SEQUENCES

Arithmetic Sequence : a, $a+d$, $a+2d$ ...... $a+(n-1)d$

T_1 T_2 T_3 T_n

Ex 3 pg 140 / Ex 4 p141 / Q10: pg 141 If $T_n = n(n+2)$ verify NOT arithmetic seq.

ARITHMETIC SERIES $a + a+d + a+2d + \dots + a+(n-1)d$

$$S_n = a + a+d + a+2d + \dots\dots a+(n-1)d$$
$$S_n = a+(n-1)d + a+(n-2)d + a+(n-3)d \dots\dots + a$$

$2S_n = [2a+(n-1)d]n \Rightarrow \boxed{S_n = \frac{n}{2}(2a+(n-1)d)}$ Arithmetic!

Given $S_n = n^2 - 4n$, find an expression for T_n and determine if seq is arithmetic Ex 3 p 146

Ex 4 p146 (i) Use Σ notation to represent $2+6+10+14+\dots$ for 45 terms p 148 Ex 5

Q12 pg 150 find sum of 1st n even 1st n odd

Q13 p150 In arithmetic seq $T_{21} = 31$, $S_{20} = 320$ find S_{10}

S_∞ for arithmetic can't be found - either its $\pm\infty$ - as its either >>> increasing or decreasing ($d-$) ($d+$)

Geometric SEQUENCE:

a, ar, ar^2, ar^3 .... ar^{n-1}

T_1 T_2 T_3 T_4 T_n

Arithmetic : $T_n - T_{n-1} = d$

Geometric : $T_n \div T_{n-1} = r$

Arithmetic $T_n - T_{n-1} = T_x - T_{x-1} = k$

Geometric $T_n \div T_{n-1} = T_x \div T_{x-1} = k$

$3, x, x+6$ are 1st 3 terms in geometric sequence & (+ve) terms find x, T_{10} Ex3 p152

Product of 1st 3 terms of GS is 216, their sum is 21. Given that $r<1$, find the 1st 3 terms. Ex 4 p152

$\frac{a}{r}$, a, ar = 3 terms

EXPONENTIAL SEQUENCES

$T_n = Aa^{n-1}$ = Geometric

Bouncing ball → loses 2/3 of height on each bounce. Dropped from 27m. Find height of ball after 10th bounce

Ex 6 pg 154 write out the 1st few terms of the sequence

To show sequence is Geometric Show $\frac{T_{n+1}}{T_n} = \text{constant}$

Arithmetic $T_{n+1} - T_n = \text{constant}$

GEOMERIC SERIES:

$$S_n = a + ar + ar^2 + \dots\dots ar^{n-1}$$
$$rS_n = ar + ar^2 + \dots\dots ar^{n-1} + ar^n$$
$$S_n - rS_n = a - ar^n$$

Geometric series $S_n = \frac{a(1-r^n)}{1-r}$

** In a GS $T_3 = 32$, $T_6 = 4$. Find a and r and hence find S_8

E2 p157

$\lim_{n \to \infty} \frac{a(1-r^n)}{1-r} = -\infty$ if $r > 1$

$\lim_{n \to \infty} \frac{a(1-r^n)}{1-r} = \frac{a}{1-r}$ iff $|r| < 1$

ie. if r is a fraction

S_∞ G.S. $= \frac{a}{1-r}$ $|r| < 1$

Write a recurring decimal as a fraction:

$0.\dot{3} = 0.3333 = \frac{3}{10} + \frac{3}{100} + \cdots$ infinite = GS.

$a = 3/10$ $r = 1/10$

$S_\infty = \frac{a}{1-r} = \frac{0.3}{1-0.1} = \frac{3/10}{9/10} = \frac{1}{3}$

Write $0.\dot{2}\dot{3}$ as a fraction in form a/b $a,b \in \mathbb{N}$

* Ex 4 p160

If you can check answers Do! @ end !!!

LINEAR PATTERN = ARITHMETIC - can be written as $T_n = an + b$

$a =$ common difference

QUADRATIC PATTERN can be written as $T_n = an^2 + bn + c$

$2a =$ the second difference

$2, 5, 10, 17, 26, \cdots$ Find T_n?

3 5 7 9

2 2 2 ← 2nd Common diff = $2a$

quadratic ⟶ $2 = 2a$ $a = 1$

so $T_n = an^2 + bn + c$

$T_n = 1n^2 + bn + c$

we know $T_1, T_2 \cdots$

so $T_1 = 1(1)^2 + b(1) + c = 2$ → $1 + b + c = 2$ → $b + c = 1$

$T_2 = 1(2)^2 + b(2) + c = 5$ → $4 + 2b + c = 5$ → $2b + c = 1$

$-b = 0, c = 1$

So $T_n = 1n^2 + 0n + 1$ or $n^2 + 1$ — CHECK!

CUBIC PATTERN → $T_n = an^3 + bn^2 + cn + d$

$6a =$ third difference

Write a formula for nth term of $4, 31, 98, 223, 424$