

SEQUENCES

Arithmetic Sequence :  $a, a+d, a+2d, \dots, a+(n-1)d$   
 $T_1 \quad T_2 \quad T_3 \quad T_n$

Q1 (Ex 3 pg 140) Ex 4 p141 Q10 pg 141 If  $T_n = n(n+2)$  verify NOT arithmetic seq.

ARITHMETIC SERIES  $a + a+d + a+2d + \dots + a+(n-1)d$

$$S_n = a + a+d + a+2d + \dots + a+(n-1)d$$

$$S_n = a+(n-1)d + a+(n-2)d + a+(n-3)d + \dots + a$$

$$2S_n = [2a+(n-1)d]n \Rightarrow S_n = \frac{n}{2} (2a+(n-1)d) \text{ Arithmetic!}$$

Given  $S_n = n^2 - 4n$ , find an expression for  $T_n$  and determine if seq is arithmetic Ex 3 p 146

Ex 4 p 146 (i) Use  $\Sigma$  notation to represent  $2+6+10+14+\dots$  for 45 terms  
 p 148 Ex 5

Q12 pg 150  
 find sum of  
 1st n even  
 1st n odd

Q13 p 150 In arithmetic seq  $T_{21} = 31, S_{20} = 320$   
 find  $S_{10}$

for arithmetic can't be found - either  $\pm \infty$  as its either  $\Rightarrow$  increasing or decreasing (d- / d+)

Geometric SEQUENCE:

$a, ar, ar^2, ar^3, \dots, ar^{n-1}$   
 $T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_n$

Arithmetic :  $T_n - T_{n-1} = d$   
 Geometric :  $T_n \div T_{n-1} = r$

Arithmetic  $T_n - T_{n-1} = T_x - T_{x-1} = k$

Geometric  $T_n \div T_{n-1} = T_x \div T_{x-1} = k$

3, x, x+6 are 1st 3 terms in geometric sequence of (+ve) terms find x,  $T_{10}$  Ex 3 p 152

Product of 1st 3 terms of GS is 216, their sum is 21. Given that  $r < 1$ , find the 1st 3 terms. Ex 4 p 152

$\frac{a}{r}, a, ar = 3 \text{ terms}$

EXPONENTIAL SEQUENCES

$T_n = Aa^{n-1} = \text{Geometric}$

Bouncing ball  $\rightarrow$  loses  $\frac{2}{3}$  of height on each bounce. Dropped from 27m. Find height of ball after 10th bounce

Ex 6 pg 154

To show sequence is

Geometric Show  $\frac{T_{n+1}}{T_n} = \text{constant}$

write out the 1st few terms of the sequence

Arithmetic  $T_{n+1} - T_n = \text{constant}$

GEOMETRIC SERIES:

$$S_n = a + ar + ar^2 + \dots + ar^{n-1}$$

$$rS_n = ar + ar^2 + \dots + ar^{n-1} + ar^n$$

$S_n - rS_n = a - ar^n$  | Geometric series |  $S_n = \frac{a(1-r^n)}{1-r}$



\*\* In a GS  $T_3 = 32$ ,  $T_6 = 4$ . Find  $a$  and  $r$  and hence find  $S_8$

$$\lim_{n \rightarrow \infty} \frac{a(1-r^n)}{1-r} = -\infty \text{ if } r > 1$$

$$\lim_{n \rightarrow \infty} \frac{a(1-r^n)}{1-r} = \frac{a}{1-r} \text{ iff } |r| < 1$$

$$S_{\infty} \text{ G.S.} = \frac{a}{1-r} \quad |r| < 1$$

Write a recurring decimal as a fraction:  $0.\dot{3} = 0.3333 = \frac{3}{10} + \frac{3}{100} + \dots = \text{infinite G.S.}$

Write  $0.\dot{2}\dot{3}$  as a fraction in form  $a/b$   $a, b \in \mathbb{N}$

\* Ex 4 p160

$$a = 3/10 \quad r = 1/10$$

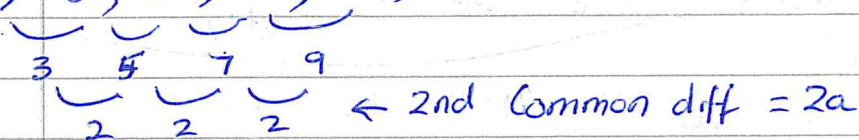
$$S_{\infty} = \frac{a}{1-r} = \frac{0.3}{1-0.1} = \frac{3/10}{9/10} = \frac{1}{3}$$

If you can check answers Do! @ end!!!

LINEAR PATTERN = ARITHMETIC - can be written as  $T_n = an + b$   
 $a = \text{common difference}$

QUADRATIC PATTERN can be written as  $T_n = an^2 + bn + c$   
 $2a = \text{the second difference}$

2, 5, 10, 17, 26, ... Find  $T_n$ ?



quadratic  $\rightarrow 2 = 2a \quad a = 1$

$$\text{So } T_n = an^2 + bn + c$$

$$T_n = 1n^2 + bn + c$$

we know  $T_1, T_2 \dots$

$$\text{so } T_1 = 1(1)^2 + b(1) + c = 2 \rightarrow 1 + b + c = 2 \rightarrow b + c = 1$$

$$T_2 = 1(2)^2 + b(2) + c = 5 \rightarrow 4 + 2b + c = 5 \rightarrow 2b + c = 1$$

$$-b = 0, c = 1$$

$$\text{So } T_n = 1n^2 + 0n + 1 \text{ or } n^2 + 1 \text{ - CHECK!}$$

CUBIC PATTERN  $\rightarrow T_n = an^3 + bn^2 + cn + d$

$6a = \text{third difference}$

Write a formula for  $n$ th term of 4, 31, 98, 223, 424