Fundamental Principle of Counting: If one task can be done m ways and then a 2nd task (following the first) can be done n ways, then the no of ways of doing the 1st task followed by the 2nd is $m \times n$.

A permutation is an arrangement of a no. of objects in a certain order (ABC, ACB, BCA, BAC, CBA, CAB = Permutation)

$n! = n(n-1)\cdots 1$

FILL in the restriction first!!!

Six books - sci on left extreme: $1 \times 5 \times 4 \times 3 \times 2 \times 1$

SCOTLAND - vowels together? OA

really 6 choices as OA tied together

$6 \times 5 \times 4 \times 3 \times 2 \times 1$ but AO

$+ 6 \times 5 \times 4 \times 3 \times 2 \times 1$ E3 P3

Ex4: How many 4 digit nos using 0, 2, 5, 7, 8 that are >5000 (ii) odd

>5000 ⇒ 1st letter = 5 or 7, 8 — 3 choices

odd ends in 5, 7 (2 choices)

3	4	3	2	= >5000

3	3	2	2	= odd

n objects taking r at a time (Permutations) nPr $7P_3 = 7 \times 6 \times 5$

$$\frac{n(n-1)\cdots 1}{(n-r)!} = \frac{n!}{(n-r)!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$$ leaving $7 \times 6 \times 5$

In how many ways can the word IRELAND be arranged if 3 vowels come together. 7 letters but 3 tied together. EAI as one

so 5 4 3 2 1 ← $5P_3 \times 3!$

but EAI can come together 3! ways So

A security lock requires a 4-digit code. Digits can be 0, 1, 2, 3, 4, 5 but can't start with 0. How may different codes if all digits differe.

(ii) First digit is 5, none repeated

Combinations: selecting from a given set (selecting team - order does NOT matter)

ABCD : ABC = ACB.

so for n items choose r it is $nPr \div r!$ (since each r items can be arranged r ways but in Combinations different arrangements are not classed as different selection

$$7C_3 = \frac{7P_3}{3!} = \frac{7 \times 6 \times 5}{4 \times 3 \times 2 \times 1 \times 3!}$$

$$^nC_r = \frac{n!}{r!(n-r)!}$$

$$nCr = \frac{(n)(n-1)\cdots(n-r+1)(n-r)!}{r!(n-r)!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$$

$$\binom{n}{r} = \binom{n}{n-r}$$

Ex 2 p224 How many ways can a group of 5 be selected from 10 people if 2 particular people are never together → (Total no of ways − no of ways where they ARE together)

$10C_5 - 8C_3 = 56$ Group

Combinations from 2 sets (one m items, second n items)

How many ways to choose 4 men and 3 women from 7 men & 5 women: $\binom{7}{4} \times \binom{5}{3}$

How many ways can a comittee of 6 be chosen from 5 teachers & 8 students if there must be more teachers than students on each Committee

4 teachers, 2s → $\binom{5}{4} \times \binom{8}{2}$ +
5 of teachers, 1s → $\binom{5}{5} \times \binom{8}{1}$

5 points on a plane no 3 are collinear. How many different $\triangle$'s can be formed using these points as vertices?

Find $n \in \mathbb{N}$ if $\binom{n+1}{2} = 28$

2 of the sides are labelled X, Y. How many $\triangle$'s have XY as a side?

Probability of an event = $\frac{\text{no of successful outcomes}}{\text{total no. of possible outcomes}}$

$P(\bar{E}) = 1 - P(E)$

Sample space = set of all possible outcom

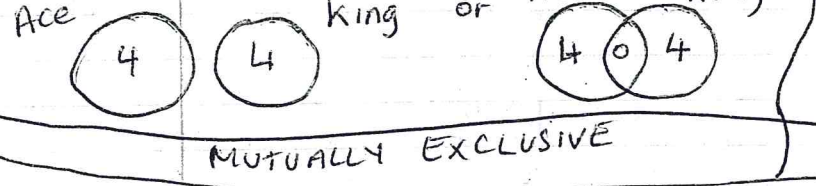
Relative frequency = 'guess' at the probability estimate = $\frac{\text{No. sucessful trials}}{\text{total no of trials}}$

Expected frequency = probability × no of trials (Q11 P 23

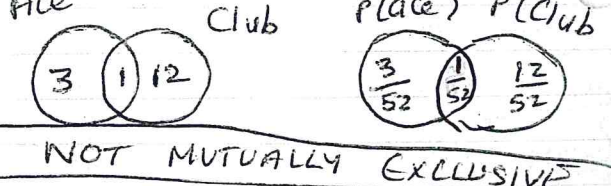
Mutually exclusive EVENTS: P(Ace) P(king) drawing Ace drawing King (cannot occur together)

Not Mutally exclusive P(ace), P(club) drawing Ace drawing Club.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ → 0 for mutally exclusive



MUTUALLY EXCLUSIVE



NOT MUTUALLY EXCLUSIVE

$P(A \text{ and } B) = P(A \cap B)$, $P(A \text{ OR } B) = P(A \cup B)$!!!

Exhaustive events – if $P(A) + P(B) = 1$

Bag contains 8 red, 12 blue, unknown gr
$P(G) = \frac{1}{5}$ find # green

A, B are mutally exclusive $P(A) = 3/7$, $P(B) = \frac{1}{5}$
find $P(A \cup B)$

INDEPENDENT EVENTS: outcomes do not affect each other
e.g. toss a coin and roll a dice.

For Independent events: $P(A) \times P(B) \times P(C) = P(A \text{ and } B \text{ and } C)$

Q8 throwing 2 dice independent events? Whats P(4 or more on each die)?

$\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \left(\frac{1}{4}\right)$

Gambler must throw a 6 with a single dice to win.
What is prob that he wins on his third attempt.

= FFS Fail, Fail, Success. P(Fail) = 5/6 P(Success = 1/6)

$= \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$

3 pupils A, B, C have birthdays in same week.
What is prob. the 3 birthday fall on (i) Same day
(ii) Monday (iii) 3 different days

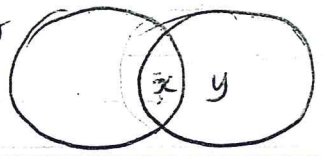
E4 P 32

3 people selected at random what is prob that none of their birthdays falls on Sun, (ii) Prob only one of birthdays falls on Sunday, (iii) At least one of the birthdays falls on a Sunday. Q15 P 37

Conditional probability outcome of one depends on outcome of other

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

Prob. of A given B



$P(A|B) = x/y+x$

So P(A and B) = $P(A \cap B) = P(A|B)\,P(B)$

Q20 P41 TREE DIAGRAMS Ex 2 P 105 Do

12 beads, 5 Yel, 7 green. remove bead, do NOT return. Second bead removed, do NOT return 3rd bead removed, Draw Tree Diag. Find P (exactly 2 green beads)

Expected Value:

Throw dice → output = 1, 2, 3, 4, 5, 6, Throw 10 times
Expected value = average value of o/p
so P(1) = 1/6 P(2) = 1/6 P(3) = 1/6 P(4) = 1/6 P(5) = 1/6 P(6) = 1/6
So I will turn up 1/6 of 10 times ...etc

4 Coins tossed x = no of heads
x = random variable / outcome
x can be discrete or continuous

$E(x) = 1\times\frac{1}{6} + 2\times\frac{1}{6} + 3\times\frac{1}{6} + 4\times\frac{1}{6} + 5\times\frac{1}{6} + 6\times\frac{1}{6} = \frac{21}{6} = 3\,1/2$ = Expected Value on average

$E(x) = P(x) \times x$ where x = outcome/random variable

Ex 2 pg 111 Q11, 17 p 113, 115

Ex 2 Given that 10% of apples are bad. Find prob that in a box of 6 apples
1.) There is No bad apple
2.) There is just 1 bad apple
3) There is at least one bad one

BINOMIAL DISTRIBUTION: (Bernoulli trials)

1.) Fixed no. of trials
2.) Each trial has 2 possible outcomes S, F or
3.) Trials are independent
4.) Prob of success in each trial is same

$P(r \text{ successes in } n \text{ trials}) = \binom{n}{r} p^r q^{n-r}$

$p = P(S)$, $q = P(f)$

Probability that horse A will win any given race is 2/5. What is prob that, in 5 races, A will win (i) exactly 3 races (ii) the 1st, 3rd, 5th and lose rest

Probability of k success on nth Bernoulli trial.

must have $(k-1)$ successes in $(n-1)$ trials & then one success on the n^{th} trial.

$(k-1)$ successes in $(n-1)$ trials $= \binom{n-1}{k-1} \cdot p^{k-1} q^{n-(k-1)}$ (then $\times p$)

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p119 Q16: A fair dice is thrown repeatedly. (i) Find prob. of getting 2 fives in 1st ten throws, (ii) Find probability of getting the 3rd five on 11th throw

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Q20 p120! Independent events = outcome of one does not affect outcome of other

*REM 2 events are independent if $P(A) \times P(B) = P(A \text{ and } B)$

$P(A) \times P(B) = P(A \cap B)$

*Rem $P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$ (-or-)

*REM $P(A|B) = \frac{P(A \cap B)}{P(B)}$ 'given'

Ex 2,3 on p121

Q14 p123

$P(\text{Event occurring at least once}) = 1 - P(\text{Event NOT occurring at all})$

Probability involving Permutations and Combinations: Ex 1,2 pg 125

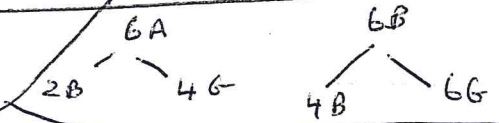
In class 6A, 2 Boys, 4 girls study music. In 6B, 4 boys and 6 girls study Music. 2 pupils are chosen at random from each of the 2 classes.

(i) How many ways can the 4 pupils be selected? $\binom{6}{2} \times \binom{10}{2}$

(ii) Calc. the prob. that the 4 chosen consist of 2 boys from 6A, 2 girls from 6B

(iii) Calc. the prob. that the 4 pupils are of same gender.

Rem prob = $\frac{\text{no of possible outcomes}}{\text{total no of outcomes}}$



Ex 2 p125: 3 cards drawn at random, without replacement from 52

(i) Prob that 3 cards are J♠, Q♣, K♣ 1J, 1Q, 1K, 49 others

$3! \times \left(\frac{1}{52}\right)\left(\frac{1}{51}\right)\left(\frac{1}{50}\right)$ (JQK, QJK, KQJ.....) OR $\frac{\binom{1}{1}\binom{1}{1}\binom{1}{1}}{\binom{52}{3}}$ $\frac{\text{no of ways of selecting each card}}{\text{total no of ways of selecting 3 cards from 52}}$

(ii) Prob that 3 cards are Aces:

4 Aces, others

$\frac{\binom{4}{3}}{\binom{52}{3}}$ ← no of ways of selecting 3 Aces from 4 / total no of ways of selecting 3 from 52

(iii) Prob that 2 are red, 1 is club

26 red, 13 clubs

$\frac{\binom{26}{2}\binom{13}{1}}{\binom{52}{3}}$ $\frac{\text{no of ways of selecting 2 red \& 1 club}}{\text{total no of ways of selecting 3 cards from 52}}$

(iv) Prob of all same colour 26 red, 26 black

$\frac{\binom{26}{3} + \binom{26}{3}}{\binom{52}{3}}$