55^\circ \\
 & \Rightarrow |\angle YPZ| + \angle a + \angle b = 180^\circ \\
 & \Rightarrow |\angle YPZ| + 55^\circ = 180^\circ \\
 & \Rightarrow |\angle YPZ| = 180^\circ - 55^\circ = 125^\circ
 \end{aligned}$$

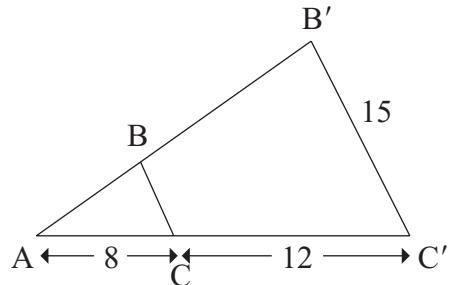


(iii) Yes because P is the point of intersection of the bisectors of the angles $\angle XYZ$ and $\angle XZY$.

Q8. (i) Scale factor $= \frac{20}{8} = 2.5$

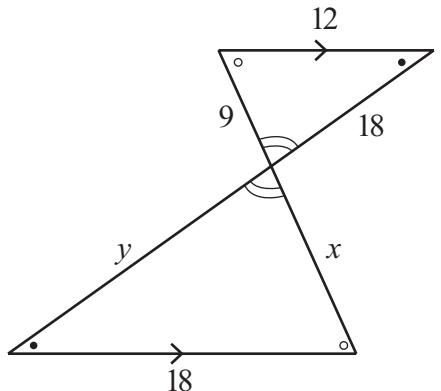
$$\begin{aligned}
 \text{(ii)} \quad & \frac{|BC|}{15} = \frac{8}{20} \Rightarrow 20|BC| = 120 \\
 & \Rightarrow |BC| = \frac{120}{20} = 6
 \end{aligned}$$

$$\text{(iii)} \quad |AB| : |AB'| = 2 : 5$$



Q9. Similar Triangles with equal angles marked.

$$\begin{aligned}
 \Rightarrow \frac{x}{9} &= \frac{18}{12} = \frac{y}{18} \\
 \Rightarrow \frac{x}{9} &= \frac{18}{12} \quad \text{and} \quad \frac{18}{12} = \frac{y}{18} \\
 \Rightarrow 12x &= (9)(18) = 162 \quad \Rightarrow 12y = (18)(18) = 324 \\
 \Rightarrow x &= \frac{162}{12} = 13.5 \quad \Rightarrow y = \frac{324}{12} = 27
 \end{aligned}$$



Q10. (i) Parallel lines $\Rightarrow \frac{|AB|}{|BC|} = \frac{|DE|}{|CD|} = \frac{7}{7} = 1 \Rightarrow |DE| = |CD| = 8 \text{ cm}$

(ii) $\Delta s BCD$ and ACE are similar $\Rightarrow \frac{|BC|}{|AC|} = \frac{|BD|}{|AE|}$
 $\Rightarrow \frac{7}{14} = \frac{|BD|}{12} \Rightarrow 14|BD| = 84$
 $\Rightarrow |BD| = \frac{84}{14} = 6 \text{ cm}$

B Questions

Q1. Area $\triangle BIC = \frac{1}{2}(6)(3) = 9 \text{ cm}^2$

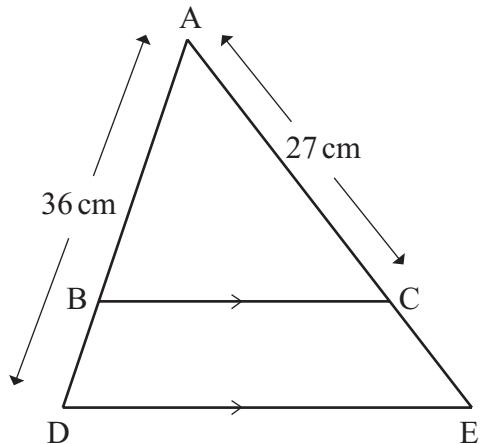
$$\begin{aligned}\text{Area } \triangle ABC &= \text{Area } \triangle BIC + \text{Area } \triangle AIC + \text{Area } \triangle AIB \\ &= \frac{1}{2}(6)(3) + \frac{1}{2}(4)(3) + \frac{1}{2}(5)(3) \\ &= 9 + 6 + 7.5 = 22.5 \text{ cm}^2\end{aligned}$$

Q2. (i) Scale factor $= \frac{4.2}{3.5} = 1.2$

(ii) $a = (1.2)(4.5) = 5.4 \text{ cm}$
 $b = \frac{7.2}{1.2} = 6 \text{ cm}$

Q3. (i) $|AB| = 2|BD|$ and $|AD| = 3|BD| = 36$
 $\Rightarrow |BD| = \frac{36}{3} = 12$
 $\Rightarrow |AB| = 2(12) = 24 \text{ cm}$

(ii) $|AC| = 2|CE| = 27$
 $\Rightarrow |CE| = \frac{27}{2} = 13.5$
 $\Rightarrow |AE| = 3|CE| = 3(13.5) = 40.5 \text{ cm}$



Q4. $|AB| = 6 \Rightarrow |BF| = \frac{1}{2}(6) = 3 \text{ cm}$

Find radius $|OB|$ using pythagoras' theorem.

$$\Rightarrow r^2 = (3)^2 + (4)^2 = 9 + 16 = 25$$

$$\Rightarrow r = \sqrt{25} = 5$$

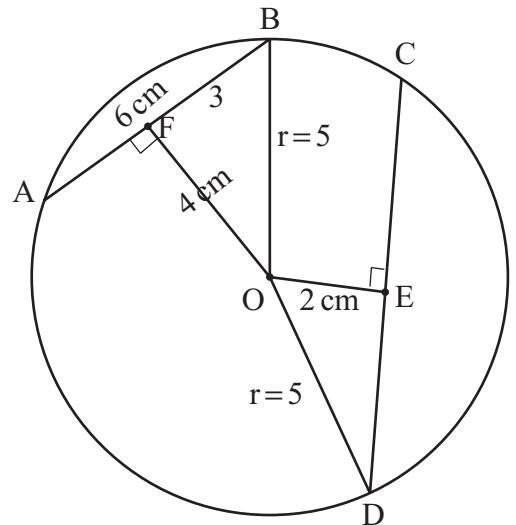
$$|OD| = 5 \Rightarrow |DE|^2 + (2)^2 = 5^2$$

$$\Rightarrow |DE|^2 + 4 = 25$$

$$\Rightarrow |DE|^2 = 25 - 4 = 21$$

$$\Rightarrow |DE| = \sqrt{21}$$

$$\Rightarrow |DC| = 2|DE| = 2\sqrt{21}$$



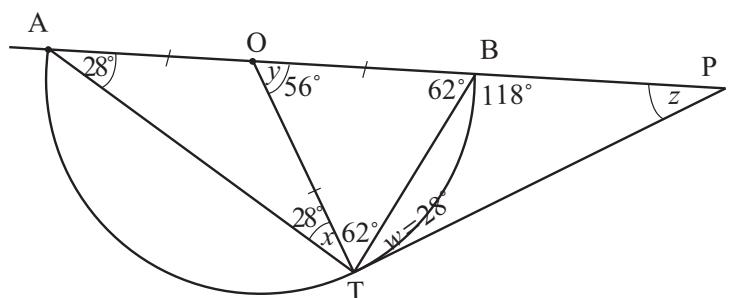
Q5. $|OA| = |OT| \Rightarrow \angle x = 28^\circ$

$$\Rightarrow \angle y = 28^\circ + \angle x = 28^\circ + 28^\circ = 56^\circ$$

$$\angle w = 28^\circ$$

$$\Rightarrow \angle z + 28^\circ + 118^\circ = 180^\circ$$

$$\Rightarrow \angle z = 180^\circ - 146^\circ = 34^\circ$$



Q6. (i) Scale factor $= k \Rightarrow k^2 = \frac{500}{20} = 25$

$$\Rightarrow k = \sqrt{25} = 5$$

Radius of small ball = $r \text{ cm}$

\Rightarrow Radius of large ball = $5r \text{ cm}$

(ii) $x^2 + (2x)^2 = (25)^2$

$$\Rightarrow x^2 + 4x^2 = 5x^2 = 625$$

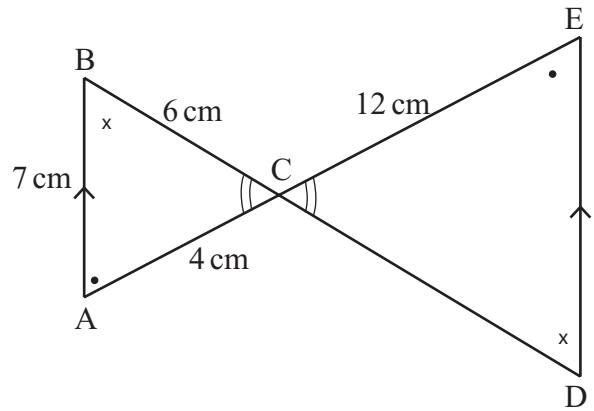
$$\Rightarrow x^2 = \frac{625}{5} = 125 \quad \Rightarrow x = \sqrt{125} = 5\sqrt{5}$$

$$\Rightarrow 2x = 10\sqrt{5}$$

$$\text{Area rectangle} = (2x)(x) = (10\sqrt{5})(5\sqrt{5}) = 250 \text{ cm}^2$$

- Q7. (i) Triangles ABC, CDE are equiangular with equal angles marked,
 \Rightarrow Triangles are similar.

$$\text{(ii)} \quad \frac{|CD|}{6} = \frac{12}{4} \Rightarrow 4|CD| = 72 \\ \Rightarrow |CD| = \frac{72}{4} = 18 \text{ cm}$$



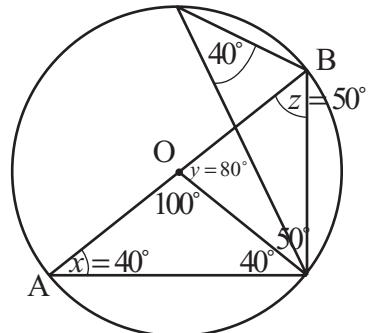
- Q8. $\angle x = 40^\circ$ (standing on the same arc)

$$\angle y = 40^\circ + 40^\circ = 80^\circ$$

$$\angle z + 40^\circ + 90^\circ = 180^\circ$$

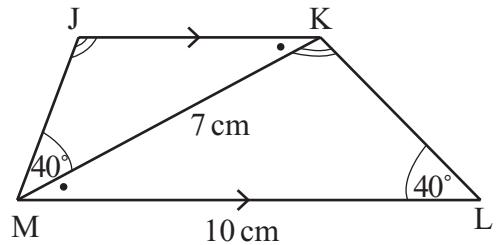
$$\angle z + 130^\circ = 180^\circ$$

$$\Rightarrow \angle z = 180^\circ - 130^\circ = 50^\circ$$

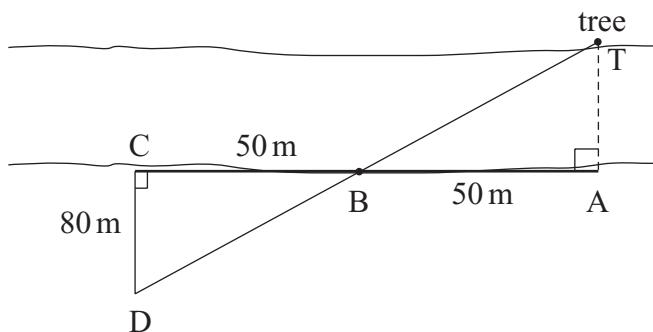


- Q9. (i) Triangles JKM, KML are equiangular with equal angles marked.
 \Rightarrow Triangles are similar.

$$\text{(ii)} \quad \frac{|JK|}{|KM|} = \frac{|KM|}{|ML|} \\ \Rightarrow \frac{|JK|}{7} = \frac{7}{10} \Rightarrow 10|JK| = (7)(7) = 49 \\ \Rightarrow |JK| = \frac{49}{10} = 4.9 \text{ cm}$$



- Q10. Triangles BAT and DCB are similar and congruent.
 \Rightarrow Width of River = $|AT| = |DC| = 80 \text{ m}$.



C Questions

Q1. (i) (a) Surface Areas = $(2)^2 : (3)^2 = 4 : 9$

(b) Volumes = $(2)^3 : (3)^3 = 8 : 27$

(ii) Edge of smaller cube = $12 \left(\frac{2}{3} \right) = 8 \text{ cm}$

(iii) Total surface area of larger cube = $54 \left(\frac{9}{4} \right) = 121.5 \text{ cm}^2$

(iv) Volume smaller cube = $108 \left(\frac{8}{27} \right) = 32 \text{ cm}^3$

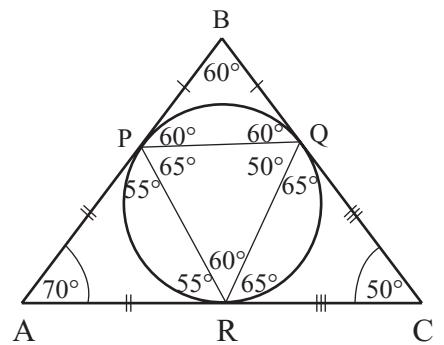
Q2. The lengths of two tangents from a point to a circle are equal.

Hence, $|PB| = |BQ|$, $|AP| = |AR|$, $|CQ| = |CR|$

(i) $|\angle PRQ| = 180^\circ - (55^\circ + 65^\circ) = 60^\circ$

(ii) $|\angle BPQ| = 60^\circ$

(iii) $|\angle PQR| = 180^\circ - (60^\circ + 65^\circ) = 55^\circ$



Q3. (i) Scale factor = $\frac{6}{2} = 3 \Rightarrow k^3 = (3)^3 = 27$

Volume small pyramid = 2.75 cm^3

\Rightarrow Volume large pyramid = $27(2.75) = 74.25 \text{ cm}^3$

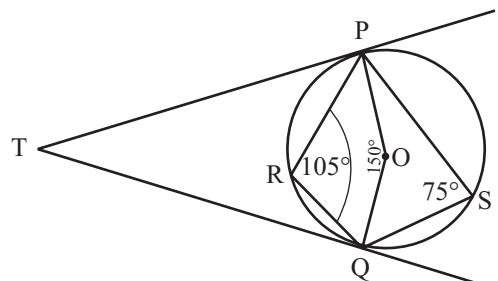
(ii) (a) $|\angle PRQ| = 105^\circ$

$|\angle PSQ| + |\angle PRQ| = 180^\circ$

$\Rightarrow |\angle PSQ| + 105^\circ = 180^\circ$

$\Rightarrow |\angle PSQ| = 180^\circ - 105^\circ = 75^\circ$

$\Rightarrow |\angle POQ| = 2(75^\circ) = 150^\circ$



(b) POQT is a cyclic quadrilateral because $|\angle TPO| + |\angle TQO| = 180^\circ$.

(c) $|\angle PTQ| + 150^\circ = 180^\circ$

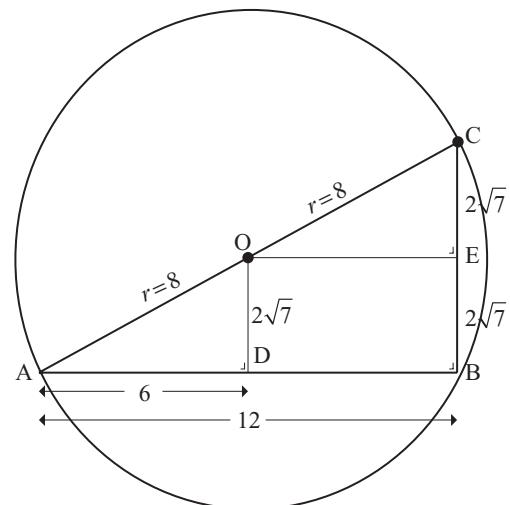
$\Rightarrow |\angle PTQ| = 180^\circ - 150^\circ = 30^\circ$

Q4. (i) $|OD|^2 + (6)^2 = (8)^2 \Rightarrow |OD|^2 = 64 - 36 = 28$
 $\Rightarrow |OD| = \sqrt{28} = 2\sqrt{7}$

(ii) $|BC|^2 + (12)^2 = (16)^2$
 $\Rightarrow |BC|^2 = 256 - 144 = 112$
 $\Rightarrow |BC| = \sqrt{112} = 4\sqrt{7}$

(iii) $|OE|^2 + (2\sqrt{7})^2 = 8^2$
 $\Rightarrow |OE|^2 + 28 = 64$
 $\Rightarrow |OE|^2 = 64 - 28 = 36$
 $\Rightarrow |OE| = \sqrt{36} = 6$

(iv) Area $\triangle ABC = \frac{1}{2}(12)(4\sqrt{7}) = 24\sqrt{7} \text{ cm}^2$



Q5. (i) Scale factor $= \frac{6}{4} = 1.5 = k \Rightarrow k^3 = (1.5)^3 = 3.375$

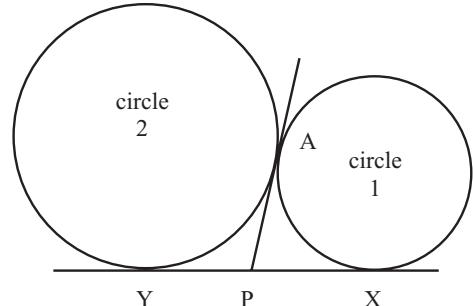
Capacity smaller jug $= 50 \text{ cm}^3 \Rightarrow$ Capacity larger jug $= 50(3.375)$
 $= 168.75 \text{ cm}^3$

(ii)(a) In $\triangle APY$, $|PA| = |PY|$ (equal tangents)
 In $\triangle APX$, $|PA| = |PX|$ (equal tangents)
 $\Rightarrow |PY| = |PX|$
 \Rightarrow Tangent at A bisects the line $[XY]$

(b) $|PA| = |PY| \Rightarrow \angle c = \angle c$
 $|PA| = |PX| \Rightarrow \angle d = \angle d$

Hence, $\triangle YAX \Rightarrow \angle c + \angle c + \angle d + \angle d = 2\angle c + 2\angle d = 180^\circ$

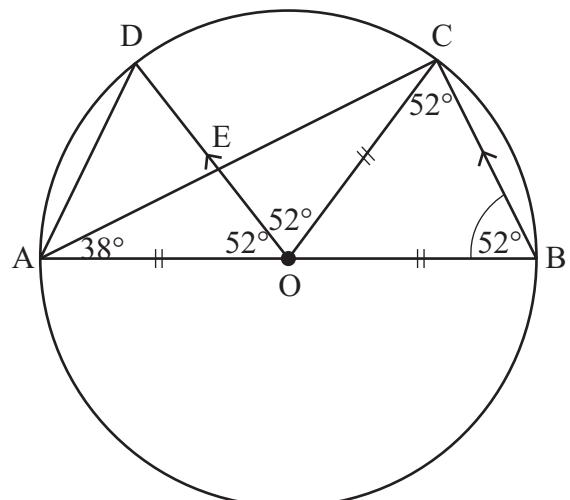
$\Rightarrow \angle c + \angle d = \frac{180^\circ}{2} = 90^\circ$
 ie $|\angle XAY| = 90^\circ$



Q6. (i)(a) $|\angle COE| = |\angle OCB| = 52^\circ$ (alternate angles)
 $|\angle AOE| = |\angle OBC| = 52^\circ$ (corresponding angles)
 $\Rightarrow |\angle AOE| = |\angle COE|$

Hence, OD bisects $\angle AOC$

(b) $|\angle COE| = 52^\circ$
 $\Rightarrow |\angle CAD| = \frac{1}{2}|\angle COE| = \frac{1}{2}(52^\circ) = 26^\circ$

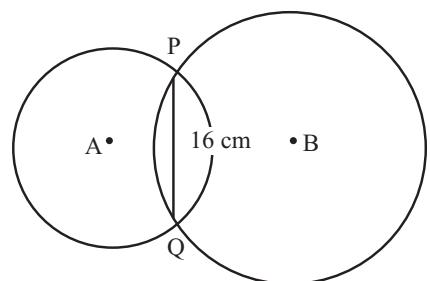


(c) In $\triangle AEO$, $|\angle AEO| + 38^\circ + 52^\circ = 180^\circ$
 $\Rightarrow |\angle AEO| = 180^\circ - 90^\circ = 90^\circ$

Hence, $OD \perp AC$.

(ii) In $\triangle APQ$, $|AQ|^2 + (8)^2 = (10)^2$
 $\Rightarrow |AQ|^2 + 64 = 100$
 $\Rightarrow |AQ|^2 = 100 - 64 = 36 \Rightarrow |AQ| = \sqrt{36} = 6 \text{ cm}$
In $\triangle BPQ$ $\Rightarrow |BQ|^2 + (8)^2 = (17)^2$
 $\Rightarrow |BQ|^2 + 64 = 289$
 $\Rightarrow |BQ|^2 = 289 - 64 = 225 \Rightarrow |BQ| = \sqrt{225} = 15 \text{ cm}$

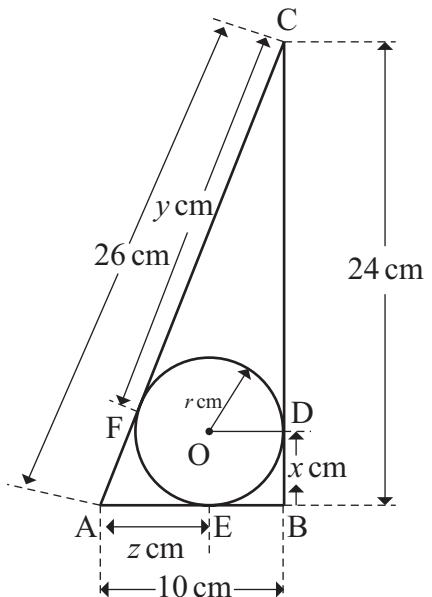
Hence, $|AB| = 6 + 15 = 21 \text{ cm}$



Q7. (i) $|CF| = |CD| = y \text{ cm}$
 $|CB| = |CD| + |DB| = y + x = 24$

(ii) $x + z = 10 \quad \text{and} \quad z + y = 26$
 $\underline{x + y = 24} \quad (\text{subtracting}) \quad \underline{z - y = -14} \quad (\text{adding})$
 $\Rightarrow z - y = -14 \quad 2z = 12$
 $\Rightarrow z = 6 \quad$
 $\Rightarrow 6 - y = -14$
 $\Rightarrow -y = -14 - 6 = -20$
 $\Rightarrow y = 20$
 $x + 6 = 10 \Rightarrow x = 10 - 6 = 4$

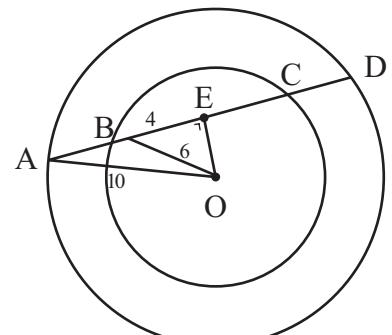
(iii) Radius $= |OD| = |EB| = 10 - 6 = 4 \text{ cm}$



Q8. (i) $|OE|^2 + |BE|^2 = |OB|^2$
 $\Rightarrow |OE|^2 + (4)^2 = (6)^2$
 $\Rightarrow |OE|^2 = 36 - 16 = 20$
 $\Rightarrow |OE| = \sqrt{20} = 2\sqrt{5}$

$$\begin{aligned} &|AE|^2 + |EO|^2 = |AO|^2 \\ &\Rightarrow |AE|^2 + (2\sqrt{5})^2 = (10)^2 \\ &\Rightarrow |AE|^2 + 20 = 100 \\ &\Rightarrow |AE|^2 = 100 - 20 = 80 \\ &\Rightarrow |AE| = \sqrt{80} = 4\sqrt{5} \end{aligned}$$

Hence, $|AB| = |AE| - |BE| = 4\sqrt{5} - 4 = 4(\sqrt{5} - 1) \text{ cm}$



- (ii) Scale factor = $k \Rightarrow k^2$ is used for area
 \Rightarrow Area enlarged square = $25k^2$
 and perimeter of enlarged square = $4(5k) = 20k$
 Hence, $25k^2 = 20k$
 $\Rightarrow 25k^2 - 20k = 0$
 $\Rightarrow 5k^2 - 4k = 0$
 $\Rightarrow k(5k - 4) = 0$
 $\Rightarrow k = \frac{4}{5}$ or $k = 0$ (not valid)

Q9. (i) Scale factor = $\frac{8}{6} = 1\frac{1}{3} = k \Rightarrow k^3 = \left(1\frac{1}{3}\right)^3 = \frac{64}{27}$

Capacity of larger tin = 252 cm^3
 \Rightarrow Capacity of small tin = $252 \div \frac{64}{27} = 106.3125 \text{ cm}^3$

- (ii) Scale factor = $k = \frac{100}{1} = 100 \Rightarrow k^2 = (100)^2 = 10,000$
 Surface area golf ball = 50 cm^2
 \Rightarrow Surface area giant ball = $50(10,000)$
 $= 500,000 \text{ cm}^2$
 $= 50 \text{ m}^2$

- Q10. $\triangle ABD$ has angles $30^\circ, 60^\circ, 90^\circ$
 and has sides in the ratio $1:\sqrt{3}:2$
 $\Rightarrow |BD| = 29 \text{ m}$
 $|AD| = 58 \text{ m}$
 and radius = $|AB| = 29\sqrt{3}$
 $= 50.229$
 $= 50.23 \text{ m}$
 Distance from circumference
 $= 58 - 50.23 = 7.77 \text{ m} = 7.8 \text{ m}$

