L.17/20

Pre-Leaving Certificate Examination, 2014

Mathematics
(Project Maths – Phase 3)

Higher Level

Marking Scheme

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Pre-Leaving Certificate Examination, 2014

Mathematics
(Project Maths – Phase 3)

Higher Level – Paper 1
Marking Scheme (300 marks)

General Instructions

There are **three** sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Structure of the Marking Scheme

Students' responses are marked according to different scales, depending on the types of response anticipated.
Scales labelled A divide students' responses into two categories (correct and incorrect).
Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on.
These scales and the marks that they generate are summarised in the following table:

Scale label	**A**	**B**	**C**	**D**	**E**
No of categories	2	3	4	5	6
5 mark scale		0, 2, 5	0, 2, 4, 5	0, 2, 3, 4, 5	
10 mark scale		0, 5, 10	0, 3, 7, 10	0, 3, 5, 8, 10	
15 mark scale		0, 7, 15	0, 5, 10,15	0, 4, 7, 11, 15	
20 mark scale					

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

E-scales (six categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response almost half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk.
Thus, for example, scale **10C*** indicates that 9 marks may be awarded.

Summary of Marks

Q.1	**(a)**		5C (0, 2, 4, 5)
	(b)		10C (0, 3, 7, 10)
	(c)		10C (0, 3, 7, 10)
			25

Q.2	**(a)**		10C (0, 3, 7, 10)
	(b)	**(i)**	5C (0, 2, 4, 5)
		(ii)	5B (0, 2, 5)
		(iii)	5B (0, 2, 5)
			25

Q.3	**(a)**		10D (0, 3, 5, 8, 10)
	(b)	**(i)**	5B (0, 2, 5)
		(ii)	5C (0, 2, 4, 5)
		(iii)	5C (0, 2, 4, 5)
			25

Q.4	**(a)**		5B (0, 2, 5)
			5B (0, 2, 5)
	(b)		5B (0, 2, 5)
	(c)		10C (0, 3, 7, 10)
			25

Q.5	**(a)**		10C (0, 3, 7, 10)
	(b)	**(i)**	10C (0, 3, 7, 10)
		(ii)	5C (0, 2, 4, 5)
			25

Q.6	**(a)**		10B (0, 5, 10)
	(b)	**(i)**	10C (0, 3, 7, 10)
		(ii)	5C (0, 2, 4, 5)
			25

Q.7	**(a)**	**(i)**	10C (0, 3, 7, 10)
		(ii)	5C (0, 2, 4, 5)
		(iii)	10D (0, 3, 5, 8, 10)
		(iv)	5C (0, 2, 4, 5)
	(b)	**(i)**	5B (0, 2, 5)
		(ii)	15C* (0, 5, 10, 15)
			50

Q.8	**(a)**	**(i)**	5B (0, 2, 5)
		(ii)	10C (0, 3, 7, 10)
		(iii)	5C (0, 2, 4, 5)
		(iv)	5B (0, 2, 5)
	(b)		5C (0, 2, 4, 5)
	(c)	**(i)**	10C (0, 3, 7, 10)
		(ii)	10C (0, 3, 7, 10)
			50

Q.9	**(a)**	**(i)**	5B* (0, 2, 5)
		(ii)	5C (0, 2, 4, 5)
		(iii)	15D (0, 4, 7, 11, 15)
	(b)	**(i)**	10C (0, 3, 7, 10)
		(ii)	10D (0, 3, 5, 8, 10)
		(iii)	5C (0, 2, 4, 5)
			50

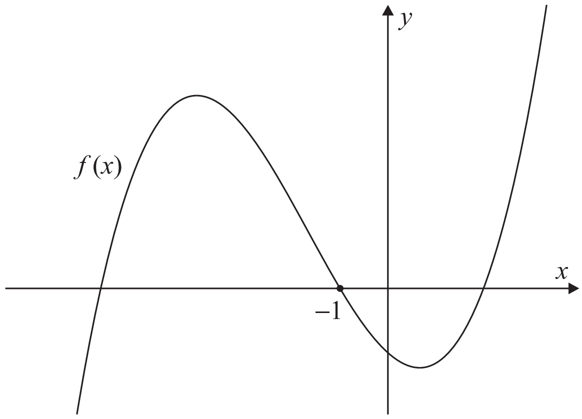
Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

The diagram shows the graph of the cubic function f, defined for $x \in \mathbb{R}$ as

$$f : x \mapsto x^3 + 5x^2 + kx - 12, \quad \text{where } k \text{ is a constant.}$$



1(a) Given that $x + 1$ is a factor of f, find the value of k. **(5C)**

❶ $x + 1$ is a factor of $x^3 + 5x^2 + kx - 12$

$$\begin{aligned}
\Rightarrow\quad & f(-1) &= 0 \\
\Rightarrow\quad & (-1)^3 + 5(-1)^2 + k(-1) - 12 &= 0 \\
\Rightarrow\quad & -1 + 5 - k - 12 &= 0 \\
\Rightarrow\quad & -8 - k &= 0 \\
\Rightarrow\quad & -k &= 8 \\
\Rightarrow\quad & k &= -8
\end{aligned}$$

<u>or</u>

❷

$$\begin{array}{r l}
 & x^2 + 4x + k - 4 \\ \hline
x + 1 \,\big) & x^3 + 5x^2 + kx - 12 \\
 & \underline{-x^3 - 1x^2} \\
 & 4x^2 + kx \\
 & \underline{-4x^2 - 4x} \\
 & (k-4)x - 12 \\
 & \underline{-(k-4)x - (k-4)} \\
 & 0
\end{array}$$

$$\begin{aligned}
\Rightarrow\quad & -(k-4) - 12 &= 0 \\
\Rightarrow\quad & -k + 4 - 12 &= 0 \\
\Rightarrow\quad & -k - 8 &= 0 \\
\Rightarrow\quad & -k &= 8 \\
\Rightarrow\quad & k &= -8
\end{aligned}$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	– –	States $x = -1$ is a root. Finds x^2 in division.
High partial credit: (4 marks)	– –	Substitutes $x = -1$ correctly. Long division correct.

1(b) Hence, or otherwise, find the co-ordinates of the other two points at which f cuts the x-axis. **(10C)**

❶ f cuts the x-axis

$$\begin{aligned}
\Rightarrow\quad & f(x) &=\ & 0\\
\Rightarrow\quad & x^3+5x^2-8x-12 &=\ & 0
\end{aligned}$$

$$\begin{array}{r|l}
 & x^2+4x-12 \\ \hline
x+1 & x^3+5x^2-8x-12 \\
 & \underline{-x^3-1x^2} \\
 & 4x^2-8x \\
 & \underline{-4x^2-4x} \\
 & -12x-12 \\
 & \underline{-12x-12} \\
 & 0
\end{array}$$

$$\begin{aligned}
\Rightarrow\quad & (x+1)(x^2+4x-12) &=\ & 0\\
\Rightarrow\quad & (x+1)(x-2)(x+6) &=\ & 0\\
\Rightarrow\quad & x-2 &=\ & 0\\
\Rightarrow\quad & x &=\ & 2\\
\Rightarrow\quad & x+6 &=\ & 0\\
\Rightarrow\quad & x &=\ & -6
\end{aligned}$$

$\Rightarrow$ f cuts the x-axis at (2, 0) and (–6, 0)

<u>or</u>

❷

$$\begin{aligned}
\Rightarrow\quad & f(x) &=\ & x^3+5x^2-8x-12\\
& f(x) &=\ & (x+1)(x^2+bx+c)\\
& &=\ & x^3+bx^2+cx+x^2+bx+c\\
& &=\ & x^3+x^2(b+1)+x(b+c)+c
\end{aligned}$$

Equating x^2 terms in both equations:

$$\begin{aligned}
& b+1 &=\ & 5\\
\Rightarrow\quad & b &=\ & 4
\end{aligned}$$

Equating x terms in both equations:

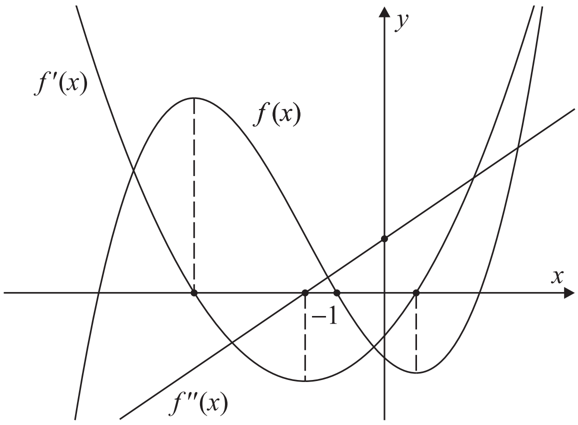
$$\begin{aligned}
\Rightarrow\quad & b+c &=\ & -8\\
\Rightarrow\quad & 4+c &=\ & -8\\
\Rightarrow\quad & c &=\ & -12\\
\Rightarrow\quad & f(x) &=\ & (x+1)(x^2+bx+c)\\
& &=\ & 0\\
\Rightarrow\quad & (x+1)(x-2)(x+6) &=\ & 0\\
\Rightarrow\quad & x-2 &=\ & 0\\
\Rightarrow\quad & x &=\ & 2\\
\Rightarrow\quad & x+6 &=\ & 0\\
\Rightarrow\quad & x &=\ & -6
\end{aligned}$$

$\Rightarrow$ f cuts the x-axis at (2, 0) and (–6, 0)

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any reasonable first step.
High partial credit: (7 marks)	– –	Co-ordinates of one point found. Two correct factors but co-ordinates of points not written down.

1(c) Sketch the graphs of $y = f'(x)$ and $y = f''(x)$, on the diagram above. **(10C)**



Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	– –	Any relevant first step. Finds $f'(x)$ <u>or</u> $f''(x)$ <u>or</u> both but no graphs.
High partial credit: (7 marks)	– –	Correct graph of $f'(x)$ <u>or</u> $f''(x)$. Correct shaped graphs not fully accurate (with minor errors).

Question 2 **(25 marks)**

2(a) $2 - 3i$, $6 + 4i$ and $-8 + pi$ are consecutive terms of a geometric sequence, where $p \in \mathbb{Z}$ and $i^2 = -1$.
Find the value of p. **(10C)**

❶ $2 - 3i$, $6 + 4i$ and $-8 + pi$ are consecutive terms of a geometric sequence

$$
\begin{aligned}
\Rightarrow\quad \frac{T_n}{T_{n-1}} &= \frac{T_{n+1}}{T_n} \\
\Rightarrow\quad \frac{6+4i}{2-3i} &= \frac{-8+pi}{6+4i} \\
\Rightarrow\quad (2-3i)(-8+pi) &= (6+4i)(6+4i) \\
\Rightarrow\quad -16 + 2pi + 24i - 3pi^2 &= 36 + 24i + 24i + 16i^2 \\
\Rightarrow\quad 3p - 16 + (2p + 24)i &= 36 - 16 + 48i \\
\Rightarrow\quad 3p - 16 &= 36 - 16 \\
\Rightarrow\quad 3p &= 36 \\
\Rightarrow\quad p &= 12
\end{aligned}
$$

or

$$
\begin{aligned}
\Rightarrow\quad 2p + 24 &= 48 \\
\Rightarrow\quad 2p &= 48 - 24 \\
&= 24 \\
\Rightarrow\quad p &= 12
\end{aligned}
$$

or

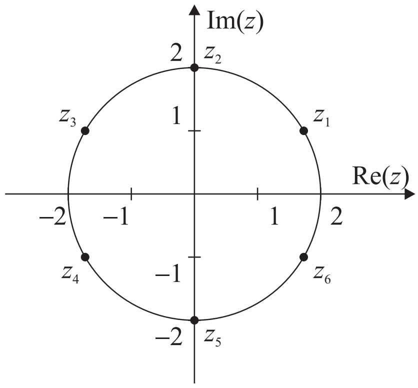
❷ $2 - 3i$, $6 + 4i$ and $-8 + pi$ are consecutive terms of a geometric sequence

$$
\begin{aligned}
\Rightarrow\quad r &= \frac{T_n}{T_{n-1}} \\
&= \frac{6+4i}{2-3i} \\
&= \frac{6+4i}{2-3i} \times \frac{2+3i}{2+3i} \\
&= \frac{12 + 18i + 8i + 12i^2}{4 + 6i - 6i - 9i^2} \\
&= \frac{26i}{13} \\
&= 2i \\
\Rightarrow\quad r &= \frac{T_{n+1}}{T_n} \\
\Rightarrow\quad T_{n+1} &= r(T_n) \\
\Rightarrow\quad -8 + pi &= 2i(6 + 4i) \\
&= 12i + 8i^2 \\
\Rightarrow\quad &= -8 + 12i \\
\Rightarrow\quad p &= 12
\end{aligned}
$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any relevant first step, *e.g.* definition of G.P. or $r = \frac{T_{n+1}}{T_n}$ or equivalent.
High partial credit: (7 marks)	–	Ratio found correctly ($2i$) or $(2 - 3i)(-8 + pi) = (6 + 4i)(6 + 4i)$.

2(b) z is a complex number such that $z^6 = a + bi$, where $i^2 = -1$.
The Argand diagram shows the six roots of z^6.



(i) Given that $z_1 = \sqrt{3} + i$ is one root, complete the table below to show the other roots of z^6. **(5C)**

z_1	z_2	z_3	z_4	z_5	z_6
$\sqrt{3} + i$	$0 + 2i$	$-\sqrt{3} + i$	$-\sqrt{3} - i$	$0 - 2i$	$\sqrt{3} - i$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	One correct root found.
High partial credit: (4 marks)	–	Only one incorrect root.

2(b) **(ii)** Write z_1 in polar form. **(5B)**

$$\sqrt{3} + i = r(\cos\theta + i\sin\theta)$$

$$r = \sqrt{(1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\tan\theta = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\Rightarrow \quad \theta = \frac{\pi}{6} \ \underline{\text{or}}\ 30^\circ$$

$$\Rightarrow \quad \sqrt{3} + i = r(\cos\theta + i\sin\theta) = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}) \ \underline{\text{or}}\ 2(\cos 30^\circ + i\sin 30^\circ)$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	r <u>or</u> θ calculated correctly.
High partial credit: (4 marks)	–	r <u>and</u> θ calculated correctly but not finished off.

2(b) (iii) Use De Moivre's theorem to find the value of *a* and the value of *b*. **(5B)**

$$
\begin{array}{llcl}
 & z^6 & = & a + bi \\
\Rightarrow & a + bi & = & [2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})]^6 \\
 & & = & 2^6(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}) \\
 & & = & 64(-1 + 0i) \\
 & & = & -64 + 0i \\
\Rightarrow & a & = & -64 \\
 & b & = & 0
\end{array}
$$

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Some correct work with De Moivre's theorem, *e.g.* $2^6(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$.

Question 3 **(25 marks)**

3(a) Solve the inequality $\frac{x+3}{x-4} < 2$, where $x \in \mathbb{R}$ and $x \neq 4$. **(10D)**

$$
\begin{array}{llll}
 & \frac{x+3}{x-4} & < & 2 \\
\Rightarrow & \frac{x+3}{x-4}(x-4)^2 & < & 2(x-4)^2 \\
\Rightarrow & (x+3)(x-4) & < & 2(x^2-8x+16) \\
\Rightarrow & x^2-x-12 & < & 2x^2-16x+32 \\
\Rightarrow & x^2-15x+44 & > & 0 \\
\Rightarrow & (x-4)(x-11) & > & 0
\end{array}
$$

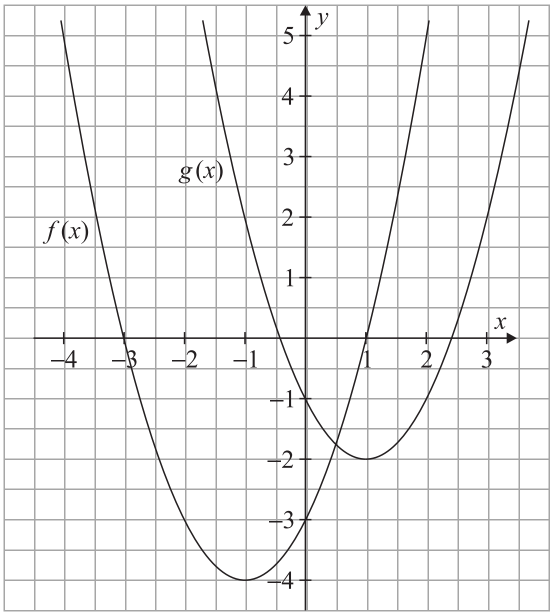
Consider:

$$
\begin{array}{llll}
 & (x-4)(x-11) & = & 0 \\
\Rightarrow & x-4 & = & 0 \\
\Rightarrow & x & = & 4 \\
\Rightarrow & x-11 & = & 0 \\
\Rightarrow & x & = & 11 \\
 & x^2-15x+44 & > & 0 \\
\Rightarrow & x & < & 4 \\
\Rightarrow & x & > & 11
\end{array}
$$

Scale 10D (0, 3, 5, 8, 10)

Low partial credit: (3 marks)	– –	Any relevant first step. Finds particular values of x for which the inequality is true.
Middle partial credit: (5 marks)	– –	x – 4 >0 only considered leading to x > 11 Multiplies by $(x-4)^2$ but errors in subsequent work.
High partial credit: (8 marks)	– –	Finds correct values of x (4 and 11) but range not stated <u>or</u> incorrect range. One case correct only, i.e. $x < 4$ <u>or</u> $x > 11$.

3(b) The diagram shows the graph of the function $f : x \mapsto x^2 + 2x - 3$.
$g(x)$ is the image of $f(x)$ by the translation $(0, 0) \rightarrow (2, 2)$.



(i) Draw a sketch of the curve $y = g(x)$ on the same diagram. **(5B)**

$$\Rightarrow \begin{array}{lll} (0, 0) & \rightarrow & (2, 2) \\ (-1, -4) & \rightarrow & (1, -2) \\ (-3, 0) & \rightarrow & (-1, 2) \\ (1, 0) & \rightarrow & (3, 2) \end{array}$$

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Any correct relevant step, e.g. image of (–3, 0) under t: $(0, 0) \rightarrow (2, 2)$.
	–	A U-shaped graph drawn with specific points indicated.

3(b) **(ii)** Express $g(x)$ in the form $(x + a)^2 + b$, where a and b are constants. **(5C)**

$$\begin{aligned} f(x) &= x^2 + 2x - 3 \\ &= x^2 + 2x + 1 - 1 - 3 \\ &= (x + 1)^2 - 4 \qquad \text{min. point } (-1, -4) \text{ (vertex)} \end{aligned}$$

$$\Rightarrow \begin{array}{lll} (0, 0) & \rightarrow & (2, 2) \\ (-1, -4) & \rightarrow & (-1 + 2, -4 + 2) \\ & & (1, -2) \qquad \text{min. point } (1, -2) \text{ (vertex)} \end{array}$$

$$\begin{array}{lll} f(x) & \rightarrow & g(x) \\ \Rightarrow g(x) & = & (x - 1)^2 - 2 \end{array}$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any relevant first step, *e.g.* substituting (1, –2) into $g(x)$.
High partial credit: (4 marks)	–	Two correct equations in a and b.
	–	Finds $f(x) = (x + 1)^2 - 4$ with some work with translating.

3(b) (iii) Hence, or otherwise, solve $g(x) = 0$. **(5C)**

$$g(x) = (x-1)^2 - 2 = 0$$
$$\Rightarrow (x-1)^2 = 2$$
$$\Rightarrow x - 1 = \pm\sqrt{2}$$
$$\Rightarrow x = 1 + \sqrt{2} \quad \underline{\text{and}} \quad x = 1 - \sqrt{2}$$

Scale 5C (0, 2, 4, 5)

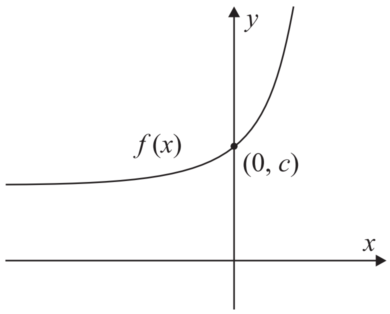
Low partial credit: (2 marks)	–	Any relevant first step, *e.g.* simplifies $g(x)$ to $x^2 - 2x - 1$.
High partial credit: (4 marks)	–	Finds only correct solution.

Question 4 **(25 marks)**

The exponential function f is defined as

$$f : \mathbb{R} \to \mathbb{R} : x \mapsto e^x + 2.$$

The diagram shows the part of the graph where f intersects the y-axis at $(0, c)$.



4(a) Determine the value of c and the range of f. **2 × (5B)**

❶ <u>Value of c</u>

$$\begin{array}{llcl} & f(x) & = & e^x + 2 \\ & (0, c) & \in & f(x) \\ \Rightarrow & e^0 + 2 & = & c \\ \Rightarrow & 1 + 2 & = & c \\ \Rightarrow & c & = & 3 \end{array}$$

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Any relevant first step, *e.g.* $x = 0$ <u>or</u> equivalent.

❷ <u>Range of f</u>

Range of f $=$ $(2, \infty)$ <u>or</u> $y > 2$

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	2 <u>or</u> ∞ written down.

4(b) State whether or not f is bijective. Give a reason for your answer. **(5B)**

Answer: f is <u>not</u> bijective

Reason: it is domain to $\mathbb{R}$ - only bijective when codomain restricted to $y > 2$

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Correct states f is not bijective but no reason given.
	–	States f is both injective and subjective.

4(c) Find the co-ordinates of the point where the tangent to the curve at $(0, c)$ crosses the x-axis. **(10C)**

	$f(x)$	$=$	$e^x + 2$
$\Rightarrow$	$f'(x)$	$=$	e^x
$\Rightarrow$	$f'(0)$	$=$	e^0
$\Rightarrow$	slope	$=$	1

<u>Equation of the tangent:</u>

	$y - y_1$	$=$	$m(x - x_1)$
$\Rightarrow$	$y - 3$	$=$	$1(x - 0)$
$\Rightarrow$	$x - y$	$=$	-3

Tangent crosses the x-axis @ $y = 0$

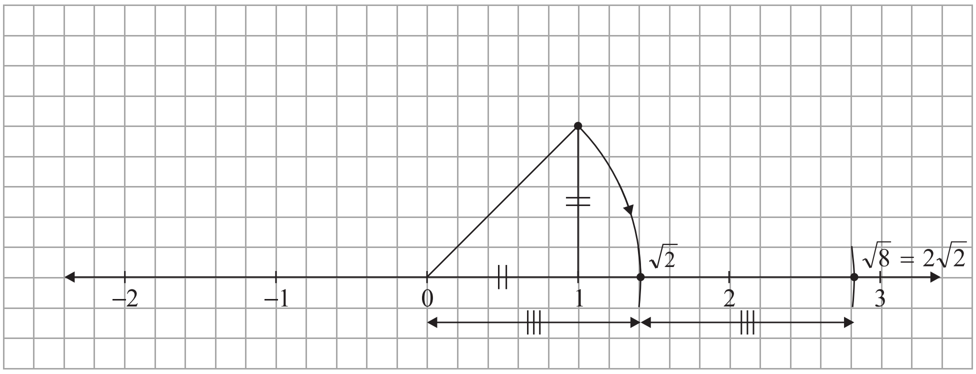
$\Rightarrow$	$x - 0$	$=$	-3
$\Rightarrow$	x	$=$	-3
$\Rightarrow$	co-ordinates of point: $(3, 0)$		

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any relevant first step.
	–	Finds $f'(x)$ <u>and stops</u>.
	–	$y - 3 = m(x - 0)$.
High partial credit: (7 marks)	–	Finds correct equation of tangent <u>and stops</u>.

Question 5 **(25 marks)**

5(a) Giving the number line below an appropriate scale, show, by construction, the position of $\sqrt{2}$ on it, using only a compass and straight edge. Hence, indicate $\sqrt{8}$ on the number line. **(10C)**



$$\begin{aligned}\sqrt{8} &= \sqrt{4} \times \sqrt{2} \\ &= 2\sqrt{2}\end{aligned}$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any relevant first step.
	–	Right-angled triangle constructed with sides of lengths 1, 1 and $\sqrt{2}$ indicated, drawn with ruler and protractor.
	–	Draws, using a protractor and ruler, a triangle with sides 1, 1 and $\sqrt{2}$, based on $\cos 45° = \dfrac{1}{\sqrt{2}}$.
High partial credit: (7 marks)	–	$\sqrt{2}$ correctly constructed and position shown on the number line <u>and stops</u>.

5(b) In an arithmetic sequence, the third term is $3\sqrt{2}+11$ and the sixth term is $3\sqrt{8}+20$.

(i) Find the first term and the common difference. **(10C)**

$$
\begin{array}{llcl}
 & T_n & = & a+(n-1)d \\
\Rightarrow & T_3 & = & a+2d \\
 & & = & 3\sqrt{2}+11 \\
\Rightarrow & a+2d & = & 3\sqrt{2}+11 \\
\Rightarrow & T_6 & = & a+5d \\
 & & = & 3\sqrt{8}+20 \\
\Rightarrow & a+5d & = & 3\sqrt{8}+20 \\
 & a+5d & = & 3\sqrt{8}+20 \\
 & a+2d & = & 3\sqrt{2}+11\ (\times -1) \\
\hline
 & a+5d & = & 3\sqrt{8}+20 \\
 & -a-2d & = & -3\sqrt{2}-11 \\
\hline
 & 3d & = & 3\sqrt{8}+20-3\sqrt{2}-11 \\
 & & = & 3(2\sqrt{2})+9-3\sqrt{2} \\
 & & = & 3\sqrt{2}+9 \\
\Rightarrow & d & = & \sqrt{2}+3 \\
 & a+2d & = & 3\sqrt{2}+11 \\
\Rightarrow & a+2(\sqrt{2}+3) & = & 3\sqrt{2}+11 \\
\Rightarrow & a & = & 3\sqrt{2}+11-2(\sqrt{2}+3) \\
 & & = & \sqrt{2}+5
\end{array}
$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any relevant first step, *e.g.* $T_3 = a+2d$ <u>or</u> $T_3 = a+5d$ (<u>or</u> both) <u>or</u> some relevant substitution into $T_n = a+(n-1)d$.
High partial credit: (7 marks)	–	Two correct equations in terms of a and d.

5(b) (ii) How many terms of the sequence are less than 100? **(5C)**

$$
\begin{array}{llcl}
 & T_n & < & 100 \\
\Rightarrow & a+(n-1)d & < & 100 \\
\Rightarrow & \sqrt{2}+5+(n-1)(\sqrt{2}+3) & < & 100 \\
\Rightarrow & \sqrt{2}+5+n(\sqrt{2}+3)-\sqrt{2}-3 & < & 100 \\
\Rightarrow & 2+n(\sqrt{2}+3) & < & 100 \\
\Rightarrow & n(\sqrt{2}+3) & < & 98 \\
\Rightarrow & n & < & \dfrac{98}{\sqrt{2}+3} \\
\Rightarrow & n & < & \dfrac{98}{\sqrt{2}+3}\times\dfrac{\sqrt{2}-3}{\sqrt{2}-3} \\
 & & < & 42-14\sqrt{2} \\
 & & < & 22.201010... \\
\Rightarrow & \text{no. of terms less than 100} & = & 22
\end{array}
$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any relevant step, *e.g.* $a+(n-1)d < 100$ <u>or</u> $T_n = \sqrt{2}+5+(n-1)(\sqrt{2}+3)$.
High partial credit: (4 marks)	–	$n = \dfrac{98}{\sqrt{2}+3}$ <u>or</u> equivalent <u>and stops</u>.

Question 6 **(25 marks)**

6(a) Differentiate $\sqrt{e^x}$ with respect to x. **(10B)**

$$f(x) = \sqrt{e^x}$$
$$= (e^x)^{\frac{1}{2}}$$
$$= e^{\frac{x}{2}}$$
$$\Rightarrow \quad f'(x) = e^{\frac{x}{2}} \times \frac{1}{2}$$
$$= \frac{1}{2}\sqrt{e^x}$$

Scale 10B (0, 5, 10)

Partial credit: (5 marks)	–	Any relevant first step, *e.g.* $f(x) = e^{\frac{x}{2}}$.
	–	Some correct differentiation.

6(b) **(i)** Given that $y = x\sin 2x$, $x \in \mathbb{R}$, find $\frac{dy}{dx}$. **(10C)**

$$y = x\sin 2x$$
$$\frac{dy}{dx} = x(2\cos 2x) + \sin 2x(1)$$
$$= 2x\cos 2x + \sin 2x$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Some correct differentiation.
High partial credit: (7 marks)	–	Product rule applied correctly but one error in differentiation.

6(b) (ii) Using your answer to part **(i)**, or otherwise, find $\displaystyle\int_0^{\frac{\pi}{2}} 2x\cos 2x\,dx$. **(5C)**

❶

$$\begin{aligned}
\frac{dy}{dx} &= 2x\cos 2x + \sin 2x \\
\Rightarrow \int_0^{\frac{\pi}{2}} \frac{dy}{dx}\,dx &= \int_0^{\frac{\pi}{2}} 2x\cos 2x\,dx + \int_0^{\frac{\pi}{2}} \sin 2x\,dx \\
\Rightarrow \int_0^{\frac{\pi}{2}} dy &= \int_0^{\frac{\pi}{2}} 2x\cos 2x\,dx + \int_0^{\frac{\pi}{2}} \sin 2x\,dx \\
\Rightarrow x\sin 2x \Big|_0^{\frac{\pi}{2}} &= \int_0^{\frac{\pi}{2}} 2x\cos 2x\,dx - \frac{\cos 2x}{2}\Big|_0^{\frac{\pi}{2}} \\
\Rightarrow \int_0^{\frac{\pi}{2}} 2x\cos 2x\,dx &= x\sin 2x \Big|_0^{\frac{\pi}{2}} + \frac{\cos 2x}{2}\Big|_0^{\frac{\pi}{2}} \\
&= \frac{\pi}{2}\sin\pi - 0 + \frac{\cos\pi}{2} - \frac{\cos 0}{2} \\
&= 0 - 0 - \frac{1}{2} - \frac{1}{2} \\
&= -1
\end{aligned}$$

<u>or</u>

** Substitution method not on syllabus but accept if given.

❷ $\displaystyle\int_0^{\frac{\pi}{2}} 2x\cos 2x\,\mathrm{d}x$:

$$\begin{aligned}
\int u\,dv &= uv - \int v\,du \\
\text{Let } u &= 2x \\
\Rightarrow \frac{du}{dx} &= 2 \\
\Rightarrow du &= 2\,dx \\
\text{Let } dv &= \cos 2x\,dx \\
\Rightarrow \int dv &= \int \cos 2x\,dx \\
\Rightarrow v &= \frac{\sin\ 2x}{2}
\end{aligned}$$

6(b) (ii) (Cont'd.)

$$\Rightarrow \int 2x\cos 2x\,\mathrm{d}x = (2x)\frac{\sin\ 2x}{2} - \frac{1}{2}\int \sin 2x.2\,dx$$

$$= x\sin 2x - \int \sin 2x dx$$

$$= x\sin 2x + \frac{\cos 2x}{2}$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} 2x\cos 2x\,dx = x\sin 2x\,\Big|_0^{\frac{\pi}{2}} + \frac{\cos 2x}{2}\,\Big|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}\sin\pi - 0 + \frac{\cos\pi}{2} - \frac{\cos 0}{2}$$

$$= 0 - 0 - \frac{1}{2} - \frac{1}{2}$$

$$= -1$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	– –	Any relevant first step. Some correct relevant integration.
High partial credit: (4 marks)	–	Correct integration ($x\sin 2x + \frac{\cos 2x}{2}$) but limits not evaluated <u>or</u> evaluated incorrectly.

DEB exams

Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section.

Question 7 **(50 marks)**

7(a) The pressure that a scuba diver encounters at a certain depth is the sum of the atmospheric and the water pressures above the diver. *Atmospheric pressure* is the pressure exerted by the air in the earth's atmosphere. *Water pressure* is the pressure exerted on the body due to the weight of water above the diver.

The pressure, P, experienced by a scuba diver is a function of the diver's depth and is given by the formula:

$$P(d) = kd + c$$

where P is the pressure on the diver in atmospheres, d is the depth of the diver below the surface of the water, in metres, and k and c are constants.
The pressure is 1 atmosphere at the surface of the water and 4 atmospheres at a depth of 30 m.

(i) Find the value of c and the value of k. **(10C)**

$$P(d) = kd + c$$

@ the surface ($d = 0$), pressure = 1 atm.

$$\begin{aligned}
\Rightarrow\quad 1 &= k(0) + c \\
\Rightarrow\quad 1 &= 0 + c \\
\Rightarrow\quad c &= 1
\end{aligned}$$

@ depth of 30 m, pressure = 4 atm.

$$\begin{aligned}
\Rightarrow\quad 4 &= k(30) + 1 \\
\Rightarrow\quad 4 &= 30k + 1 \\
\Rightarrow\quad 30k &= 4 - 1 \\
&= 3 \\
\Rightarrow\quad k &= \frac{3}{30} \\
&= \frac{1}{10}
\end{aligned}$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	– –	Any relevant first step. Finds value for c correctly.
High partial credit: (7 marks)	–	Finds correct linear equation involving k and stops or k not found correctly.

Diving instructors recommend that divers do not descend at a constant rate. The rate of descent depends on both a diver's depth and his/her experience. The recommended rate of descent specifies the maximum depth, in metres, which a diver should be at after a given time, and is given by:

$$d(t) = 1 + 10\log_e(nt)$$

where d is the depth below the surface, in metres, n is the number of dives completed in the previous 12 months and t is the time, in minutes, from the instant the diver submerges.

7(a) **(ii)** Assuming that a diver descends at the recommended rate, show that the pressure experienced by the diver is $P(t) = 1{\cdot}1 + \log_e(nt)$. **(5C)**

$$\begin{aligned}
P(d) &= kd + c \\
&= \frac{1}{10}d + 1 \\
\Rightarrow \quad P(t) &= \frac{1}{10}[1 + 10\log_e(nt)] + 1 \\
&= \frac{1}{10} + \log_e(nt) + 1 \\
&= \frac{11}{10} + \log_e(nt) \\
&= \frac{11}{10} + \log_e(nt) \\
&= 1{\cdot}1 + \log_e(nt)
\end{aligned}$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any relevant first step, *e.g.* $P(d) = k[1 + 10\log_e(nt)] + c$.
High partial credit: (4 marks)	–	Fully correct work but unable to get required answer due to errors in part (i).

7(a) (iii) Find, in terms of n, the rate of change of pressure, with respect to time, that the diver would experience descending at the recommended rate at her ultimate dive depth of 21 m. **(10D)**

$$P(t) = 1\cdot 1 + \log_e(nt)$$

$$\Rightarrow \frac{dP}{dt} = \frac{1}{nt} \times n$$

$$= \frac{1}{t}$$

@ depth of 21 m

$$d(t) = 1 + 10\log_e(nt)$$

$$\Rightarrow 1 + 10\log_e(nt) = 21$$

$$\Rightarrow 10\log_e(nt) = 21 - 1$$

$$= 20$$

$$\Rightarrow \log_e(nt) = \frac{20}{10}$$

$$= 2$$

$$\Rightarrow nt = e^2$$

$$\Rightarrow t = \frac{e^2}{n}$$

$$\Rightarrow \frac{dP}{dt} = \frac{1}{\frac{e^2}{n}}$$

$$= \frac{n}{e^2}$$

Scale 10D (0, 3, 5, 8, 10)

Low partial credit: (3 marks)	–	Any relevant first step, *e.g.* some correct differentiation or $1 + 10\log_e(nt) = 21$.
Middle partial credit: (5 marks)	–	Correct differentiation, *i.e.* $\frac{dP}{dt} = \frac{1}{t}$.
High partial credit: (8 marks)	–	Correct differentiation and $t = \frac{e^2}{n}$ found correctly and stops or not completed correctly.

7(a) (iv) If the diver ascends too quickly to the surface, she can be susceptible to decompression sickness, commonly known as the 'bends', due to the excessive rate of change of pressure as she rises.

By considering the difference in the rate of change of pressure between different stages of the diver's ascent, determine when she is most susceptible to decompression sickness. **(5C)**

@ depth of 21 m

$$\frac{dP}{dt} = \frac{n}{e^2} = 0{\cdot}135335\ldots n \cong 0{\cdot}135n$$

@ depth of 11 m

$$\frac{dP}{dt} = \frac{n}{e} = 0{\cdot}367879\ldots n \cong 0{\cdot}368n$$

@ depth of 1 m

$$\frac{dP}{dt} = \frac{n}{1} = n$$

$\Rightarrow$ Rate of change of pressure between 21 m and 11 m

$$= 0{\cdot}368n - 0{\cdot}135n = 0{\cdot}233n$$

$\Rightarrow$ Rate of change of pressure between 11 m and 1 m

$$= n - 0{\cdot}368n = 0{\cdot}632n$$

– she is more susceptible to decompression sickness as she approaches nearer to the surface due to the larger change in pressure with the same absolute change in depth over this stage of her ascent

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Correct conclusion (more susceptible nearer the surface) but no reason given or exemplified.
High partial credit: (4 marks)	–	Correct conclusion (more susceptible nearer the surface) and finds $\frac{dP}{dt}$ at depth other than 21 m but not explained fully.

7(b) A local scuba diving club is recruiting new members and is offering annual membership which includes training and the use of equipment for the first year. Payment for membership can be made by means of a single payment in advance or, alternatively, the club can arrange finance so that members can make 12 equal monthly payments of €200 at the beginning of each month, which includes interest charged at an annual equivalent rate (AER) of 4·5%.

(i) Find the rate of interest, compounded monthly, which is equivalent to an AER of 4·5%, correct to three decimal places. **(5B)**

$$
\begin{array}{llll}
 & F & = & P(1+i)^t \\
 & \text{Let } R = \text{rate per month} & & \\
\Rightarrow & 1{\cdot}045 & = & (1+R)^{12} \\
\Rightarrow & 1+R & = & \sqrt[12]{1\cdot 045} \\
\Rightarrow & R & = & \sqrt[12]{1\cdot 045} - 1 \\
 & & = & 1.003674... - 1 \\
 & & = & 0{\cdot}003674... \\
\Rightarrow & R & \cong & 0{\cdot}367\%
\end{array}
$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any relevant first step, *e.g.* some correct substitution into correct formula.
High partial credit: (4 marks)	–	R correctly found but not given as a percentage <u>or</u> not to three decimal places.

7(b) (ii) Find the cost of annual membership of the club for members who pay in advance, correct to the nearest euro. **(15C*)**

❶

$$A = \frac{200}{1\cdot 00367} + \frac{200}{(1\cdot 00367)^2} + \frac{200}{(1\cdot 00367)^3} + \dots + \frac{200}{(1\cdot 00367)^{12}}$$

$\Rightarrow$ geometric progression

$$a = \frac{200}{1\cdot 00367}$$

$$r = \frac{1}{1\cdot 00367}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow A = \frac{\frac{200}{1\cdot 00367}\left(1 - \frac{1}{1\cdot 00367^{12}}\right)}{1 - \frac{1}{1\cdot 00367}}$$

$$= \frac{200\left(1 - \frac{1}{1\cdot 00367^{12}}\right)}{0\cdot 00367}$$

$= €2{,}343\cdot 715...$

$\cong €2{,}344$

Scale 15C* (0, 5, 10, 15)

Low partial credit: (5 marks)	– – –	Any relevant step. Reference to 1·0367. Recognises geometric progression.
High partial credit: (10 marks)	–	Correct substitution of all values into sum of geometric progression.

* If final answer not rounded to the nearest euro or incorrectly rounded - deduct 1 mark from total.

or

❷

$$A = \frac{Pi(1+i)^t}{(1+i)^t - 1}$$

$$\Rightarrow 200 = \frac{P(0\cdot 00367)(1+0\cdot 00367)^{12}}{(1+0\cdot 00367)^{12} - 1}$$

$$= \frac{P(0\cdot 00367)(1\cdot 00367)^{12}}{(1\cdot 00367)^{12} - 1}$$

$= P(0\cdot 085334...)$

$$\Rightarrow P = \frac{200}{0\cdot 085334...}$$

$= €2{,}343\cdot 715...$

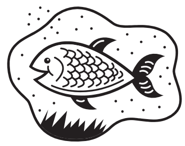
$\cong €2{,}344$

Scale 15C* (0, 5, 10, 15)

Low partial credit: (5 marks)	– –	Any relevant first step. Reference to 1·0367.
High partial credit: (10 marks)	–	Correct substitution of all values into formula for A.

* If final answer not rounded to the nearest euro or incorrectly rounded - deduct 1 mark from total.

Question 8 **(50 marks)**



Good water quality is a precondition to the development and protection of our inland waterways. Inland Fisheries Ireland (IFI) is the state agency charged with ensuring the protection and conservation of our fisheries resources, both fish stocks and their habitats. It carries out monitoring to assess the impact of pollution and to determine environmental trends.

Due to concern over declining fish stocks in a river, the water was tested over a period of time. Water samples were collected at the same time at the start of each week and analysed in a laboratory.

8(a) (i) Complete the table below, which shows the bacteria count in each of the samples taken over three weeks, where n is the number of bacteria, in thousands and correct to the nearest thousand, and w is the number of weeks from when the monitoring programme began. **(5B)**

w (number of weeks)	0	1	2	3
Number of bacteria	967	2067	4125	7784
n (number of bacteria, in thousands)	1	**2**	**4**	**8**

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	One or two correct entries in table.

8(a) (ii) Write down, in terms of w, the expected number of bacteria, in thousands, in future water samples. Explain your answer. **(10C)**

❶

$$\begin{array}{cccc} w_0 & w_1 & w_2 & w_3 \\ 1, & 2, & 4, & 8, & \ldots \end{array}$$

$\Rightarrow$ geometric series

$$a = 1$$
$$r = 2$$

$$T_n = ar^n \quad \text{(starts at } n = 0\text{)}$$
$$\Rightarrow T_w = 1(2)^w$$
$$\Rightarrow n \text{ (in thousands)} = 2^w$$

or

❷

$$F = P(1+i)^w$$
$$= 1(1+1)^w$$
$$= 2^w$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any relevant first step, *e.g.* a = 1 or $r = 2$ (or both).
High partial credit: (7 marks)	–	Gets $n = 2^{w-1}$ after correct use of formula $T_n = ar^{n-1}$.

8(a) (iii) If the growth of bacteria continued uninterrupted at the same rate, find the expected number of bacteria in a water sample collected from the river at the start of weeks 12 and 18. **(5C)**

Start of week 12:

$$
\begin{aligned}
\Rightarrow \quad w_{12} &= 11 \\
\Rightarrow \quad n &= 2^w \\
\Rightarrow \quad n_{12} &= 2^{11} \\
&= 2{,}048 \text{ thousands} \\
&= 2{,}048{,}000
\end{aligned}
$$

Start of week 18:

$$
\begin{aligned}
\Rightarrow \quad w_{18} &= 17 \\
\Rightarrow \quad n &= 2^w \\
\Rightarrow \quad n_{18} &= 2^{17} \\
&= 131{,}072 \text{ thousands} \\
&= 131{,}072{,}000
\end{aligned}
$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	– –	Any relevant first step. Only one answer given but not evaluated (*i.e.* $n_{12} = 2^{11}$ or $n_{18} = 2^{17}$).
High partial credit: (4 marks)	–	Both answers given but not evaluated (*i.e.* $n_{12} = 2^{11}$ or $n_{18} = 2^{17}$).

8(a) (iv) Using your answers to part **(iii)**, find the expected percentage increase in the number of bacteria in water samples collected from the river per week. **(5B)**

$$
\begin{array}{ccc}
w_{12} & \rightarrow & w_{18} \\
2^{11} & & 2^{17}
\end{array}
$$

$$
\begin{aligned}
F &= P(1+i)^t \\
\Rightarrow \quad 2^{17} &= 2^{11}(1+i)^6 \\
\Rightarrow \quad 2^6 &= (1+i)^6 \\
\Rightarrow \quad 2^6 &= (1+i)^6 \\
\Rightarrow \quad 1+i &= 2 \\
\Rightarrow \quad i &= 1 \\
\Rightarrow \quad \text{expected increase} &= 100\%
\end{aligned}
$$

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	– –	Any relevant first step. Correct answer given (*i.e.* 100%) but no explanation.

8(b) After monitoring the water quality for three weeks, IFI staff added regular quantities of bactericide agent to the water in the river to neutralise the bacteria. This treatment continued for five weeks and reduced the growth rate of the bacteria each week by 80%.

Estimate the bacteria count in a water sample collected three weeks after the treatment began. **(5C)**

3 weeks $\Rightarrow$ start of 4th week

$$\begin{aligned}
& w_4 && = && 3 \\
& n && = && 2^w \\
\Rightarrow\quad & n_4 && = && 2^3 \\
& && = && 2 \times 2 \times 2 \times 1{,}000 \\
& && = && 8{,}000
\end{aligned}$$

Growth rate reduced each week by 80%

$$\begin{aligned}
& F && = && P(1+i)^t \\
\Rightarrow\quad & \text{Bacteria count} && = && 8{,}000(1 + 0{\cdot}2)^3 \\
& && = && 8{,}000(1{\cdot}2)^3 \\
& && = && 8{,}000(1{\cdot}728) \\
& && = && 13{,}824
\end{aligned}$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any relevant first step, *e.g.* bacteria count after 3 weeks = 8,000 (but not $n = 8$).
High partial credit: (4 marks)	–	Correct expression for bacteria count <u>or</u> n after 3 further weeks [$8{,}000(1{\cdot}2)^3$ <u>or</u> $8(1{\cdot}2)^3$] <u>and stops</u>.

8(c) At the beginning of 2010, the river contained 6,500 trout. If the pollution source is not tackled, IFI scientists can estimate the numbers of trout in the river in future years. The numbers of trout can be projected to decline each year by 10% of the previous year's population. The formula used is $n = 6{,}500(0{\cdot}9)^t$, where n is the number of trout and t is the number of years after 2010.

(i) During what year will trout stocks in the river fall below 1300? **(10C)**

$$\begin{aligned}
& n && = && 6{,}500(0{\cdot}9)^t \\
& \underline{\text{when } n < 1300?} && && \\
\Rightarrow\quad & 6{,}500(0{\cdot}9)^t && < && 1{,}300 \\
\Rightarrow\quad & (0{\cdot}9)^t && < && \frac{1{,}300}{6{,}500} \\
& && < && 0{\cdot}2 \\
\Rightarrow\quad & \log(0{\cdot}9)^t && < && \log 0{\cdot}2 \\
\Rightarrow\quad & \log(0{\cdot}9) && < && \log 0{\cdot}2 \\
\Rightarrow\quad & t && > && \frac{\log 0{\cdot}2}{\log 0{\cdot}9} \\
\Rightarrow\quad & t && > && 15{\cdot}275 \text{ years}
\end{aligned}$$

$\therefore$ trout stocks will fall below 1,300 in the year 2025

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any relevant first step, *e.g.* $6{,}500(0{\cdot}9)^t < 1{,}300$.
High partial credit: (7 marks)	–	Correct numerical value for t (15·275 years) and stops.

8(c) (ii) Find the average number of trout in the river over the 10-year period beginning with 2014. **(10C)**

Average number of trout in the river

$$= \frac{1}{14-4}\int_{4}^{14} 6{,}500(0{\cdot}9)^t\,dt$$

$$= \frac{6{,}500}{10}\int_{4}^{14} 0{\cdot}9^t\,dt$$

$$= 650 \times \frac{0\cdot 9^t}{\ln 0\cdot 9}\Bigg|_4^{14}$$

$$= \frac{650}{\ln 0\cdot 9}[0{\cdot}9^{14} - 0{\cdot}9^{4}]$$

$$= \frac{650}{\ln 0\cdot 9}[-0{\cdot}427332...]$$

$$= 2{,}636{\cdot}337222...$$

$$\cong 2636 \text{ trout}$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any relevant first step, *e.g.* 2014 $\Rightarrow t = 4$.
High partial credit: (7 marks)	–	Correct method but one error in integration or limits.

Question 9 **(50 marks)**

A golf buggy is a small vehicle designed to carry two golfers and their golf clubs around with less effort than walking. Most golf buggies are electrically-powered as their lack of pollutants, lack of noise, and safety for pedestrians and other buggies (due to slow speeds) make them more beneficial than petrol-powered designs.

Two golfers, Paul and Rory, leave their golf buggy on a fairway to take their next shots. The handbrake on the buggy partially fails, causing it to roll backwards in a straight line.

The speed of the golf buggy, g, in metres per second, is given by

$$g(t) = 0{\cdot}5t$$

where t is the time, in seconds, from the instant the golf buggy begins to move.

9(a) **(i)** Show that the acceleration of the golf buggy is constant. **(5B*)**

❶
$$\begin{aligned} & g(t) &&= 0{\cdot}5t \\ \Rightarrow\ & g'(t) &&= 0{\cdot}5 \text{ m s}^{-2} \\ & &&= \text{constant} \end{aligned}$$

<u>or</u>

❷
$$\begin{aligned} & u + at &&= v \\ \Rightarrow\ & 0 + at &&= 0{\cdot}5t \\ \Rightarrow\ & a &&= 0{\cdot}5 \text{ m s}^{-2} \\ & &&= \text{constant} \end{aligned}$$

Scale 5B* (0, 2, 5)

Partial credit: (2 marks)	–	Any relevant first step, *e.g.* $g(1) = 0{\cdot}5$ <u>or</u> $u + at = 0{\cdot}5t$ <u>and stops</u>.

* If units omitted or incorrect - deduct 1 mark from total.
* Penalise units only once in part (a) of question.

9(a) (ii) Find the distance travelled by the golf buggy after 3 seconds. **(5C)**

❶ $s = \int_0^3 0{\cdot}5t\,dt$

$= \left.\dfrac{0\cdot 5t^2}{2}\right|_0^3$

$= \left.0{\cdot}25t^2\right|_0^3$

$= 0{\cdot}25(3)^2 - 0$

$= 0{\cdot}25(9)$

$= 2{\cdot}25$ m

<u>or</u>

❷ $s = ut + \tfrac{1}{2}at^2$

$= 0 + \dfrac{1}{2}(0{\cdot}5)(3)^2$

$= 0{\cdot}25(9)$

$= 2{\cdot}25$ m

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any relevant first step.
High partial credit: (4 marks)	–	Correct integration, i.e. $0{\cdot}25t^2$ but limits not evaluated <u>or</u> evaluated incorrectly.
	–	$s = 0 + \dfrac{1}{2}(0{\cdot}5)(3)^2$ <u>and stops</u>.

* If units omitted or incorrect - deduct 1 mark from total.
* Penalise units only once in part (a) of question.

9(a) (iii) Paul is standing 9 m in front of the golf buggy when it starts to roll backwards. He runs after the golf buggy to catch up and stop it. His speed can be approximated by using the following model, where p is in metres per second and t is the time in seconds from the instant the buggy begins to move.

$$p(t) = \begin{cases} 0, & \textit{for } 0 \le t < 0{\cdot}5 \\ -0{\cdot}6t^2 + 3{\cdot}6t - 1{\cdot}65, & \textit{for } 0{\cdot}5 \le t < 3 \\ 3{\cdot}75, & \textit{for } t \ge 3 \end{cases}$$

How long does it take Paul to reach the golf buggy? **(15D)**

Distance Paul runs:

$$\begin{aligned}
s(\text{Paul}) &= 0 + \int_{0.5}^{3} (-0{\cdot}6t^2 + 3{\cdot}6t - 1{\cdot}65)\,dt + 3{\cdot}75(t-3) \\
&= -0{\cdot}6\frac{t^3}{3} + 3{\cdot}6\frac{t^2}{2} - 1{\cdot}65t \Big|_{0.5}^{3} + 3{\cdot}75(t-3) \\
&= -0{\cdot}2t^3 + 1{\cdot}8t^2 - 1{\cdot}65t \Big|_{0.5}^{3} + 3{\cdot}75t - 11{\cdot}25 \\
&= [-0{\cdot}2(3)^3 + 1{\cdot}8(3)^2 - 1{\cdot}65(3)] - [-0{\cdot}2(0{\cdot}5)^3 + 1{\cdot}8(0{\cdot}5)^2 \\
&\quad - 1{\cdot}65(0{\cdot}5)] + 3{\cdot}75t - 11{\cdot}25 \\
&= -5{\cdot}4 + 16{\cdot}2 - 4{\cdot}95 + 0{\cdot}025 - 0{\cdot}45 + 0{\cdot}825 + 3{\cdot}75t - 11{\cdot}25 \\
&= -5 + 3{\cdot}75t
\end{aligned}$$

Paul will catch the buggy:

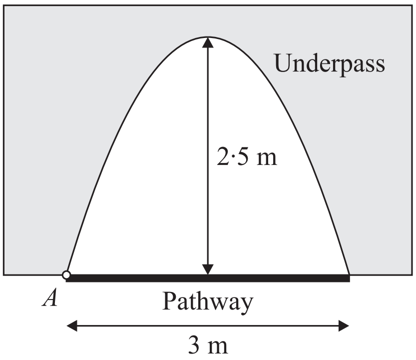
$$\begin{aligned}
& s(\text{Paul}) &&= s(\text{buggy}) + 9 \\
& s(\text{buggy}) &&= 0{\cdot}25t^2 \\
\Rightarrow\ & -5 + 3{\cdot}75t &&= 0{\cdot}25t^2 + 9 \\
\Rightarrow\ & 0{\cdot}25t^2 - 3{\cdot}75t + 9 + 5 &&= 0 \\
\Rightarrow\ & 0{\cdot}25t^2 - 3{\cdot}75t + 14 &&= 0 \\
\Rightarrow\ & t^2 - 15t + 56 &&= 0 \\
\Rightarrow\ & (t-7)(t-8) &&= 0 \\
\Rightarrow\ & t - 7 &&= 0 \quad \underline{\text{or}} \quad t - 8 = 0 \\
\Rightarrow\ & t &&= 7 \quad\quad\quad\ \ t = 8 \\
\Rightarrow\ & t &&= 7 \text{ seconds}
\end{aligned}$$

Scale 15D (0, 4, 7, 11, 15)

Low partial credit: (4 marks)	–	Any relevant first step.
	–	Some correct integration to find distance that Paul runs.
Middle partial credit: (7 marks)	–	Integration fully correct but limits not evaluated or not evaluated correctly.
High partial credit: (11 marks)	–	Correct answer for s(Paul) <u>and stops</u>.
	–	Correct answer for s(buggy) and error in limits only for s(Paul) <u>and stops</u>.

9(b) To complete the back 9 holes of the course, golfers need to cross a roadway using an underpass which has a cross-sectional opening, as shown in the diagram.

The height of the underpass is defined by the equation $f(x) = ax^2 + bx + c$.
The pathway through the underpass has a width of 3 m and a maximum height of 2·5 m at the centre of the pathway, as shown.



(i) Using point A as the origin, write three equations in a, b and c. **(10C)**

Points on curve of underpass: (0, 0), (1·5, 2·5), (3, 0)

$$f(x) = ax^2 + bx + c$$

$(0, 0) \in f(x)$

$$\Rightarrow \quad 0 = a(0)^2 + b(0) + c$$
$$\Rightarrow \quad 0 = c$$

$(1{\cdot}5, 2{\cdot}5) \in f(x)$

$$\Rightarrow \quad 2{\cdot}5 = a(1{\cdot}5)^2 + b(1{\cdot}5) + c$$
$$\Rightarrow \quad 2{\cdot}5 = 2{\cdot}25a + 1{\cdot}5b + 0$$
$$\Rightarrow \quad 2{\cdot}5 = 2{\cdot}25a + 1{\cdot}5b$$

$(3, 0) \in f(x)$

$$\Rightarrow \quad 0 = a(3)^2 + b(3) + c$$
$$\Rightarrow \quad 0 = 9a + 3b + 0$$
$$\Rightarrow \quad 0 = 9a + 3b$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any relevant first step.
High partial credit: (7 marks)	–	Any two correct equations in a, b and c.

9(b) (ii) Hence, find $f(x)$. **(10D)**

$$\begin{array}{lrcl}
 & 2{\cdot}25a + 1{\cdot}5b & = & 2{\cdot}5\ (\times 4) \\
 & \underline{9a \quad + \quad 3b} & = & \underline{0}\ (\times -1) \\
\Rightarrow & 9a \quad + \quad 6b & = & 10 \\
 & \underline{-9a \quad - \quad 3b} & = & \underline{0} \\
\Rightarrow & 3b & = & 10 \\
\Rightarrow & b & = & \frac{10}{3} \\
 & 9a + 3b & = & 0 \\
\Rightarrow & 9a + 3(\frac{10}{3}) & = & 0 \\
\Rightarrow & 9a + 10 & = & 0 \\
\Rightarrow & 9a & = & -10 \\
\Rightarrow & a & = & -\frac{10}{9} \\
\Rightarrow & f(x) & = & -\frac{10}{9}x^2 + \frac{10}{3}x
\end{array}$$

Scale 10D (0, 3, 5, 8, 10)

Low partial credit: (3 marks)	–	Any relevant step to solution.
Middle partial credit: (5 marks)	–	Evaluates one unknown correctly.
High partial credit: (8 marks)	–	Evaluates *a* and *b* correctly and stops. [does not find $f(x)$].

9(b) (iii) The management committee of the golf club is considering the purchase of new golf buggies which have a width of 140 cm and an overall height of 180 cm above ground level.

In practice, would you recommend the purchase of these new golf buggies in view of clearance limitations in the underpass? Justify your answer. **(5C)**

$$140 \text{ cm} = 1{\cdot}4 \text{ m}$$
$$180 \text{ cm} = 1{\cdot}8 \text{ m}$$

If the buggy drives through the centre of the tunnel

$\Rightarrow$ Distance from point A to side of buggy

$$= \frac{3 - 1{\cdot}4}{2}$$
$$= \frac{1{\cdot}6}{2}$$
$$= 0{\cdot}8 \text{ m}$$

$$f(x) = -\frac{10}{9}x^2 + \frac{10}{3}x$$
$$\Rightarrow f(0{\cdot}8) = -\frac{10}{9}(0{\cdot}8)^2 + \frac{10}{3}(0{\cdot}8)$$
$$= \frac{-6{\cdot}4 + 24}{9}$$
$$= \frac{17{\cdot}6}{9}$$
$$= 1{\cdot}955555...$$
$$\therefore \text{Clearance} = 1{\cdot}955555... - 1{\cdot}8$$
$$= 0{\cdot}155555...$$
$$\cong 1{\cdot}56 \text{ m} \quad \underline{\text{or}} \quad 15{\cdot}6 \text{ cm}$$

– purchase of new golf buggies not recommended as distance, given best position clearance is unacceptable/not safe.

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	– –	Any relevant step. Correctly evaluates $f(0{\cdot}8)$ <u>or</u> $f(2{\cdot}2)$ <u>and stops</u>.
High partial credit: (4 marks)	– –	Correctly evaluates clearance <u>and stops</u>. Recommendation omitted.

Pre-Leaving Certificate Examination, 2014

Mathematics
(Project Maths – Phase 3)

Higher Level – Paper 2
Marking Scheme (300 marks)

General Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	2 questions

Students must answer **all eight** questions, as follows:

In Section A, answer:

Questions 1 to 5 and
either Question 6A **or** Question 6B.

In Section B, answer Questions 7 and 8.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Structure of the Marking Scheme

Students' responses are marked according to different scales, depending on the types of response anticipated.
Scales labelled A divide students' responses into two categories (correct and incorrect).
Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on.
These scales and the marks that they generate are summarised in the following table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scale		0, 2, 5	0, 2, 4, 5	0, 2, 3, 4, 5	
10 mark scale		0, 5, 10	0, 3, 7, 10	0, 3, 5, 8, 10	
15 mark scale		0, 7, 15	0, 5, 10,15	0, 4, 7, 11, 15	
20 mark scale					

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

E-scales (six categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response almost half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk.
Thus, for example, *scale* **10C*** indicates that 9 marks may be awarded.

Summary of Marks

Q.1	**(a)**	**(i)**	5C (0, 2, 4, 5)
		(ii)	10C (0, 3, 7, 10)
	(b)		10C* (0, 3, 7, 10)
			25

Q.2	**(a)**		10C (0, 3, 7, 10)
	(b)	**(i)**	5C (0, 2, 4, 5)
		(ii)	5C (0, 2, 4, 5)
	(c)		5C (0, 2, 4, 5)
			25

Q.3	**(a)**		5C (0, 2, 4, 5)
	(b)	**(i)**	10D (0, 3, 5, 8, 10)
		(ii)	10D (0, 3, 5, 8, 10)
			25

Q.4	**(a)**		5B (0, 2, 5)
	(b)	**(i)**	5B (0, 2, 5)
		(ii)	5C (0, 2, 4, 5)
	(c)		10D (0, 3, 5, 8, 10)
			25

Q.5	**(a)**		10C (0, 3, 7, 10)
	(b)		10C (0, 3, 7, 10)
	(c)		5C (0, 2, 4, 5)
			25

Q.6A	**(a)**		5B (0, 3, 5)
	(b)		5B (0, 3, 5)
			5B (0, 3, 5)
			10C (0, 4, 8, 10)
			25

OR

Q.6B	**(a)**		10C (0, 4, 8, 10)
	(b)		15C (0, 5, 10, 15)
			25

Q.7	**(a)**		10D (0, 3, 5, 8, 10)
	(b)		5C (0, 2, 4, 5)
	(c)	**(i)**	5B (0, 3, 5)
		(ii)	10D (0, 3, 5, 8, 10)
		(iii)	5B (0, 3, 5)
	(d)	**(i)**	10D (0, 3, 5, 8, 10)
		(ii)	10D (0, 3, 5, 8, 10)
	(e)		10D (0, 3, 5, 8, 10)
	(f)		10D (0, 3, 5, 8, 10)
			75

Q.8	**(a)**	**(i)**	5B (0, 2, 5)
		(ii)	5B (0, 2, 5)
		(iii)	5C (0, 2, 4, 5)
	(b)	**(i)**	5B (0, 2, 5)
		(ii)	5B (0, 2, 5)
		(iii)	5C (0, 2, 4, 5)
		(iv)	5B (0, 2, 5)
	(c)	**(i)**	5B (0, 2, 5)
		(ii)	5B (0, 2, 5)
		(iii)	10C (0, 4, 8, 10)
	(d)		15D (0 4, 7 11, 15)
			75

Pre-Leaving Certificate Examination, 2014

Mathematics
(Project Maths – Phase 3)

Higher Level – Paper 2
Marking Scheme (300 marks)

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

1(a) The volume, in litres, of petrol purchased per visit at a filling station last month was recorded and analysed. The random variable X, where X is the volume of petrol purchased, follows a normal distribution with mean 37 and standard deviation 9. A customer is chosen at random.

(i) Find $P(X \leq 45)$. **(5C)**

$$\mu = 37, \ \sigma = 9$$

$$z = \frac{x - \mu}{\sigma}$$

$$\Rightarrow \quad z_{45} = \frac{45 - 37}{9} = \frac{8}{9} = 0{\cdot}888888\ldots$$

$$\therefore \quad P(X \leq 45) = P(z \leq 0{\cdot}888888\ldots) \cong P(z \leq 0{\cdot}89) = 0{\cdot}8133$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any relevant step.
High partial credit: (4 marks)	–	$z_{45} = \frac{8}{9}$ <u>or</u> $0{\cdot}888888\ldots$ <u>and stops</u>.

1(a) (ii) Find $P(30 \le X \le 45)$. **(10C)**

$$\mu = 37, \ \sigma = 9$$

$$z = \frac{x-\mu}{\sigma}$$

$$\Rightarrow z_{30} = \frac{30-37}{9} = -\frac{7}{9} = -0{\cdot}777777... \cong -0{\cdot}78$$

$$\begin{aligned} P(30 \le X \le 45) &\cong P(-0{\cdot}78 \le z \le 0{\cdot}89) \\ &= P(z \le 0{\cdot}89) - P(z \le -0{\cdot}78) \\ &= 0{\cdot}8133 - [1 - P(z \le 0{\cdot}78)] \\ &= 0{\cdot}8133 - [1 - 0{\cdot}7823] \\ &= 0{\cdot}8133 - 1 + 0{\cdot}7823 \\ &= 0{\cdot}5956 \end{aligned}$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any relevant step.
High partial credit: (7 marks)	–	$P(z \le 0{\cdot}78) = 0{\cdot}7823$ <u>and stops</u>.

1(b) The cost of petrol remained constant at €1·49 per litre during this period of time. Given that 90% of customers spent more than €N on petrol, find the value of N, correct to the nearest euro. **(10C*)**

$$P(z \ge a) = 0{\cdot}9$$

$$\Rightarrow z \ge -1{\cdot}28$$

$$z = \frac{x-\mu}{\sigma}$$

$$\Rightarrow \frac{x-\mu}{\sigma} = -1{\cdot}28$$

$$\mu = 37, \ \sigma = 9$$

$$\Rightarrow \frac{x-37}{9} \ge -1{\cdot}28$$

$$\Rightarrow x - 37 \ge 9(-1{\cdot}28) \ge -11{\cdot}52$$

$$\Rightarrow x \ge 37 - 11{\cdot}52 \ge 25{\cdot}48 \text{ litres}$$

$$\Rightarrow N = 1{\cdot}49 \times 25{\cdot}48 = 37{\cdot}9652 \cong €38$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any relevant step.
High partial credit: (7 marks)	–	$x \ge 25{\cdot}48$ <u>and stops</u>.

* If final answer not rounded to the nearest euro <u>or</u> incorrectly rounded - deduct 1 mark from total.

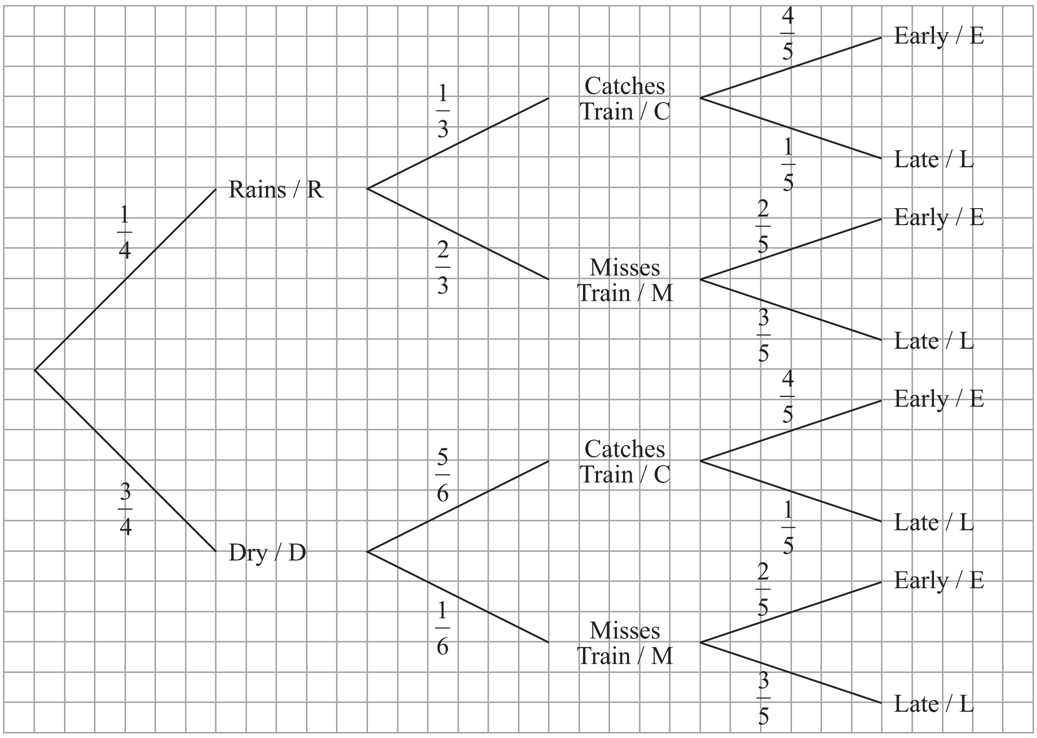
Question 2 **(25 marks)**

Jim travels to work by train every morning. The probability that it rains (R) on a given morning is $\frac{1}{4}$.

If it rains, the probability that Jim misses his train is $\frac{2}{3}$. If it does not rain, the probability that Jim catches (C) his train is $\frac{5}{6}$. If he catches his train, the probability that he is early (E) for work is $\frac{4}{5}$.

However, if he misses his train, the probability that he is late for work is $\frac{3}{5}$.

2(a) Using the partially completed tree diagram, or otherwise, show clearly all the possible outcomes. Indicate the probability of each event on the branches on your diagram. **(10C)**



Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any relevant first step, *e.g.* any new correct probability.
High partial credit: (7 marks)	–	All required probabilities calculated correctly but tree diagram not fully completed.

2(b) Using your diagram, or otherwise, find the probability that, on a given morning,

(i) it rains and Jim is still early for work, **(5C)**

$$\begin{aligned} P(\text{rains, Jim early}) &= [\tfrac{1}{4} \times \tfrac{2}{3} \times \tfrac{2}{5}] + [\tfrac{1}{4} \times \tfrac{1}{3} \times \tfrac{4}{5}] \\ &= \tfrac{1}{15} + \tfrac{1}{15} \\ &= \tfrac{2}{15} \end{aligned}$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any one of the two possibilities in above solution, *e.g.* $\frac{1}{4} \times \frac{2}{3} \times \frac{2}{5}$ <u>or</u> $\frac{1}{4} \times \frac{1}{3} \times \frac{4}{5}$.
High partial credit: (4 marks)	–	Both possibilities as above but not finished correctly.

2(b) **(ii)** Jim is early for work. **(5C)**

$$\begin{aligned} P(\text{Jim early}) &= P(\text{rains, Jim early}) + P(\text{dry, Jim early}) \\ &= \frac{2}{15} + [\frac{3}{4} \times \frac{1}{6} \times \frac{2}{5}] + [\frac{3}{4} \times \frac{5}{6} \times \frac{4}{5}] \\ &= \frac{2}{15} + \frac{1}{20} + \frac{1}{2} \\ &= \frac{8 + 3 + 30}{60} \\ &= \frac{41}{60} \end{aligned}$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any one of the new possibilities in above solution, *e.g.* $\frac{3}{4} \times \frac{1}{6} \times \frac{2}{5}$ <u>or</u> $\frac{3}{4} \times \frac{5}{6} \times \frac{4}{5}$.
High partial credit: (4 marks)	–	Both possibilities as above and answer to part (b)(i) but not finished correctly.

2(c) Find the probability that Jim is early for work on at least three out of the five days of a given week, assuming that each of these days is independent of each other. **(5C)**

$$\begin{aligned} P(\text{Jim early}) &= \frac{41}{60} \\ \Rightarrow \quad P(\text{Jim late}) &= 1 - \frac{41}{60} \\ &= \frac{19}{60} \\ \text{No. of days, } n &= 5 \end{aligned}$$

<u>Bernouilli trial:</u>

P(Jim early on at least 3 days)

$$\begin{aligned} &= \binom{5}{3}\left(\frac{41}{60}\right)^3\left(\frac{19}{60}\right)^2 + \binom{5}{4}\left(\frac{41}{60}\right)^4\left(\frac{19}{60}\right)^1 + \binom{5}{5}\left(\frac{41}{60}\right)^5\left(\frac{19}{60}\right)^0 \\ &= 0{\cdot}31996... + 0{\cdot}34522... + 0{\cdot}14899... \\ &= 0{\cdot}8142... \end{aligned}$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any relevant first step, *e.g.* $p = \frac{41}{60}$, $q = 1 - \frac{41}{60} = \frac{19}{60}$.
High partial credit: (4 marks)	–	Any two of the three possibilities correctly evaluated.

Question 3 **(25 marks)**

3(a) l is the line $x + 2y - 6 = 0$. The line k is perpendicular to l and its y-intercept is -7.
Find the equation of the line k. **(5C)**

$$
\begin{array}{llcl}
 & l:\ x + 2y - 6 & = & 0 \\
\Rightarrow & 2y & = & -x + 3 \\
\Rightarrow & y & = & -\frac{1}{2}x + 3 \\
 & y & = & mx + c \\
 & m_l & = & -\frac{1}{2} \\
 & k \perp l & & \\
\Rightarrow & m_k \times m_l & = & -1 \\
\Rightarrow & m_k & = & 2
\end{array}
$$

$$
\begin{array}{ll}
 & y\text{-intercept is } -7 \\
\Rightarrow & \text{Point on } k\text{: } (0, -7)
\end{array}
$$

❶ Equation of k:

$$
\begin{array}{llcl}
 & y - y_1 & = & m_k(x - x_1) \\
\Rightarrow & y - (-7) & = & 2(x - 0) \\
\Rightarrow & y + 7 & = & 2x \\
\Rightarrow & k:\ 2x - y & = & 7
\end{array}
$$

or

❷ Equation of k:

$$
\begin{array}{llcl}
 & y & = & mx + c \\
\Rightarrow & -7 & = & 2(0) + c \\
\Rightarrow & c & = & -7 \\
\Rightarrow & c & = & -7 \\
\Rightarrow & k:\ y & = & 2x - 7 \\
\Rightarrow & k:\ 2x - y & = & 7
\end{array}
$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any relevant first step, *e.g.* $m_l = -\frac{1}{2}$.
High partial credit: (4 marks)	–	Correct slope ($m_k = 2$) and point (c value) but one error in equation.

3(b) *m* and *n* are two lines which pass through the point of intersection of the lines *l* and *k*.

(i) (6, –3) is a point on *m*. Find the equation of *m*. **(10D)**

$$\begin{array}{lrcl} & l:\ x+2y & = & 6 \\ & k:\ \underline{2x-\ y} & = & \underline{7}\ (\times 2) \\ & x+2y & = & 6 \\ & \underline{4x-2y} & = & \underline{14} \\ & 5x & = & 20 \\ \Rightarrow & x & = & 4 \end{array}$$

$$\begin{array}{lrcl} & x+2y & = & 6 \\ \Rightarrow & 4+2y & = & 6 \\ \Rightarrow & 2y & = & 6-4 \\ & & = & 2 \\ \Rightarrow & y & = & 1 \\ \Rightarrow & l \cap k & = & (4, 1) \end{array}$$

or

$$\begin{array}{lrcl} & 2x-\ y & = & 7 \\ \Rightarrow & 2(4)-y & = & 7 \\ \Rightarrow & -y & = & 7-8 \\ & & = & -1 \\ \Rightarrow & y & = & 1 \\ & l \cap k & = & (4, 1) \end{array}$$

$$\begin{array}{rcl} \text{Slope of } m & = & \dfrac{y_2 - y_1}{x_2 - x_1} \\ & = & \dfrac{-3-1}{6-4} \\ & = & \dfrac{-4}{2} \\ & = & -2 \end{array}$$

❶ Equation of *m*:

$$\begin{array}{lrcl} & y - y_1 & = & m_m(x - x_1) \\ \Rightarrow & y-(-3) & = & -2(x-6) \\ \Rightarrow & y+3 & = & -2x+12 \\ \Rightarrow & m:\ 2x+y-9 & = & 0 \end{array}$$

or

❷ Equation of *m*:

$$\begin{array}{lrcl} & y & = & mx + c \\ \Rightarrow & -3 & = & 2(-6)+c \\ \Rightarrow & -c & = & -12+3 \\ & & = & -9 \\ \Rightarrow & c & = & 9 \\ \Rightarrow & m:\ y & = & -2x-9 \\ \Rightarrow & k:\ 2x+y-9 & = & 0 \end{array}$$

or

❸ Equation of *m*:

$$\begin{array}{lrcl} & l + \lambda k & = & 0 \\ \Rightarrow & x+2y-6+\lambda(2x-y-7) & = & 0 \end{array}$$

(6, –3) is a point on *m*

$$\begin{array}{lrcl} \Rightarrow & 6+2(-3)-6+\lambda[2(6)-(-3)-7] & = & 0 \\ \Rightarrow & 6-6-6+\lambda[12+3-7] & = & 0 \\ \Rightarrow & -6+8\lambda & = & 0 \\ \Rightarrow & 8\lambda & = & 6 \\ \Rightarrow & \lambda & = & \dfrac{6}{8} \\ & & = & \dfrac{3}{4} \\ \Rightarrow & x+2y-6+\dfrac{3}{4}(2x-y-7) & = & 0 \\ \Rightarrow & 4x+8y-24+6x-3y-21 & = & 0 \\ \Rightarrow & 10x+5y-45 & = & 0 \\ \Rightarrow & 2x+y-9 & = & 0 \end{array}$$

Scale 10D (0, 3, 5, 8, 10)			
	Low partial credit: (3 marks)	–	Any relevant step.
	Middle partial credit: (5 marks)	–	Point of intersection of l and k found correctly <u>and stops</u>.
	High partial credit: (8 marks)	–	Point of intersection of l and k and slope of m found correctly <u>and stops</u>.

3(b) (ii) Find the two possible equations of n if the angle between m and n is 45°. **(10D)**

Line m:

$$2x + y = 9$$
$$\text{Slope of } m,\ m_m = -2$$
$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$
$$\Rightarrow \tan 45° = \pm\frac{m - (-2)}{1 + (m)(-2)}$$
$$\Rightarrow 1 = \pm\frac{m + 2}{1 - 2m}$$
$$\Rightarrow \frac{m + 2}{1 - 2m} = \pm 1$$
$$\Rightarrow m + 2 = \pm 1(1 - 2m)$$

$$\Rightarrow m + 2 = 1 - 2m$$
$$\Rightarrow m + 2m = 1 - 2$$
$$\Rightarrow 3m = -1$$
$$\Rightarrow m = -\frac{1}{3}$$

<u>or</u>

$$m + 2 = -(1 - 2m) = -1 + 2m$$
$$\Rightarrow m - 2m = -1 - 2$$
$$\Rightarrow -m = -3$$
$$\Rightarrow m = 3$$

❶ Equation of n:

$l \cap k$: (4, 1)

$$y - y_1 = m_m(x - x_1)$$
$$\Rightarrow y - 1 = -\frac{1}{3}(x - 4)$$
$$\Rightarrow 3y - 3 = -x + 4$$
$$\Rightarrow x + 3y - 7 = 0$$

<u>or</u>

$$y - 1 = 3(x - 4) = 3x - 12$$
$$\Rightarrow 3x - y - 11 = 0$$

<u>or</u>

❷ Equation of n:

$l \cap k$: (4, 1)

$$y = mx + c$$
$$\Rightarrow 1 = -\frac{1}{3}(4) + c$$
$$\Rightarrow 3 = -4 + 3c$$
$$\Rightarrow 3c = 3 + 4 = 7$$
$$\Rightarrow c = \frac{7}{3}$$
$$\Rightarrow y = mx + c$$
$$\Rightarrow y = -\frac{1}{3}x + \frac{7}{3}$$
$$\Rightarrow x + 3y - 7 = 0$$

<u>or</u>

$$\Rightarrow 1 = 3(4) + c = 12 + c$$
$$\Rightarrow -c = 12 - 1 = 11$$
$$\Rightarrow c = -11$$
$$\Rightarrow y = mx + c$$
$$\Rightarrow y = 3x - 11$$
$$\Rightarrow 3x - y - 11 = 0$$

or

❸ Equation of n:

$$
\begin{aligned}
& l + \lambda k &&= 0\\
\Rightarrow\ & x + 2y - 6 + \lambda(2x - y - 7) &&= 0\\
\Rightarrow\ & x(1 + 2\lambda) + y(2 - \lambda) - 6 - 7\lambda &&= 0\\
\Rightarrow\ & \text{Slope of } n = \frac{-(1 + 2\lambda)}{2 - \lambda}\\
& \phantom{\text{Slope of } n} = \frac{1 + 2\lambda}{\lambda - 2}
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow\ & \text{Slope of } n &&= -\frac{1}{3}\\
\Rightarrow\ & \frac{1 + 2\lambda}{\lambda - 2} &&= -\frac{1}{3}\\
\Rightarrow\ & 3(1 + 2\lambda) &&= -1(\lambda - 2)\\
\Rightarrow\ & 3 + 6\lambda &&= -\lambda + 2\\
\Rightarrow\ & 6\lambda + \lambda &&= 2 - 3\\
\Rightarrow\ & 7\lambda &&= -1\\
\Rightarrow\ & \lambda &&= -\frac{1}{7}
\end{aligned}
$$

or

$$
\begin{aligned}
\Rightarrow\ & \text{Slope of } n &&= 3\\
\Rightarrow\ & \frac{1 + 2\lambda}{\lambda - 2} &&= 3\\
\Rightarrow\ & 1 + 2\lambda &&= 3(\lambda - 2)\\
& &&= 3\lambda - 6\\
\Rightarrow\ & 2\lambda - 3\lambda &&= -6 - 1\\
\Rightarrow\ & -\lambda &&= -7\\
\Rightarrow\ & \lambda &&= 7
\end{aligned}
$$

$$
\begin{aligned}
& x + 2y - 6 + \lambda(2x - y - 7) &&= 0\\
& \lambda = -\frac{1}{7}\\
\Rightarrow\ & x + 2y - 6 + -\frac{1}{7}(2x - y - 7) &&= 0\\
\Rightarrow\ & 7x + 14y - 42 - 2x + y + 7 &&= 0\\
\Rightarrow\ & 5x + 15y - 35 &&= 0\\
\Rightarrow\ & x + 3y - 7 &&= 0
\end{aligned}
$$

$$
\begin{aligned}
& \lambda = 7\\
\Rightarrow\ & x + 2y - 6 + 7(2x - y - 7) &&= 0\\
\Rightarrow\ & x + 2y - 6 + 14x - 7y - 49 &&= 0\\
\Rightarrow\ & 15x - 5y - 55 &&= 0\\
\Rightarrow\ & 3x - y - 11 &&= 0
\end{aligned}
$$

Scale 10D (0, 3, 5, 8, 10)

Low partial credit: (3 marks)	–	Any relevant step.
Middle partial credit: (5 marks)	–	Correct formula fully substituted for θ and slope of 2.
High partial credit: (8 marks)	–	Both slopes found correctly and stops.

Question 4 **(25 marks)**

c_1 is the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ and l is the line $2x - y + 5 = 0$.
c_1 and l intersect at the points P and Q.

4(a) Write down the centre and radius-length of c_1. **(5B)**

❶ Equation of a circle:
$x^2 + y^2 + 2gx + 2fy + c = 0$
centre $= (-g, -f)$
c_1: $x^2 + y^2 - 2x - 4y - 20 = 0$

$$\Rightarrow \text{centre} = \left(\frac{-2}{-2}, \frac{-4}{-2}\right) = (1, 2)$$

❷
$$\begin{aligned}\text{radius} &= \sqrt{g^2 + f^2 - c} \\ &= \sqrt{(-1)^2 + (-2)^2 - (-20)} \\ &= \sqrt{1 + 4 + 20} \\ &= \sqrt{25} \\ &= 5\end{aligned}$$

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Centre <u>or</u> radius found correctly only.

4(b) **(i)** Find the perpendicular distance from the centre of c_1 to the line l, in surd form. **(5B)**

$\perp$ distance from centre of c_1 to l

$$\begin{aligned} &= \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right| \\ &= \left|\frac{2(1) - 1(2) + 5}{\sqrt{(2)^2 + (-1)^2}}\right| \\ &= \left|\frac{2 - 2 + 5}{\sqrt{4 + 1}}\right| \\ &= \frac{5}{\sqrt{5}} \\ &= \sqrt{5}\end{aligned}$$

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Some substitution into correct formula.

4(b) (ii) Hence, or otherwise, find $|PQ|$, in surd form. **(5C)**

❶ Radius of $c_1 = 5$

$\perp$ distance from centre of c_1 to $l = \sqrt{5}$

Using Theorem of Pythagoras

$$
\begin{aligned}
& |PO|^2 + (\sqrt{5})^2 + && = (5)^2 \\
\Rightarrow\ & |PO|^2 && = (5)^2 - (\sqrt{5})^2 \\
& && = 25 - 5 \\
& && = 20 \\
\Rightarrow\ & |PO| && = \sqrt{20} \\
& && = 2\sqrt{5} \\
\Rightarrow\ & |PQ| && = 2(2\sqrt{5}) \\
& && = 4\sqrt{5}
\end{aligned}
$$

<u>or</u>

$c_1 \cap l$:

$$
\begin{aligned}
& x^2 + y^2 - 2x - 4y - 20 && = 0 \\
& 2x - y + 5 && = 0 \\
\Rightarrow\ & y && = 2x + 5 \\
& x^2 + (2x+5)^2 - 2x - 4(2x+5) - 20 && = 0 \\
\Rightarrow\ & x^2 + 4x^2 + 20x + 25 - 2x - 8x - 40 && = 0 \\
\Rightarrow\ & 5x^2 + 10x - 15 && = 0 \\
\Rightarrow\ & x^2 + 2x - 3 && = 0 \\
\Rightarrow\ & (x+3)(x-1) && = 0 \\
\Rightarrow\ & x + 3 && = 0 \quad \text{or} \quad x - 1 = 0 \\
\Rightarrow\ & x && = -3 \quad\quad\quad\ \ x = 1 \\
& y && = 2x + 5 \\
\Rightarrow\ & y && = 2(-3) + 5 \quad\quad y = 2(1) + 5 \\
& && = -6 + 5 \quad\quad\quad\ \ = 2 + 5 \\
& && = -1 \quad\quad\quad\quad\quad\ \ = 7 \\
\Rightarrow\ & P(-3, -1),\ Q(1, 7) && \\
& |PQ| && = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
& && = \sqrt{(1 - (-3))^2 + (7 - (-1))^2} \\
& && = \sqrt{(4)^2 + (8)^2} \\
& && = \sqrt{16 + 64} \\
& && = \sqrt{80} \\
& && = \sqrt{16}\sqrt{5} \\
& && = 4\sqrt{5}
\end{aligned}
$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any relevant step.
High partial credit: (4 marks)	–	$|PO|$ found correctly <u>and stops</u> <u>or</u> co-ordinates of P and Q found correctly but error in distance formula.

4(c) [PQ] is the diameter of the circle c_2.
Find the equation of c_2. . **(10D)**

$$
\begin{array}{lll}
 & |PQ| & = & 4\sqrt{5} \\
\Rightarrow & \text{radius of } c_2 & = & 2\sqrt{5} \\
 & \text{centre of } c_2 & = & \text{midpoint of } [PQ]
\end{array}
$$

$c_1 \cap l$:

$$
\begin{array}{llll}
 & x^2 + y^2 - 2x - 4y - 20 & = & 0 \\
 & 2x - y + 5 & = & 0 \\
\Rightarrow & y & = & 2x + 5 \\
 & x^2 + (2x+5)^2 - 2x - 4(2x+5) - 20 & = & 0 \\
\Rightarrow & x^2 + 4x^2 + 20x + 25 - 2x - 8x - 40 & = & 0 \\
\Rightarrow & 5x^2 + 10x - 15 & = & 0 \\
\Rightarrow & x^2 + 2x - 3 & = & 0 \\
\Rightarrow & (x+3)(x-1) & = & 0
\end{array}
$$

$$
\begin{array}{llll|lll}
\Rightarrow & x + 3 & = & 0 \quad \underline{\text{or}} & x - 1 & = & 0 \\
\Rightarrow & x & = & -3 & x & = & 1 \\
 & y & = & 2x + 5 & & & \\
\Rightarrow & y & = & 2(-3) + 5 & y & = & 2(1) + 5 \\
 & & = & -6 + 5 & & = & 2 + 5 \\
 & & = & -1 & & = & 7
\end{array}
$$

$\Rightarrow \quad P(-3, -1),\ Q(1, 7)$

$$
\begin{array}{llll}
 & \text{midpoint of } [PQ] & = & \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right) \\
 & & = & \text{centre of } c_2 \\
\Rightarrow & \text{centre of } c_2 & = & \left(\dfrac{-3+1}{2}, \dfrac{-1+7}{2}\right) \\
 & & = & \left(\dfrac{-2}{2}, \dfrac{6}{2}\right) \\
 & & = & (-1, 3)
\end{array}
$$

Eqn of c_2:

$$
\begin{array}{llll}
 & (x-h)^2 + (y-k)^2 & = & r^2 \\
\Rightarrow & (x-(-1))^2 + (y-3)^2 & = & (2\sqrt{5})^2 \\
\Rightarrow & (x+1)^2 + (y-3)^2 & = & 20
\end{array}
$$

Scale 10D (0, 3, 5, 8, 10)

Low partial credit: (3 marks)	–	Any relevant step.
Middle partial credit: (5 marks)	–	Co-ordinates of P and Q found correctly <u>and stops</u>.
High partial credit: (8 marks)	–	Centre of c_2 found correctly <u>and stops</u>.

Question 5 **(25 marks)**

5(a) Prove that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$. **(10C)**

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B-\sin A\sin B}$$

$$= \frac{\dfrac{\sin A\cos B}{\cos A\cos B}+\dfrac{\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B}{\cos A\cos B}-\dfrac{\sin A\sin B}{\cos A\cos B}}$$

$$= \frac{\dfrac{\sin A}{\cos A}+\dfrac{\sin B}{\cos B}}{1-\dfrac{\sin A}{\cos A}.\dfrac{\sin B}{\cos B}}$$

$$= \frac{\tan A+\tan B}{1-\tan A\tan B}$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any relevant first step.
High partial credit: (7 marks)	–	$\tan(A+B)=\dfrac{\dfrac{\sin A\cos B}{\cos A\cos B}+\dfrac{\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B}{\cos A\cos B}-\dfrac{\sin A\sin B}{\cos A\cos B}}$ <u>and stops</u> <u>or</u> errors mades preventing correct finish.

5(b) Show that the value of $\tan 75° = 2+\sqrt{3}$. **(10C)**

$$\tan 75° = \tan(45°+30°)$$

$$= \frac{\tan 45°+\tan 30°}{1-\tan 45°\tan 30°}$$

$$= \frac{1+\dfrac{1}{\sqrt{3}}}{1-1\times\dfrac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{3+2\sqrt{3}+1}{3-1}$$

$$= \frac{4+2\sqrt{3}}{2}$$

$$= 2+\sqrt{3}$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Any relevant first step.
High partial credit: (7 marks)	–	$\tan 75° = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ <u>and stops</u>.

5(c) Hence, or otherwise, find the value of tan 15°, in surd form. **(5C)**

❶

$$\begin{aligned}\tan 15^\circ &= \frac{\sin 15^\circ}{\cos 15^\circ}\\ &= \frac{\cos(90^\circ - 15^\circ)}{\sin(90^\circ - 15^\circ)}\\ &= \frac{\cos 75^\circ}{\sin 75^\circ}\\ &= \frac{1}{\tan 75^\circ}\\ &= \frac{1}{2+\sqrt{3}}\\ &= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}\\ &= \frac{2-\sqrt{3}}{4-3}\\ &= \frac{2-\sqrt{3}}{1}\\ &= 2-\sqrt{3}\end{aligned}$$

<u>or</u>

❷

$$\begin{aligned}\tan 15^\circ &= \tan(45^\circ - 30^\circ)\\ &= \frac{\tan 45^\circ - \tan 30^\circ}{1+\tan 45^\circ \tan 30^\circ}\\ &= \frac{1-\frac{1}{\sqrt{3}}}{1+1\times\frac{1}{\sqrt{3}}}\\ &= \frac{\sqrt{3}-1}{\sqrt{3}+1}\\ &= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}\\ &= \frac{3-2\sqrt{3}+1}{3-1}\\ &= \frac{4-2\sqrt{3}}{2}\\ &= 2-\sqrt{3}\end{aligned}$$

<u>or</u>

❸ $\tan 15^\circ = \tan(60^\circ - 45^\circ)$

❹ $\tan 15^\circ = \tan(75^\circ - 60^\circ)$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any relevant step.
High partial credit: (4 marks)	–	$\tan 15^\circ = \frac{1}{2+\sqrt{3}}$ <u>or</u> $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ <u>or</u> equivalent <u>and stops</u>.

Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

6A(a) Explain the difference between *similar triangles* and *congruent triangles*. **(5B)**

- *similar triangles* have exactly the same angles (equiangular), corresponding sides not equal
- *congruent triangles* are identical triangles, *i.e.* corresponding sides and angles are all equal

Scale 5B (0, 2 5)

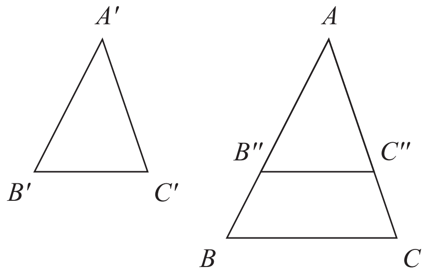
Partial credit: (2 marks)	– One term explained correctly <u>and stops</u>.

6A(b) Prove that, if two triangles ΔABC and $\Delta A'B'C'$ are similar, then their sides are proportional, in order:

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}.$$ **(5B, 5B, 10C)**

Given:

Two triangles ΔABC and $\Delta A'B'C'$ with $|\angle A| = |\angle A'|$, $|\angle B| = |\angle B'|$ and $|\angle C| = |\angle C'|$.



Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Diagram <u>or</u> Given correct.

To prove:

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}.$$

Construction:

Mark C″ on $[CA]$ such that $|AC''| = |A'C'|$.
Mark B'' on $[AB]$ such that $|B''A| = |B'A'|$.
Join $[B''C'']$.

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Construction not explicit.

Proof:

❶ $\Delta AB''C''$ and $\Delta A'B'C'$ are congruent as

$|AC''| = |A'C'|$,
$|\angle B''AC''| = |\angle B'A'C'|$
and $|AB''| = |A'B'|$ ... SAS

❷ $\therefore$ $|\angle AB''C''| = |\angle A'B'C'|$ ... corresponding $\angle$'s in congruent Δ's

❸ But $|\angle A'B'C'| = |\angle ABC|$
$\Rightarrow$ $|\angle AB''C''| = |\angle ABC|$
$\Rightarrow$ $B''C'' \parallel BC$

❹ $\therefore$ $\dfrac{|AB|}{|AB''|} = \dfrac{|AC|}{|AC''|}$ ... a line drawn parallel to 3rd side of a triangle divides the other two sides in the same ratio

❺ $\therefore$ $\dfrac{|AB|}{|A'B'|} = \dfrac{|AC|}{|A'C'|}$

Similarly,

❻ $\dfrac{|AB|}{|A'B'|} = \dfrac{|BC|}{|B'C'|}$

$\therefore$ $\dfrac{|AB|}{|A'B'|} = \dfrac{|BC|}{|B'C'|} = \dfrac{|CA|}{|C'A'|}$

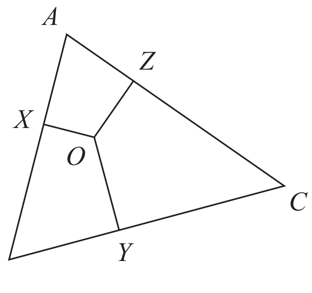
Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	One correct statement.
High partial credit: (7 marks)	–	One missing step <u>or</u> steps in incorrect order.

OR

Question 6B

O is a point inside an acute-angled triangle ABC where $AB \perp OX$, $BC \perp OY$ and $CA \perp OZ$.



6B(a) Prove that $|BY|^2 - |YC|^2 = |OB|^2 - |OC|^2$. **(10C)**

$$
\begin{aligned}
|BY|^2 &= |OB|^2 - |OY|^2 \\
|YC|^2 &= |OC|^2 - |OY|^2 \\
\Rightarrow \quad |BY|^2 - |YC|^2 &= |OB|^2 - |OY|^2 - (|OC|^2 - |OY|^2) \\
&= |OB|^2 - |OY|^2 - |OC|^2 + |OY|^2 \\
&= |OB|^2 - |OC|^2
\end{aligned}
$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Some correct relevant application of Pythagoras's theorem.
High partial credit: (7 marks)	–	Pythagoras's theorem correctly applied in ΔOBY and ΔOCY but conclusion not reached.

6B(b) Hence, or otherwise, deduce that $|AX|^2 + |BY|^2 + |CZ|^2 = |XB|^2 + |YC|^2 + |ZA|^2$. **(15C)**

$$
\begin{aligned}
|AX|^2 - |XB|^2 &= |OA|^2 - |OB|^2 \\
|BY|^2 - |YC|^2 &= |OB|^2 - |OC|^2 \\
\underline{|CZ|^2 - |ZA|^2} &= \underline{|OC|^2 - |OA|^2} \\
\Rightarrow \quad |AX|^2 + |BY|^2 + |CZ|^2 & \\
- (|XB|^2 + |YC|^2 + |ZA|^2) &= 0 \\
\Rightarrow \quad |AX|^2 + |BY|^2 + |CZ|^2 &= |XB|^2 + |YC|^2 + |ZA|^2
\end{aligned}
$$

Scale 15C (0, 5, 10, 15)

Low partial credit: (5 marks)	–	Correct expression using Pythagoras's theorem for $|AX|^2$ <u>or</u> $|CZ|^2$.
High partial credit: (10 marks)	–	Correct expression using Pythagoras's theorem for $|AX|^2$, $|XB|^2$, $|CZ|^2$ and $|ZA|^2$ <u>and stops</u> <u>or</u> not finished.

Section B	Contexts and Applications	150 marks

Answer **Question 7** and **Question 8** from this section.

Question 7 **(75 marks)**

The London Eye is the most popular paid tourist attraction in Britain, with over 3·5 million visitors annually.

The design principle behind the London Eye is very similar to that of a Ferris wheel but it is actually an observation wheel. The wheel's 32 passenger capsules are attached to the external circumference of the wheel and are rotated by electric motors. Each excursion lasts the length of time it takes to complete one revolution.

The observation wheel has a diameter of 120 m and the centre of each passenger capsule is 4 m from the external circumference of the wheel. The centre of a passenger capsule is 6 m above ground level at the point that passengers enter the capsule.

The wheel does not usually stop to allow passengers on, as the rate of movement is slow enough to allow visitors to walk on and off the moving capsules. The wheel can complete 2·5 revolutions per hour if it does not stop.

7(a) What is the maximum height that the centre of a passenger capsule reaches above ground level and how long does it take a capsule to reach this height if the wheel is not stopped? **(10D)**

❶ $h_{max} = 6 + 4 + 120 + 4$

$= 134$ m (above ground level)

❷ speed $=$ 2·5 revolutions per hour

$\Rightarrow$ time for 1 rev. $= \dfrac{60}{2{\cdot}5}$

$= 24$ minutes

h_{max} occurs after ½ revolution

$\Rightarrow$ time to reach $h_{max} = \dfrac{24}{2}$

$= 12$ minutes

Scale 10D (0, 3, 5, 8, 10)

Low partial credit: (3 marks)	–	Any relevant first step in finding the maximum height or time.
Middle partial credit: (5 marks)	–	Correct height and stops.
High partial credit: (8 marks)	–	Time correct and one error in height. – Height correct and substantiallt correct work towards finding the time.

7(b) Find the angle of rotation of a passenger capsule from the point of entry to the capsule, after 5 minutes. **(5C)**

1 revoloution takes 24 minutes
After 1 minute:

$$\text{Angle, } \theta = \frac{360°}{24} = 15°$$

After 5 minutes:

$$\text{Angle, } \theta = 15° \times 5 = 75°$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any relevant first step, *e.g.* 360° = full revolution <u>or</u> 5 min. = $\frac{1}{12}$ hr.
High partial credit: (4 marks)	–	15° per minute <u>and stops</u>.

7(c) **(i)** Write down, in terms of t, the angle of rotation of a passenger capsule at any given time, where t is the time, in minutes, after a person enters the capsule, if the wheel is not stopped. **(5B)**

1 revoloution takes 24 minutes

$$\text{Angular speed} = \text{1 revolution per 24 minutes} = \frac{360}{24} = 15° \text{ per minute}$$

$\Rightarrow$ Angle of rotation after t minutes

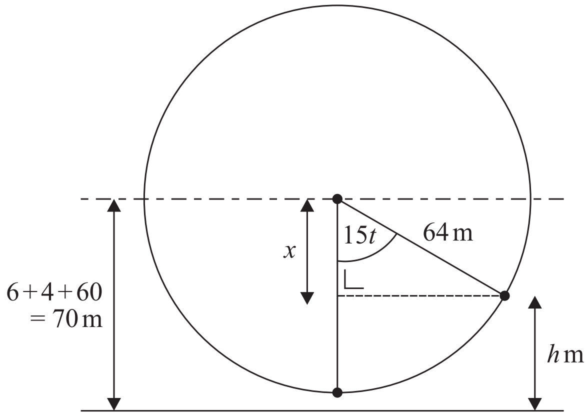
$$\text{Angle, } \theta = 15t$$

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Angle turned through in 1 minute <u>and stops</u>.

7(c) (ii) Hence, by drawing a sketch, or otherwise, find an expression for the height, h, in metres, of the centre of the capsule above ground level, in terms of t. **(10D)**

Diagram



Expression for h

$$\cos 15t = \frac{x}{64}$$

$$\Rightarrow \quad x = 64\cos 15t$$

$$h = 70 - x$$

$$\Rightarrow \quad h(t) = 70 - 64\cos 15t$$

Scale 10D (0, 3, 5, 8, 10)

Low partial credit: (3 marks)	–	Any relevant first step, *e.g.* radius = 64 m <u>or</u> height found at specific time.
Middle partial credit: (5 marks)	–	Answer includes $h = 70 \pm$ expression, *i.e.* 70 involved in height expression.
High partial credit: (8 marks)	–	One small error in height answer, *e.g.* $70 - 60\cos 15t$.

7(c) (iii) The wheel does not normally stop to allow passengers to embark and disembark and, as such, the speed of the wheel is constant. Suggest a reason why this is desirable. **(5B)**

Any 1:

- easier on the motors and turning parts if maintained at a constant speed and so will last longer //
- better passenger comfort as the observation wheel is not stopping and starting, *i.e.* no sudden jerks, *etc*. //
- more efficient way of getting large numbers onto the observation wheel // *etc.*

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Reason vague <u>or</u> incomplete.

7(d) (i) Complete the table below to show the height of the centre of a passenger capsule above ground level, at 2-minute intervals, for the time period shown. **(10D)**

Time	0	2	4	6	8	10	12
Height of passenger capsule	**6**	**14·6**	**38**	**70**	**102**	**125·4**	**134**

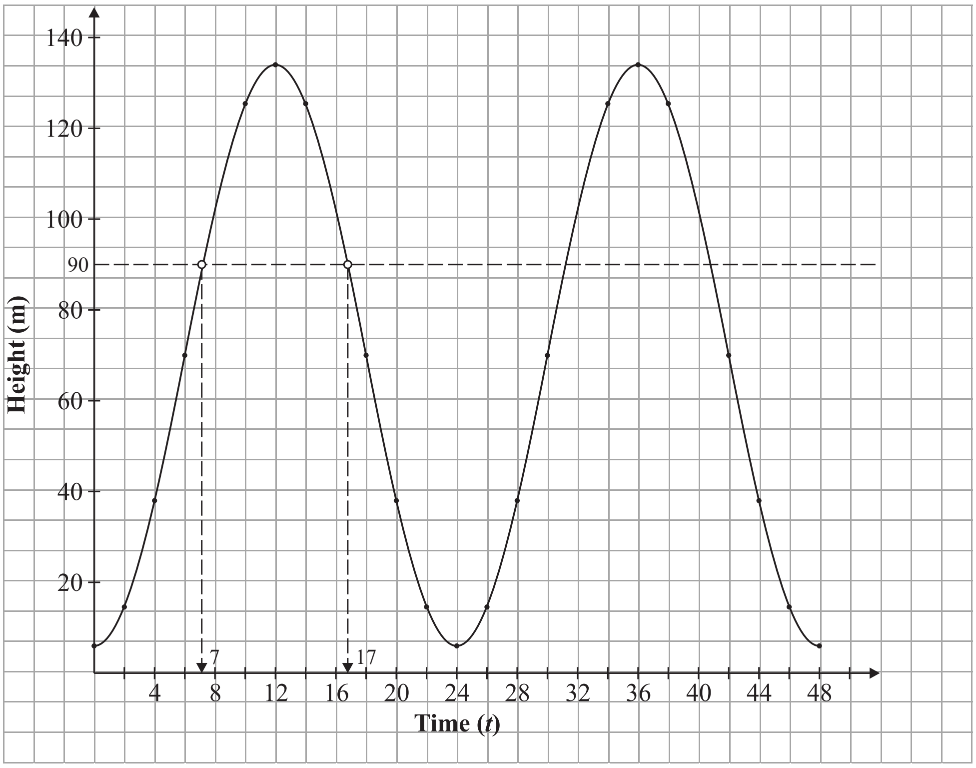
$h = 70 - 64\cos 15t$

t	0	2	4	6	8	10	12
$\cos 15t$	1	0·866...	0·5	0	–0·5	–0·866...	–1
70 $-64\cos 15t$	70 –64	70 –55·424	70 –32	70 0	70 32	70 55·424	70 64
h	6	14·576	38	70	102	125·424	134

Scale 10D (0, 3, 5, 8, 10)

Low partial credit: (3 marks)	–	1 or 2 correct entries.
Middle partial credit: (5 marks)	–	3 or 4 correct entries.
High partial credit: (8 marks)	–	5 or 6 correct entries.

7(d) (ii) Using your results from part **(i)** above, draw the graph to show the height of the centre of a passenger capsule above ground level for two complete rotations. **(10D)**



Scale 10D (0, 3, 5, 8, 10)

Low partial credit: (3 marks)	–	Correctly scaled axes with at least 2 points plotted correctly.
Middle partial credit: (5 marks)	–	Correct graph in the range $0 \le t \le 12$.
High partial credit: (8 marks)	–	Correct graph for 1 revolution only.

7(e) The most spectacular views of the city can be seen when the height of the centre of a capsule exceeds 90 m above ground level. Using your graph, or otherwise, calculate the length of time during the excursion for which these views are possible. Give your answer correct to the nearest minute. **(10D)**

$$h = 70 - 64\cos 15t$$
$$\Rightarrow \quad 70 - 64\cos 15t > 90$$
$$\Rightarrow \quad -64\cos 15t > 90 - 70$$
$$\Rightarrow \quad -64\cos 15t > 20$$
$$\Rightarrow \quad \cos 15t < -\frac{20}{64}$$
$$\Rightarrow \quad \cos 15t < -0{\cdot}3125$$

<u>Consider</u>

$$\cos 15t = 0{\cdot}3125$$
$$\Rightarrow \quad 15t = \cos^{-1}(0{\cdot}3125)$$
$$= 71{\cdot}790043...$$

as $\cos\theta < 0$

$\Rightarrow$ $90° < \theta < 180°$ <u>or</u> $180° < \theta < 270°$

$$\Rightarrow \quad 15t < 180 - 71{\cdot}790043...$$
$$< 108{\cdot}209956...$$
$$\Rightarrow \quad t < \frac{108{\cdot}209956...}{15}$$
$$< 7{\cdot}.213997...$$
$$\cong 7$$

$$\Rightarrow \quad 15t < 180 + 71{\cdot}790043...$$
$$\Rightarrow \quad < 251{\cdot}790043...$$
$$\Rightarrow \quad t < \frac{251{\cdot}790043...}{15}$$
$$< 16.786002...$$
$$\cong 17$$

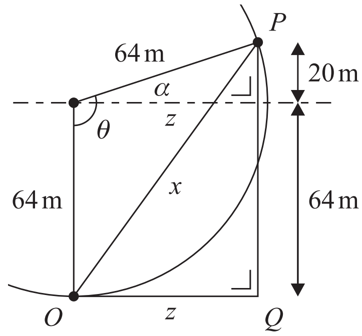
$\Rightarrow$ most spectacular views available between 7 and 17 minutes

$\Rightarrow$ length of time that most spectacular views available is 10 minutes

Scale 10D (0, 3, 5, 8, 10)

Low partial credit: (3 marks)	–	Horizontal line drawn at 90 m on graph <u>and stops</u>.
	–	$70 - 64\cos 15t = 90$ <u>or</u> equivalent.
Middle partial credit: (5 marks)	–	One correct value for t (nearest minute) from graph <u>and stops</u>.
	–	One correct solution by calculation <u>and stops</u>.
High partial credit: (8 marks)	–	Two correct value for t (nearest minute) from graph <u>and stops</u>, *i.e.* no conclusion.
	–	Two correct solutions by calculation <u>and stops</u>, *i.e.* no conclusion.

7(f) Find the distance, in a straight line, from the point of entry to a passenger capsule at the start of an excursion to the centre of a capsule 90 m above ground level. Give your answer correct to one decimal place. **(10D)**



❶

$$
\begin{aligned}
\sin\alpha &= \frac{90-(64+6)}{64} \\
&= \frac{20}{64} \\
&= 0{\cdot}3125 \\
\Rightarrow \quad \alpha &= \sin^{-1}(0{\cdot}3125) \\
&= 18{\cdot}209956... \\
&\cong 18{\cdot}21^\circ \\
\Rightarrow \quad \theta &= 90^\circ + 18{\cdot}21^\circ \\
&= 108{\cdot}21^\circ
\end{aligned}
$$

$$
\begin{aligned}
a^2 &= b^2 + c^2 - 2bc\cos A \\
\Rightarrow \quad x^2 &= 64^2 + 64^2 - 2(64)(64)\cos 108{\cdot}21^\circ \\
&= 4{,}096 + 4{,}096 - 8{,}192(-0{\cdot}312500...) \\
&= 8{,}192 + 2{,}560{\cdot}005858... \\
&= 10{,}752{\cdot}005858... \\
\Rightarrow \quad x &= \sqrt{10{,}752{\cdot}005858...} \\
&= 103{\cdot}691879... \\
&\cong 103{\cdot}7 \text{ m}
\end{aligned}
$$

<u>or</u>

❷ Using Pythagoras's theorem

$$
\begin{aligned}
d^2 &= 64^2 - 20^2 \\
&= 4{,}096 - 400 \\
&= 3{,}696 \\
\Rightarrow \quad d &= \sqrt{3{,}696}
\end{aligned}
$$

Using Pythagoras's theorem

$$
\begin{aligned}
\Rightarrow \quad x^2 &= d^2 + 84^2 \\
&= 3{,}696 + 7{,}056 \\
&= 10{,}752 \\
\Rightarrow \quad x &= \sqrt{10{,}752} \\
&= 103{\cdot}691851... \\
&\cong 103{\cdot}7 \text{ m} \\
&= 3{,}696 + 7{,}056 \\
&= 10{,}752
\end{aligned}
$$

Scale 10D (0, 3, 5, 8, 10)

Low partial credit: (3 marks)	–	Any relevant first step, *e.g.* correct triangle drawn <u>or</u> mention of 20 <u>or</u> 84.
Middle partial credit: (5 marks)	–	Angle θ found correctly and stops <u>or</u> d found correctly <u>and stops</u>.
High partial credit: (8 marks)	– –	Cosine formula correctly substituted and stops or error therein. Pythagoras applied correctly to ΔPCQ but error in finding x.

Question 8 **(75 marks)**

The 'milk round' is the term used to describe the practice by which companies visit third-level colleges and universities each year, in order to advertise possible opportunities within their organisations and to recruit final-year students.

Students who are interested in joining these companies may be interviewed. In many cases, they receive job offers prior to their final-year results based on their interview and a review of their previous years' exam results.

A group of students wants to conduct a study to examine how closely students' final-year results match their previous years' results. They obtain a list of the overall results for all the graduates who finished their final year in 2013 and who completed a particular business module as part of their course.

They decided to further analyse the data from a sample of 20 graduates. The data is shown in the table below.

Student	2nd Year (%)	3rd Year (%)	Final Year (%)
1	66	55	76
2	92	71	83
3	42	40	58
4	40	28	52
5	68	55	47
6	53	55	74
7	70	82	85
8	87	69	78
9	55	44	58
10	49	41	62
11	63	65	72
12	90	69	72
13	20	51	72
14	55	79	68
15	70	85	86
16	40	22	54
17	51	66	74
18	42	48	66
19	72	74	77
20	75	76	78

8(a) **(i)** Suggest a reason why companies might offer jobs to third-level students before they have completed their final-year exams. **(5B)**

Any 1:

- easy access to a large number of potential employees //
- seek competitive advantage over rivals by recruiting the best students //
- colleges facilitate the process and foreign firms may not attract students to travel abroad to be interviewed // *etc*.

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Reason vague <u>or</u> incomplete.

8(a) **(ii)** Explain the terms ‘sample’ and ‘population’ in the context of this question. **(5B)**

❶	Sample	–	a subset of the population that is representative of the population
❷	Population	–	the set of all students who finished their final year in 2013 <u>and</u> who completed a particular business module

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	One correct explanation only.

8(a) **(iii)** What method of sampling should be used to select the 20 graduates? Explain your answer. **(5C)**

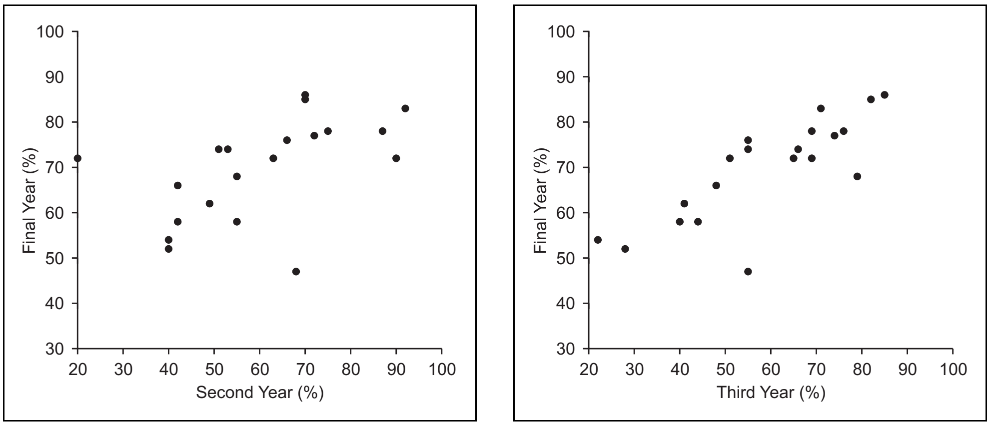
Any 1:

- stratified random sampling - to ensure that students from all courses who took the particular module are represented in the sample
- quota sampling - to ensure that all subgroups are presented, *e.g.* sex (male, female), race, ethnic background, *etc*.

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Answer only with no explanation.

8(b) The students are interested in the relationships between the graduates' performance in their previous college exams and their final-year exams. They produce the following scatter plots.



(i) Estimate the correlation coefficients of each scatter plot. **(5B)**

A: Second Year Results vs. Final Year Results
Correlation coefficient = 0·5126

** Accept figure between 0·4 and 0·7.

B: Third Year (%) vs. Final Year (%)
Correlation coefficient = 0·8018

** Accept figure between 0·6 and 0·9.

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Reasonably estimate for one correlation coefficient.

8(b) **(ii)** What can you conclude from the scatter plots and the correlation coefficients? **(5B)**

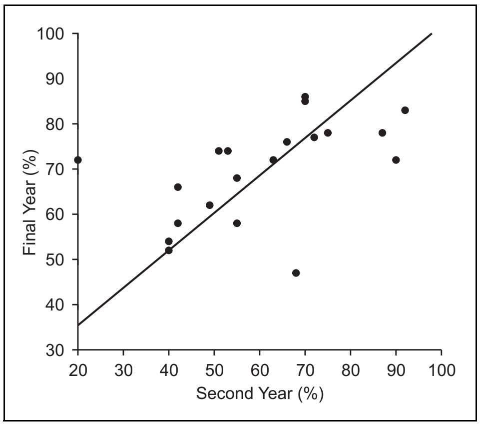
A: Second Year Results vs. Final Year Results
Correlation coefficient = 0·5126
⇒ Moderate positive correlation (tending towards low)

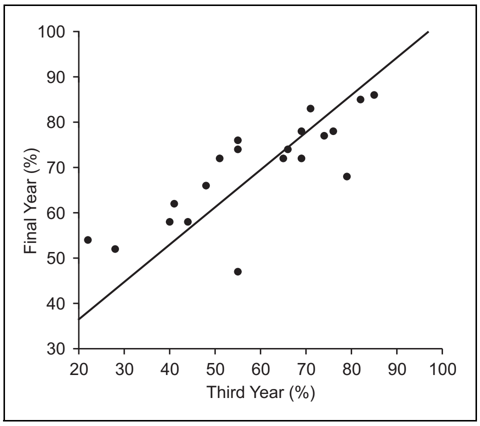
B: Third Year Results vs. Final Year Results
Correlation coefficient = 0·8018
⇒ Strong positive correlation

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Correct conclusion for one of the scatter plots only.

8(b) **(iii)** Sketch the line of best fit in each of the scatter plots above. **(5C)**





Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Straight line on one graph but clearly not the line of best fit.
High partial credit: (4 marks)	–	One line of best fit drawn correct to the eye.

(iv) By taking suitable readings from the diagrams, or otherwise, calculate the slope of the line of best fit in each plot. **(5B)**

A: Second Year Results vs. Final Year Results

$\bar{x} = 60 \qquad a = 51{\cdot}2$

$\bar{y} = 69{\cdot}6 \qquad b = 0{\cdot}306$

$\Rightarrow \quad y = 0{\cdot}306x + 51{\cdot}2$

B: Third Year Results vs. Final Year Results

$\bar{x} = 58{\cdot}75 \qquad a = 0{\cdot}5046$

$\bar{y} = 69{\cdot}6 \qquad b = 39{\cdot}95$

$\Rightarrow \quad y = 0{\cdot}5046x + 39{\cdot}95$

Scale 5B (0, 2, 5)

Low partial credit: (2 marks)	–	Slope formula correctly substituted for one plot only - accept student's points.

** Answer will depend on student's line of best fit - marks should be awarded for correct method been applied.

8(b) **(v)** Is there any significance to the difference between the two correlation coefficients? In your answer you should refer to the scatter plots and to an appropriate measure of central tendency. **(5C)**

- stronger correlation between Third Year results and Final Year results than between Second Year Results and Final Year results
- appropriate measure of central tendency

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Correct conclusion with no reference to an appropriate measure of central tendency.
High partial credit: (4 marks)	–	Correct conclusion with vague or incomplete reference to an appropriate measure of central tendency.

8(c) On further analysis of the students who completed the business module in 2013 as part of their course, it was found that:

- of the 450 students who graduated, 125 of them were part-time students
- of the students who studied part-time, 8 were repeat students
- 304 students were neither part-time nor repeat students.

(i) What is the probability that a student selected at random from all the students was part-time? **(5B)**

$$\text{P(part-time)} = \frac{125}{450} = \frac{5}{18}$$

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Uses a relevant number.
	–	Writes $\frac{\#E}{\#S}$ or equivalent.
	–	Identifies “number of outcomes of interest = ...” or “total number of outcomes = ...”.

8(c) (ii) What is the probability that a student selected at random from all the students who were part-time was also a repeat student? **(5B)**

P(repeat given part-time)

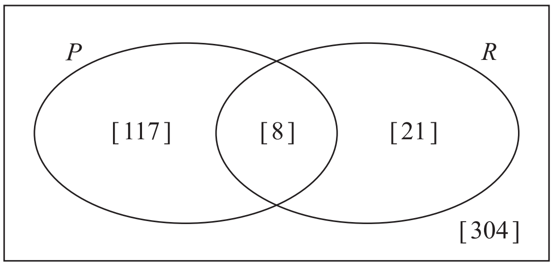
$$= \frac{8}{125}$$

Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Uses a relevant number.
	–	Writes $\frac{\# E}{\# S}$ <u>or</u> equivalent.
	–	Identifies "number of outcomes of interest = ..." or "total number of outcomes = ...".

8(c) (iii) Find the probability that a student selected at random from all the repeat students was also a part-time student. **(10C)**

Venn Diagram



P(part-time given repeat)

$$= \frac{8}{8+21}$$

$$= \frac{8}{29}$$

Scale 10C (0, 3, 7, 10)

Low partial credit: (3 marks)	–	Venn diagram with some correct substitution.
	–	Calculates total number of repeat students <u>and stops</u>.
High partial credit: (7 marks)	–	Fully correct Venn diagram <u>or</u> equivalent but error in probability, *e.g.* $\frac{8}{21}$.

8(d) The students wish to compare the results from the sample with other final-year students in that year. The university published the following results for the graduate class of 2013 who completed this module: "70% of students achieved a 2.1 honours degree or higher. That is, 70% students got an overall result of 60 percent or more."

Use an hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to conclude that the sample results are in line with the published statistics. Write the null hypothesis and the alternative hypothesis and state your conclusion clearly. **(15D)**

70% of students achieved a 2.1 honours degree or higher

H_0: Null Hypothesis:
Percentage of students who achieved a 2.1 honours degree or higher is 70%

H_1: Alternative Hypothesis:
Percentage of students who achieved a 2.1 honours degree or higher is not 70%.

Sample Proportion:

$$\hat{P} = \frac{15}{20} = 0{\cdot}75$$

Margin of Error:

$$\frac{1}{\sqrt{N}} = \frac{1}{\sqrt{20}} = 0{\cdot}223606... \cong 0{\cdot}2236$$

Confidence Interval:

$$\hat{P} - \frac{1}{\sqrt{N}} < P < \hat{P} + \frac{1}{\sqrt{N}}$$

$$\Rightarrow \quad 0{\cdot}75 - 0{\cdot}2236 < P < 0{\cdot}75 + 0{\cdot}2236$$

$$\Rightarrow \quad 0{\cdot}5264 < P < 0{\cdot}9736$$

Since 70% = 0·7 is within this interval, we accept the Null Hypothness - the percentage of students who achieved a 2.1 honours degree or higher is 70%

Scale 15D (0, 4, 7, 11, 15)

Low partial credit: (4 marks)	–	One relevant step, *e.g.* null hypothesis, margin of error, sample proportion stated only and stops.
Middle partial credit: (7 marks)	– –	Null hypothesis, margin of error and sample proportion stated and stops. Substantive work with one or more critical omissions.
High partial credit: (11 marks)	– –	Fails to accept null/alternative hypothesis correctly. Fails to contextualise answer, *e.g.* 'Accept null hypothesis' and stop.