Question 1

(a) Prove that $\cos 2A = \cos^2 A - \sin^2 A$.

(b) The diagram shows part of the circular end of a running track with three running lanes shown. The centre of each of the circular boundaries of the lanes is at O.

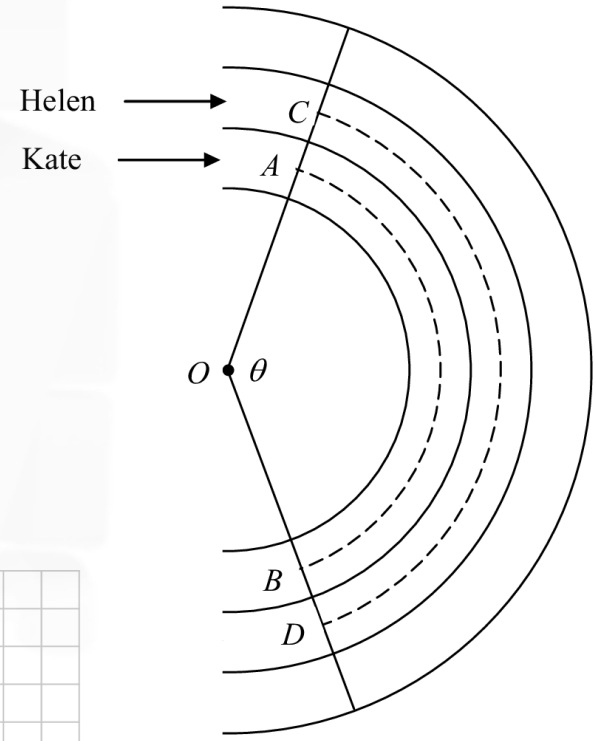
Kate runs in the middle of lane 1, from A to B as shown.

Helen runs in the middle of lane 2, from C to D as shown.

Helen runs 3 m further than Kate.

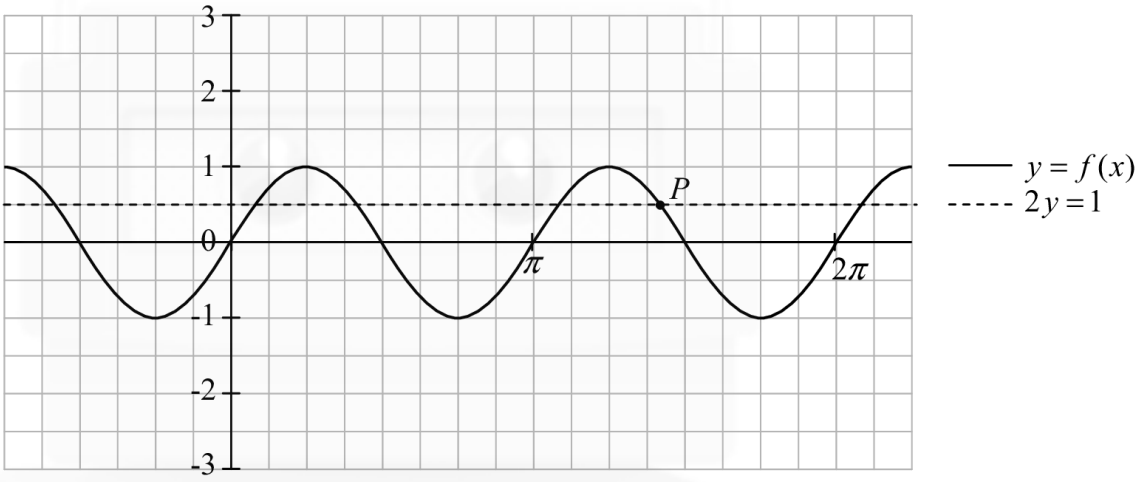
$|\angle AOB| = |\angle COD| = \theta$ radians.

If each lane is 1·2 m wide, find θ.



Question 5 **(25 marks)**

The diagram below shows the graph of the function $f : x \mapsto \sin 2x$. The line $2y = 1$ is also shown.



(a) On the same diagram above, sketch the graphs of $g : x \mapsto \sin x$ and $h : x \mapsto 3\sin 2x$. Indicate clearly which is g and which is h.

(b) Find the co-ordinates of the point P in the diagram.

Question 8 **(50 marks)**

A stand is being used to prop up a portable solar panel. It consists of a support that is hinged to the panel near the top, and an adjustable strap joining the panel to the support near the bottom.

By adjusting the length of the strap, the angle between the panel and the ground can be changed.

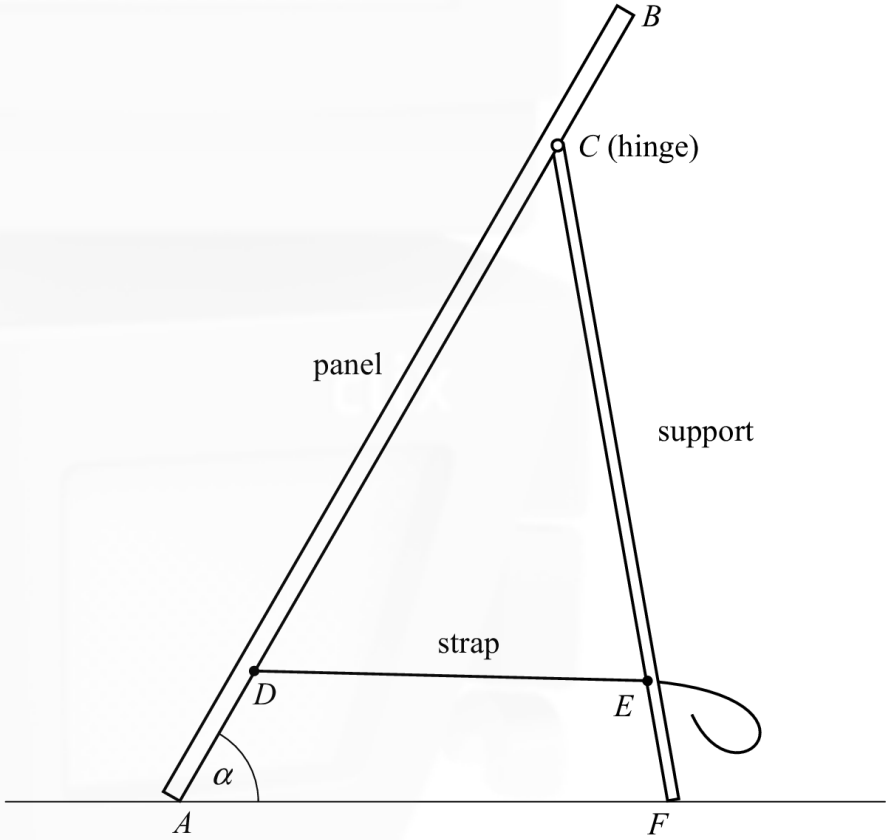
The dimensions are as follows:

$|AB| = 30$ cm

$|AD| = |CB| = 5$ cm

$|CF| = 22$ cm

$|EF| = 4$ cm.



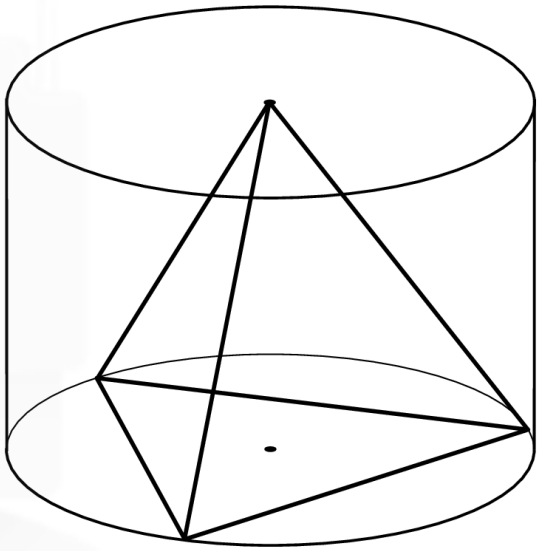
(a) Find the length of the strap $[DE]$ such that the angle α between the panel and the ground is 60°.

(b) Find the maximum possible value of α, correct to the nearest degree.

Question 9 **(25 marks)**

A regular tetrahedron has four faces, each of which is an equilateral triangle.

A wooden puzzle consists of several pieces that can be assembled to make a regular tetrahedron. The manufacturer wants to package the assembled tetrahedron in a clear cylindrical container, with one face flat against the bottom.



If the length of one edge of the tetrahedron is $2a$, show that the volume of the smallest possible cylindrical container is $\left(\frac{8\sqrt{6}}{9}\right)\pi a^3$.

Question 5 **(25 marks)**

(a) In a triangle ABC, the lengths of the sides are a, b and c. Using a formula for the area of a triangle, or otherwise, prove that

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}.$$

(b) In a triangle XYZ, $|XY| = 5$ cm, $|XZ| = 3$ cm and $|\angle XYZ| = 27°$.

(i) Find the two possible values of $|\angle XZY|$. Give your answers correct to the nearest degree.

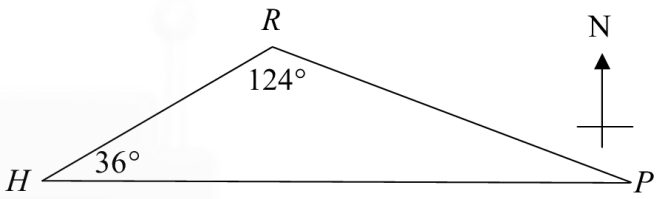
(ii) Draw a sketch of the triangle XYZ, showing the two possible positions of the point Z.

(c) In the case that $|\angle XZY| < 90°$, write down $|\angle ZXY|$, and hence find the area of the triangle XYZ, correct to the nearest integer.

$|\angle ZXY| =$

Question 8 **(30 marks)**

(a) A port P is directly east of a port H. To sail from H to P, a ship first sails 80 km, in the direction shown in the diagram, to the point R before turning through an angle of 124° and sailing 110 km directly to P.

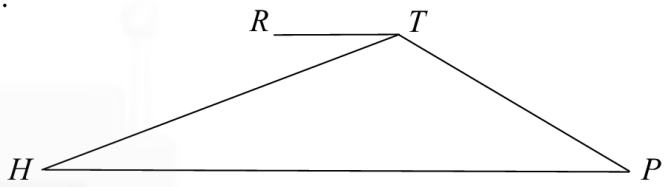


(i) Find the distance from R to HP.

(ii) Calculate $|HP|$.

(b) The point T is directly east of the point R.
$|HT| = 110$ km and $|TP| = 80$ km.

Find $|RT|$.



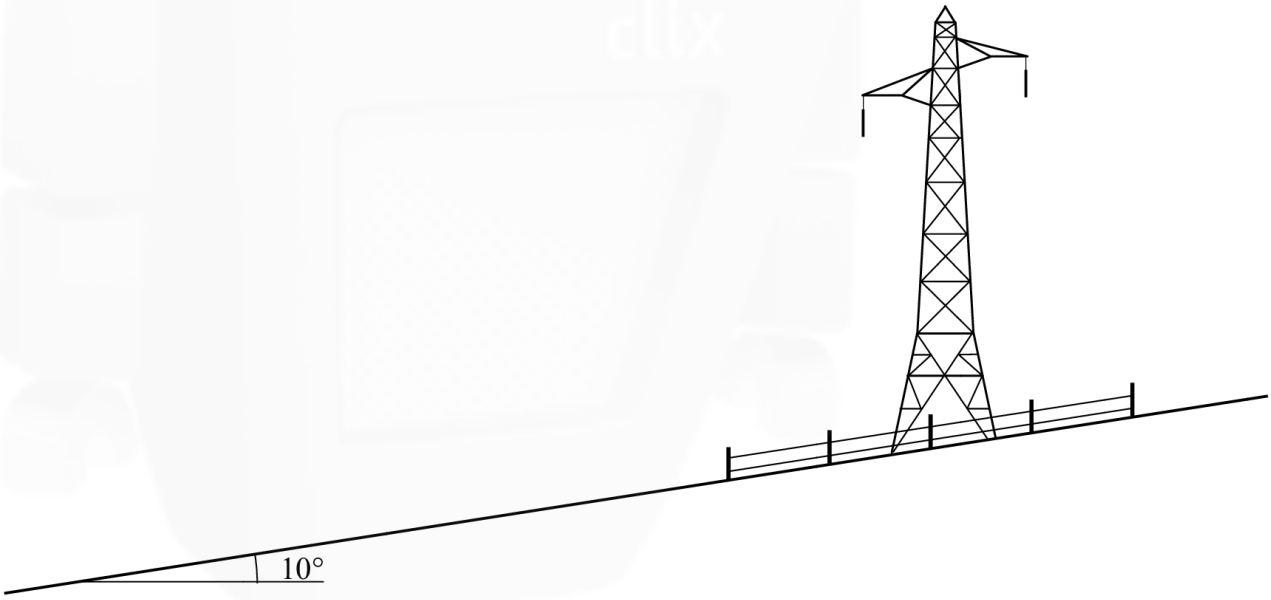
Question 8 **(75 marks)**

(a) Two surveyors want to find the height of an electricity pylon. There is a fence around the pylon that they cannot cross for safety reasons. The ground is inclined at an angle. They have a clinometer (for measuring angles of elevation) and a 100 metre tape measure. They have already used the clinometer to determine that the ground is inclined at 10° to the horizontal.

(i) Explain how they could find the height of the pylon.

Your answer should be illustrated on the diagram below. Show the points where you think they should take measurements, write down clearly what measurements they should take, and outline briefly how these can be used to find the height of the pylon.

Diagram:

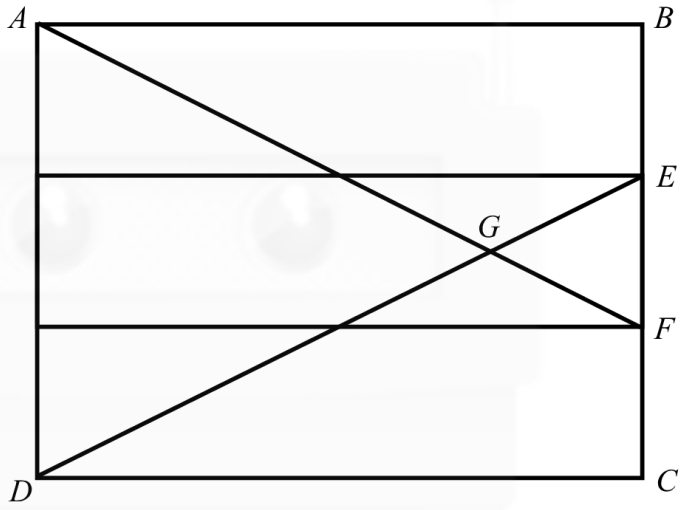


Measurements to be taken:

Procedure used to find the height:

(ii) Write down possible values for the measurements taken, and use them to show how to find the height of the pylon. (That is, find the height of the pylon using your measurements, and showing your work.)

(b) Anne is having a new front gate made and has decided on the design below.



The gate is 2 metres wide and 1·5 metres high. The horizontal bars are 0·5 metres apart.

(i) Calculate the common length of the bars $[AF]$ and $[DE]$, in metres, correct to three decimal places.

(ii) In order to secure the bar $[AF]$ to $[DE]$, the manufacturer needs to know:

- the measure of the angle EGF, and
- the common distance $|AG| = |DG|$.

Find these measures. Give the angle correct to the nearest degree and the length correct to three decimal places.

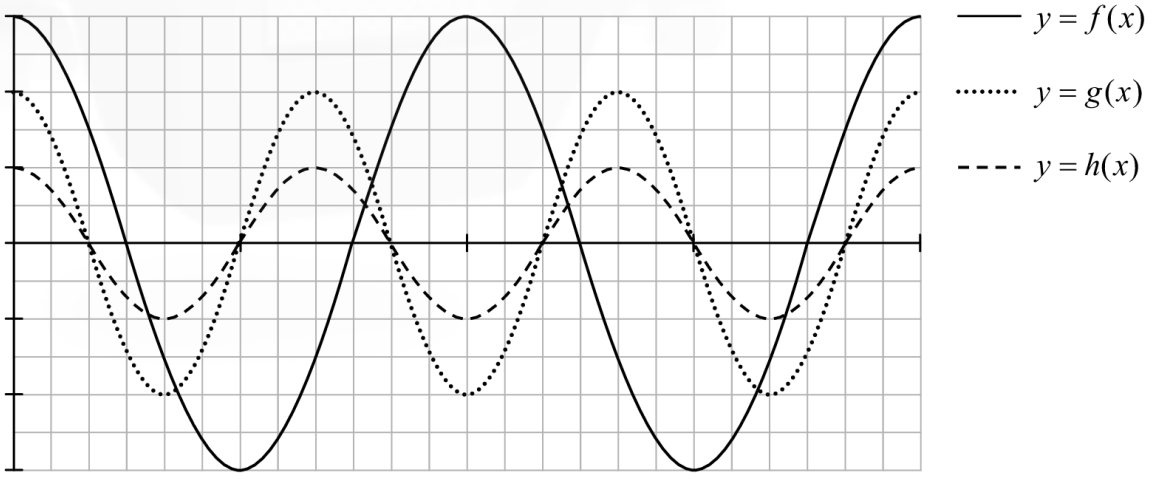
Question 9

(a) Solve the equation $\cos 3\theta = \frac{1}{2}$, for $\theta \in \mathbb{R}$, (where θ is in radians).

(b) The graphs of three functions are shown on the diagram below. The scales on the axes are not labelled. The three functions are:

$$x \to \cos 3x$$
$$x \to 2\cos 3x$$
$$x \to 3\cos 2x$$

Identify which function is which, and write your answers in the spaces below the diagram.

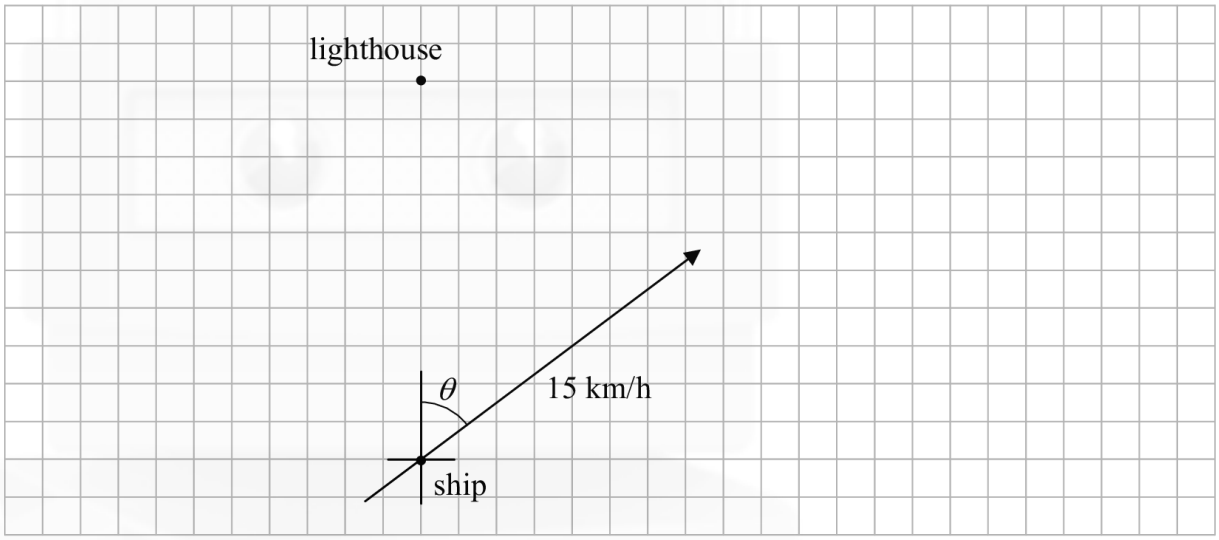


$f : x \to$ _____ $g : x \to$ _____ $h : x \to$ _____

(c) Label the scales on the axes in the diagram in part **(b)**.

Question 10

A ship is 10 km due South of a lighthouse at noon.
The ship is travelling at 15 km/h on a bearing of θ, as shown below, where $\theta = \tan^{-1}\left(\frac{4}{3}\right)$.

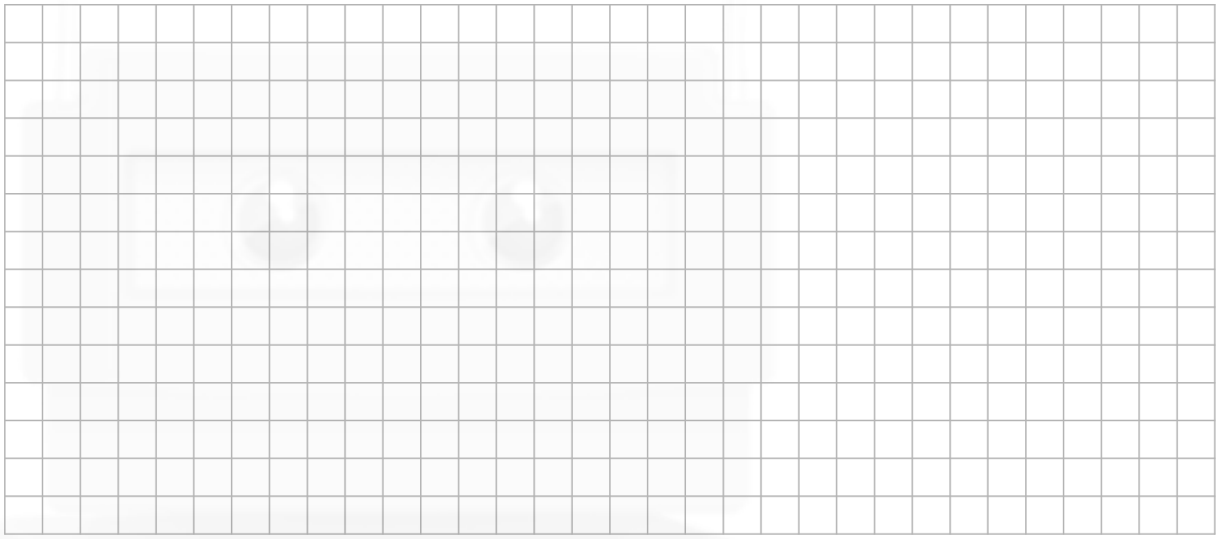


(a) On the diagram above, draw a set of co-ordinate axes that takes the lighthouse as the origin, the line East-West through the lighthouse as the x-axis, and kilometres as units.

(b) Find the equation of the line along which the ship is moving.

(c) Find the shortest distance between the ship and the lighthouse during the journey.

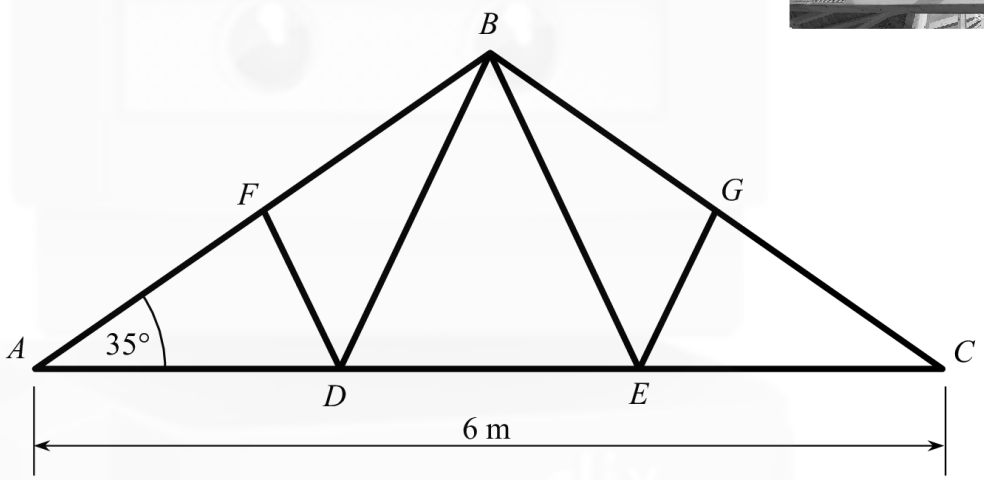
(d) At what time is the ship closest to the lighthouse?



(e) Visibility is limited to 9 km. For how many minutes in total is the ship visible from the lighthouse?

(b) Roofs of buildings are often supported by frameworks of timber called *roof trusses*.

A quantity surveyor needs to find the total length of timber needed in order to make the triangular truss shown below.



The length of $[AC]$ is 6 metres, and the pitch of the roof is 35°, as shown.
$|AD| = |DE| = |EC|$ and $|AF| = |FB| = |BG| = |GC|$.

(i) Calculate the length of $[AB]$, in metres, correct to two decimal places.

(ii) Calculate the total length of timber required to make the truss.