PROOFS & CONSTRUCTIONS $1 \rightarrow 15$ * De Moivres Theorem

* Constructions 16 – 22 *

- use terms
theorem, proof, axiom, Corollary, Converse, implies
is equivalent to, if and only if, proof by contradiction

* prove theorems 11, 12, 13 which lay the proper foundation for theorem of Pythagorus (*4, *6, *9, *14, *19 from JC)

- investigate theorems 7, 8, 11, 12, 13, 16, 17, 18, 20, 21 & Coroll 1–6.
& use to solve problems

* Derive trignometric formulae 1, 2, 3, 4, 5, 6, 7, 9

- Apply trignometric formulae 1–24

Print out

↔ commutative, associative, distributive laws & relationship between them.

* Geometrically construct $\sqrt{2}$, $\sqrt{3}$

* Prove $\sqrt{2}$ is not rational (by contradiction)

* Proof by induction : simple identities $(1+2+\cdots n$ etc$)$
simple inequalities $(n! > 2^n,\ 2^n \geq n^2 \quad n \geq 4)$
$(1+x)^n \geq 1+nx \quad (x > -1)$

factorising : $4^n - 1$ has 3 as a factor

* Derive the formula for the sum to ∞ of a geometric series by considering the limit of a sequence of partial ~~terms~~ sums.

* Derive formula for a mortgage repayment (geometric series)

List of Theorems you need to know (not prove)

Th 1: Vertically opposite angles are equal

Th 2: In an isosceles triangle the angles opposite the equal sides are equal.

Conversely, If 2 angles are equal then the Δ is isosceles.

Th 3: ~~Alternate~~ If a transversal makes equal alternate angles on two lines then the lines are parallel

Conversely: If the two lines are parallel then alternate angles are equal

Th 4: The angles in any Δ sum to 180°

Th 5: Two lines are parallel if and only if, for any transversal, corresponding angles are equal.

~~Th 6~~ Conversely: If the two lines are parallel then corresponding angles are equal

Th 6: Each exterior angle of a Δ is equal to the sum of the interior opposite angles

Th 7: The angle opposite the greater of 2 sides & a Δ is greater than the angle opposite the lesser side.

Conversely if angle ~~A~~ > angle 2 then the side opposite angle 1 is > side opp angle 2

Th 8: Two sides of a Δ are together greater than the third

Theorem 9: In a parallelogram, opposite sides are equal and opposite angles are equal.

** Corollary: a diagonal divides a parallelogram into two congruent triangles.

** Converse is false

Th 10: The diagonals of a parallelogram bisect each other

Converse: If the diagonals of a quadrilateral bisect each other then the quadrilateral is a parallelogram.

Th 11: If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

Th 12: In ΔABC, If a line l parallel to BC cuts $[AB]$ in ratio $[s:t]$, then it also cuts $[AC]$ in same ratio

~~Th 13~~ Converse is true: (ΔABC) If line l cuts AB and AC in same ratio, then it is parallel to BC.

Th 13: If two Δ's, ΔABC and $\Delta A'B'C'$ are similar, then their sides are proportional in order:

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|AC|}{|A'C'|}$$

Converse: If $\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|AC|}{|A'C'|}$ then ΔABC, $\Delta A'B'C'$ are similar

(8) False = Converse of 'Congruent Δ's have equal areas'

Theorem 14: In a right angled Δ, the square of the hypotenuse = sum of the squares of the opposite two sides.

Theorem 15: (Converse to Pythagoras): If the square of one side of a Δ is the sum of the squares of the other two, then the angle opposite the first side is a right angle.

Theorem 16: For a Δ, base times height does not depend on the choice of base.

Theorem 17: A diagonal of a parallelogram bisects the area

Theorem 18: The area of a parallelogram = base × height

Theorem 19: The angle at centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.

Corollary: angle at ~~e~~ edge of O standing on diameter = 90°.

Corollary: All angles at edge of O, standing on same arc are equal.

Corollary: If ABCD is a cyclic quadrilateral, then opp angles sum to 180°. (+ Converse)

Theorem 20: (1) Each tangent is $\perp$ to the radius that goes ~~throug~~ to the point of contract.
(2) If P lies on a circle s, and a line l through P is $\perp$ to the radius to P, then l is a tangent to S.

* Proof by Contradiction * * look in T&T4

Corollary 6: If 2 circles share a common tangent line at one point, then the two centres and that point are collinear.

Theorem 21: (1) The perpendicular from the centre to a chord, bisects the chord.
(2) The perpendicular bisector of a chord passes through the centre.