Complex numbers $i = \sqrt{-1}$

$\sqrt{-3} = \sqrt{-1}\sqrt{3} = i\sqrt{3}$

$\sqrt{-9} = \sqrt{-1}\sqrt{+9} = \pm 3i$

$\mathbb{C} = \{\text{Complex no's}\}$

Solve $x^2 + 25 = 0 \rightarrow x = \sqrt{-5} = \pm 5i$

Solve $x^2 + 2x + 2 = 0$

$z = a + ib$ (RE, IM)

$6i + 2j + 3i - 4j = ?$

Q9 p 101

Simplify i^{30}

$4i^3 + 7i^9 = bi$?

$(6i + 2j)(3i - 4j) = ?$

$z = a + ib$

Complex Conjugate $\bar{z} = a - ib$

$z\bar{z} = \text{REAL!}$

$(a+ib)(a-ib) = a^2 - i^2 b^2$
$= a^2 + b^2$

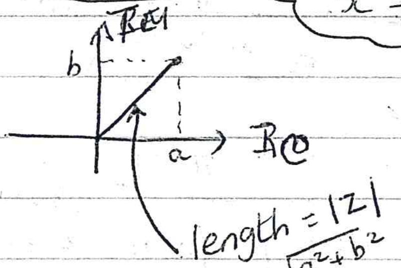
Given $(z+i)(2-i) = 3 - 4i$ find z in form $a+ib$

express $\sqrt{5+12i}$ in form $a + bi$

Find x and y if
$x + 2i + 2(3 - 5yi) = 8 - 13i$

ARGAND DIAGRAM:

$z = a + ib$

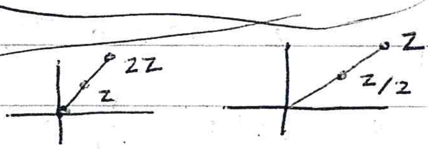


$z = a + ib$
Modulus of $z = |z|$
$|z| = \sqrt{a^2 + b^2}$

evaluate
$\left|\frac{3+i}{-2-3i}\right|$ ← Q9 p107

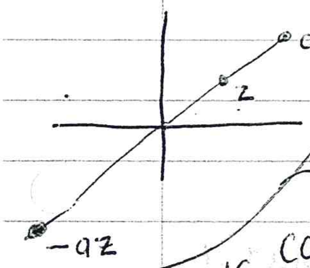
Q11 p107

Q15 p107

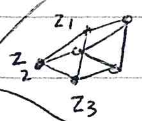


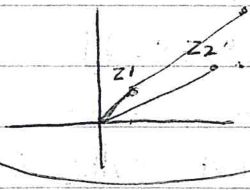
Complex no × Real + $z \times a$ = stretch or shrink $|z|$

Complex no × $-a$ → stretch or shrink and reflect $|z|$ thro (0,0)



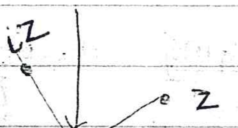
$z_1 + z_2 = a + ib + c + id = (a+c) + (b+d)i$





= translation of plane

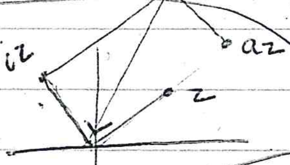
MULTIPLYING COMPLEX no's



rotat
$z \times i^2 = 180$
$z \times i^3 = 270$
$z \times i^4 = 360$
$= z \times 1$

$z \times i$ = rotation of 90° anticlockwise

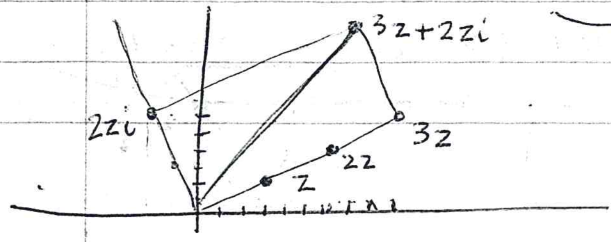
$z \times (a + i) = za + zi$
stretch + rotate



Q6 pg 113

$z(3+2i) = z.3 + z.2i$

Stretch z + stretch z then rotate
translation



(Conjugate Roots Theorem : z = complex no

If z is a root of $az^n + bz^{n-1} + \dots dz + c$ where $a, b, c, d \dots$ are $\in \mathbb{R}$

then $\bar{z}$ is also a root of this equation COEFF'TS REAL!!

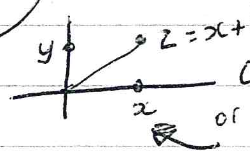
if $z = 1+5i$ is a root of $az^2+bz+c=0$ $a,b,c \in \mathbb{R}$
find a,b,c

Solve $z = 1^{1/3}$
given roots are $\alpha, \beta, 1$! Q9, Q13 p117
find cube roots of unity

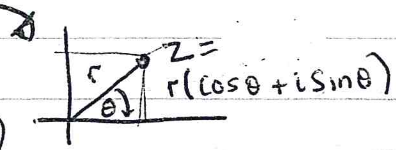
Given that $z = 1+2i$ is a root of $z^3 - z^2 + 3z + 5 = 0$
Find the 2 other roots
ex pg 115

Show $(1+2i)$ is a root of eqn
$z^2 + (-1+5i)z + 14 - 7i = 0$
why is the conjugate of $1+2i$
NOT a root ?

POLAR FORM:



CARTESIAN or RECTANGULAR FORM



$r = \sqrt{x^2+y^2}$

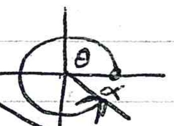
$\theta = \tan^{-1} x/y$

✗ use rad on calculator when using π !!

Express $z = 12(\cos \pi/6 + i \sin \pi/6)$ in form $x+iy$ P118 Ex1

Express $(-1 + i\sqrt{3})$ in form $r(\cos\theta + i\sin\theta)$
P118 Ex2

θ = angle referen from $+x$ axis



angle $= \theta$ or $-\alpha$

θ = argument !!!

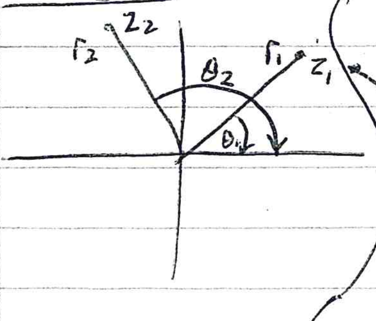
Draw diagrams !!

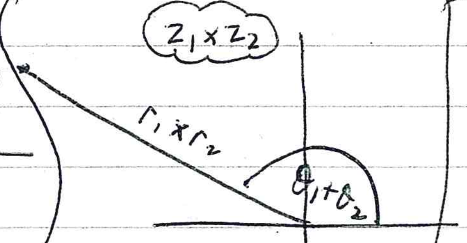
r = modulus

Products and Quotients of Complex no's :

$$z_1 z_2 = r_1(\cos\theta_1 + i\sin\theta_1) \cdot r_2(\cos\theta_2 + i\sin\theta_2)$$
$$= r_1 r_2 [\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$$

So Multiplying means the r's get multiplied, θ's get added





Dividing :

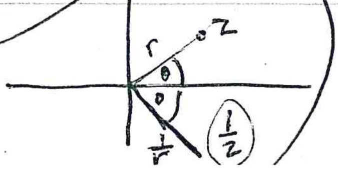
$$\frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2 \cos\theta_2 + i\sin\theta_2}$$

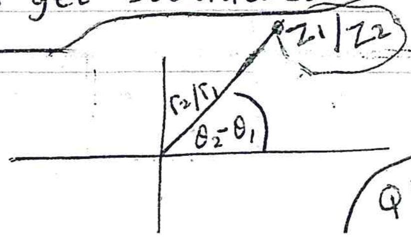
$$= \frac{r_1}{r_2}\left(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right)$$

So dividing means the r's get divided, θ's get subtracted

$z = r(\cos\theta + i\sin\theta)$

$$\frac{1}{z} = \frac{1}{r}\left(\cos(-\theta) + i\sin(-\theta)\right)$$





Q11 p123

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

write $(1+\sqrt{3}i)$ in Polar form then find value of $(1+\sqrt{3}i)^9$

P125E

Q4 p 125 Q9 p126

$\cos\theta - i\sin\theta = \cos(-\theta) + i\sin(-\theta)$!!

$$\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^8 = \left(\cos\left(-\frac{\pi}{3}\right) + i\sin(-\pi/3)\right)^8$$
$$= \left(\cos\left(\frac{-8\pi}{3}\right) + i\sin\left(\frac{-8\pi}{3}\right)\right)$$

Express $\cos 2\theta$ in terms of θ using de moivres theorem:

$$(\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$$
$$\underline{\cos^2\theta} + 2i\cos\theta\sin\theta - \underline{\sin^2\theta} = \underline{\cos 2\theta} + i\sin\theta$$

RE RE RE

RE = RE

$$\cos^2\theta - \sin^2\theta = \cos 2\theta \qquad \text{but } \sin^2\theta = 1 - \cos^2\theta$$
$$\cos 2\theta = \cos^2\theta - (1 - \cos^2\theta)$$
$$= 2\cos^2\theta - 1$$

Express $\sin 3\theta$ in terms of θ using de moivres theorem

$$(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$$

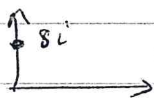
$$\cos^3\theta + \underline{3i\cos^2\theta\sin\theta} + 3i^2\cos\theta\sin^2\theta + \underline{i^3\sin^3\theta} = \cos 3\theta + \underline{i\sin 3\theta}$$

IM = IM

$$3\cos^2\theta\sin\theta - \sin^3\theta = \sin 3\theta$$
$$3(1-\sin^2\theta)\sin\theta - \sin^3\theta = \sin 3\theta$$
$$3\sin\theta - 4\sin^3\theta = \sin 3\theta$$

Find the nth root of a complex no using de Moivre

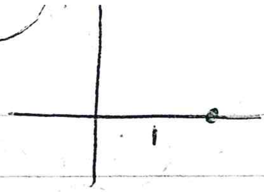
$$z^3 = 8i \Rightarrow z = (8i)^{1/3}$$



$= 8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$$8i = 8(\cos(\pi/2 + 2n\pi) + i\sin(\pi/2 + 2n\pi))$$
$$z = (8i)^{1/3} = 8^{1/3}(\cos(\pi/6 + 2n\pi/3) + i\sin(\pi/6 + 2n\pi/3))$$
$$= 2(\cos(\pi/6 + 2n\pi/3) + i\sin(\pi/6 + 2n\pi/3))$$
$$n = 0 \quad = 2(\cos(\pi/6 + 0) + i\sin(\pi/6 + 0))$$
$$= 2(\sqrt{3}/2 + i\,1/2) = \sqrt{3} + i$$
$$n = 1 \quad = 2(\cos 5\pi/6 + i\sin 5\pi/6) = -\sqrt{3} + i$$
$$n = 2 \quad = 2(\cos 9\pi/6 + i\sin 9\pi/6) = -2i$$

using de Moivre's theorem



$z^3 = 1 \quad = 1(\cos 0 + i \sin 0)$

$z = 1^{1/3} \quad = 1\left(\cos 0 + 2n\pi \quad + i \sin 0 + 2n\pi\right)^{1/3}$

$\quad = 1\left(\cos 2n\pi/3 \quad + i \sin 2n\pi/3\right)$

$n = 0 \quad = 1(\cos 0 + i \sin 0) = 1$

$n = 1 \quad = 1\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$n = 2 \quad = 1\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$