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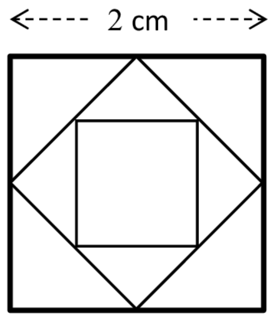
Question 1

(b) A square has sides of length 2 cm. The midpoints of the sides of this square are joined to form another square. This process is continued.
The first three squares in the process are shown below.
Find the sum of the perimeters of the squares if this process is continued indefinitely.
Give your answer in the form $a + b\sqrt{c}$ cm, where a, b, and $c \in \mathbb{N}$.



Question 9 **(40 marks)**

The depth of water, in metres, at a certain point in a harbour varies with the tide and can be modelled by a function of the form

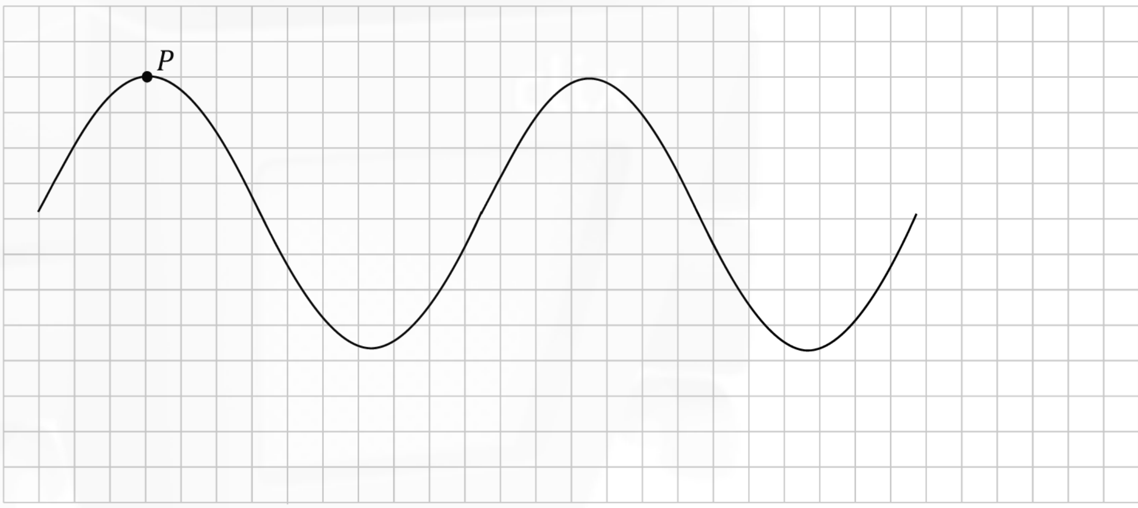
$$f(t) = a + b\cos ct$$

where t is the time in hours from the first high tide on a particular Saturday and a, b, and c are constants. (**Note**: ct is expressed in radians.)

On that Saturday, the following were noted:

- The depth of the water in the harbour at high tide was 5·5 m
- The depth of the water in the harbour at low tide was 1·7 m
- High tide occurred at 02:00 and again at 14:34.

(a) Use the information you are given to add, as accurately as you can, labelled and scaled axes to the diagram below to show the graph of f over a portion of that Saturday.
The point P should represent the depth of the water in the harbour at high tide on that Saturday morning.



(b) **(i)** Find the value of a and the value of b.

(ii) Show that $c = 0{\cdot}5$, correct to 1 decimal place.

(c) Use the equation $f(t) = a + b\cos ct$ to find the times on that Saturday **afternoon** when the depth of the water in the harbour was exactly 5·2 m.
Give each answer correct to the nearest minute.

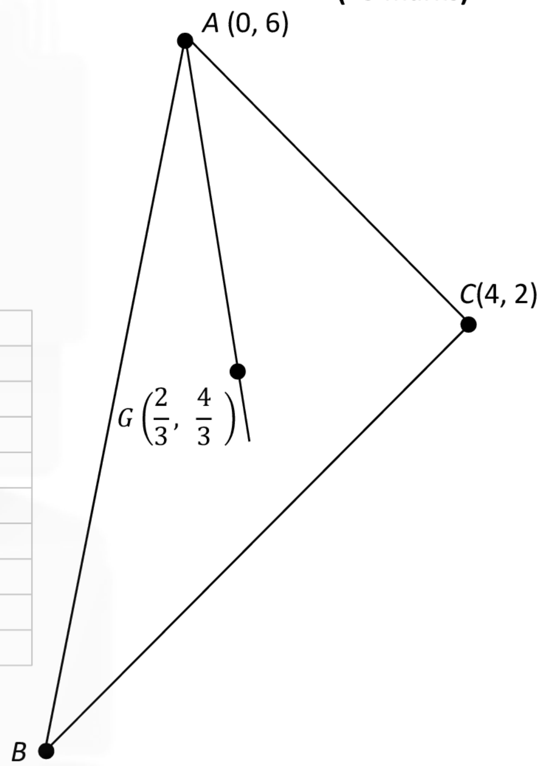
Question 3 **(25 marks)**

ABC is a triangle where the co-ordinates of *A* and *C* are (0, 6) and (4, 2) respectively.

$G\left(\frac{2}{3}, \frac{4}{3}\right)$ is the centroid of the triangle *ABC*.

AG intersects *BC* at the point *P*.

$|AG| : |GP| = 2 : 1$.



(a) Find the co-ordinates of *P*.

(b) Find the co-ordinates of *B*.

(c) Prove that *C* is the orthocentre of the triangle *ABC*.

Question 9 **(50 marks)**

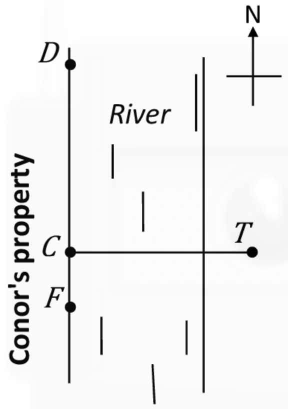


Figure 1

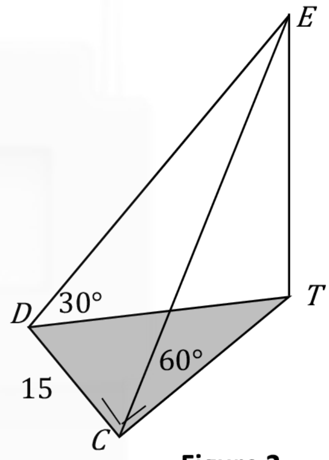


Figure 2

Conor's property is bounded by the straight bank of a river, as shown in **Figure 1** above.
T is the base of a vertical tree that is growing near the opposite bank of the river.
$|TE|$ is the height of the tree, as shown in **Figure 2** above.
From the point C, which is due west of the tree, the angle of elevation of E, the top of the tree, is $60°$.
From the point D, which is 15 m due north of C, the angle of elevation of E is $30°$ (see **Figure 2**).
The land on both sides of the river is flat and at the same level.

(a) Use triangle ECT, to express $|TE|$ in the form $\sqrt{a}|CT|$ metres, where $a \in \mathbb{N}$.

(b) Show that $|TE|$ may also be expressed as $\sqrt{\dfrac{225+|CT|^2}{3}}$ metres.

(c) Hence find $|CT|$, the distance from the base of the tree to the bank of the river at Conor's side. Give your answer correct to 1 decimal places.

(d) Find $|TE|$, the height of the tree. Give your answer correct to 1 decimal place.

(e) The tree falls across the river and hits the bank at Conor's side at the point F. Find the maximum size of the angle FTC. Give your answer in degrees, correct to 1 decimal place.

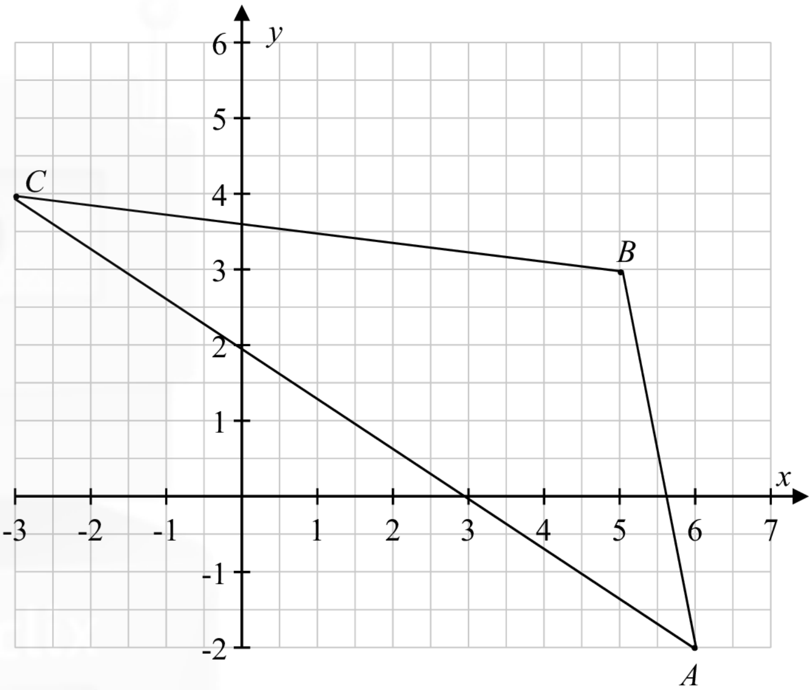
(f) If the tree was equally likely to fall in any direction, find the probability that it would hit the bank at Conor's side, when it falls.
Give your answer as a percentage, correct to 1 decimal place.

previous	page	running

Question 5

The points $A(6, -2)$, $B(5, 3)$ and $C(-3, 4)$ are shown on the diagram.

(a) Find the equation of the line through B which is perpendicular to AC.



(b) Use your answer to part **(a)** above to find the co-ordinates of the orthocentre of the triangle ABC.

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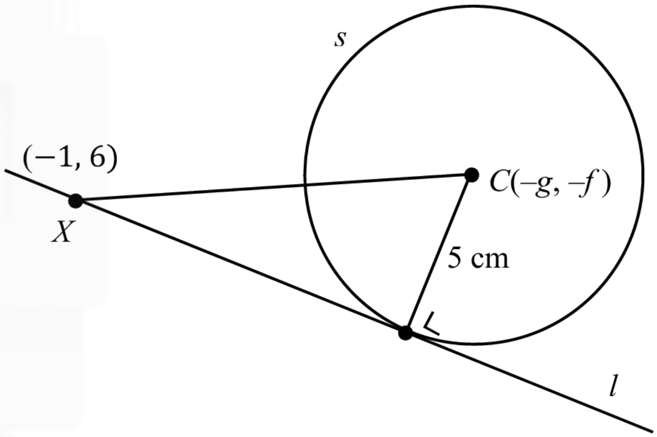
Question 6

A point X has co-ordinates $(-1, 6)$ and the slope of the line XC is $\frac{1}{7}$.

(a) Find the equation of XC. Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.

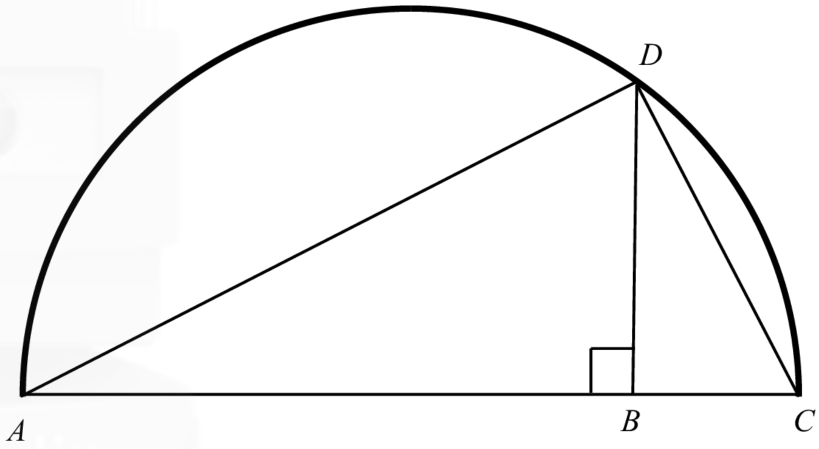


(b) C is the centre of a circle s, of radius 5 cm. The line l: $3x + 4y - 21 = 0$ is a tangent to s and passes through X, as shown. Find the equation of one such circle s.

Question 7

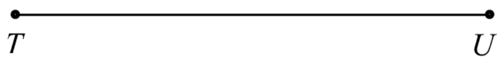
The diagram shows a semi-circle standing on a diameter $[AC]$, and $[BD] \perp [AC]$.

(a) **(i)** Prove that the triangles *ABD* and *DBC* are similar.



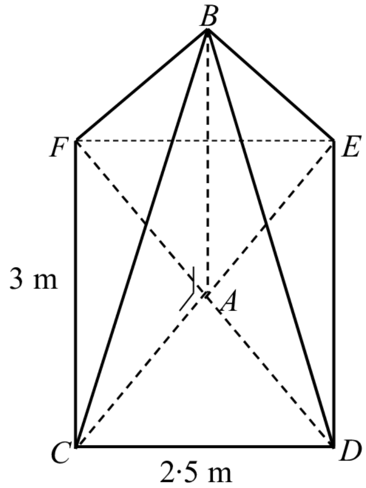
(ii) If $|AB| = x$, $|BC| = 1$, and $|BD| = y$, write y in terms of x.

(b) Use your result from part **(a)(ii)** to **construct** a line segment equal in length (in centimetres) to the square root of the length of the line segment $[TU]$ which is drawn below.



Question 8

A glass Roof Lantern in the shape of a pyramid has a rectangular base $CDEF$ and its apex is at B as shown. The vertical height of the pyramid is $|AB|$, where A is the point of intersection of the diagonals of the base as shown in the diagram.
Also $|CD| = 2 \cdot 5$ m and $|CF| = 3$ m.



(a) **(i)** Show that $|AC| = 1 \cdot 95$ m, correct to two decimal places.

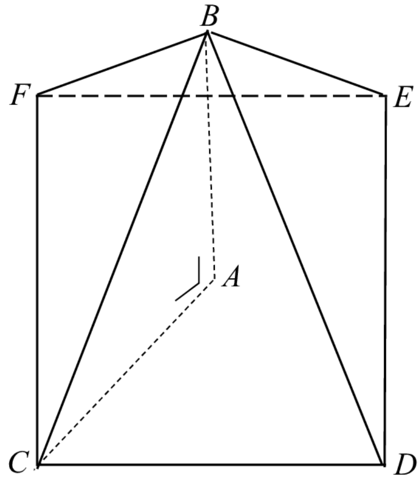
(ii) The angle of elevation of B from C is $50°$ (i.e. $|\angle BCA| = 50°$).
Show that $|AB| = 2 \cdot 3$ m, correct to one decimal place.

(iii) Find $|BC|$, correct to the nearest metre.

(iv) Find $|\angle BCD|$, correct to the nearest degree.

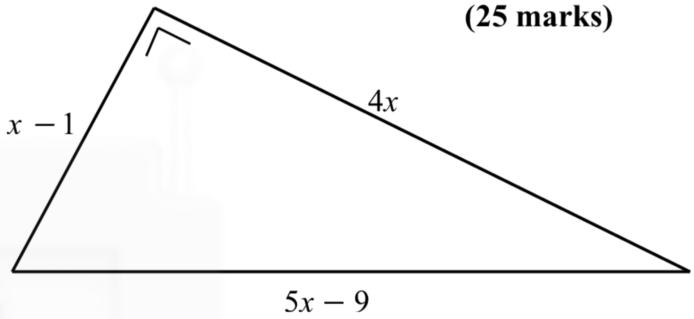
(v) Find the area of glass required to glaze all four triangular sides of the pyramid. Give your answer correct to the nearest m^2.

(b) Another Roof Lantern, in the shape of a pyramid, has a square base $CDEF$. The vertical height $|AB| = 3$ m, where A is the point of intersection of the diagonals of the base as shown.
The angle of elevation of B from C is $60°$
(i.e. $|\angle BCA| = 60°$).
Find the length of the side of the square base of the lantern.
Give your answer in the form $\sqrt{a}$ m, where $a \in \mathbb{N}$.



Question 5 **(25 marks)**

(a) **(i)** The lengths of the sides of a right-angled triangle are given by the expressions $x - 1$, $4x$, and $5x - 9$, as shown in the diagram.
Find the value of x.



(ii) Verify, with this value of x, that the lengths of the sides of the triangle above form a pythagorean triple.

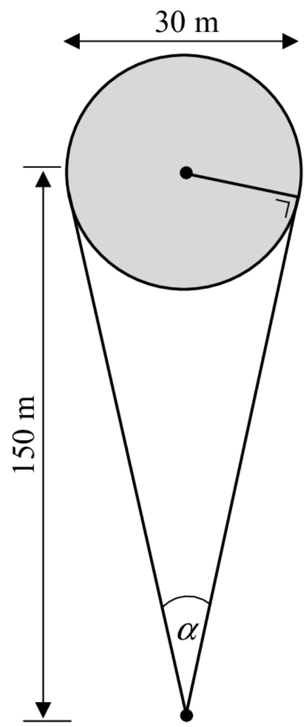
Question 5 **(25 marks)**

(a) Prove that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$.

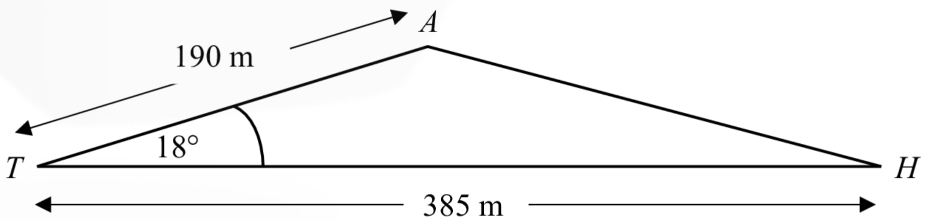
(b) Find all the values of x for which $\sin(3x)=\dfrac{\sqrt{3}}{2}$, $0\le x\le 360$, x in degrees.

Question 9 **(45 marks)**

(a) Joan is playing golf. She is 150 m from the centre of a circular green of diameter 30 m. The diagram shows the range of directions in which Joan can hit the ball so that it could land on the green. Find α, the measure of the angle of this range of directions. Give your answer, in degrees, correct to one decimal place.



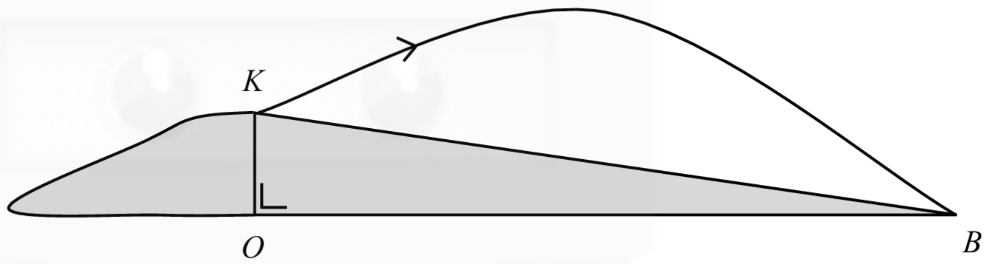
(b) At the next hole, Joan, at T, attempts to hit the ball in the direction of the hole H. Her shot is off target and the ball lands at A, a distance of 190 metres from T, where $|\angle ATH| = 18°$. $|TH|$ is 385 metres. Find $|AH|$, the distance from the ball to the hole, correct to the nearest metre.



(c) At another hole, where the ground is not level, Joan hits the ball from K, as shown. The ball lands at B. The height of the ball, in metres, above the horizontal line OB is given by

$$h = -6t^2 + 22t + 8$$

where t is the time in seconds after the ball is struck and h is the height of the ball.



(i) Find the height of K above OB.

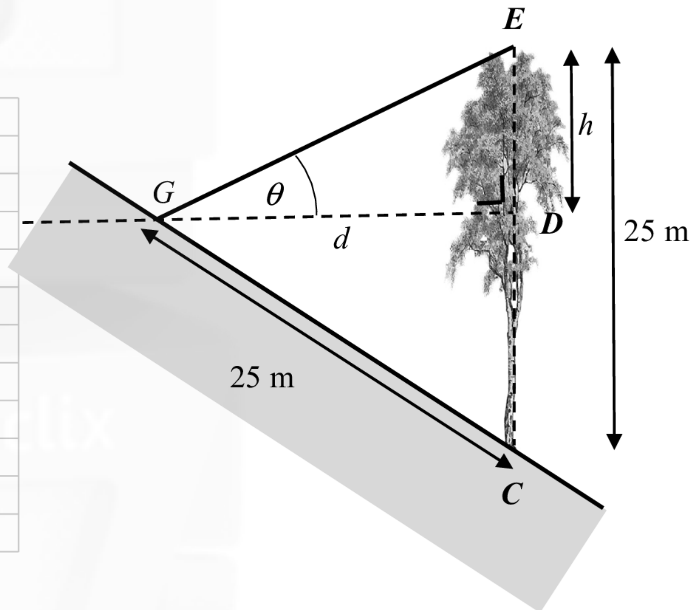
(ii) The horizontal speed of the ball over the straight distance $[OB]$ is a constant $38\ \text{m s}^{-1}$. Find the angle of elevation of K from B, correct to the nearest degree.

(d) At a later hole, Joan's first shot lands at the point G, on ground that is sloping downwards, as shown. A vertical tree, $[CE]$, 25 metres high, stands between G and the hole. The distance, $|GC|$, from the ball to the bottom of the tree is also 25 metres.

The angle of elevation at G to the top of the tree, E, is θ, where $\theta = \tan^{-1}\frac{1}{2}$.

The height of the top of the tree above the horizontal, GD, is h metres and $|GD| = d$ metres.

(i) Write d and $|CD|$ in terms of h.



$d =$

$|CD| =$

(ii) Hence, or otherwise, find h.