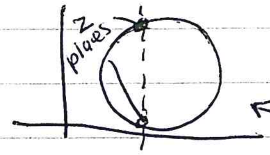
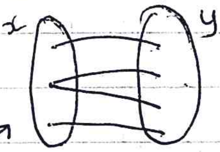
Functions :

A relation is where there is a rule mapping one point of a couple onto the other.

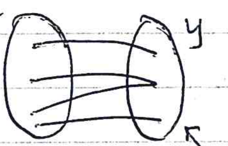
e.g. $\{(x,y) : y = 2x+1,\ x \in \{1,2,3,4\}\}$ is relation $\{(1,3),(2,5),(3,7),(4,9)\}$

A relation is a <u>function</u> is when for each x you only have one y value.

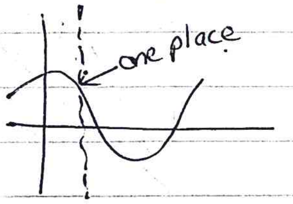




NOT a function



is a function



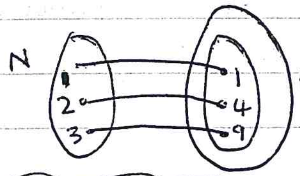
Domain = set of inputs (x's) Codomain = set containing y's

Range = set of outputs (y's) $\subset$ Codomain

$\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{R}\setminus\mathbb{Z}$

$f: \mathbb{N} \rightarrow \mathbb{N} : x \rightarrow x^2$ (input, rule)

TOT 7 p9 Q16



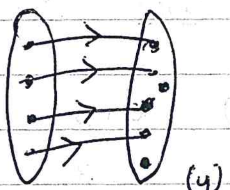
Domain $= \{1,2,3,\ldots\} = \mathbb{N}$, Codomain $= \mathbb{N}$, Range $= \{1,4,9,16 \ldots\}$

Composite functions :

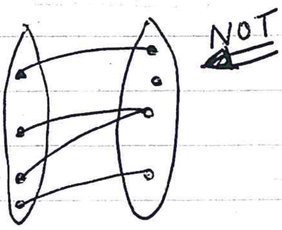
$fg(x) = f(g(x)) = f\circ g(x)$, $f(f(x)) = f^2(x) = f\circ f(x)$

Q10 p12, Q17, pg 13

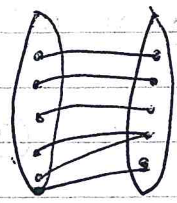
Injective (function)



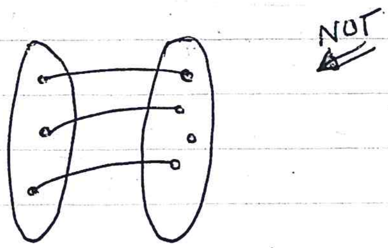
every output has a <u>unique</u> input (x)



Surjective (function)



every element of Codomain is in range

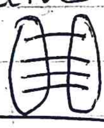


Q19 14, 15 pg 18, 19 TOT 7

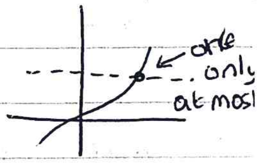
Bijective function

is both surjective <u>and</u> injective

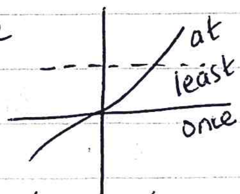
one → one



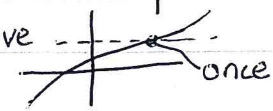
Injective test



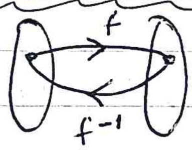
Surjective test



Bijective

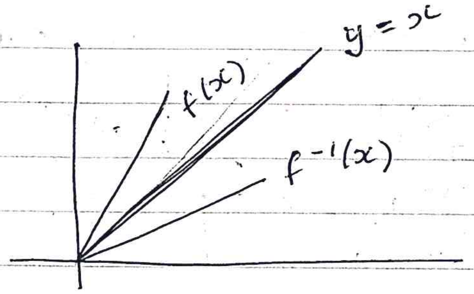


Inverse functions: $f(x) = 3x - 2$, $y = 3x - 2$

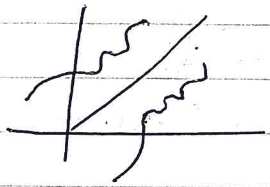


$3x = y + 2$

$x = \frac{y+2}{3} \Rightarrow f^{-1}(x) = \frac{x+2}{3}$

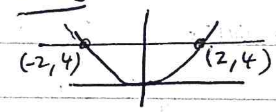


$f^{-1}(x)$ is a reflection of $f(x)$ (in line $y = x$)



* A function only has an inverse iff it is Bijective

e.g x^2



$f^{-1}(x^2)$ will have 2 different o/ps for same i/p so will not be a function.

You could restrict the domain to $\mathbb{R}^+$

Q12, 14, 19 pg 23

Limits, Continuity: when ~~limit~~ $f(a) = \frac{0}{0}$ we cannot say it is defined - $f(x)$ is undefined for $x = a$

We can find the limit as $x \to \infty$ by dividing by the highest power of ~~x~~ x on top & bottom of fraction.

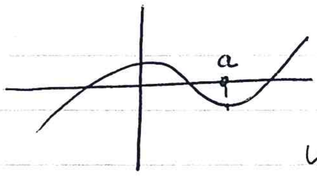
e.g $f(x) = \frac{x^2 - 9}{x - 3}$ $f(3) = \frac{0}{0}$ so it does not exist

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \frac{(x-3)(x+3)}{(x-3)} = x + 3 = 3$$

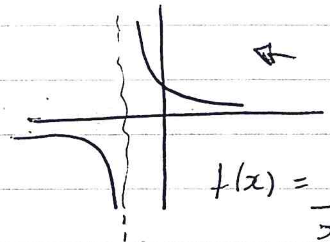
$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{1 - 9/x^2}{\frac{1}{x} - 3/x^2} = \frac{1}{0} = \text{undef}$$ $\boxed{\frac{1}{\infty} \approx 0}$

$$\lim_{x \to \infty} \frac{3x^2 - x}{2x^2 + 3x} = \lim_{x \to \infty} \frac{3 - 1/x}{2 + 3/x} = \frac{3}{2}$$

Continuity



<u>Continuous</u> @ $x = a$ since graph can be drawn thro $(a, f(a))$ without a break

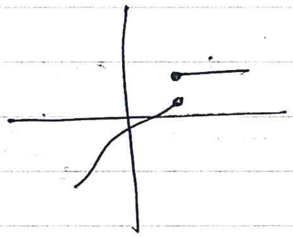


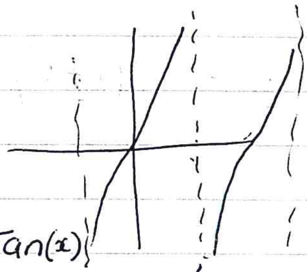
NOT

Continuous

$f(x) = \frac{1}{x+1}$

asymptote





Tan(x)

asymptote

A function is continuous at $x = a$ if $\lim_{x \to a} f(x) = f(a)$

Q2 pg 28, Q13 pg 29, Q7 p29,

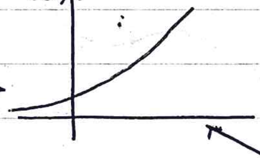
Sketching Graphs :

x^2 $-x^2$ x^3 $-x^3$

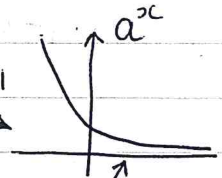
Q9 p35, Q14 p36

Q16, 20 p37 & Q21 ⊛

Exponential : $y = a^x$ $a > 1$

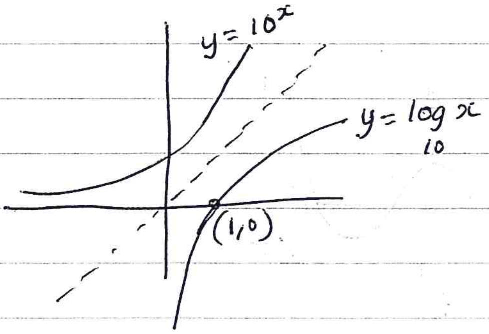


$0 < a < 1$



asymptote

Logs : Inverse function of exponentials



$y = 10^x$

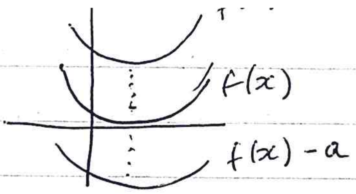
$\log_{10} y = x$ so $f^{-1}(x) = \log_{10} x$

$y = e^x$

$\ln y = x$ so $f^{-1}(x) = \ln x$

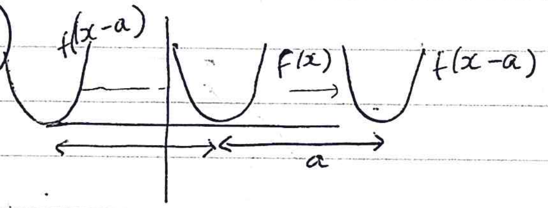
p42 Q3, 4, 15, 19

$y = f(x) \pm a$



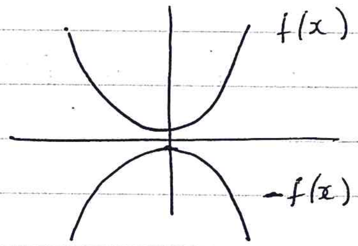
translation ↑ up/down

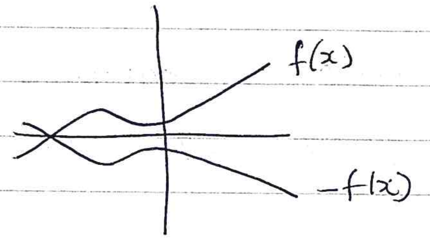
$y = f(x \pm a)$



translation left/right

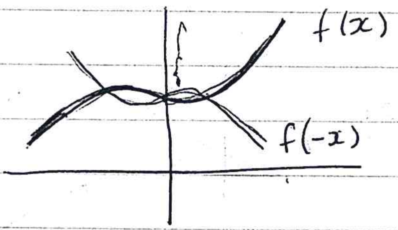
$y = -f(x)$





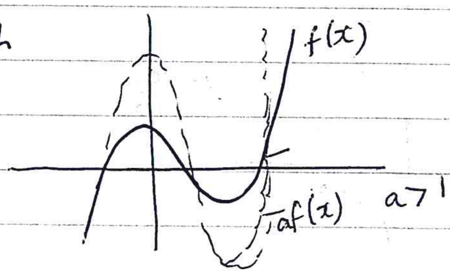
$y = -f(x)$ is a reflection in x axis

$y = f(-x)$



reflection in y axis

$y = af(x)$ Doesn't change where $f(x)$ crosses x axis $[f(x)=0]$
since $a.(0) = 0$
but does stretch the graph



Q4 p47
Q7 p48
Q10 p48