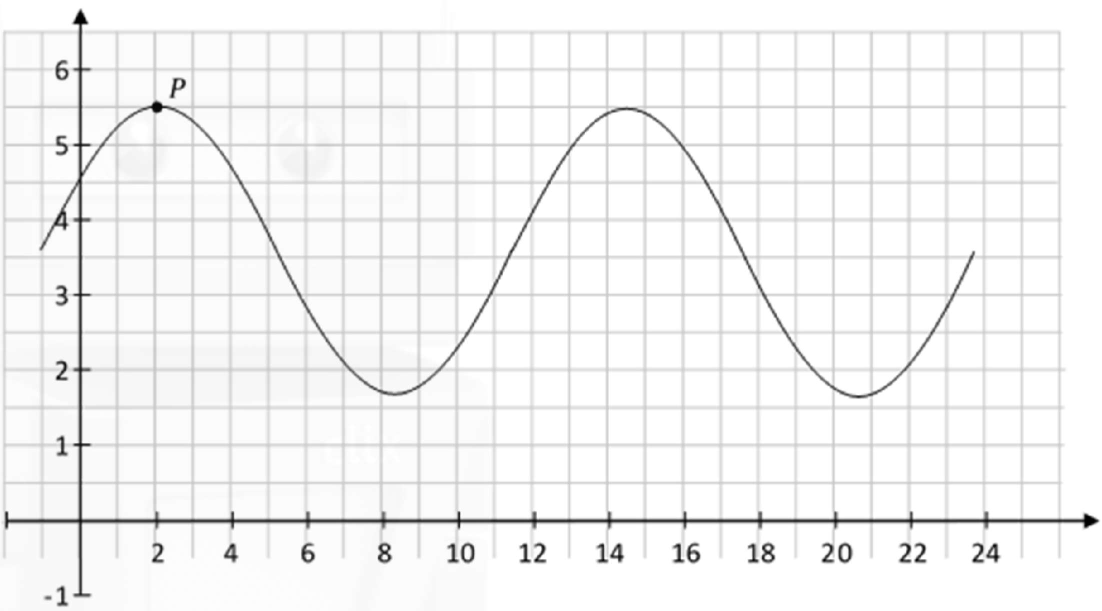
MarkingScheme

TrigPart1H

Question 1 (2017)

		$4(2) + 4\sqrt{2} + 4 + \cdots\cdots$ $a = 8 \quad r = \frac{1}{\sqrt{2}}$ $S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{8}{1 - \frac{1}{\sqrt{2}}}$ $S_\infty = \frac{8}{1 - \frac{1}{\sqrt{2}}} \cdot \frac{1 + \frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}}$ $S_\infty = \frac{8\left(1 + \frac{1}{\sqrt{2}}\right)}{\frac{1}{2}}$ $S_\infty = 16 + 8\sqrt{2}$	**Scale 10C (0, 5, 8, 10)** *Low Partial Credit:* • length of one side of new square *High Partial Credit:* • S_∞ fully substituted • Correct work with one side only

(a)		
(a)		**Scale 20C (0, 10, 18, 20)** *Low Partial Credit:* • Vertical axis drawn • Horizontal axis drawn. *High Partial Credit:* • Horizontal axis fully scaled and positioned **OR** • Vertical axis fully scaled Use relevant portions of axes **Note:** *P* can be on vertical axis

(b) (i)	$f(t) = a + b\cos ct$ $Range: \; [(a+b), (a-b)]$ $a + b = 5{\cdot}5 \qquad a - b = 1{\cdot}7$ $a = 3{\cdot}6 \qquad b = 1{\cdot}9$	**Scale 10C (0, 5, 8, 10)** *Low Partial Credit:* • one equation in a and b • Range in terms of a and b *High Partial Credit:* • a or b found **Note**: Accept correct answer without work
(b) (ii)	Time between two successive high tides is: $12\frac{34}{60}$ hours $\text{period} = 12\frac{34}{60}$ $\text{period} = \frac{2\pi}{c}$ $c = \frac{2\pi}{12\frac{34}{60}} = 0{\cdot}4999 = 0{\cdot}5$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • Period identified **or** $\frac{2\pi}{c}$ **or** 12·34 *High Partial Credit:* • equation in c with some substitution
(c)	$5{\cdot}2 = a + b\cos ct$ $5{\cdot}2 = 3{\cdot}6 + 1{\cdot}9\cos 0{\cdot}5t$ $0{\cdot}5t = \cos^{-1}\frac{1{\cdot}6}{1{\cdot}9} = 0{\cdot}569621319$ $0{\cdot}5t = 0{\cdot}5696$ $t = 1{\cdot}139$ hours (before and after high tide at 14:34) Time = 1 hour 8 minutes Times: (14:34) ± 1 hour 8 min ⇒ 13:26 and 15:42	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • equation with some substitution *High Partial Credit:* • solution for t **Note***:* Low partial at most if formula not used

(a)	$A(0,6) \to G\left(\frac{2}{3}, \frac{4}{3}\right)$ $\to P\left(\frac{2}{3}+\frac{1}{2}\left(\frac{2}{3}\right), \frac{4}{3}+\frac{1}{2}\left(\frac{-14}{3}\right)\right)$ $= \left(\frac{3}{3}, -\frac{3}{3}\right)$ $P = (1, -1)$ **or** $P = (x, y)$ $\left(\frac{2x+1(0)}{3}, \frac{2y+6}{3}\right) = \left(\frac{2}{3}, \frac{4}{3}\right)$ $x = 1, \quad y = -1$ **or** $P = (x, y)$ $\left(\frac{3(\frac{2}{3})-1(0)}{3-1}, \frac{3\left(\frac{4}{3}\right)-1(6)}{3-1}\right)$ $= \left(\frac{2}{2}, \frac{-2}{2}\right) = (1, -1)$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • $P\left(\frac{4}{3}, -\frac{10}{3}\right)$ or equivalent, i.e ratio 1:1 • $\frac{2}{3}$ or $\frac{1}{3}$ identified as part of change in x ordinate • $-\frac{14}{3}$ or $-\frac{7}{3}$ identified as part of change in y ordinate • Ratio formula with some substitution *High Partial Credit:* • one relevant co-ordinate of P found
(b)	$C(4,2) \to P(1, -1) \to B(1-3, -1-3)$ $= (-2, -4)$ $B(x,y) \to \left(\frac{4+x}{2}, \frac{2+y}{2}\right) = (1, -1)$ $x = -2, \quad y = -4$ $B = (-2, -4)$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • P as mid-point of BC *High Partial Credit:* • one relevant co-ordinate of B found ***Note:*** Accept (–2, –4) without work Accept correct graphical solution

<table>
<tr>
<td>(c)</td>
<td>

$AC \perp BC$

$AC = \frac{2-6}{4-0} = -1$

$BC = \frac{2+4}{4+2} = 1$

$-1 \times 1 = -1$

lines are perpendicular

or

$\text{Slope AB} = 5.$

$\text{Altitude from C}: y - 2 = -\frac{1}{5}(x-4)$

$\rightarrow x + 5y = 14 \ldots.(\text{i}).$

$\text{Slope AC} = -1.$

$\text{Altitude from B}:$

$y + 4 = 1(x+2)$

$\rightarrow x - y = 2 \ldots\ldots(\text{ii})$

$\rightarrow \text{Solving (i)and (ii)}$

$x = 4$

$y = 2$

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<td>

Scale 10C (0, 4, 5, 10)
Low Partial Credit:

- Identifies significance of right-angled triangle
- one equation of perpendicular from vertex to opposite side found

High Partial Credit:

- slope of AC and slope of BC found but no conclusion
- two equations of perpendiculars from vertex to opposite side found

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</table>

(a)	$\tan 60° = \frac{\lvert TE\rvert}{\lvert CT\rvert}$ $\sqrt{3}\lvert CT\rvert = \lvert TE\rvert$	**Scale 10B (0, 5, 10)** *Partial Credit:* • tan 60° • effort to express $\lvert TE\rvert$ in terms of another side of the triangle
(b)	$\tan 30° = \frac{\lvert TE\rvert}{\lvert DT\rvert}$ $\lvert TE\rvert = \lvert DT\rvert \frac{1}{\sqrt{3}}$ $\lvert TE\rvert = \frac{\sqrt{225+\lvert CT^2\rvert}}{\sqrt{3}}$ $\lvert TE\rvert = \sqrt{\frac{225+\lvert CT\rvert^2}{3}}$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • tan 30° • Use of Pythagoras for $\lvert DT\rvert$ • Effort at expressing $\lvert DT\rvert$ in terms of another side of ΔDET *High Partial Credit:* • $\lvert TE\rvert = \lvert DT\rvert \frac{1}{\sqrt{3}}$
(c)	$\sqrt{3}\lvert CT\rvert = \sqrt{\frac{225+\lvert CT\rvert^2}{3}}$ $\lvert CT\rvert = \sqrt{\frac{225}{8}}$ $= 5{\cdot}3033$ m $= 5{\cdot}3$ m	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • equates both expressions *High Partial Credit:* • Isolate $\lvert CT\rvert$ in equation

(d)	$$\|TE\| = \sqrt{3}\|CT\| = 9{\cdot}17986 \text{ m} = 9{\cdot}2 \text{ m}$$	**Scale 10B (0, 5, 10)** *Low Partial Credit* • Substitution into formula for $\|TE\|$
(e)	$$\cos\theta = \frac{\|CT\|}{\|FT\|} = \frac{\|CT\|}{\|TE\|} = \frac{\|CT\|}{\sqrt{3}\|CT\|} = \frac{1}{\sqrt{3}}$$ $$\theta = 54{\cdot}7$$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • Some relevant substitution for $\cos\theta$ *High Partial Credit:* • Formula for $\cos\theta$ substituted in terms of $\|CT\|$
(f)	$$P = \frac{(54{\cdot}7)(2)}{360}$$ $$= 0{\cdot}3038$$ $$= 30{\cdot}4$$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • (Answer to **part (e)**)×2 • 360° *High Partial Credit:* • P fully formulated

<table>
<tr><th>Q1</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$$\text{Slope } AC = -\frac{2}{3}$$
$$\text{perp. slope} = \frac{3}{2}$$
$$y - 3 = \frac{3}{2}(x - 5)$$
$$3x - 2y = 9$$</td><td>Scale 10C (0, 3, 7, 10)
Low Partial Credit
• slope formula with some relevant substitution
• 3 = 5m+c
• $y - y_1 = m(x - x_1)$ with $x_1$or y_1 or both substituted

High Partial Credit
• perpendicular slope
• equation of line through B parallel to AC</td></tr>
<tr><td>(b)</td><td>Point of intersection of the altitudes
$$\text{Slope } AB = \frac{3+2}{5-6} = -\frac{5}{1}$$
$$\text{perp. slope} = \frac{1}{5}$$
$$y - 4 = \frac{1}{5}(x + 3)$$
$$x - 5y + 23 = 0$$
Orthocentre:
$$3x - 2y = 9 \cap x - 5y = -23$$
$$\Rightarrow y = 6 \quad x = 7$$
$$(7, 6)$$

or

If BC chosen:
$$\text{Slope } BC = \frac{3-4}{5+3} = -\frac{1}{8}$$
$$\text{perp. slope} = 8$$
Equation of altitude: $y + 2 = 8(x - 6)$
Equation: $8x - y = 50$
Orthocentre:
$$3x - 2y = 9 \cap 8x - y = 50$$
$$\Rightarrow y = 6 \quad x = 7$$
$$(7, 6)$$</td><td>Scale 15D (0, 4, 7,11,1 5)
Low Partial Credit
• demonstration of understanding of orthocentre (e.g. mentions altitude)
• slope formula with some relevant substitution
• altitude from part (a)

Mid Partial Credit
• equation of an altitude other than (a)
• some relevant substitution towards finding a second altitude and altitude from (a)
• correct construction

High Partial Credit
• two correct altitudes
• correct construction with orthocentre (7, 6)</td></tr>
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<table>
<tr><th>Q2</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$$y - 6 = \frac{1}{7}(x + 1)$$
$$x - 7y + 43 = 0$$</td><td>Scale 10C (0, 3, 7, 10)
Low Partial Credit:
• equation of line formula with some relevant substitution

High Partial Credit:
• equation of line not in required form</td></tr>
<tr><td>(b)</td><td>$$D = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$
$$D = \frac{|3(-g) + 4(-f) - 21|}{\sqrt{3^2 + 4^2}}$$
$$25 = |-3g - 4f - 21|$$
$$-3g - 4f - 21 = \pm 25$$
$$\Rightarrow 3g + 4f = -46 \quad \ldots \text{(i)}$$
$$\text{and } 3g + 4f = 4 \quad \ldots \text{(ii)}$$

But $(-g, -f) \in x - 7y + 43 = 0$
$$\Rightarrow -g + 7f + 43 = 0 \quad \ldots \text{(iii)}$$
$$\Rightarrow g = 7f + 43$$

Solving : $g = 7f + 43$ and $3g + 4f = -46$
$$f = -7 \text{ and } g = -6$$
Centre (6, 7)
$$(x - 6)^2 + (y - 7)^2 = 25$$

or
Solving: $g = 7f + 43$ and $3g + 4f = 4$
$$f = -5 \text{ and } g = 8$$
Centre (- 8, 5)

$$(x + 8)^2 + (y - 5)^2 = 25$$</td><td>Scale 15D (0, 4, 7,11,1 5)
Low Partial Credit
• some correct substitution into relevant formula (line, circle, perpendicular distance).

Mid Partial Credit
• one relevant equation in g and f
• (either(i) or (ii) or (iii))

High Partial Credit
• two relevant equations (either (i) and (iii) or (ii) and (iii))</td></tr>
</table>

<table>
<tr><th>Q4</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>$|\angle ABD| = |\angle CBD| = 90°$…….(i)
$|\angle BDC| + |\angle BCD| = 90°$ …angles in triangle sum to $180°$
$|\angle ADB| + |\angle BDC| = 90°$ …. angle in semicircle
$|\angle ADB| + |\angle BDC| = |\angle BDC| + |\angle BCD|$
$|\angle ADB| = |\angle BCD|$……..(ii)
$\therefore$ Triangles are equiangular (or similar)
or

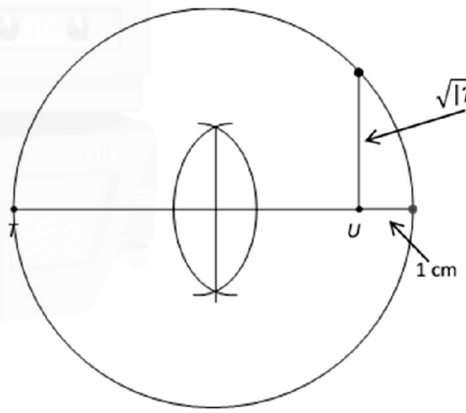
$|\angle ABD| = |\angle CBD| = 90°$…….(i)
$|\angle DAB| = |\angle DAC|$ same angle $\Rightarrow |\angle ADB| = |\angle DCA|$ (reasons as above) which is also $\angle DCB$………(ii)</td><td>Scale 15C (0, 5, 10, 15)
Low Partial Credit
• identifies one angle of same size in each triangle

High Partial Credit
• identifies second angle of same size in each triangle
• implies triangles are similar without justifying (ii) in model solution or equivalent</td></tr>
<tr><td>(a)
(ii)</td><td>$\frac{y}{1} = \frac{x}{y}$
$\Rightarrow y^2 = x$
$y = \sqrt{x}$
or
$|AD|^2 + |DC|^2 = |AC|^2$
$|AD| = \sqrt{x^2 + y^2}$
$|DC| = \sqrt{y^2 + 1}$
$x^2 + y^2 + y^2 + 1 = (x + 1)^2$
$2y^2 = 2x$
$y = \sqrt{x}$

Or

$\frac{\sqrt{x^2 + y^2}}{\sqrt{y^2 + 1}} = \frac{y}{1} \Rightarrow x^2 + y^2 = y^2(y^2 + 1)$
$y^4 = x^2 \Rightarrow y^2 = x \Rightarrow y = \sqrt{x}$</td><td>Scale 5C (0, 2, 4, 5)
Low Partial Credit
• one set of corresponding sides identified
• indicates relevant use of Pythagoras

High Partial Credit
• corresponding sides fully substituted
• expression in y^2 or y^4, i.e. fails to finish</td></tr>
</table>

(b)	Construction 	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • perpendicular line drawn at *U* or *T* • relevant use of 1 cm length • mid point of incorrect extended segment constructed *High Partial Credit* • correct mid-point constructed

Question 8 (2016)

Q7	Model Solution – 55 Marks	Marking Notes
(a) **(i)**	$\lvert EC\rvert^2 = 3^2 + 2{\cdot}5^2 = 15{\cdot}25$ $\lvert EC\rvert = \sqrt{15{\cdot}25}$ $\lvert EC\rvert = 3{\cdot}905$ $\Rightarrow \lvert AC\rvert = 1{\cdot}9525$ $= 1{\cdot}95$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • Pythagoras with relevant substitution *High Partial Credit* • $\lvert EC\rvert$ correct • $\lvert AC\rvert = \frac{1}{2}\sqrt{15{\cdot}25}$
(a) **(ii)**	$\tan 50° = \frac{\lvert AB\rvert}{1{\cdot}95}$ $\lvert AB\rvert = 1{\cdot}95(1{\cdot}19175) = 2{\cdot}23239$ $\lvert AB\rvert = 2{\cdot}3$	Scale 10B (0, 5, 10) *Partial Credit* • tan formulated correctly
(a) **(iii)**	$\lvert BC\rvert^2 = 1{\cdot}95^2 + 2{\cdot}3^2$ $\lvert BC\rvert = 3 \cdot 015377$ $\lvert BC\rvert = 3$ Also: $\sin 40° = \frac{1{\cdot}95}{\lvert BC\rvert}$ **or** $\cos 40° = \frac{2{\cdot}3}{\lvert BC\rvert}$ **or** $\cos 50° = \frac{1{\cdot}95}{\lvert BC\rvert}$ **or** $\sin 50° = \frac{2{\cdot}3}{\lvert BC\rvert}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • Pythagoras with relevant substitution *High Partial Credit* • Pythagoras fully substituted • $\lvert BC\rvert = \frac{1{\cdot}95}{\sin 40°}$ (i.e. $\lvert BC\rvert$isolated)
(a) **(iv)**	$3^2 = 3^2 + 2{\cdot}5^2 - 2(3)(2{\cdot}5)\cos \propto$ $15\cos \propto = 6{\cdot}25$ $\propto = 65°$ **or** $\cos \propto = \frac{1{\cdot}25}{3}$ $\propto = 65°$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • cosine rule with some relevant substitution • cosine ratio with some relevant substitutions • identifies three sides of triangle *BCD* *High Partial Credit* • cosine rule with full relevant substitutions • cosine ratio with full relevant substitutions

(a) **(v)**	$A = 2\times$ isosceles triangle $+2\times$ equilateral triangle $= 2\times\left[\frac{1}{2}(2{\cdot}5)(3)\sin 65°\right] +$ $2\times\left[\frac{1}{2}(3)(3)\sin 60°\right]$ $= 14{\cdot}59$ $A=15$	Scale 10D (0,3,5,8,10) *Low Partial Credit* • recognises area of 4 triangles *Mid Partial Credit* • Area of 1 triangle correct *High Partial Credit* • area of isosceles triangle and equilateral triangle **Note**: Area = 4 isosceles or 4 equilateral triangles merit *HPC* at most
(b)	$\tan 60° = \frac{3}{\lvert CA\rvert}$ $\Rightarrow \lvert CA\rvert = \sqrt{3}$ $\lvert CE\rvert = 2\sqrt{3}$ $x^2 + x^2 = \left(2\sqrt{3}\right)^2$ $x = \sqrt{6}$	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • effort at Pythagoras but without $\lvert CA\rvert$ (or $\lvert CE\rvert$) • $\lvert CA\rvert$ found *High Partial Credit* • $\lvert CE\rvert = 2\sqrt{3}$

Question 9 (2016)

Q5	**Model Solution – 25 Marks**	**Marking Notes**
(a) **(i)**	$(5x-9)^2 = (x-1)^2 + (4x)^2$ $8x^2 - 88x + 80 = 0$ $x^2 - 11x + 10 = 0$ $(x-1)(x-10) = 0$ $x = 1$ or $x = 10$ $x = 10$	Scale 10D (0, 2, 5, 8, 10) *Low Partial Credit* • any use of Pythagoras *Mid Partial Credit* • fully correct substitution *High Partial Credit* • both roots correct
(a) **(ii)**	Sides=9, 40, 41 $9^2 + 40^2 = 41^2$ $81 + 1600 = 1681$ $1681 = 1681$	Scale 5B (0, 2, 5) *Partial Credit* • 9 or 40 or 41 • using 1 or -10 from candidates work

(a) Prove that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$.

$$\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}$$
$$=\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B-\sin A\sin B}$$
$$=\frac{\dfrac{\sin A\cos B}{\cos A\cos B}+\dfrac{\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B}{\cos A\cos B}-\dfrac{\sin A\sin B}{\cos A\cos B}}$$
$$=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

or

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\dfrac{\sin A}{\cos A}+\dfrac{\sin B}{\cos B}}{1-\dfrac{\sin A\sin B}{\cos A\cos B}}=$$
$$\frac{\dfrac{\sin A\cos B+\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B-\sin A\sin B}{\cos A\cos B}}=\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B-\sin A\sin B}=$$
$$\frac{\sin(A+B)}{\cos(A+B)}=\tan(A+B)$$

(b) Find all the values of x for which $\sin(3x)=\dfrac{\sqrt{3}}{2}$, $0\le x\le 360$, x in degrees.

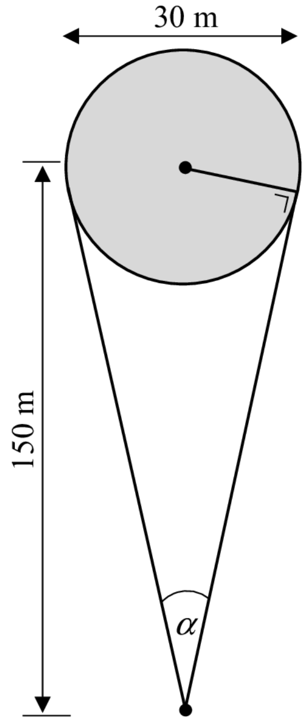
$$\sin 3x=\frac{\sqrt{3}}{2}$$
$$\Rightarrow 3x=60°,\quad 120°,\quad 420°,\quad 480°,\quad 780°,\quad 840°$$
$$\Rightarrow x=20°,\quad 40°,\quad 140°,\quad 160°,\quad 260°,\quad 280°$$

or

$3x=60°+n(360°),\ n\in\mathbb{Z}$ or $3x=120°+n(360°),\ n\in\mathbb{Z}$
$x=20°+n(120°),\ n\in\mathbb{Z}$ or $x=40°+n(120°),\ n\in\mathbb{Z}$

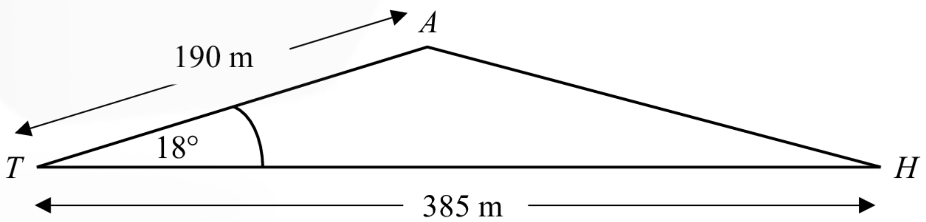
$n=0\Rightarrow x=20°$ or $x=40°$
$n=1\Rightarrow x=140°$ or $x=160°$
$n=2\Rightarrow x=260°$ or $x=280°$

(a) Joan is playing golf. She is 150 m from the centre of a circular green of diameter 30 m. The diagram shows the range of directions in which Joan can hit the ball so that it could land on the green. Find α, the measure of the angle of this range of directions. Give your answer, in degrees, correct to one decimal place.



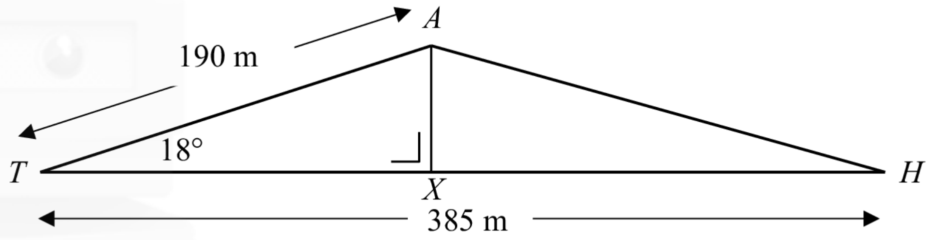
$$\sin \tfrac{1}{2}\alpha = \frac{15}{150} = 0\cdot 1$$
$$\Rightarrow \tfrac{1}{2}\alpha = 5\cdot 739^\circ$$
$$\Rightarrow \alpha = 11\cdot 478^\circ$$
$$\alpha = 11\cdot 5^\circ$$

(b) At the next hole, Joan, at T, attempts to hit the ball in the direction of the hole H. Her shot is off target and the ball lands at A, a distance of 190 metres from T, where $|\angle ATH| = 18^\circ$. $|TH|$ is 385 metres. Find $|AH|$, the distance from the ball to the hole, correct to the nearest metre.



$$|AH|^2 = 190^2 + 385^2 - 2(190)(385)\cos 18^\circ$$
$$= 36100 + 148225 - 139139\cdot 5683$$
$$= 45185\cdot 4317$$
$$|AH| = 212\cdot 57 = 213$$

or



Draw AX perpendicular to TH

triangle ATX: $\sin 18° = \frac{|AX|}{190} \Rightarrow |AX| = 58\cdot71$

$\cos 18° = \frac{|TX|}{190} \Rightarrow |TX| = 180\cdot7$

$\Rightarrow |XH| = 204\cdot3$

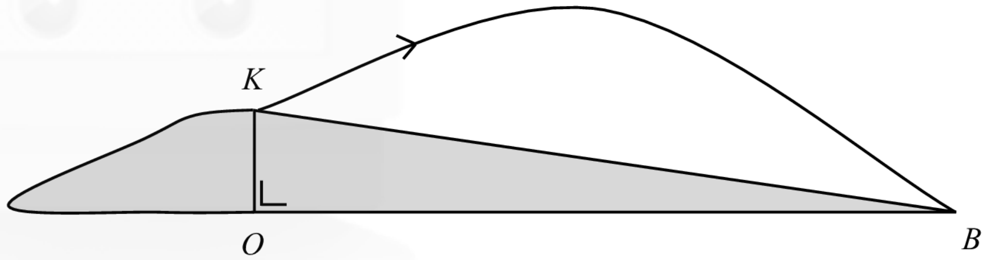
$\Rightarrow |AH|^2 = (58\cdot71)^2 + (204\cdot3)^2$

$\Rightarrow |AH| = 212\cdot566 = 213$

(c) At another hole, where the ground is not level, Joan hits the ball from K, as shown. The ball lands at B. The height of the ball, in metres, above the horizontal line OB is given by

$$h = -6t^2 + 22t + 8$$

where t is the time in seconds after the ball is struck and h is the height of the ball.



(i) Find the height of K above OB.

$h = -6t^2 + 22t + 8$
$t = 0 \Rightarrow h = 8 \text{ m}$

(ii) The horizontal speed of the ball over the straight distance $[OB]$ is a constant 38 m s^{-1}. Find the angle of elevation of K from B, correct to the nearest degree.

$$h = 0 \Rightarrow -6t^2 + 22t + 8 = 0$$
$$\Rightarrow (t-4)(-6t-2) = 0$$
$$\Rightarrow t = 4, \quad t = -\tfrac{1}{3}$$

$t = 4 \Rightarrow |OB| = 38 \times 4 = 152 \text{ m}$

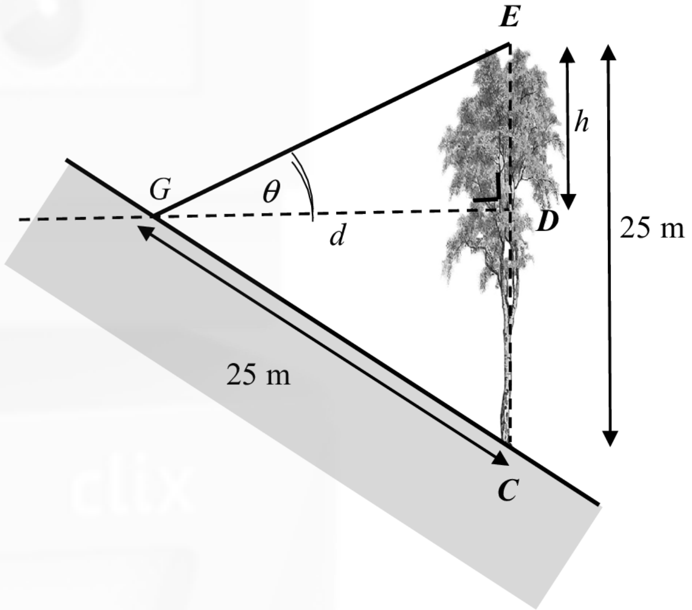
$$\tan|\angle OBK| = \frac{8}{152} = \frac{1}{19} \quad \Rightarrow \quad |\angle OBK| = 3\cdot 01° = 3°$$

(d) At a later hole, Joan's first shot lands at the point G, on ground that is sloping downwards, as shown. A vertical tree, $[CE]$, 25 metres high, stands between G and the hole. The distance, $|GC|$, from the ball to the bottom of the tree is also 25 metres.

The angle of elevation at G to the top of the tree, E, is θ, where $\theta = \tan^{-1}\frac{1}{2}$.

The height of the top of the tree above the horizontal, GD, is h metres and $|GD| = d$ metres.

(i) Write d and $|CD|$ in terms of h.



$$\tan\theta = \frac{h}{d} = \frac{1}{2}$$
$$\Rightarrow d = 2h$$

$$|CD| = 25 - h$$

(ii) Hence, or otherwise, find h.

$$d^2 + |CD|^2 = 25^2$$
$$(2h)^2 + (25-h)^2 = 25^2$$
$$4h^2 + 625 - 50h + h^2 = 625$$
$$5h^2 - 50h = 0$$
$$h = 0, \quad h = 10$$
$$h = 10 \text{ m}$$

or

$$\theta = \tan^{-1}\tfrac{1}{2} = 26\cdot565°$$
$$\Rightarrow \quad |GED| = 63\cdot435°$$
$$\Rightarrow \quad |CGE| = 63\cdot435°$$
$$\Rightarrow \quad |CGD| = 63\cdot435° - 26\cdot565° = 36\cdot87°$$
$$\sin 36\cdot87 = \frac{25-h}{25} = 0\cdot6$$
$$\Rightarrow 25 - h = 15$$
$$\Rightarrow h = 10 \text{ m}$$

or

$$|\angle GCE = 53\cdot14°| \Rightarrow \sin 53\cdot14° = \frac{2h}{25}$$
$$\Rightarrow 0\cdot8 = \frac{2h}{25} \Rightarrow h = 10 \text{ m}$$