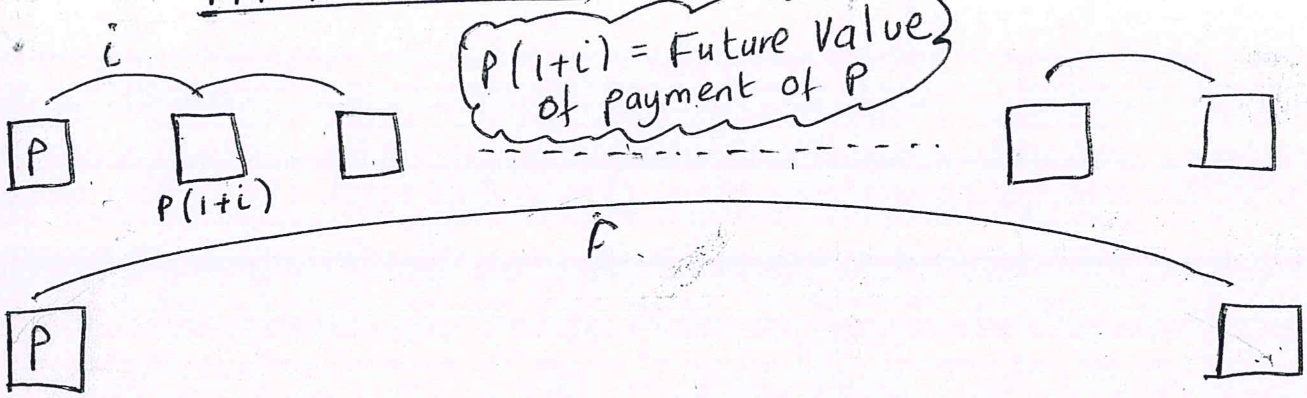
Financial Maths



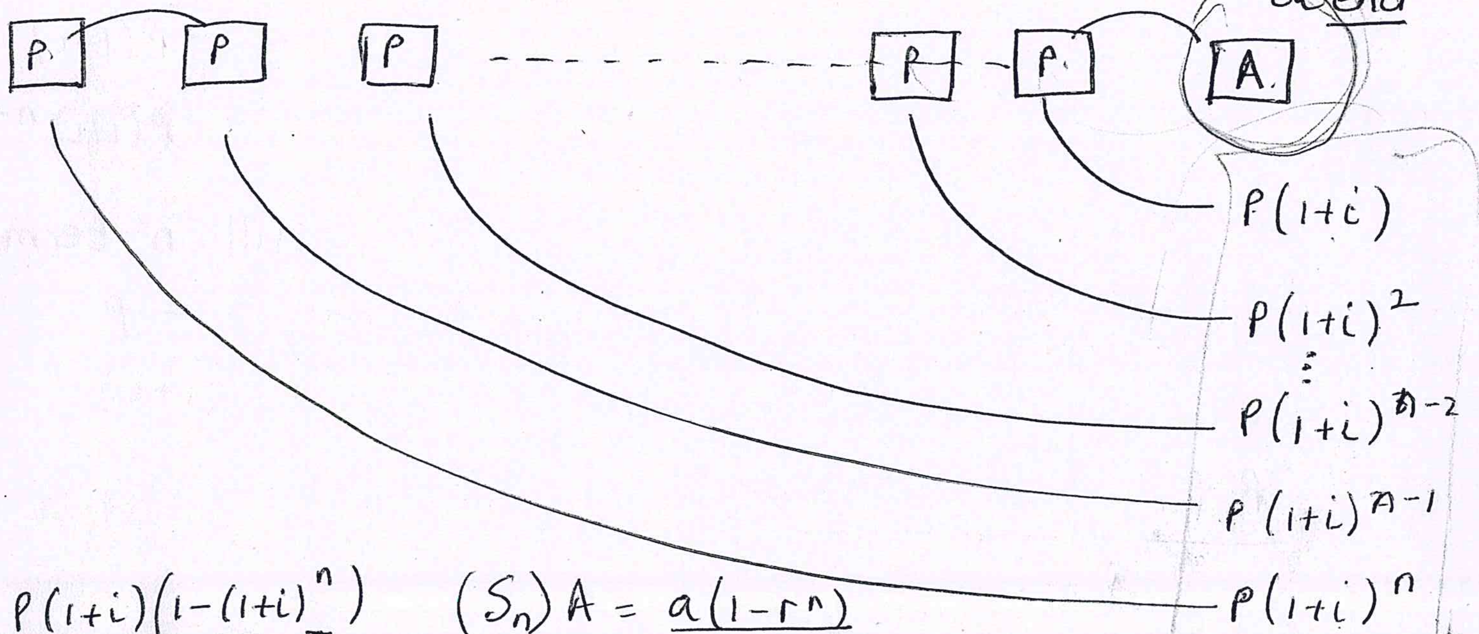
What rate per month is same as r for a year

$$P(1+r) = P(1+i)^{12} \qquad (1+r) = (1+i)^{12}$$

$$\sqrt[12]{1+r} = 1+i \qquad i = (1+r)^{1/12} - 1$$

In reverse: What rate per <u>year</u> is equivelent to i per month $\quad 1+r = (1+i)^{12} \quad r = (1+i)^{12} - 1$

Pensions, Annuities etc: You will put in $\boxed{P}$ at start every time period for a certain no. (n) of time periods to get out A <u>at end</u>



$$= \frac{P(1+i)\left(1-(1+i)^n\right)}{-i}$$

OR

$$= \frac{P(1+i)\left((1+i)^n - 1\right)}{i}$$

$$(S_n)\ A = \frac{a(1-r^n)}{1-r}$$

$$A = \frac{P(1+i)\left(1-(1+i)^n\right)}{1-(1+i)}$$

Don't bother tidying up if you just want A

G. Series
n terms
$a = P(1+i)$
$r = (1+i)$

$$A = \frac{P(1+i)(1-(1+i)^n)}{1-(1+i)} = \frac{P(1+i)(1-(1+i)^n)}{-i}$$

$$\text{So} \quad Ai = P(1+i)((1+i)^n - 1)$$

$$\boxed{P = \frac{Ai}{(1+i)((1+i)^n - 1)}}$$ for payments put in at start of ~~year~~ timeframe

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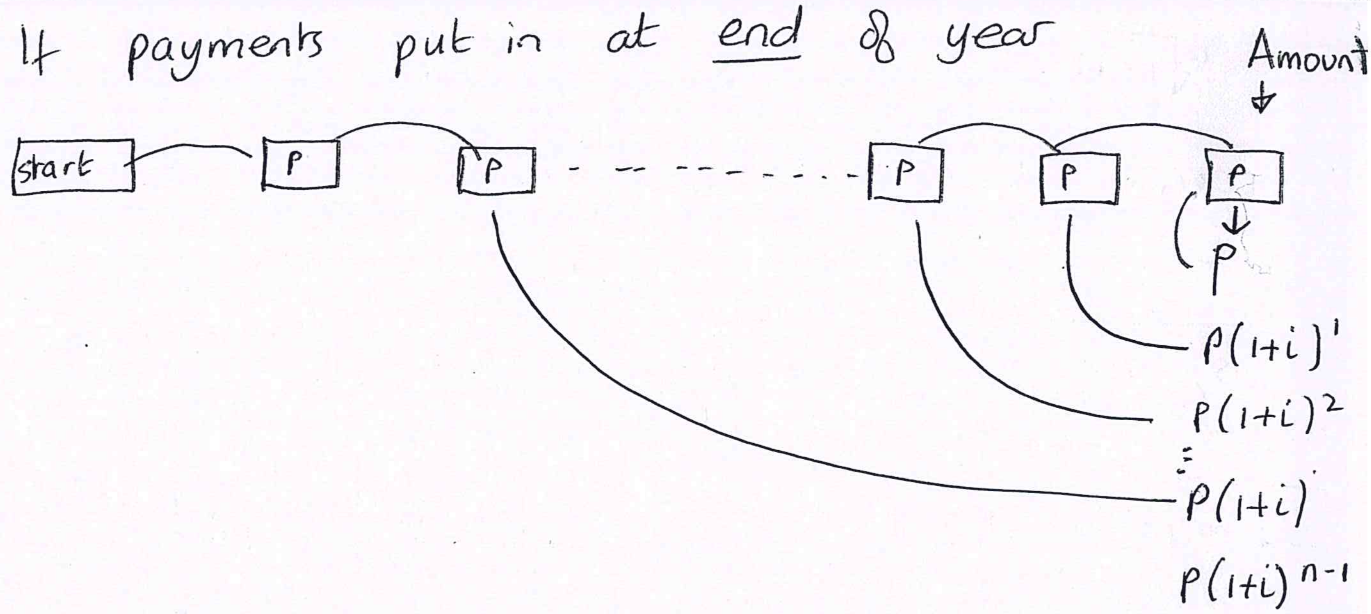
If payments put in at <u>end</u> of year



$$S_n = \frac{P(1-(1+i)^n)}{1-(1+i)} = A$$

still n terms
but $a = P$
$r = (1+i)$

$$S_n = \frac{P(1-(1+i)^n)}{-i} = A$$

$$\boxed{A = \frac{P((1+i)^n - 1)}{i}} \Rightarrow \boxed{P = \frac{Ai}{(1+i)^n - 1}}$$

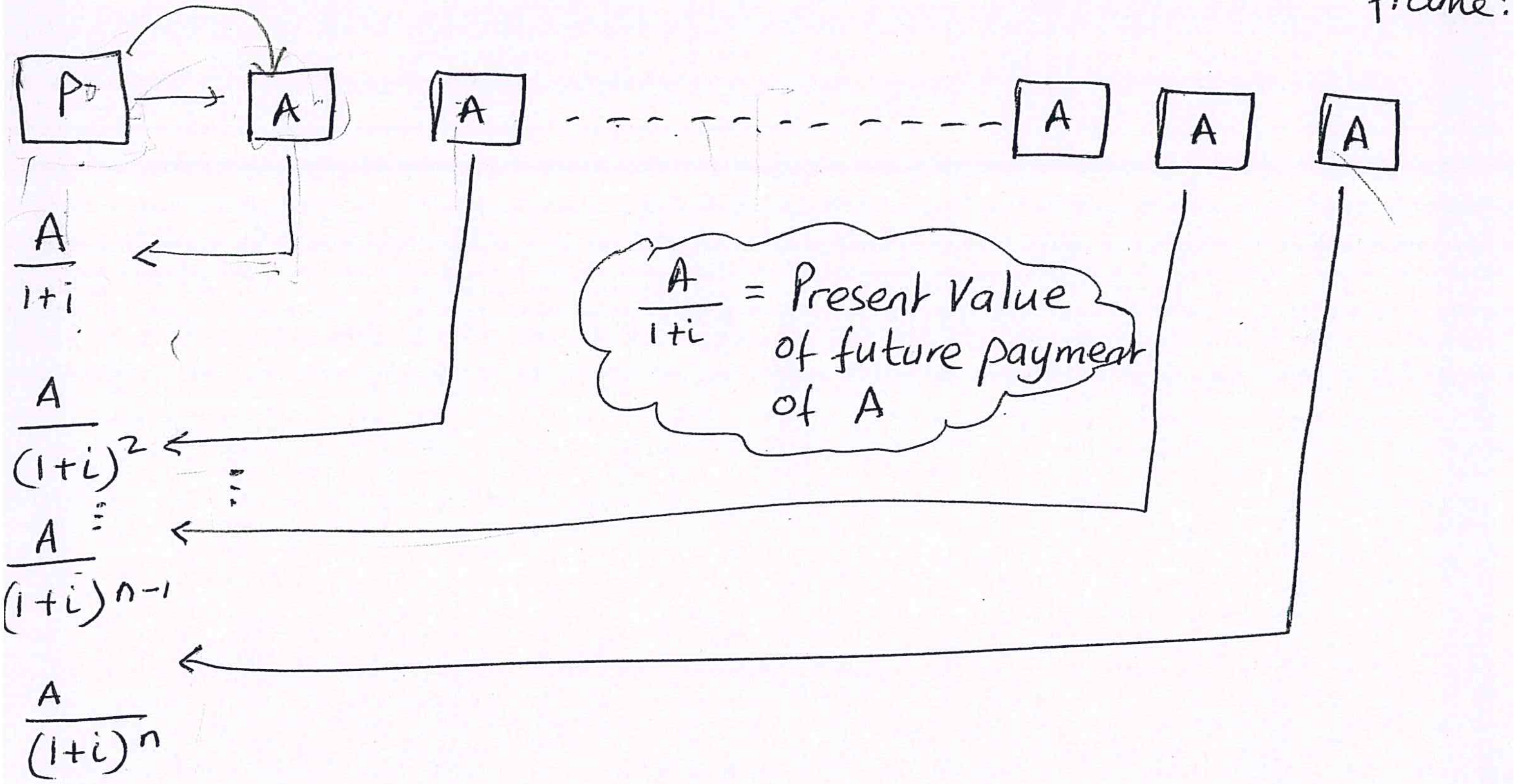
for payments put in at <u>end</u> of timeframe

For a Pension RePayment or Mortgage Repayment or Loan Repayment

You Put in P at the start and you get out

$\boxed{A}$ payments per timeframe over a fixed amount

of timeframes (n) If you receive A at end of each time frame:



So LOAN, Mortgage, Pension Value = P

G. Series

$$P = \frac{A}{1+i} + \frac{A}{(1+i)^2} + \cdots \frac{A}{(1+i)^n}$$

$$\left\{ \begin{array}{l} a = \dfrac{A}{1+i} \\ r = \dfrac{1}{1+i} \end{array} \right. \quad n \text{ terms}$$

$$P(S_n) = \frac{\frac{A}{1+i}\left(1 - \left(\frac{1}{1+i}\right)^n\right)}{1 - \frac{1}{1+i}} = P$$

(Don't bother tidying if you want to find P)

To Derrive Mortgage formula (formula to calc the repayment value A) :

$$P = \frac{\frac{A}{1+i}\left(1-\left(\frac{1}{1+i}\right)^n\right)}{1-\frac{1}{1+i}} \times \frac{1+i}{1+i} = \frac{A\left(1-\frac{1}{(1+i)^n}\right)}{1+i-1}$$

$$P = \frac{A\left(1-\frac{1}{(1+i)^n}\right)}{i}$$

$$A = \frac{Pi}{1-\frac{1}{(1+i)^n}} \times \frac{(1+i)^n}{(1+i)^n}$$

$$A = \frac{Pi(1+i)^n}{(1+i)^n - 1}$$

for Mortgage (P) payments (A) at <u>end</u> of each time frame

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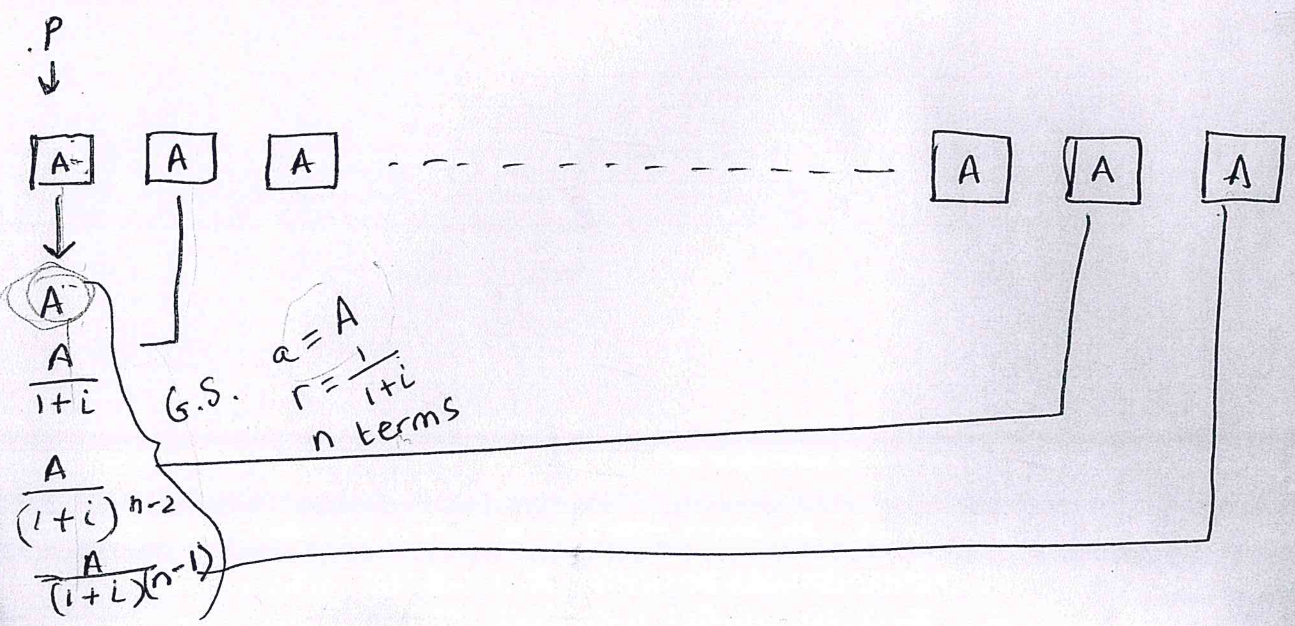
If the Repayments (A) were made at <u>start</u> of every month/timeframe :



Then $S_n(P) = \dfrac{A\left(1 - \dfrac{1}{(1+i)^n}\right)}{1 - \dfrac{1}{1+i}}$ 6

$$P = \frac{A(1+i)\left(1 - \dfrac{1}{(1+i)^n}\right)}{1+i-1}$$

$$P = \frac{A(1+i)\left(1 - \dfrac{1}{(1+i)^n}\right)}{i}$$

$$A = \frac{Pi}{(1+i)\left(1 - \dfrac{1}{(1+i)^n}\right)} \times \frac{(1+i)^n}{(1+i)^n}$$

$$A = \frac{Pi\,(1+i)^n}{(1+i)\left((1+i)^n - 1\right)}$$

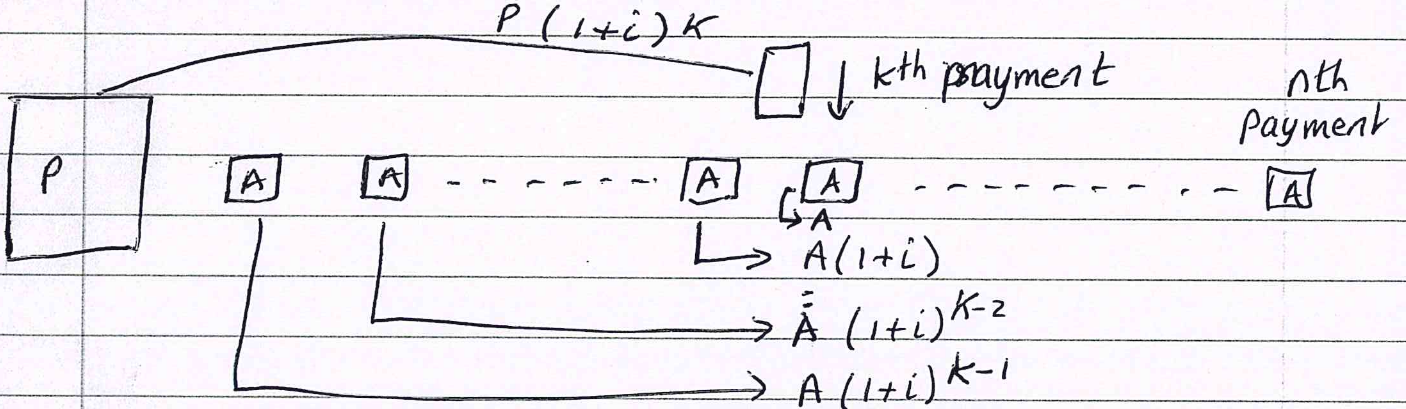
Balance on a loan / Mortgage

If you got a question about a person who wanted to pay the balance of a loan/mortgage after paying k installments then the way you work out the balance is as follows:

You work out the 'future value' at end timeframe k of each of the payments they have made, sum them together as a Geometric Series

Then you work out the 'future value' of the loan P at end of timeframe k $(= P(1+i)^K)$

then you subtract the sum of the G.S. from the value of the loan and that is the balance due on the loan.



G.S. n terms

$a = A$

$r = (1+i)$

$$S_n = \frac{A(1-(1+i)^K)}{1-(1+i)} \quad \frac{\times -1}{\times -1}$$

$$S_n = \frac{A((1+i)^K - 1)}{i}$$

Balance after k payments made at end of each timefram

$$B = P(1+i)^K - \frac{A((1+i)^K - 1)}{i}$$

* note this is also what a person would get if they cashed in their pension after K payments.