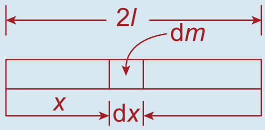
Chapter 14 Exercise 14A

Q. 1.



(i) $\rho = \dfrac{m}{2l}$, $dm = \rho\, dx$

$dI = x^2\, dm$

$\Rightarrow I = \rho \int x^2\, dx$

$\Rightarrow I = \dfrac{m}{3(2l)}\left[x^3\right]_0^{2l}$

$\Rightarrow I = \dfrac{m8l^3}{6l}$

$\Rightarrow I = \dfrac{4ml^2}{3}$ **QED**

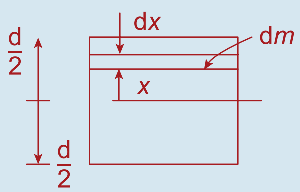
(ii) $mk^2 = I$

$\Rightarrow k = \sqrt{\dfrac{I}{m}}$

$= \sqrt{\dfrac{4\cancel{m}l^2}{3\cancel{m}}}$

$= \sqrt{\dfrac{4l^2}{3}}$ **OR** $\dfrac{2l}{\sqrt{3}}$

Q. 2.



$\rho = \dfrac{m}{d^2}$

(i) $dI = x^2\, dm$,

$\Rightarrow I = \dfrac{m}{d}\displaystyle\int_{-\frac{d}{2}}^{\frac{d}{2}} x^2\, dx$

$= \dfrac{m}{3d}\left[x^3\right]_{\frac{-d}{2}}^{\frac{d}{2}}$

$= \dfrac{m}{3d}\left[\dfrac{d^3}{8} + \dfrac{d^3}{8}\right]$

$= \dfrac{2md^3}{24d}$

$I = \dfrac{md^2}{12}$ **QED**

$dm = \rho\, dA$

$= \rho d\, dx$

$= \dfrac{m}{d^2} d\, dx$

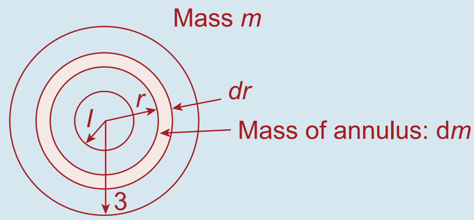
$= \dfrac{m}{d} dx$

(ii) $k = \sqrt{\dfrac{I}{m}}$

$= \sqrt{\dfrac{\cancel{m}d^2}{12\cancel{m}}}$

$\Rightarrow k = \dfrac{d}{2\sqrt{3}}$

Q. 3.



$\rho = \dfrac{\text{Mass}}{\text{Area}}$

$= \dfrac{m}{\pi(3^2 - 1^2)}$

$= \dfrac{m}{8\pi}$

$dm = \rho\, dA$

$= \dfrac{m}{8\cancel{\pi}} 2\cancel{\pi} r\, dr$

$= \dfrac{mr}{4} dr$

So, $dI = r^2\, dm$

$\Rightarrow dI = \dfrac{m}{4} r^3\, dr$

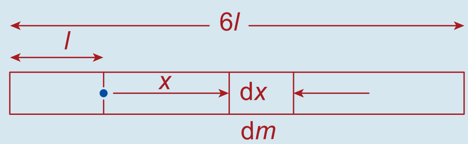
$\Rightarrow I = \dfrac{m}{4}\displaystyle\int_1^3 r^3\, dr$

$= \dfrac{m}{16}\left[r^4\right]_1^3$

$= \dfrac{m}{16}[81 - 1]$

$\Rightarrow I = 5m$ **QED**

Q. 4.



$dI = x^2\, dm$ $\rho = \dfrac{m}{6l}$

But $dm = \rho\, dx$

$= \dfrac{m}{6l} dx$

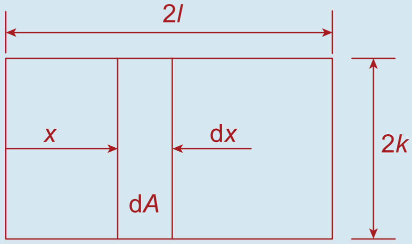
$\therefore I = \dfrac{m}{6l}\displaystyle\int_{-l}^{5l} x^2\, dx$

$= \dfrac{m}{18l}\left[x^3\right]_{-l}^{5l}$

$= \dfrac{m}{18l}[125l^3 + l^3]$

$= 7ml^2$

Q. 5.



$$\rho = \frac{3m}{4kl}$$

$$dm = \rho\, dA$$

$$= \frac{3m}{{}_2\cancel{4}\cancel{k}l} \cancel{2}\cancel{k}\, dx$$

$$= \frac{3m\, dx}{2l}$$

now $dI = x^2\, dm$

$$\Rightarrow I = \frac{3m}{2l}\int_0^{2l} x^2\, dx$$

$$\Rightarrow I = \frac{\cancel{3}m}{\cancel{3}(2)l}\left[x^3\right]_0^{2l}$$

$$= \frac{8ml^{\cancel{3}\,2}}{2\cancel{l}}$$

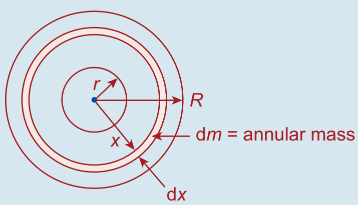
$$\Rightarrow I = 4ml^2 \quad \textbf{QED}$$

$$k = \sqrt{\frac{I}{m}}$$

$$k = \sqrt{\frac{4\cancel{m}l^2}{3\cancel{m}}} \qquad \text{Noting, here mass} = 3m$$

$$= \frac{2l}{\sqrt{3}}$$

Q. 6.



$$\rho = \frac{\text{Mass}}{\text{Area}}$$

$$= \frac{m}{\pi(R^2 - r^2)}$$

$$dm = \rho\, dA$$

$$= \frac{m}{\cancel{\pi}(R^2 - r^2)} 2\cancel{\pi}x\, dx$$

$$= \frac{2m}{R^2r^2} x\, dx$$

now, by Definition $dI = x^2\, dm$

$$\Rightarrow dI = \frac{2m}{R^2 - r^2}(x^3)\, dx$$

$$\Rightarrow I = \frac{2m}{R^2 - r^2}\int_r^R x^3\, dx$$

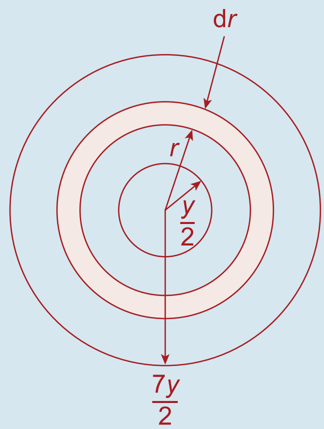
$$= \frac{2m}{4(R^2 - r^2)}\left[x^4\right]_r^R$$

$$= \frac{m}{2(R^2 - r^2)}(R^4 - r^4)$$

Noting that
$R^4 - r^4 = (R^2 - r^2)(R^2 + r^2)$

$$= \frac{m}{2}(R^2 + r^2) \quad \textbf{QED}$$

Q. 7.



Mass, $8m$

$$\rho = \frac{8m}{\pi\left(\frac{49y^2}{4} - \frac{y^2}{4}\right)} = \frac{8m}{\pi(48)y^2}(4)$$

$$\Rightarrow \rho = \frac{2m}{3\pi y^2}$$

$$dm = \rho\, dA$$

$$= \rho 2\pi r\, dr$$

$$= \frac{4m}{3y^2}\, dr$$

Now $dI = r^2\, dm$

$$\Rightarrow I = \frac{4m}{3y^2}\int_{\frac{y}{2}}^{\frac{7y}{2}} r^3\, dr$$

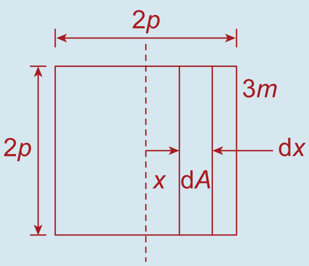
$$= \frac{\cancel{4}m}{\cancel{4}(3y^2)}\left[r^4\right]_{\frac{y}{2}}^{\frac{7y}{2}}$$

$$= \frac{m}{3y^2}\left[\frac{2{,}401}{16}y^4 - \frac{y^4}{16}\right]$$

$$= \frac{m}{3y^2}\left[\frac{2{,}400y^4}{16}\right]$$

$$\Rightarrow I = 50my^2 \quad \textbf{QED}$$

Q. 8.



$\rho = \frac{3m}{4p^2}$, $dA = 2p\ dx$

$dm = \rho\ dA$

$\Rightarrow dm = \frac{3m}{4p^2} 2p(dx)$

$= \frac{3m\ dx}{2p}$

now $dI = x^2\ dm$

$\Rightarrow I = \frac{3m}{2p} \int_{-p}^{p} x^2\ dx$

$= \frac{\not{3}m}{\not{3}(2p)} \left[x^3\right]_{-p}^{p}$

$= \frac{m}{2p}[p^3 + p^3]$

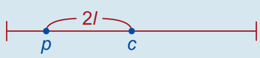
$= mp^2$ **QED**

Exercise 14B

Q. 1. (i) $I = \frac{1}{3}(m)(3l)^2 = 3ml^2$

(ii) $I = \frac{4}{3}(m)(3l)^2 = 12ml^2$

(iii) $I_p = I_c + md^2$



$= 3ml + m(2l)^2$

$= 7ml^2$

Q. 2. (a) (i) $I_A = \frac{1}{3}(m)l^2 = \frac{1}{3}ml^2$

(ii) $I_B = \frac{1}{3}(m)(2l)^2 = \frac{4}{3}ml^2$

(iii) $I_C = I_A + I_B$ (by perpendicular axis theorem)

$= \frac{5}{3}ml^2$

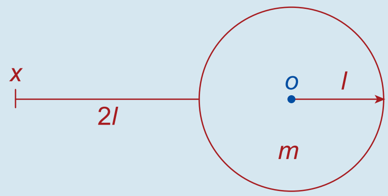
(iv) $I_p = I_c + md^2$

$= \frac{5}{3}ml^2 + m(2l)^2$

$= \frac{17}{3}ml^2$

(b) Only (iv) and by $mr^2 = (3m)(2l)^2 = 12ml^2$

Q. 3. (i)



Rod: I_X: $\frac{4}{3}ml^2$

Disc: I_o: $\frac{1}{2}ml^2$

Parallel Axes:

$(I_X = I_o + mr^2)$

$\Rightarrow I_X = \frac{ml^2}{2} + m(3l)^2$

$\Rightarrow I_X = \frac{19ml^2}{2}$

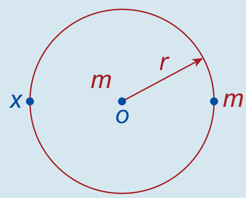
$I_{Total} = I_{Rod} + I_{Disc}$

$= \frac{19ml^2}{2} + \frac{4ml^2}{3}$

$= \frac{(57 + 8)}{6}ml^2$

$\Rightarrow I = \frac{65}{6}ml^2$

(ii)



Parallel Axes: $(I_X = I_o + mr^2)$

Disc:

$I_X = \frac{mr^2}{2} + mr^2$

$\Rightarrow I_X = \frac{3mr^2}{2}$

Pt Mass:

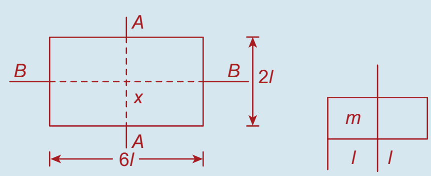
$I_X = m(2r)^2$

$\Rightarrow I_X = 4mr^2$

$I_{Total} = \frac{3mr^2}{2} + 4mr^2$

$= \frac{11mr^2}{2}$

(iii)



For Rectangle:

$I = \frac{ml^2}{3}$

$I_{AA}: \frac{m(3l)^2}{3}, \quad I_{BB} = \frac{m(l)^2}{3}$

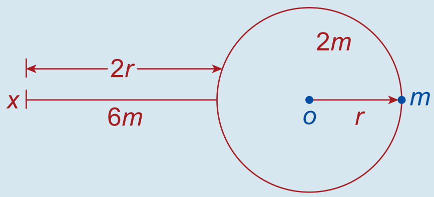
Perpendicular Axes Theorem:

$I_X = I_{AA} + I_{BB}$

$\Rightarrow I_X = \frac{9ml^2}{3} + \frac{ml^2}{3}$

$= \frac{10ml^2}{3}$

(iv)



Pt mass:

$I_X = m(4r)^2 = 16mr^2$

Disc:

$(I_X = I_O + mr^2)$

$= \frac{(2m)r^2}{2} + 2m(3r)^2$

$= 19mr^2$

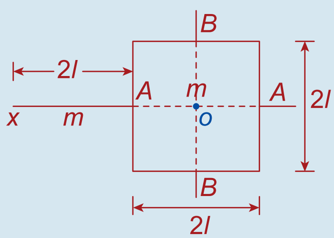
Rod:

$I_X = \frac{4}{3}(6m)r^2$

$= 8mr^2$

$I_{Total} = 19mr^2 + 8mr^2 + 16mr^2$
$= 43mr^2$

(v)



Lamina:

$I_{AA} = I_{BB} = \frac{ml^2}{3}$

$\Rightarrow \perp$ Axes $I_o = I_{AA} + I_{BB}$

$\Rightarrow I_o = \frac{2ml^2}{3}$

$\|$ Axes $(I_X = I_o + mr^2)$

$\Rightarrow I_o = \frac{2ml^2}{3} + m(3l)^2$

$\Rightarrow I_X = \frac{29ml^2}{3}$

Rod:

$I_X = \frac{4}{3} ml^2$

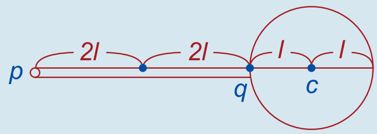
System:

$I_{Total} = I_{Rod} + I_{Lamina}$

$= \frac{29ml^2}{3} + \frac{4ml^2}{3}$

$= 11ml^2$

Q. 4.



Rod:

$I_p = \frac{4}{3}(m)(2l)^2$

$= \frac{16}{3} ml^2$

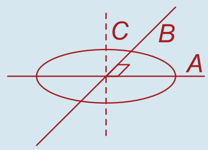
$= 5\frac{1}{3} ml^2$

Disc: $I_p = I_c + md^2$

$= \frac{1}{2}(2m)(l)^2 + (2m)(5l)^2 = 51\ ml^2$

Total $= 5\frac{1}{3}ml^2 + 51\ ml^2 = 56\frac{1}{3}ml^2$

Q. 5. (i)



By perpendicular axis theorem

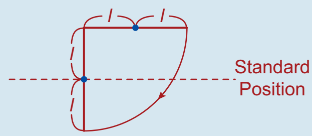
$I_A + I_B = I_C$

(ii) But since $I_A = I_B$ (by symmetry)

$\therefore I_A = \frac{1}{2}I_C$

$\therefore I_A = \frac{1}{2}\left(\frac{1}{2}mr^2\right) = \frac{1}{4} mr^2$

Q. 6. $I_p = \frac{4}{3}ml^2$



$mgh_1 + \frac{1}{2} I \omega_1^2 = mgh_2 + \frac{1}{2} I \omega_2^2$

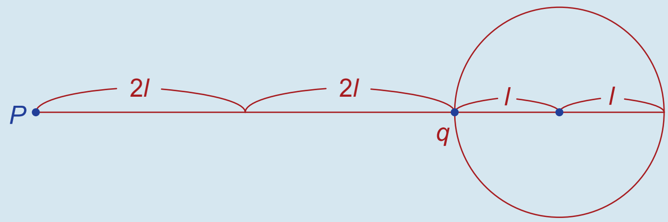
$\Rightarrow mg(1) + \frac{1}{2}I(0) = mg(0) + \frac{1}{2}I \omega_2^2$

$\Rightarrow I \omega_2^2 = 2mg$

$\Rightarrow \frac{4}{3}ml^2 \omega_2^2 = 2mg$

$\Rightarrow \omega_2 = \sqrt{\frac{3g}{2l}}$

Q. 7. (i)



Rod:

$$I_P = \frac{4}{3}m(2l)^2$$

$$= \frac{16}{3}ml^2$$

Disc:

$$I_P = I_C + md^2$$

$$= \frac{1}{2}ml^2 + m(5l)^2$$

$$= \frac{51}{2}ml^2$$

$$\text{Total} = \frac{16}{3}ml^2 + \frac{51}{2}ml^2$$

$$= \frac{185}{6}ml^2$$

(ii) $$\underbrace{mgh}_{\text{Rod}} + \underbrace{mgh}_{\text{Disc}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{System}} = \underbrace{mgh}_{\text{Rod}} + \underbrace{mgh}_{\text{Disc}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{System}}$$

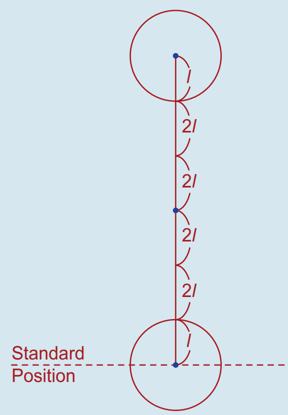
$$mg(7l) + mg(10l) + \frac{1}{2}I(0)^2 = mg(3l) + mg(0) + \frac{1}{2}I\omega^2$$

$$\Rightarrow I\omega^2 = 28mgl$$

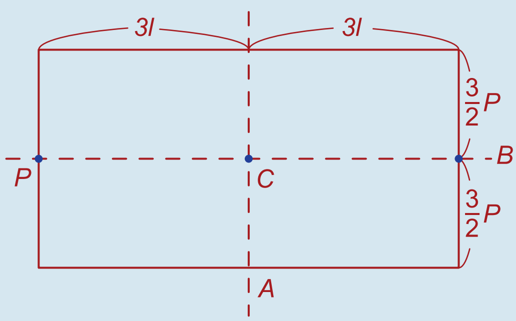
$$\Rightarrow \frac{185}{6}ml^2\,\omega^2 = 28\,mgl$$

$$\Rightarrow \omega = \sqrt{\frac{168g}{185l}}$$

$$\therefore v = \omega r = 5l\sqrt{\frac{168g}{185l}}$$



Q. 8. **The rectangular lamina:**



$$I_A = \frac{1}{3}m(3l)^2$$

$$= 3ml^2$$

$$I_B = \frac{1}{3}m\left(\frac{3}{2}l\right)^2$$

$$= \frac{3}{4}ml^2$$

$$I_C = I_A + I_B$$

$$= \frac{15}{4}ml^2$$

$$I_P = I_C + md^2$$

$$= \frac{15}{4}ml^2 + m(3l)^2$$

$$= \frac{51}{4}ml^2$$

The point mass: $I_p = mr^2$

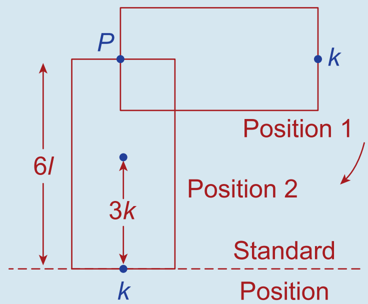
$= m(6l)^2$

$= 36ml^2$

The system $I_p = \frac{51}{4}ml^2 + 36ml^2$

$= \frac{195}{4}ml^2$

$$\underbrace{mgh}_{\text{Lamina}} + \underbrace{mgh}_{\text{Point mass}} + \underbrace{\tfrac{1}{2}I\omega^2}_{\text{System}} = \underbrace{mgh}_{\text{Lamina}} + \underbrace{mgh}_{\text{Point mass}} + \underbrace{\tfrac{1}{2}I\omega^2}_{\text{System}}$$



$$mg(6l) + mg(6l) + \frac{1}{2}I(0)^2 = mg(3l) + mg(0) + \frac{1}{2}I\omega^2$$

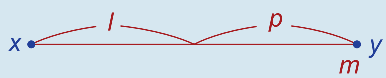
$$\Rightarrow I\omega^2 = 18mgl$$

$$\Rightarrow \frac{195}{4}ml^2\omega^2 = 18mgl$$

$$\Rightarrow \omega = \sqrt{\frac{72g}{195l}} = \sqrt{\frac{24g}{65l}}$$

Speed of $k = \omega r = 6l\sqrt{\frac{24g}{65l}}$

Q.9.



The rod:

$I_x = \frac{4}{3}(3m)l^2 = 4ml^2$

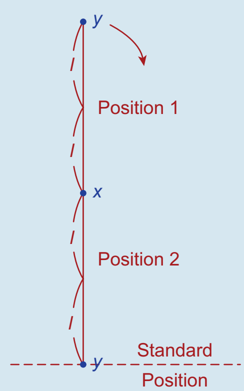
The point mass:

$I_x = m(2l)^2 = 4ml^2$

The system:

$I_x = 8ml^2$

$$\underbrace{mgh}_{\text{Rod}} + \underbrace{mgh}_{\text{Point mass}} + \underbrace{\tfrac{1}{2}I\omega^2}_{\text{System}} = \underbrace{mgh}_{\text{Rod}} + \underbrace{mgh}_{\text{Point mass}} + \underbrace{\tfrac{1}{2}I\omega^2}_{\text{System}}$$



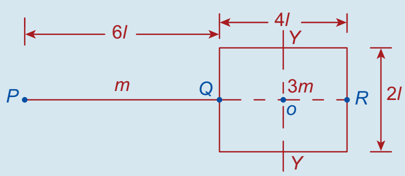
$$(3m)g(3l) + mg(4l) + \frac{1}{2}I\left(\frac{g}{l}\right) = (3m)gl + mg(0) + \frac{1}{2}I\omega^2$$

$$\Rightarrow 9mgl + 4mgl + \frac{1}{2}(8ml^2)\left(\frac{g}{l}\right) = 3mgl + \frac{1}{2}(8ml^2)\omega^2$$

$$\Rightarrow 4ml^2\omega^2 = 14mgl$$

$$\Rightarrow \omega = \sqrt{\frac{7g}{2l}}$$

Q. 10. (i)



Rod:

$I_P = \frac{4}{3}m(3l)^2$

$= 12ml^2$

Lamina:

$I_{QR} = \frac{3m(2l)^2}{3} = 4ml^2$

$I_{yy} = \frac{3ml^2}{3} = ml^2$

⊥ Axes: $I_O = I_{YY} + I_{QR}$

$= 5ml^2$

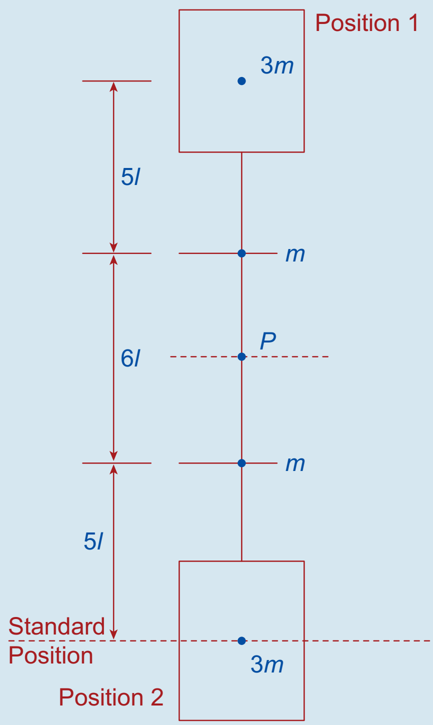
Then $(I_P = I_O + mr^2)$, (|| Axes Theorem)

$$I_P = 5ml^2 + 3m(8l)^2$$
$$I_P = 197ml^2$$

System:

$$I_{Total} = I_{Rod} + I_{Lamina}$$
$$= 12ml^2 + 197ml^2$$
$$= 209ml^2$$

(ii)



Energy Conserved:

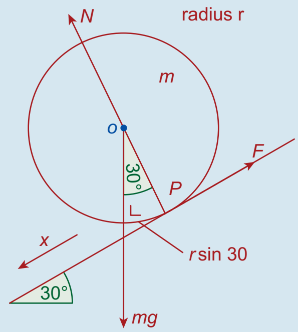
$$P.E._{①} + K.E._{①} = P.E._{②} + K.E._{②}$$
$$mg(11l) + 3mg(16l)$$
$$= mg(5l) + 3mg(0) + \frac{1}{2}I\omega_{②}^2$$
$$\Rightarrow 54mgl = \frac{1}{2}I\omega_{②}^2$$

But $I = 209ml^2$

$$\therefore \omega_{②}^2 = \frac{108mgl}{209ml^2}$$
$$\Rightarrow \omega_{②} = \sqrt{\frac{108g}{209l}}$$

Exercise 14C

Q. 1.



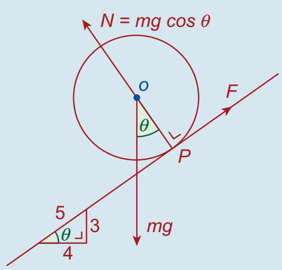
(i) Gain in K.E. = Loss in P.E.

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh \qquad \ldots \omega = \frac{v}{r}$$
$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = mg(120 \sin 30°)$$
$$\Rightarrow \frac{1}{2}v^2 + \frac{1}{4}v^2 = 60g$$
$$\Rightarrow \frac{3}{4}v^2 = 60g$$
$$\Rightarrow v^2 = 80g$$
$$\Rightarrow v = \sqrt{80g}$$
$$= 28 \text{ m/s}$$

(ii) $v^2 = u^2 + 2as$

$$\Rightarrow a = \frac{v^2 - u^2}{2s} = \frac{28^2 - 0^2}{2(120)} = \frac{49}{15}$$
$$v = u + at$$
$$\Rightarrow t = \frac{v-u}{a} = \frac{28-0}{\left(\frac{49}{15}\right)} = \frac{60}{7} \text{ s.}$$

Q. 2. (i)



Gain in K.E. = Loss in P.E.

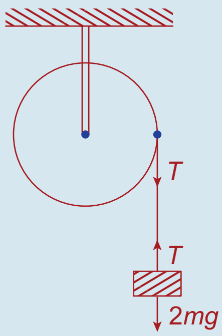
$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$
$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right) = mg(s \sin \theta)$$
$$\Rightarrow \frac{1}{2}v^2 + \frac{1}{4}v^2 = gs\left(\frac{3}{5}\right) \qquad \ldots\text{multiply by 20}$$

$$\Rightarrow 10v^2 + 5v^2 = 12gs$$
$$\Rightarrow 15v^2 = 12gs$$
$$\Rightarrow v^2 = \frac{4gs}{5}$$
$$v^2 = u^2 + 2as$$
$$\Rightarrow \frac{4gs}{5} = 0 + 2as$$
$$\Rightarrow 4g = 10a$$
$$\Rightarrow a = \frac{2}{5}g$$

(ii) On the point of slipping

$$\Rightarrow mg\sin\theta - F = ma$$
$$\Rightarrow \cancel{m}g\left(\frac{3}{\cancel{5}}\right) - \mu\,\cancel{m}g\left(\frac{4}{\cancel{5}}\right) = \cancel{m}\left(\frac{2g}{\cancel{5}}\right)$$
$$\Rightarrow \mu = \frac{1}{4} \quad \textbf{QED}$$

Q. 3. **Forces:**



Firstly, we find the speed of the $2m$ mass after it has fallen a distance h.

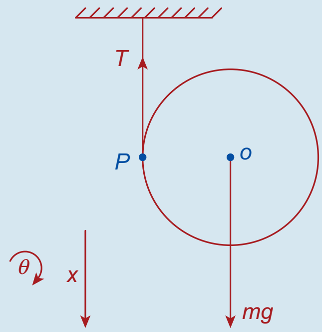
Gain in K.E. = Loss in P.E.

$$\Rightarrow \underset{\text{mass}}{\frac{1}{2}mv^2} + \underset{\text{disc}}{\frac{1}{2}I\omega^2} = \underset{\text{mass}}{mgh}$$
$$\Rightarrow \frac{1}{2}(2m)v^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right) = 2mgh$$
$$\Rightarrow v^2 + \frac{1}{4}v^2 = 2gh$$
$$\Rightarrow \frac{5}{4}v^2 = 2gh$$
$$\Rightarrow v^2 = \frac{8gh}{5}$$
$$v^2 = u^2 + 2as$$
$$\Rightarrow a = \frac{v^2 - u^2}{2s}$$
$$= \frac{\frac{8gh}{5} - 0}{2h}$$
$$= \frac{4}{5}g \text{ ms}^{-2}$$

Looking at the forces on the $2m$ mass:

$$F = ma$$
$$\Rightarrow 2mg - T = 2m\left(\frac{4g}{5}\right)$$
$$\Rightarrow 10mg - 5T = 8mg$$
$$\Rightarrow 5T = 2mg$$
$$\Rightarrow T = \frac{2}{5}mg \text{ N}$$

Q. 4.



(i) Gain in K.E. = Loss in P.E.

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgs$$
$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right) = mgs$$
$$\Rightarrow \frac{1}{2}v^2 + \frac{1}{4}v^2 = gs$$
$$\Rightarrow \frac{3}{4}v^2 = gs$$
$$\Rightarrow v^2 = \frac{4}{3}gs$$
$$\Rightarrow v = \sqrt{\frac{4}{3}gs}$$

(ii) $v^2 = u^2 + 2as$

$$\Rightarrow a = \frac{v^2 - u^2}{2s} = \frac{\frac{4}{3}gs - 0}{2s} = \frac{2}{3}g$$

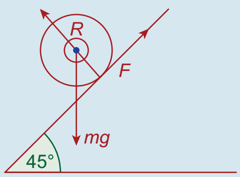
(iii) $F = ma$

$$\Rightarrow mg - T = m\left(\frac{2}{3}g\right)$$
$$\Rightarrow 3mg - 3T = 2mg$$
$$\Rightarrow 3T = mg$$
$$\Rightarrow T = \frac{1}{3}mg$$

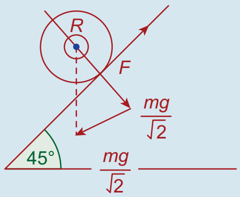
Q. 5. (i) $R = 2$ and $r = 1$

$$\therefore I = \frac{1}{2}m(4 + 1) = \frac{5}{2}m$$

Forces:



Resolved:



Gain in K.E. = Loss in P.E.

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{5}{2}m\right)\left(\frac{v^2}{r^2}\right) = mg(s \sin 45°)$$

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{5}{2}m\right)\left(\frac{v^2}{4}\right) = mg\left(\frac{s}{\sqrt{2}}\right)$$

$$\ldots\text{multiply by } \frac{16\sqrt{2}}{m}$$

$$\Rightarrow 8v^2\sqrt{2} + 5v^2\sqrt{2} = 16gs$$

$$\Rightarrow 13v^2\sqrt{2} = 16gs$$

$$\Rightarrow v^2 = \frac{16gs}{13\sqrt{2}}$$

$$v^2 = u^2 + 2as$$

$$\Rightarrow a = \frac{v^2 - u^2}{2s} = \frac{\frac{16gs}{13\sqrt{2}} - 0}{2s}$$

$$= \frac{8g}{13\sqrt{2}} \text{ m/s}^2$$

(ii) $F = ma$ (assume annulus is on the point of slipping)

$$\Rightarrow \frac{mg}{\sqrt{2}} - \mu\left(\frac{mg}{\sqrt{2}}\right) = m\left(\frac{8g}{13\sqrt{2}}\right)$$

$$\ldots\text{multiply by } \frac{13\sqrt{2}}{mg}$$

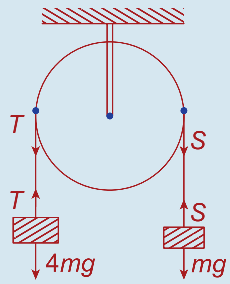
$$\Rightarrow 13 - 13\mu = 8$$

$$\Rightarrow 13\mu = 5$$

$$\Rightarrow \mu = \frac{5}{13}$$

This is the least value of μ that will prevent slipping $\therefore \mu \geq \frac{5}{13}$

Q. 6. Forces:



(i) Gain in K.E. = Loss in P.E.

$$\Rightarrow \underset{m \text{ mass}}{\frac{1}{2}mv^2} + \underset{4m \text{ mass}}{\frac{1}{2}mv^2} + \underset{\text{disc}}{\frac{1}{2}I\omega^2} = \underset{4m \text{ mass}}{mgh} - \underset{m \text{ mass}}{mgh}$$

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}(4m)v^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right)$$

$$= 4mgh - mgh \quad \ldots\text{multiply by } \frac{4}{m}$$

$$\Rightarrow 2v^2 + 8v^2 + v^2 = 12gh$$

$$\Rightarrow 11v^2 = 12gh$$

$$\Rightarrow v^2 = \frac{12}{11}gh$$

$$v^2 = u^2 + 2as$$

$$\Rightarrow a = \frac{v^2 - u^2}{2s} = \frac{\frac{12}{11}gh - 0}{2h} = \frac{6}{11}g$$

(ii) $4m$ mass:

$F = ma$

$$\Rightarrow 4mg - T = 4m\left(\frac{6}{11}g\right)$$

$$\Rightarrow 44mg - 11T = 24mg$$

$$\Rightarrow 11T = 20mg$$

$$\Rightarrow T = \frac{20}{11}mg$$

m mass:

$F = ma$

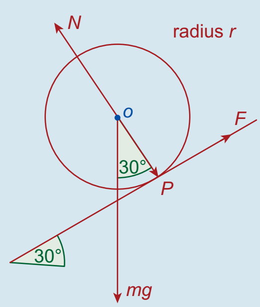
$$\Rightarrow S - mg = m\left(\frac{6}{11}g\right)$$

$$\Rightarrow 11S - 11mg = 6mg$$

$$\Rightarrow 11S = 17mg$$

$$\Rightarrow S = \frac{17}{11}mg$$

Q. 7.



Gain in K.E. = Loss in P.E.

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\left(\frac{v^2}{r^2}\right) = mg(s \sin 30°)$$

…assuming it rolls a distance s downhill

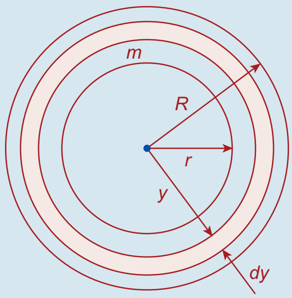
$$\Rightarrow \frac{1}{2}v^2 + \frac{1}{2}v^2 = \frac{gs}{2}$$

$$\Rightarrow v^2 = \frac{gs}{2}$$

$$v^2 = u^2 + 2as$$

$$\Rightarrow a = \frac{v^2 - u^2}{2s} = \frac{\frac{gs}{2} - 0}{2s} = \frac{1}{4}g$$

Q. 8.



$$\rho = \frac{m}{\pi(R^2 - r^2)}$$

$$dI = y^2\,dm$$

$$= \rho y^2\,dA$$

$$= 2\pi\rho\, y^2(y)\,dy$$

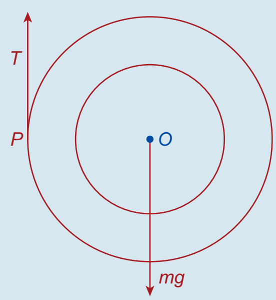
$$\Rightarrow I = \frac{2\pi m}{\pi(R^2 - r^2)}\int_r^R y^3 dy$$

$$= \frac{2m}{4(R^2 - r^2)}\left[y^4\right]_r^R$$

$$= \frac{m}{2(R^2 - r^2)}(R^4 - r^4)$$

(i) $$= \frac{m(R^2 - r^2)(R^2 + r^2)}{2(R^2 - r^2)}$$

$$\Rightarrow I = \frac{m}{2}(R^2 + r^2) \quad \textbf{QED}$$



$$\Rightarrow I_p = \frac{m}{2}(R^2 + r^2) + mR^2$$

$$\Rightarrow I_p = \frac{m}{2}(3R^2 + r^2)$$

NZL: $\Sigma L = I\ddot{\theta}$

About p: ↻

$$mgR = \frac{m}{2}(3R^2 + r^2)\,\ddot{\theta}$$

(ii) Gain in K.E. = Loss in P.E.

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\left[\frac{1}{2}m(R^2 + r^2)\right]\left(\frac{v^2}{R^2}\right) = mgh$$

…multiply by $\frac{4R^2}{m}$

$$\Rightarrow 2v^2R^2 + (R^2 + r^2)v^2 = 4R^2gh$$

$$\Rightarrow v^2(2R^2 + R^2 + r^2) = 4R^2gh$$

$$\Rightarrow v^2(3R^2 + r^2) = 4R^2gh$$

$$\Rightarrow v^2 = \frac{4R^2gh}{3R^2 + r^2}$$

$$v^2 = u^2 + 2as$$

$$\Rightarrow a = \frac{v^2 - u^2}{2s}$$

$$= \frac{\frac{4R^2gh}{3R^2 + r^2} - 0}{2h}$$

$$= \frac{2R^2g}{3R^2 + r^2}$$

(iii) $F = ma$

$$\Rightarrow mg - T = m\left(\frac{2R^2g}{3R^2 + r^2}\right)$$

$$\Rightarrow T = mg\left(1 - \frac{2R^2}{3R^2 + r^2}\right)$$

$$= mg\left(\frac{3R^2 + r^2 - 2R^2}{3R^2 + r^2}\right)$$

$$= mg\left(\frac{R^2 + r^2}{3R^2 + r^2}\right)$$

Exercise 14D

Q. 1. **Rod:** $I = \frac{4ml^2}{3}$ Length: $2l$

mass: m

For the Compound Pendulum:

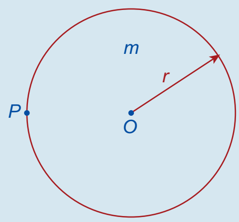
$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

Here $h = l$:

$$\therefore T = 2\pi\sqrt{\frac{4ml^2}{3mgl}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{4l}{3g}}$$

Q. 2. **Disc:**



$$I_O = \frac{mr^2}{2}$$

$$I_P = \frac{mr^2}{2} + mr^2$$

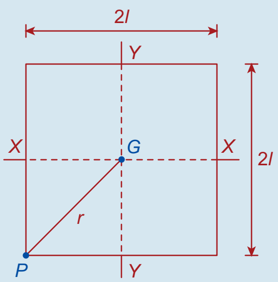
$$\Rightarrow I_P = \frac{3mr^2}{2}$$

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$= 2\pi\sqrt{\frac{3mr^2}{2mgr}}$$

$$= 2\pi\sqrt{\frac{3r}{2g}}$$

Q. 3.



mass: m

$I_{XX} = I_{YY}$ (Square Lamina)

$I_{XX} = \frac{ml^2}{3}$ (Standard Formula)

Perpendicular Axes:

$$I_G = I_{xx} + I_{yy}$$

$$\Rightarrow I_G = 2I_{xx}$$

$$\Rightarrow I_G = \frac{2ml^2}{3}$$

Parallel Axes:

$I_P = I_G + mr^2$

Here $r = \sqrt{2}l$

So $I_P = \frac{2ml^2}{3} + m(\sqrt{2}l)^2$

$$\Rightarrow I_P = \frac{8ml^2}{3}$$

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$= 2\pi\sqrt{\frac{8ml^2}{3mg\sqrt{2}l}}$$

$$= 2\pi\sqrt{\frac{8l}{3\sqrt{2}g}}$$

Q. 4. **The rod:**

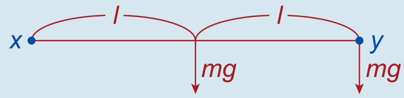
$I_x = \frac{4}{3}ml^2$

The system:

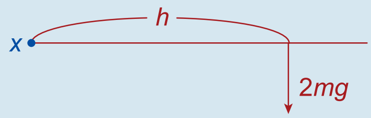
$I_x = \frac{16}{3}ml^2$

The point mass: $I_x = m(2l)^2 = 4ml^2$

Forces:



Resultant:



Taking moments about x:

$mg(l) + mg(2l) = 2mgh$

$$\Rightarrow h = \frac{3}{2}l$$

The mass of the system is $2m$.

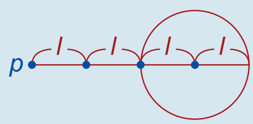
$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$= 2\pi\sqrt{\frac{\frac{16ml^2}{3}}{(2m)g\left(\frac{3l}{2}\right)}}$$

$$= 2\pi\sqrt{\frac{16l}{9g}}$$

$$= \frac{8}{3}\pi\sqrt{\frac{l}{g}}$$

Q. 5.



The rod:

$I_P = \frac{4}{3}ml^2$

The disc:

$I_P = I_C + md^2$

$= \frac{1}{3}(2m)l^2 + (2m)(3l)^2$

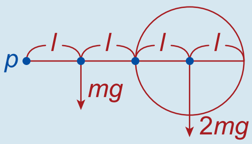
$= 19ml^2$

The system:

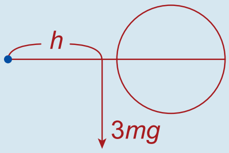
$I_P = \frac{4}{3}ml^2 + 19ml^2 = \frac{61}{3}ml^2$

To find *h*:

Forces:



Resultant:



$mg(l) + 2mg(3l) = 3mg(h)$

$$\Rightarrow h = \frac{7}{3}l$$

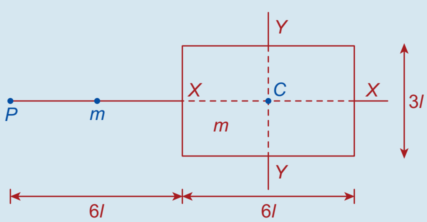
The mass of the system is $3m$.

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$= 2\pi\sqrt{\frac{\frac{61ml^2}{3}}{(3m)g\left(\frac{7l}{3}\right)}}$$

$$= 2\pi\sqrt{\frac{61l}{21g}}$$

Q. 6. (i)



Rod:

$$I_P = \frac{4}{3}m(3l)^2$$

$$\Rightarrow I_P = 12ml^2$$

Lamina:

$$I_{XX} = \frac{m}{3}\left(\frac{3l}{2}\right)^2$$

$$= \frac{3ml^2}{4}$$

$$I_{YY} = \frac{m}{3}(3l)^2$$

$$= 3ml^2$$

⊥ Axes:

$$I_C = I_{XX} + I_{YY}$$

$$= \frac{15ml^2}{4}$$

II Axes:

$$I_P = I_C + mr^2$$

$$\Rightarrow I_P = \frac{15ml^2}{4} + m(9l)^2$$

$$= \frac{339ml^2}{4}$$

$$\therefore I_{Total} = I_{Rod} + I_{Lamina}$$

$$= 12ml^2 + \frac{339ml^2}{4}$$

$$= \frac{387ml^2}{4}$$

Find Position of Centre of Gravity G:

Moments about *p*:

$m(3l) + m(9l) = 2mh$

$$\Rightarrow h = 6l$$

$$\text{So } T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$= 2\pi\sqrt{\frac{387ml^2}{4(2m)g(6l)}}$$

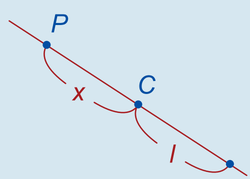
$$= 2\pi\sqrt{\frac{129l}{16g}}$$

(ii) **Simple Pendulum Equivalent:**

$$2\pi\sqrt{\frac{k}{g}} = 2\pi\sqrt{\frac{129l}{16g}}$$

$$\Rightarrow k = \frac{129l}{16}$$

Q. 7. $I_P = I_C + md^2$



$$= \frac{1}{3}ml^2 + mx^2$$

$h = x \quad m = m$

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$= 2\pi\sqrt{\frac{\frac{1}{3}ml^2 + mx^2}{mgx}}$$

$$= 2\pi\sqrt{\frac{7l}{6g}}$$

$$\Rightarrow \frac{\frac{1}{3}l^2 + x^2}{x} = \frac{7l}{6}$$

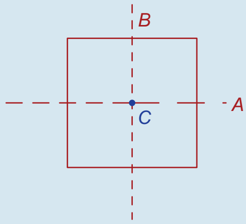
$$\Rightarrow 2l^2 + 6x^2 = 7lx$$

$$\Rightarrow 6x^2 - 7lx + 2l^2 = 0$$

$$\Rightarrow (3x - 2l)(2x - l) = 0$$

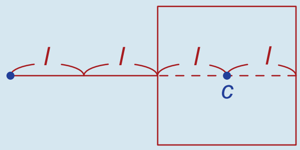
$$\Rightarrow x = \frac{2}{3}l \quad \textbf{OR} \quad x = \frac{1}{2}l$$

Q. 8. (a)



$$I_C = I_A + I_B$$
$$= \frac{1}{3}ml^2 + \frac{1}{3}ml^2$$
$$= \frac{2}{3}ml^2$$

(b) (ii)



The rod:

$$I_p = \frac{4}{3}ml^2$$

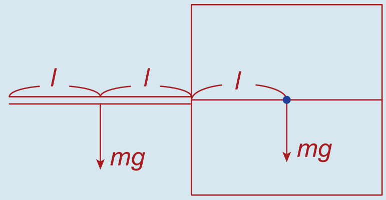
The lamina:

$$I_p = I_c + md^2$$
$$= \frac{2}{3}ml^2 + m(3l)^2$$
$$= \frac{29}{3}ml^2$$

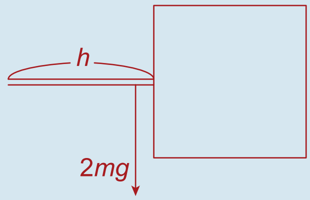
The system:

$$I_p = \frac{4}{3}ml^2 + \frac{29}{3}ml^2$$
$$= 11ml^2$$

(ii) **Forces:**



Resultant:



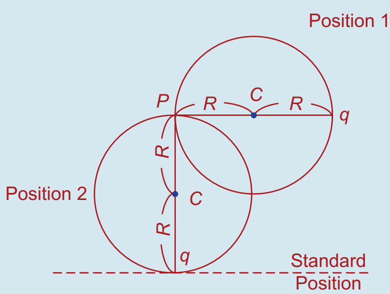
Taking moments about *p*:

$$mg(l) + mg(3l) = 2mgh$$
$$\Rightarrow h = 2l$$

The mass of the system is $2m$.

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$
$$= 2\pi\sqrt{\frac{11ml^2}{(2m)g(2l)}}$$
$$= 2\pi\sqrt{\frac{11l}{4g}}$$

Q. 9. (i)



The disc:

$$I_p = I_c + md^2$$
$$= \frac{1}{2}mR^2 + m = \frac{3}{2}mR^2$$

The point mass:

$$I_p = (2m)(2R)^2 = 8mR^2$$

The system:

$$I_p = \frac{3}{2}mR^2 + 8mR^2 = \frac{19}{2}mR^2$$

$$\underbrace{mgh}_{\text{Disc}} + \underbrace{mgh}_{\text{Point Mass}} + \underbrace{\tfrac{1}{2}I\omega^2}_{\text{System}} = \underbrace{mgh}_{\text{Disc}} + \underbrace{mgh}_{\text{Point Mass}} + \underbrace{\tfrac{1}{2}I\omega^2}_{\text{System}}$$

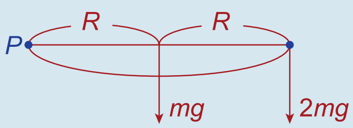
$$mg(2R) + (2m)g(2R) + \frac{1}{2}I(0)^2 = mg(R) + (2m)g(0) + \frac{1}{2}\left(\frac{19}{2}mR^2\right)\omega^2$$

$$5mgR = \frac{19}{4}mR^2\omega^2$$

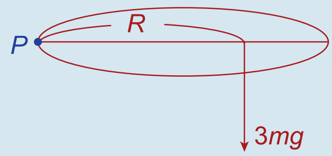
$$\Rightarrow \omega = \frac{\sqrt{20g}}{19R}$$

(ii) **To find h:**

Forces:



Resultant:



$$mg(R) + 2mg(2R) = 3mg(h)$$

$$\Rightarrow h = \frac{5}{3}R$$

The mass of the system is $3m$.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\frac{19mR^2}{2}}{(3m)g\frac{5R}{3}}} = 2\pi\sqrt{\frac{19R}{10g}}$$

If this equals $2\pi\sqrt{\frac{l}{g}}$, then $l = \dfrac{19R}{10}$

Q. 10. (i) **The rod:**

$$I_a = \frac{4}{3}(3m)p^2$$
$$= 4mp^2$$

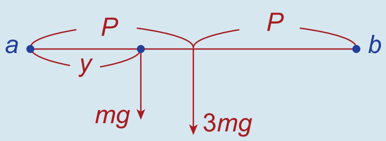
The point mass:

$$I_a = my^2$$

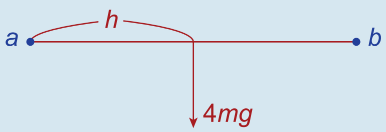
The system $= 4mp^2 + my^2$

(ii) **To find h:**

Forces



Resultant



Taking moments about a:

$$mgy + 3mgp = 4mgh$$

$$\Rightarrow h = \frac{y + 3p}{4}$$

The mass of the system is $4m$.

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$= 2\pi\sqrt{\frac{4mp^2 + my^2}{4mg\left(\frac{y+3p}{4}\right)}}$$

$$= 2\pi\sqrt{\frac{4p^2 + y^2}{g(y + 3p)}}$$

But this equals $2\pi\sqrt{\dfrac{40p}{33g}}$

$$\therefore \frac{4p^2 + y^2}{y + 3p} = \frac{40p}{33}$$

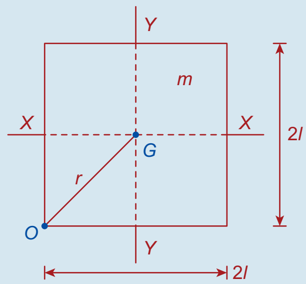
$$\Rightarrow 33y^2 - 40py + 12p^2 = 0$$

$$\Rightarrow (3y - 2p)(11y - 6p) = 0$$

$$\Rightarrow y = \frac{2p}{3} \quad \textbf{OR} \quad \frac{6p}{11}$$

Q. 11. (i) Standard Proof

(ii)



$$I_{XX} = I_{YY}$$

$$= \frac{m}{3}l^2$$

Perpendicular Axes:

$$I_G = I_{xx} + I_{yy}$$

$$I_G = \frac{2ml^2}{3}$$

Parallel Axes:

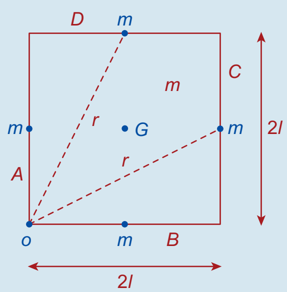
$$I_O = I_G + mr^2$$

where $r = \sqrt{2}l$

$$\Rightarrow I_O = \frac{2ml^2}{3} + m(\sqrt{2}l)^2$$

$$\Rightarrow I_O = \frac{8ml^2}{3}$$

(iii)



Lamina:

From (ii) $I_O = \frac{8ml^2}{3}$

Rods A and B:

$$I_O = \frac{4}{3}ml^2$$

Rods C and D:

$$r = \sqrt{4l^2 + l^2}$$

$$\Rightarrow r = \sqrt{5}l$$

$$I_O = I_{\text{Mid point}} + mr^2$$

$$= \frac{ml^2}{3} + m(\sqrt{5}l)^2$$

$$\Rightarrow I_O = \frac{16ml^2}{3}$$

So $I_{\text{Total}} = \frac{8ml^2}{3} + \frac{2(4ml^2)}{3} + \frac{2(16ml^2)}{3}$

$\Rightarrow I_{\text{Total}} = 16ml^2$ **QED**

(iv) **Note:** By symmetry, centre of gravity of system is at G

$\therefore h = r$ in part(ii)

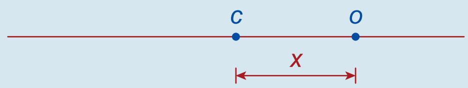
$$\therefore T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$T = 2\pi\sqrt{\frac{16ml^2}{5m\,g\sqrt{2}l}}$$

$\Rightarrow T = 2\pi\sqrt{\frac{8\sqrt{2}l}{5g}}$ **QED**

Exercise 14E

Q. 1.



Length $2l$

mass m

$I_C = \frac{ml^2}{3}$ $\quad I_O = I_C + mx^2$, (| | Axes)

$$I_O = \frac{ml^2}{3} + mx^2$$

$$= \frac{m(l^2 + 3x^2)}{3}$$

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$T = 2\pi\sqrt{\frac{m(l^2 + 3x^2)}{3mgx}} = 2\pi\left[\frac{l^2x^{-1}}{3g} + \frac{x}{g}\right]^{\frac{1}{2}}$$

$$\frac{dT}{dx} = \cancel{2}\pi\frac{1}{\cancel{2}}\left[\frac{l^2x^{-1}}{3g} + \frac{x}{g}\right]^{-\frac{1}{2}}\left[-\frac{l^2}{3gx^2} + \frac{1}{g}\right]$$

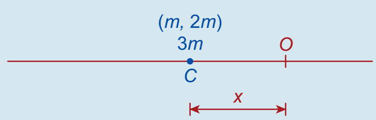
$$\frac{dT}{dx} = \pi\frac{\left[\frac{1}{g} - \frac{l^2}{3gx^2}\right]}{\left[\frac{l^2}{3gx} + \frac{x}{g}\right]^{\frac{1}{2}}} = 0 \text{ for minimum } T$$

$$\therefore \frac{l^2}{3gx^2} = \frac{1}{g}$$

$$\Rightarrow x^2 = \frac{l^2}{3}$$

$\Rightarrow x = \frac{l}{\sqrt{3}}$ $\quad$ For minimum T

Q. 2.



Pt Mass: $2m$

$$I_O = 2mx^2$$

Rod: m

$$I_C = \frac{ml^2}{3}$$

||Axes:

$$I_O = I_C + mx^2$$

$$= \frac{ml^2}{3} + mx^2$$

$$I_{Total} = I_{Mass} + I_{Rod}$$

$$= 2mx^2 + \frac{ml^2}{3} + mx^2$$

$$= \frac{9mx^2 + ml^2}{3}$$

$$\left[T = 2\pi\sqrt{\frac{I}{mgh}}\right]$$

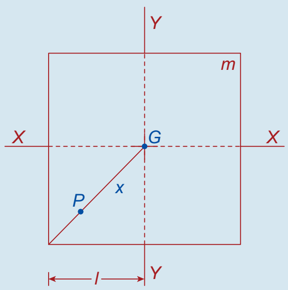
$$\Rightarrow T = 2\pi\sqrt{\frac{9mx^2 + ml^2}{3(3m)gx}}$$

$$\Rightarrow T = 2\pi\left[\frac{x}{g} + \frac{l^2}{9gx}\right]^{\frac{1}{2}}, \qquad \text{For Min } T, \frac{dT}{dx} = 0$$

$$\frac{dT}{dx} = \frac{2\pi}{2}\left[\frac{x}{g} + \frac{l^2}{9gx}\right]^{\frac{-1}{2}}\left[\frac{1}{g} - \frac{l^2}{9gx^2}\right]$$

$$\therefore \frac{1}{g} = \frac{l^2}{9gx^2} \qquad \text{For Min } T \Rightarrow x = \frac{l}{3}$$

Q. 3. **Square:**



$$I_{xx} = I_{yy}$$

$$= \frac{ml^2}{3}$$

⊥ Axes:

$$I_G = ZI_{xx}$$

$$= \frac{2ml^2}{3}$$

||Axes:

$$I_p = I_G + mx^2$$

$$= \frac{2ml^2 + 3mx^2}{3}$$

$$\left[T = 2\pi\sqrt{\frac{I}{mgh}}\right]$$

$$= 2\pi\sqrt{\frac{2ml^2 + 3mx^2}{3mgx}}$$

$$= 2\pi\left[\frac{2l^2}{3gx} + \frac{x}{g}\right]^{\frac{1}{2}}$$

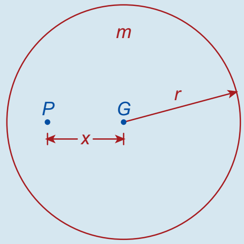
$$\frac{dT}{dx} = \frac{2\pi}{2}\left[\frac{2l^2}{3gx} + \frac{x}{g}\right]^{\frac{-1}{2}}\left[\frac{-2l^2}{3gx^2} + \frac{1}{g}\right]$$

$$\frac{dT}{dx} = 0 \qquad \text{For minimum } T$$

$$\therefore \frac{2l^2}{3gx^2} = \frac{1}{g}$$

$$x = \sqrt{\frac{2}{3}}\,l \qquad \textbf{QED}$$

Q. 4.



Disc:

$$I_G = \frac{mr^2}{2}$$

||Axes:

$$I_p = \frac{mr^2}{2} + mx^2$$

$$= \frac{mr^2 + 2mx^2}{2}$$

$$\left[T = 2\pi\sqrt{\frac{I}{mgh}}\right]$$

$$\Rightarrow T = 2\pi\sqrt{\frac{mr^2 + 2mx^2}{2mgx}} \qquad \ldots ①$$

If $x = \frac{r}{2}$, we get

$$T = 2\pi\sqrt{\frac{mr^2 + \frac{mr^2}{2}}{mgr}}$$

$$= 2\pi\sqrt{\frac{3r}{2g}}$$

For minimum T, $\frac{dt}{dx} = 0$

From ①

$$T = 2\pi\left[\frac{r^2}{2gx} + \frac{x}{g}\right]^{\frac{1}{2}} \qquad \ldots ②$$

$$\frac{dT}{dx} = \frac{2\pi}{2}\left[\frac{r^2}{2gx} + \frac{x}{g}\right]^{-\frac{1}{2}}\left[-\frac{r^2}{2gx^2} + \frac{1}{g}\right]$$

$$\frac{dT}{dx} = 0$$

$$\Rightarrow \frac{r^2}{2gx^2} + \frac{1}{g} \Rightarrow x = \frac{r}{\sqrt{2}} \qquad \textbf{QED}$$

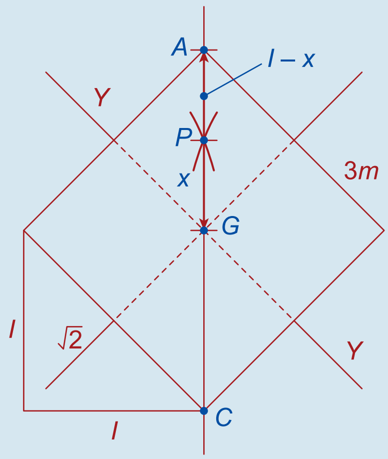
From ①

$$T = 2\pi\sqrt{\frac{mr^2 + \frac{2mr^2}{2}}{\frac{2mgr}{\sqrt{2}}}}$$

$$= 2\pi\sqrt{\frac{r}{g}\sqrt{2}} < 2\pi\sqrt{\frac{3r}{2g}}$$

since $\sqrt{2} < \frac{3}{2}$

Q. 5. Square Lamina:



(i) $I_{YY} = \frac{3m}{3}\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{m}{2}$

$I_G = 2I_{YY}$ (⊥ Axes)

$\Rightarrow I_G = m$

||Axes:

$I_P = I_G + 3mx^2$

$\Rightarrow I_P = m + 3mx^2$

Pt. Mass: A: $I_P = m(1 - x)^2$

C: $I_P = m(1 + x)^2$

$I_{Total} = I_{Lamina} + I_{Pt. Mass}$

$= m + 3mx^2 + m(1 - 2x + x^2)$

$+ m(1 + 2x + x^2)$

$= 3m + 5mx^2$

(ii) $T = 2\pi\sqrt{\frac{I}{mgh}}$, By symmetry centre of gravity of system is at G.

$\Rightarrow T^2 = 4\pi^2\left[\frac{m(3 + 5x^2)}{5mgx}\right]$

$\Rightarrow T^2 = \frac{4\pi^2}{5g}\left[\frac{3 + 5x^2}{x}\right]$ **QED**

(iii) $\therefore \frac{d(T)^2}{dx}$

$= \frac{4\pi^2}{5g}\left\{\frac{1}{x^2}\left[x(10x) - (3 + 5x^2)(1)\right]\right\}$

$= \frac{4\pi^2}{5gx^2}[5x^2 - 3]$

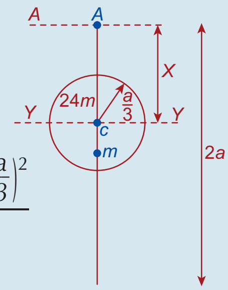
For Minimum T, $\frac{dT^2}{dx} = 0$

$\therefore 5x^2 = 3$

$\Rightarrow x = \sqrt{\frac{3}{5}}$

Q. 6. (a) Standard Proof

(b) Standard Proof



(c) **Rod:** $I_A = \frac{4}{3}ma^2$

Disc: $I_{AA} = \frac{24m\left(\frac{a}{3}\right)^2}{4}$

$I_{YY} = \frac{2ma^2}{3}$

|| Axes: $I_{AA} = \frac{2ma^2}{3} + 24mx^2$

$\therefore I_{Total} = I_{Rod} + I_{Disc}$

$= \frac{4}{3}ma^2 + \frac{2ma^2}{3} + 24mx^2$

(i) $\Rightarrow I_{TOTAL} = 2m(a^2 + 12x^2)$ **QED**

(ii) **Note:** In this question the plane of the disc is *perpendicular* to the plane in which the rod moves, i.e. as shown in diagram the disc moves *Into* the page.

Now, $T = 2\pi\sqrt{\frac{I}{mgh}}$

$\therefore$ We need to find h, (the position of centre of gravity)

Taking moments about A**:**

$m(a) + 24mx = 25mh$

$\Rightarrow h = \frac{a + 24x}{25}$

So $T = 2\pi\sqrt{\frac{2m(a^2 + 12x^2)(25)}{25m(g)(a + 24x)}}$

$\Rightarrow T = 2\pi\sqrt{\frac{2(a^2 + 12x^2)}{g(a + 24x)}}$

Now, Miniumum T occurs for $\frac{(a^2 + 12x^2)}{(a + 24x)}$ minimised

$\therefore \frac{1}{(a + 24x)^2}[(a + 24x)(24x) - (a^2 + 12x^2)(24)]$

$\Rightarrow (a + 24x)(24x) = (a^2 + 12x^2)(24)$

$\Rightarrow a + 24x^2 = a^2 + 12x^2$

$12x^2 + ax - a^2 = 0$

$\Rightarrow (3x + a)(4x - a) = 0$

$\Rightarrow x = \frac{a}{4}$ For minimum T **QED**