Chapter 7 Exercise 7A

Q. 1. (i) $\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{p}{-10} = \frac{-3}{5}$

$\Rightarrow p = 6$ m/s

The new velocity $= 6\vec{j}$ m/s

(ii) $\vec{I} = M\vec{v} - M\vec{u}$

$= 2(6\vec{j}) - 2(-10\vec{j})$

$= 32\vec{j}$ Ns

(iii) $\text{K.E.}_{\text{before}} = \frac{1}{2}(2)(-10)^2$

$= 100$ J

$\text{K.E.}_{\text{after}} = \frac{1}{2}(2)(6)^2$

$= 36$ J

Loss $= 100 - 36$

$= 64$ J

Q. 2. (i) $\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{v}{-20} = -0.5$

$\Rightarrow v = 10$ m/s

(ii) $\vec{I} = M\vec{v} - M\vec{u}$

$= (0.2)(10) - (0.2)(-20)$

$= 2 + 4 = 6$ Ns

(iii) $\text{K.E.}_{\text{before}} = \frac{1}{2}(0.2)(-20)^2$

$= 40$ J

$\text{K.E.}_{\text{after}} = \frac{1}{2}(0.2)(10)^2$

$= 10$ J

Loss $= 40 - 10$

$= 30$ J

Q. 3. (i) To find speed at impact:

$v^2 = u^2 + 2as$

$\Rightarrow v^2 = 0^2 + 2(9.8)(2.5)$

$\Rightarrow v = 7$ m/s

(ii) $\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{p}{-7} = -\frac{4}{7}$

$\Rightarrow p = 4$

$= 4$ m/s

(iii) $\vec{I} = M\vec{v} - M\vec{u}$

$= (1)(4\vec{j}) - (1)(-7\vec{j})$

$= 11\vec{j}$ Ns

(iv) Loss $= \frac{1}{2}Mu^2 - \frac{1}{2}Mv^2$

$= \frac{1}{2}(1)(49) - \frac{1}{2}(1)(16)$

$= 16\frac{1}{2}$ J

Q. 4. (i) $v^2 = u^2 + 2as$

$\Rightarrow v^2 = 0^2 + 2(9.8)(10)$

$\Rightarrow v = 14$ m/s

(ii) $\vec{I} = M\vec{v} - M\vec{u}$

$= (6)(\vec{0}) - (6)(-14\vec{j})$

$= 84\vec{j}$ Ns

(iii) Loss $= \frac{1}{2}Mu^2 - \frac{1}{2}Mv^2$

$= \frac{1}{2}(6)(14)^2 - \frac{1}{2}(6)(0)^2$

$= 588$ J

Q. 5. (i) $u = 0, \quad a = 9.8, \quad s = 22.5$

$v = \sqrt{u^2 + 2as}$

$= \sqrt{0 + 2(9.8)(22.5)}$

$= 21$ m/s

(ii) $\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{v_1}{-21} = -\frac{5}{7}$

$\Rightarrow v_1 = 15$ m/s

(iii) $\vec{I} = M\vec{v}_1 - M\vec{v}$

$= (0.1)(15) - (0.1)(-21)$

$= 3.6$ Ns

(iv) $\text{K.E.}_{\text{before}} = \frac{1}{2}(0.1)(21)^2$

$= 22.05$ J

$\text{K.E.}_{\text{after}} = \frac{1}{2}(0.1)(15)^2$

$= 11.25$ J

Loss $= 22.05 - 11.25$

$= 10.8$ J

Q. 6. (i) **Before** **(Mass)** **After**

$5\vec{i} - 8\vec{j}$ 2kg $5\vec{i} + p\vec{j}$

$\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{p}{-8} = -\frac{3}{4}$

$\Rightarrow p = 6$

Ans $= 5\vec{i} + 6\vec{j}$

(ii) $\frac{1}{2}Mu^2 - \frac{1}{2}Mv^2 = \frac{1}{2}(2)(25 + 64) - \frac{1}{2}(2)(25 + 36)$

$= 28$ J

(iii) $\vec{I} = M\vec{v} - M\vec{u}$

$= 2(5\vec{i} + 6\vec{j}) - 2(5\vec{i} - 8\vec{j})$

$= 28\vec{j}$ Ns

Q. 7. (i) **Before** **(Mass)** **After**

$8\vec{i} - 15\vec{j}$ 0.2 kg $8\vec{i} + p\vec{j}$

$\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{p}{-15} = -\frac{2}{5}$

$\Rightarrow 5p = 30 \quad \Rightarrow \quad p = 6$

$\Rightarrow$ New velocity $= 8\vec{i} + 6\vec{j}$ m/s

(ii) Speed before $= \sqrt{8^2 + (-15)^2}$

$= 17$ m/s

Speed after $= \sqrt{8^2 + 6^2}$

$= 10$ m/s

$\Rightarrow$ Fall in speed $= 17 - 10$

$= 7$ m/s

(iii) $\text{K.E.}_{\text{before}} = \frac{1}{2}(0.2)(17)^2$

$= 28.9$ J

$\text{K.E.}_{\text{after}} = \frac{1}{2}(0.2)(10)^2$

$= 10$ J

Loss $= 28.9 - 10$

$= 18.9$ J

(ii) $\text{K.E.}_{\text{before}} = \frac{1}{2}(4)(64 + 36)$

$= 200$ J

$\text{K.E.}_{\text{after}} = \frac{1}{2}(4)(64 + 16)$

$= 160$ J

% Loss $= \frac{40}{200} \times \frac{100}{1}$

$= 20\%$

(iii) $\tan A = \frac{6}{8} = \frac{3}{4}, \quad \tan B = \frac{4}{8} = \frac{1}{2}$

$\frac{\tan B}{\tan A} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$

Q. 8. (i) $\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{4}{-6} = -e$

$\Rightarrow e = \frac{2}{3}$

Q. 9. (i) **Before** **(Mass)** **After**

$4\vec{i} + 3\vec{j}$ 0.2 $p\vec{i} + 3\vec{j}$

$\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{p}{4} = -0.75$

$\Rightarrow p = -3$

$\Rightarrow$ Velocity after impact $= -3\vec{i} + 3\vec{j}$ m/s

(ii) $\text{K.E.}_{\text{before}} = \frac{1}{2}(0.2)(4^2 + 3^2)$

$= 2.5$ J

$\text{K.E.}_{\text{after}} = \frac{1}{2}(0.2)((-3)^2 + 3^2)$

$= 1.8$ J

Loss $= 2.5 - 1.8$

$= 0.7$ J

% loss $= \frac{0.7}{2.5} \times 100 = 28\%$

Q. 10. (i) $u = 0, \quad a = 9.8, \quad s = 40$

$v = \sqrt{u^2 + 2as}$

$= \sqrt{0 + 2(9.8)(40)}$

$= 28$ m/s

(ii) Let v_1 = speed directly after impact with ground.

$\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{v_1}{-28} = -\frac{1}{2}$

$\Rightarrow v_1 = 14$ m/s

(iii) $u = 14, \quad a = -9.8, \quad v = 0$

$s = \frac{v^2 - u^2}{2a}$

$= \frac{0 - 14^2}{2(-9.8)}$

$= 10$ m

Q. 11. (i) $u = 0, \quad a = g, \quad s = h$

$v = \sqrt{u^2 + 2as}$

$= \sqrt{0 + 2gh}$

$= \sqrt{2gh}$

(ii) Let v_1 = the speed with which it first rises from the ground.

$\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{v_1}{-\sqrt{2gh}} = -e$

$\Rightarrow v_1 = e\sqrt{2gh}$

(iii) $u = e\sqrt{2gh}, \quad a = -g, \quad v = 0$

$s = \frac{v^2 - u^2}{2a}$

$= \frac{0 - 2e^2gh}{-2g} = e^2h$

Q. 12.

Before	**(Mass)**	**After**
$u \cos A\vec{i} + u \sin A\vec{j}$	M	$v \cos B\vec{i} + v \sin B\vec{j}$

$u \cos A = u \cos B$ … **Equation 1** ($\vec{i}$-velocity remains the same)

$\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{u \sin B}{-u \sin A} = -e$

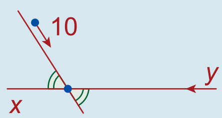
$\Rightarrow eu \sin A = u \sin B$ … **Equation 2**

Dividing **2** by **1** gives : $e \tan A = \tan B$

$\Rightarrow e = \frac{\tan B}{\tan A}$

Q. 13. (i) $\vec{u} = 20\left(\frac{3}{5}\right)\vec{i} - 20\left(\frac{4}{5}\right)\vec{j}$

$= 12\vec{i} - 16\vec{j}$



Before	**(Mass)**	**After**
$12\vec{i} - 16\vec{j}$	M	$12\vec{i} + p\vec{j}$

$\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{p}{-16} = -\frac{3}{4} \quad \Rightarrow \quad p = 12$

New Velocity $= 12\vec{i} + 12\vec{j}$

New Speed $= \sqrt{144 + 144}$

$= 12\sqrt{2}$ m/s

(ii) $\vec{I} = M\vec{v} - M\vec{u}$

$= M(12\vec{i} + 12\vec{j}) - M(12\vec{i} - 16\vec{j})$

$= 28M\vec{j}$ Ns

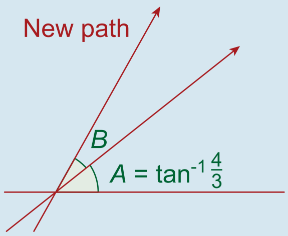
Magnitude $= 28M$ Ns

(iii) $\frac{1}{2}Mu^2 - \frac{1}{2}Mv^2$

$= \frac{1}{2}M(144 + 256) - \frac{1}{2}M(144 + 144)$

$= 56M$ J

(iv)



$\tan B = \frac{12}{12}$

$\tan B = 1$

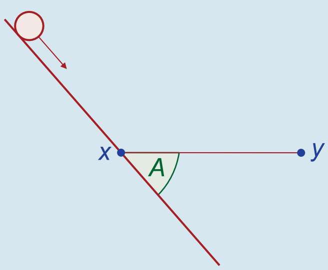
$\Rightarrow B = 45°$

$\tan(A + B) = \pm\frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \pm\frac{\frac{4}{3} + 1}{1 - \left(\frac{4}{3}\right)(1)} = \pm 7$

$\tan(A + B) = -7$ (Since $A + B > 90°$)

Q. 14. (i)



$\tan A = \frac{7}{24}$

$\Rightarrow \begin{cases} \cos A = \frac{24}{25} \\ \sin A = \frac{7}{25} \end{cases}$

$\vec{u} = 25\cos A\vec{i} - 25\sin A\vec{j}$

$= 24\vec{i} - 7\vec{j}$

Before	**(Mass)**	**After**
$24\vec{i} - 7\vec{j}$	0.1	$24\vec{i} + p\vec{j}$

$\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{p}{-7} = -\frac{3}{7} \Rightarrow p = 3$

$\Rightarrow$ New Velocity $= 24\vec{i} + 3\vec{j}$

New Speed $= \sqrt{24^2 + 3^2}$

$= 24.2$ m/s

(ii) $\vec{I} = M\vec{v} - M\vec{u}$

$= 0.1(24\vec{i} + 3\vec{j}) - 0.1(24\vec{i} - 7\vec{j})$

$= 1\vec{j}$ Ns

Magnitude = 1 Ns

(iii) $\text{K.E.}_{\text{before}} = \frac{1}{2}(0.1)(24^2 + (-7)^2)$

$= 31.25$ J

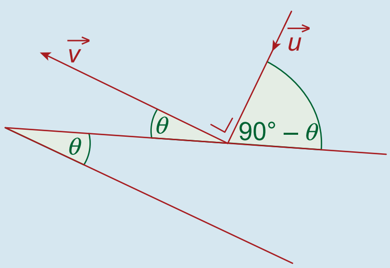
$\text{K.E.}_{\text{after}} = \frac{1}{2}(0.1)(24^2 + 3^2)$

$= 29.25$ J

Loss $= 31.25 - 29.25$

$= 2$ J

Q. 15.



$\vec{u} = -u\cos(90° - \theta)\vec{i} - u\sin(90° - \theta)\vec{j}$

$= -u\sin\theta\vec{i} - u\cos\theta\vec{j}$

$\vec{v} = -v\cos\theta\vec{i} + v\sin\theta\vec{j}$

Before	**(Mass)**	**After**
$-u\sin\theta\vec{i} - u\cos\theta\vec{j}$	M	$-v\cos\theta\vec{i} + u\sin\theta\vec{j}$

$\vec{i}$ – velocity is unchanged

$\Rightarrow -u\sin\theta = -v\cos\theta$

$\Rightarrow u\sin\theta = v\cos\theta$ … **Equation 1**

$\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{v\sin\theta}{-u\cos\theta} = \frac{-2}{3}$

$\Rightarrow 3v\sin\theta = 2u\cos\theta$ … **Equation 2**

Dividing **2** by **1** gives:

$\frac{3v}{u} = \frac{2u}{v}$

$\Rightarrow v^2 = \frac{2}{3}u^2$

$\Rightarrow \frac{1}{2}Mv^2 = \frac{2}{3}\left(\frac{1}{2}Mu^2\right)$

$\Rightarrow \frac{2}{3}$ of the energy is preserved.

$\Rightarrow \frac{1}{3}$ of the energy has been lost.

Exercise 7B

Q. 1. (i)

Before	**(Mass)**	**After**
$6\vec{i}$	2	$p\vec{i}$
$4\vec{i}$	1	$q\vec{i}$

$2(6) + 1(4) = 2(p) + 1(q)$

$\Rightarrow 2p + q = 16$ … **Equation 1**

$$\frac{p - q}{6 - 4} = -\frac{1}{2}$$

$\Rightarrow 2p - 2q = -2$

$\Rightarrow p - q = -1$ … **Equation 2**

Adding equations **1** and **2** we get:

$3p = 15$

$\Rightarrow p = 5 \quad \Rightarrow \quad q = 6$

$\Rightarrow$ Velocity of 2 kg sphere after impact is $5\vec{i}$ m/s

Velocity of 1 kg sphere after impact is $6\vec{i}$ m/s

(ii) $\text{K.E.}_{\text{before}} = \frac{1}{2}(2)(6)^2 + \frac{1}{2}(1)(4)^2$

$= 44$ J

$\text{K.E.}_{\text{after}} = \frac{1}{2}(2)(5)^2 + \frac{1}{2}(1)(6)^2$

$= 43$ J

Loss $= 44 - 43$

$= 1$ J

Q. 2. (i)

Before	**(Mass)**	**After**
$2\vec{i}$	1	$p\vec{i}$
$-6\vec{i}$	1	$q\vec{i}$

$1(2) + 1(-6) = 1(p) + 1(q)$

$\Rightarrow p + q = -4$ … **Equation 1**

$$\frac{p - q}{2 + 6} = -\frac{3}{4}$$

$\Rightarrow 4p - 4q = -24$

$\Rightarrow p - q = -6$ … **Equation 2**

Adding equations **1** and **2** we get

$2p = -10$

$\Rightarrow p = -5$

$\Rightarrow q = 1$

$\Rightarrow$ velocities after impact are $-5\vec{i}$ m/s and $\vec{i}$ m/s

$\Rightarrow$ speeds after impact are 5 m/s and 1 m/s

(ii) $\vec{I} = M\vec{v} - M\vec{u}$

$= 1(-5\vec{i}) - 1(2\vec{i})$

$= -7\vec{i}$ Ns … impulse imparted to first sphere

$\Rightarrow$ Impulse imparted to second sphere $= 7\vec{i}$ Ns

$\Rightarrow$ Magnitude of impulse imparted to each sphere $= 7$ Ns.

(iii) $\text{K.E.}_{\text{before}} = \frac{1}{2}(1)(2)^2 + \frac{1}{2}(1)(-6)^2$

$= 20$ J

$\text{K.E.}_{\text{after}} = \frac{1}{2}(1)(-5)^2 + \frac{1}{2}(1)(1)^2$

$= 13$ J

$\Rightarrow$ Loss $= 20 - 13$

$\Rightarrow = 7$ J

$\Rightarrow$ % Loss $= \frac{7}{20} \times 100 = 35\%$

Q. 3. (i)

Before	**(Mass)**	**After**
$10\vec{i}$	3	$0\vec{i}$
$\vec{i}$	5	$q\vec{i}$

$3(10) + 5(1) = 3(0) + 5(q)$

$\Rightarrow q = 7$

$$\frac{0 - q}{10 - 1} = -e$$

$\Rightarrow e = \frac{q}{9} = \frac{7}{9}$

(ii) 7 m/s

(iii) $\vec{I}_1 = M\vec{v_1} - M\vec{u_1}$

$= 3(0\vec{i}) - 3(10\vec{i})$

$= -30\vec{i}$ Ns

$\vec{I_2} = M\vec{v_2} - M\vec{u_2}$

$= 5(7\vec{i}) - 5(\vec{i}) = 30\vec{i}$ Ns

(iv) $u = 7, \quad v = 0, \quad s = 2, \quad a = ?$

$v^2 = u^2 + 2as$

$\Rightarrow 0 = 49 + 2a(2)$

$\Rightarrow a = \frac{-49}{4} = -12.25 \text{ m/s}^2$

$\therefore$ Deceleration $= -12.25 \text{ m/s}^2$

Q. 4. (i)

Before	**(Mass)**	**After**
5	2	v
0	1	$3v$

$2(5) + 1(0) = 2(v) + 1(3v)$

$\Rightarrow 5v = 10 \quad \Rightarrow \quad v = 2$

(ii) $\dfrac{v - 3v}{5 - 0} = -e$

$\Rightarrow \dfrac{-4}{5} = -e \quad \Rightarrow \quad e = \dfrac{4}{5}$

(iii) $\text{K.E.}_{\text{before}} = \frac{1}{2}(2)(5)^2 + \frac{1}{2}(1)(0)^2$

$= 25 \text{ J}$

$\text{K.E.}_{\text{after}} = \frac{1}{2}(2)(2)^2 + \frac{1}{2}(1)(6)^2$

$= 22 \text{ J}$

Loss $= 25 - 22 = 3$ J

% Loss $= \dfrac{3}{25} \times 100 = 12\%$

Q. 5. (i)

Before	**(Mass)**	**After**
$6\vec{i}$	5	$p\vec{i}$
$-4\vec{i}$	3	$q\vec{i}$

$5(6) + 3(-4) = 5p + 3q$

$\Rightarrow 5p + 3q = 18$

$\dfrac{p - q}{6 + 4} = -\dfrac{1}{3}$

$\Rightarrow 3p - 3q = -10$

Solving these gives $p = 1,\ q = \dfrac{13}{3}$.

(ii) Loss $= \frac{1}{2}Mu^2 - \frac{1}{2}Mv^2$

$= \frac{1}{2}(5)(6)^2 - \frac{1}{2}(5)(1)^2$

$= 87\frac{1}{2}$ J

(iii) $M\vec{v} - M\vec{u} = 3\left(\frac{13}{3}\vec{i}\right) - 3(-4\vec{i})$

$= 25\vec{i}$ Ns

Q. 6. (i)

Before	**(Mass)**	**After**
$10\vec{i}$	2	$p\vec{i}$
$0\vec{i}$	3	$q\vec{i}$

$2(10) + 3(0) = 2p + 3q$

$\Rightarrow 2p + 3q = 20$

$\dfrac{p - q}{10 - 0} = -\dfrac{1}{2}$

$\Rightarrow p - q = -5$

Solving these gives $p = 1,\ q = 6$.

Answer: 1 m/s and 6 m/s.

(ii) $\text{K.E.}_{\text{Before}} = \frac{1}{2}(2)(10)^2 + \frac{1}{2}(3)(0)^2$

$= 100$ J

$\text{K.E.}_{\text{After}} = \frac{1}{2}(2)(1)^2 + \frac{1}{2}(3)(6)^2$

$= 55$ J

Loss $= 100 - 55 = 45$ J

(iii) To find the acceleration:

$v = u + at$

$\Rightarrow 0 = 6 + a(2)$

$\Rightarrow a = -3$

To find the force:

$F = Ma$

$\Rightarrow F = (3)\,(-3) = -9$ N

(i.e. it is a resistance force of 9 N)

To find the distance:

$v^2 = u^2 + 2as$

$\Rightarrow 0 = (6)^2 + 2(-3)s$

$\Rightarrow s = 6$ m

To find work done:

$W = Fs = 9(6) = 54$ J

To find power:

Power $= \dfrac{\text{Work}}{\text{Time}}$

$= \dfrac{54}{2} = 27$ W

Q. 7. (i)

	Before	**(Mass)**	**After**
A:	$4\vec{i}$	M	$p\vec{i}$
B:	$0\vec{i}$	M	$q\vec{i}$

$M(4) + M(0) = Mp + Mq$

$\Rightarrow p + q = 4$

$\dfrac{p - q}{4 - 0} = -\dfrac{1}{2}$

$\Rightarrow p - q = -2$

Solving these gives $p = 1,\ q = 3$.
Speed of B is 3 m/s.

(ii)

	Before	(Mass)	After
B:	$3\vec{i}$	M	$a\vec{i}$
C:	$0\vec{i}$	M	$b\vec{i}$

$M(3) + M(0) = Ma + Mb$

$\Rightarrow a + b = 3$

$\frac{a-b}{3-0} = -\frac{1}{2}$

$\Rightarrow a - b = -\frac{3}{2}$

Solving these gives $a = \frac{3}{4}, b = 2\frac{1}{4}$.

Speed of B is $\frac{3}{4}$ m/s.

Yes, because A will catch up with B, since $v_A > v_B$

Q. 8. (i)

	Before	(Mass)	After
A:	v	4	0
B:	$-v$	2	p

$4(v) + 2(-v) = 4(0) + 2(p)$

$\Rightarrow 2p = 2v \quad \Rightarrow \quad p = v$

Speed of B after impact is the same as the speed before but in the opposite direction.

(ii) $\frac{0-p}{v-(-v)} = -e$

$\Rightarrow \frac{-v}{2v} = -e \quad \Rightarrow \quad e = \frac{1}{2}$

(iii) $\text{K.E.}_{\text{before}} = \frac{1}{2}(4)(v)^2 + \frac{1}{2}(2)(-v)^2$

$= 2v^2 + v^2 = 3v^2$

$\text{K.E.}_{\text{after}} = \frac{1}{2}(4)(0)^2 + \frac{1}{2}(2)(v)^2$

$= v^2$

$\text{Loss} = 3v^2 - v^2 = 2v^2$

$\% \text{ Loss} = \frac{2v^2}{3v^2} \times 100 = 66\frac{2}{3}\%$

Q. 9. (i)

Before	(Mass)	After
$2\vec{i}$	M	$11k\vec{i}$
$\vec{i}$	M	$13k\vec{i}$

$M(2) + M(1) = M(11k) + M(13k)$

$\Rightarrow k = \frac{1}{8}$

$\therefore$ Their speeds will be $\frac{11}{8}$ and $\frac{13}{8}$.

(ii) $\frac{\frac{11}{8} - \frac{13}{8}}{2-1} = -e$

$\Rightarrow e = \frac{1}{4}$

Q. 10. (i)

	Before	(Mass)	After
A:	$6\vec{i}$	M	$p\vec{i}$
B:	$0\vec{i}$	M	$q\vec{i}$

$M(6) + M(0) = Mp + Mq$

$\Rightarrow p + q = 6$

$\frac{p-q}{6-0} = \frac{-2}{3}$

$\Rightarrow p - q = -4$

Solving these gives $p = 1, q = 5$

Their speeds are $(1, 5, 0)$ m/s.

(ii)

	Before	(Mass)	After
B:	$5\vec{i}$	M	$a\vec{i}$
C:	$0\vec{i}$	M	$b\vec{i}$

$M(5) + M(0) = Ma + Mb$

$\Rightarrow a + b = 5$

$\frac{a-b}{5-0} = \frac{-2}{3}$

$\Rightarrow a - b = \frac{-10}{3}$

Solving these gives $a = \frac{5}{6}, b = \frac{25}{6}$.

Their speeds now are $\left(1, \frac{5}{6}, \frac{25}{6}\right)$ m/s.

(iii)

	Before	(Mass)	After
A:	$1\vec{i}$	M	$c\vec{i}$
B:	$\frac{5}{6}\vec{i}$	M	$d\vec{i}$

$M(1) + M\left(\frac{5}{6}\right) = Mc + Md$

$\Rightarrow c + d = \frac{11}{6}$

$\frac{c-d}{1-\frac{5}{6}} = -\frac{2}{3}$

$\Rightarrow c - d = -\frac{1}{9}$

Solving these gives: $c = \frac{31}{36}, d = \frac{35}{36}$

Their speeds are $\left(\frac{31}{36}, \frac{35}{36}, \frac{25}{6}\right)$ m/s

Since $v_A < v_B < v_C$, there will be no further collisions.

Q. 11.

Before	**(Mass)**	**After**
$10\vec{i}$	10	$p\vec{i}$
$-5\vec{i}$	50	$q\vec{i}$

$10(10) + 50(-5) = 10p + 50q$

$\Rightarrow p + 5q = -15$

$\frac{p - q}{10 + 5} = -\frac{1}{2}$

$\Rightarrow 2p - 2q = -15$

Solving these gives $p = -\frac{35}{4}, q = -\frac{5}{4}$.

The speeds are $8\frac{3}{4}$ m/s and $1\frac{1}{4}$ m/s.

$\vec{I_1} = M\vec{v} - M\vec{u}$

$= 10\left(-\frac{35}{4}\right) - 10(10)$

$= -187.5$ Ns

The magnitude of the impulse = 187.5 Ns

Q. 12. (a) (i) Newton's law of restitution: For two bodies impinging directly, their relative velocity after impact is equal to a constant (e) times their relative velocity before impact and in the opposite direction.

(ii) In a closed system the total momentum will be conserved.

(b) $v^2 = u^2 + 2as$

$\Rightarrow v^2 = 0^2 + 2(9.8)(19.6)$

$\Rightarrow v = 19.6$

Rebound speed $= (0.8)(19.6)$

$= 15.68$

$v^2 = u^2 + 2as$

$\Rightarrow 0^2 = (15.68)^2 + 2(-9.8)s$

$\Rightarrow s = 12.544$ m

(c)

Gun	Bullet	Gun	Bullet

$M_1u_1 + M_2u_2 = M_1v_1 + M_2v_2$

$\Rightarrow (2)(0) + (0.01)0 = (2)v_1 + (0.01)(300)$

$\Rightarrow v_1 = -1.5$ m/s

$\Rightarrow$ Initial speed of the gun = 1.5 m/s

To find acceleration:

$v^2 = u^2 + 2as = (0)^2$

$= (-1.5)^2 + 2a(0.05)$

$\Rightarrow a = -22.5$ m/s

$F = ma$

$\Rightarrow F = (2)(-22.5)$

$= -45\ N$

A constant force of 45 N is required.

Q. 13. (i)

Before	**(Mass)**	**After**
$u\vec{i}$	M	$0\vec{i}$
$-v\vec{i}$	$3M$	$q\vec{i}$

$M(u) + 3M(-v) = M(0) + 3M(q)$

$\Rightarrow q = \frac{u - 3v}{3}$

$\frac{0 - q}{u + v} = -e$

$\Rightarrow e = \frac{q}{u + v}$

$= \frac{u - 3v}{3u + 3v}$ **QED**

(ii) $e \geqslant 0$

$\therefore \frac{u - 3v}{3u + 3v} \geqslant 0$

$\therefore u - 3v \geqslant 0$

$\therefore u \geqslant 3v$ **QED**

Q. 14.

Before	**(Mass)**	**After**
$5\vec{i}$	1	$p\vec{i}$
$\vec{i}$	2	$q\vec{i}$

$1(5) + 2(1) = 1(p) + 2(q)$

$\Rightarrow p + 2q = 7$

$\Rightarrow p = 7 - 2q$

$\frac{p - q}{5 - 1} = -e$

$\Rightarrow -4e = p - q$

$= (7 - 2q) - q$

$= 7 - 3q$

$\therefore e = \frac{3q - 7}{4}$

If there are to be no more collisions $v_B \leqslant v_C$

$\therefore q \leqslant 3$

If $q \leqslant 3$ then $e = \frac{3q - 7}{4} \leqslant \frac{3(3) - 7}{4} = \frac{1}{2}$

Answer: MAX Value $= \frac{1}{2}$

Q. 15.

Before	**(Mass)**	**After**
$u_1\vec{i}$	3	$v_1\vec{i}$
$u_2\vec{i}$	4	$q\vec{i}$

1: $3u_1 + 4u_2 = 3v_1 + 4q$

$$\Rightarrow q = \frac{1}{4}(3u_1 + 4u_2 - 3v_1)$$

2: $\dfrac{v_1 - q}{u_1 - u_2} = -e$

$\Rightarrow v_1 - q = -eu_1 + eu_2$

But $q = \frac{1}{4}(3u_1 + 4u_2 - 3v_1)$

$\therefore v_1 - \frac{1}{4}(3u_1 + 4u_2 - 3v_1) = -eu_1 + eu_2$

$\Rightarrow 4v_1 - 3u_1 - 4u_2 + 3v_1 = -4eu_1 + 4eu_2$

$\Rightarrow 7v_1 = u_1(3 - 4e) + 4u_2(1 + e)$ **QED**

$I = 3v_1 - 3u_1$. But $v_1 = \frac{1}{7}(u_1(3 - 4e) + 4u_2(1 + e))$

$\therefore I = \frac{3}{7}(u_1(3 - 4e) + 4u_2(1 + e)) - 3u_1$

$= \frac{3}{7}(3u_1 - 4eu_1 + 4u_2 + 4eu_2 - 7u_1)$

$= \frac{3}{7}(-4u_1 - 4eu_1 + 4u_2 + 4eu_2)$

$= \frac{12}{7}(-u_1 - eu_1 + u_2 + eu_2)$

$= \frac{12}{7}(-u_1(1 + e) + u_2(1 + e))$

$= \frac{12}{7}(1 + e)(u_2 - u_1)$ **QED**

Q. 16. (i)

	Before	**(Mass)**	**After**
P:	$4u$	$3m$	$2u$
Q:	$2u$	$5m$	q

$3m(4u) + 5m(2u) = 3m(2u) + 5m(q)$ … divide by m

$\Rightarrow 12u + 10u = 6u + 5q$

$\Rightarrow 5q = 16u$

$\Rightarrow q = \dfrac{16u}{5}$

$\dfrac{2u - q}{4u - 2u} = -e$

$\Rightarrow \dfrac{2u - q}{2u} = -e$

$\Rightarrow 2u - q = -2eu$

$\Rightarrow q = 2u(1 + e)$ … but $q = \dfrac{16u}{5}$

$\Rightarrow 2u(1 + e) = \dfrac{16u}{5}$ … multiply by $\dfrac{5}{2u}$

$\Rightarrow 5(1 + e) = 8$

$\Rightarrow 1 + e = 1.6$

$\Rightarrow e = 0.6$

(ii) Let the velocity of P after impact be p.

$3m(4u) + 5m(2u) = 3m(p) + 5m(q)$

$\Rightarrow 3p + 5q = 22u$ … **Equation 1**

$\dfrac{p - q}{4u - 2u} = -e$

$\Rightarrow p - q = -2eu$ … multiply by -3

$\Rightarrow -3p + 3q = 6eu$ … **Equation 2**

Adding equations **1** and **2** we get

$8q = 22u + 6eu$

$\Rightarrow 8q = 2u(11 + 3e)$

$\Rightarrow q = \frac{u}{4}(11 + 3e)$ … minimum value occurs when $e = 0$

$\Rightarrow q = \frac{u}{4}(11) = 2.75u$

Q. 17. (i)

Before	**(Mass)**	**After**
u	$4m$	p
0	$2m$	q

$4m(u) + 2m(0) = 4m(p) + 2m(q)$ … divide by $2m$

$\Rightarrow 2p + q = 2u$ … **Equation 1**

$$\frac{p - q}{u - 0} = -e$$

$\Rightarrow p - q = -eu$ … **Equation 2**

Adding equations **1** and **2** we get

$3p = u(2 - e)$

$\Rightarrow p = \frac{u}{3}(2 - e)$ … speed of $4m$ sphere after impact.

$q = p + eu$ … from **Equation 1**

$\Rightarrow q = \frac{u}{3}(2 - e) + eu$

$\Rightarrow q = \frac{2u}{3} - \frac{eu}{3} + eu$

$\Rightarrow q = \frac{2u - eu + 3eu}{3}$

$\Rightarrow q = \frac{2u + 2eu}{3}$

$\Rightarrow q = \frac{2u}{3}(1 + e)$

(ii) $\text{K.E.}_{\text{before}} = \frac{1}{2}(4m)(u)^2$

$= 2mu^2$

$\text{K.E.}_{\text{after}} = \frac{1}{2}(4m)(p)^2 + \frac{1}{2}(2m)(q)^2$

$= 2m\left[\frac{u^2}{9}(2 - e)^2\right] + m\left[\frac{4u^2}{9}(1 + e)^2\right]$

$= \frac{2mu^2}{9}[4 - 4e + e^2 + 2(1 + 2e + e^2)]$

$= \frac{2mu^2}{9}[3e^2 + 6]$

$= \frac{2mu^2}{3}(e^2 + 2)$

$\text{Loss} = 2mu^2 - \frac{2mu^2}{3}(e^2 + 2)$

$= \frac{2mu^2}{3}(3 - e^2 - 2)$

$= \frac{2mu^2}{3}(1 - e^2)$

Q. 18. (i)

Before	**(Mass)**	**After**
$2u$	m	p
0	m	q

$m(2u) + m(0) = m(p) + m(q)$ … divide by m

$\Rightarrow p + q = 2u$ … **Equation 1**

$$\frac{p - q}{2u - 0} = -e$$

$\Rightarrow p - q = -2eu$ … **Equation 2**

Adding equations **1** and **2** we get

$2p = 2u(1 - e)$

$\Rightarrow p = u(1 - e)$ … speed of 1st sphere after impact

$q = p + 2eu$ … from **Equation 1**

$\Rightarrow q = u(1 - e) + 2eu$

$\Rightarrow q = u - eu + 2eu$

$\Rightarrow q = u + eu$

$\Rightarrow q = u(1 + e)$ … speed of second sphere after impact

(ii) Firstly, find how long it takes for sphere to hit wall.

speed $= u(1 + e)$

distance = 1 metre

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{1}{u(1 + e)} \text{ s}$$

Find the distance travelled by other sphere in this time.

speed $= u(1 - e)$

$$\text{time} = \frac{1}{u(1 + e)}$$

distance = speed × time

$$= u(1 - e)\left[\frac{1}{u(1 + e)}\right]$$

$$= \frac{1 - e}{1 + e}$$

$$\Rightarrow \text{distance apart} = 1 - \frac{1 - e}{1 + e}$$

$$= \frac{1 + e - 1 + e}{1 + e}$$

$$= \frac{2e}{1 + e}$$

(iii) Find the speed of sphere after colliding with wall.

$$\frac{\text{NEW}}{\text{OLD}} = -e$$

$$\Rightarrow \frac{v}{u(1 + e)} = -e$$

$$\Rightarrow v = -eu(1 + e)$$

$$\vec{v_A} = u(1 - e)$$

$$\vec{v_B} = -eu(1 + e)$$

$$\vec{v_{AB}} = \vec{v_A} - \vec{v_B}$$

$$= u(1 - e) + eu(1 + e)$$

$$= u - eu + eu + e^2u$$

$$= u(1 + e^2)$$

$$\text{time} = \frac{\text{relative distance}}{\text{relative speed}}$$

$$= \frac{\frac{2e}{1 + e}}{u(1 + e^2)}$$

$$= \frac{2e}{u(1 + e)(1 + e^2)}$$

Look at how far the sphere has moved away from the wall in this time.

distance = speed × time

$$\Rightarrow \text{distance} = eu(1 + e)\left[\frac{2e}{u(1 + e)(1 + e^2)}\right]$$

$$= \frac{2e^2}{1 + e^2}$$

Q. 19. (i) Gain in K.E. = Loss in P.E.

$$\Rightarrow \frac{1}{2}mv^2 = mgh$$

$$\Rightarrow \frac{1}{2}(0.6)v^2 = (0.6)g(0.5\cos 60°)$$

$$\Rightarrow v^2 = \frac{g}{2}$$

$$\Rightarrow v = \sqrt{\frac{g}{2}} = 2.21 \text{ m/s}$$

(ii)

Before	**(Mass)**	**After**
$\sqrt{\frac{g}{2}}$	0.6	p
0	0.8	q

$$0.6\left(\sqrt{\frac{g}{2}}\right) + 0.8(0) = 0.6(p) + 0.8(q)$$

… multiply by 5

$$\Rightarrow 3p + 4q = 3\sqrt{\frac{g}{2}}$$ … **Equation 1**

$$\frac{p - q}{\sqrt{\frac{g}{2}} - 0} = -\frac{1}{11}$$

$$\Rightarrow 11p - 11q = -\sqrt{\frac{g}{2}}$$ … **Equation 2**

Eq. 1 (×11): $33p + 44q = 33\sqrt{\frac{g}{2}}$

Eq. 2 (× −3): $-33p + 33q = 3\sqrt{\frac{g}{2}}$ add

$$77q = 36\sqrt{\frac{g}{2}}$$

$$\Rightarrow q = \frac{36}{77}\sqrt{\frac{g}{2}}$$

$$= 1.03 \text{ m/s}$$

$$p = \frac{3\sqrt{\frac{g}{2}} - 4q}{3}$$ … from **Equation 1**

$$\Rightarrow p = 0.84 \text{ m/s}$$

Q. 20. (i)

Before	**(Mass)**	**After**
u	m	p
0	$3m$	q

$m(u) + 3m(0) = m(p) + 3m(q)$ … divide by m

$\Rightarrow p + 3q = u$ … **Equation 1**

$$\frac{p-q}{u-0} = -e$$

$\Rightarrow -p + q = eu$ … **Equation 2**

Adding equations **1** and **2** we get

$4q = u(1 + e)$

$\Rightarrow q = \frac{u}{4}(1 + e)$ … speed of 2nd sphere after collision

$p = q - eu$ … from **Equation 2**

$\Rightarrow p = \frac{u}{4}(1 + e) - eu$

$\Rightarrow p = \frac{u + eu - 4eu}{4} = \frac{u - 3eu}{4}$

$\Rightarrow p = \frac{u}{4}(1 - 3e)$ … speed of 1st sphere after collision

(ii) $K.E._{before} = \frac{1}{2}mu^2$

$$K.E._{after} = \frac{1}{2}mp^2 + \frac{1}{2}(3m)q^2$$

$$= \frac{m}{2}\left[\frac{u^2}{16}(1 - 6e + 9e^2)\right] + \frac{3m}{2}\left[\frac{u^2}{16}(1 + 2e + e^2)\right]$$

$$= \frac{mu^2}{32}[1 - 6e + 9e^2 + 3 + 6e + 3e^2]$$

$$= \frac{mu^2}{32}[12e^2 + 4]$$

$$= \frac{mu^2}{8}(3e^2 + 1)$$

$$\text{Loss} = \frac{mu^2}{2} - \frac{mu^2}{8}(3e^2 + 1)$$

$$= \frac{mu^2}{2}\left[1 - \frac{3e^2}{4} - \frac{1}{4}\right]$$

$$= \frac{mu^2}{2}\left[\frac{4 - 3e^2 - 1}{4}\right]$$

$$= \frac{mu^2}{8}(3 - 3e^2)$$

$$= \frac{3mu^2}{8}(1 - e^2)$$

(iii) Let $e = \frac{1}{4}$

$$\Rightarrow \text{Loss} = \frac{3mu^2}{8}\left(\frac{15}{16}\right) = \frac{45mu^2}{128}$$

$$\%\ \text{Loss} = \frac{\frac{45mu^2}{128}}{\frac{mu^2}{2}} \times 100$$

$$= \frac{45}{64} \times 100 = 70\%$$

(iv) Loss in K.E. is maximised when $e = 0$

$$\Rightarrow \text{Loss} = \frac{3mu^2}{8}$$

$$\%\ \text{Loss} = \frac{\frac{3mu^2}{8}}{\frac{mu^2}{2}} \times 100$$

$$= \frac{3}{4} \times 100$$

$$= 75\%$$

Exercise 7C

Q. 1. (i)

Before	**(Mass)**	**After**
$4\vec{i} + 3\vec{j}$	M	$p\vec{i} + 3\vec{j}$
$\vec{i} + 2\vec{j}$	M	$q\vec{i} + 2\vec{j}$

$M(4) + M(1) = M(p) + M(q)$

$\Rightarrow p + q = 5$

$\frac{p - q}{4 - 1} = -\frac{1}{3}$

$\Rightarrow p - q = -1$

Solving these gives $p = 2, q = 3$.

The new velocities are $2\vec{i} + 3\vec{j}$ and $3\vec{i} + 2\vec{j}$

(ii) $K.E._{before} = \frac{1}{2}(M)(4^2 + 3^2) + \frac{1}{2}(M)(1^2 + 2^2)$

$= 15M$ J

$K.E._{after} = \frac{1}{2}M(2^2 + 3^2) + \frac{1}{2}M(3^2 + 2^2)$

$= 13M$ J

Loss $= 15M - 13M = 2M$ J

Q. 2.

Before	**(Mass)**	**After**
$3\vec{i} + 4\vec{j}$	2	$p\vec{i} + 4\vec{j}$
$-4\vec{i} + 3\vec{j}$	3	$q\vec{i} + 3\vec{j}$

$2(3) + 3(-4) = 2(p) + 3(q)$

$\Rightarrow 2p + 3q = -6$

$\frac{p - q}{3 + 4} = -\frac{3}{7}$

$\Rightarrow p - q = -3$

Solving these gives $p = -3, q = 0$

(i) Their velocities are $-3\vec{i} + 4\vec{j}, 0\vec{i} + 3\vec{j}$

(ii) $K.E._{before} =$

$\frac{1}{2}(2)\,(3^2 + 4^2) + \frac{1}{2}(3)((-4)^2 + 3^2)$

$= 62\frac{1}{2}$ J

$K.E._{after} =$

$\frac{1}{2}(2)\,((-3)^2 + 4^2) + \frac{1}{2}(3)(0^2 + 3^2)$

$= 38\frac{1}{2}$ J

Loss $= 62\frac{1}{2} - 38\frac{1}{2}$

$= 24$ J

(iii) $\vec{I_1} = M\vec{v} - M\vec{u}$

$= 2(-3\vec{i} + 4\vec{j}) - 2(3\vec{i} + 4\vec{j})$

$= -12\vec{i}$ Ns

The magnitude of the impulse is 12 Ns.

Q. 3.

Before	**(Mass)**	**After**
$6\vec{i} + \vec{j}$	M	$0\vec{i} + \vec{j}$
$-2\vec{i} - 5\vec{j}$	$2M$	$q\vec{i} - 5\vec{j}$

$M(6) + 2M(-2) = M(0) + 2Mq$

$\Rightarrow q = 1$

(i) Its velocity is $\vec{i} - 5\vec{j}$.

(ii) $\frac{0 - 1}{6 + 2} = -e \quad \Rightarrow \quad e = \frac{1}{8}$

Q. 4.

Before	**(Mass)**	**After**
$5\vec{i} + 5\vec{j}$	$2M$	$p\vec{i} + 5\vec{j}$
$0\vec{i} + 0\vec{j}$	M	$q\vec{i} + 0\vec{j}$

$2M(5) + M(0) = 2Mp + Mq$

$\Rightarrow 2p + q = 10$

$\frac{p - q}{5 - 0} = -\frac{1}{2}$

$\Rightarrow 2p - 2q = -5$

Solving these gives $p = \frac{21}{2}, q = 5$

(i) Their velocities are $2\frac{1}{2}\vec{i} + 5\vec{j}; 5\vec{i} + 0\vec{j}$

(ii) $\vec{I_1} = M\vec{v} - M\vec{u}$

$= 2M\left(2\frac{1}{2}\vec{i} + 5\vec{j}\right) - 2M(5\vec{i} + 5\vec{j})$

$= -5M\vec{i}$ Ns

$\vec{I_2} = M(5\vec{i} + 0\vec{j}) - M(0\vec{i} + 0\vec{j})$

$= 5M\vec{i}$ Ns

(iii) $K.E._{before} =$

$\frac{1}{2}(2M)(5^2 + 5^2) + \frac{1}{2}(M)(0^2 + 0^2)$

$= 50M$ J

$K.E._{after} =$

$\frac{1}{2}(2M)\left(\left(\frac{5}{2}\right)^2 + 5^2\right) + \frac{1}{2}(M)(5^2 + 0^2)$

$= 43\frac{3}{4}M$ J

Percentage Loss $= \frac{6\frac{1}{4}M}{50M} \times \frac{100}{1}$

$= 12\frac{1}{2}\%$

(iv) $m_1 = \frac{5}{5} = 1,\ m_2 = \frac{5}{2\frac{1}{2}} = 2$

$\therefore \tan\theta = \pm\frac{1-2}{1+(1)(2)}$

$= \pm\frac{1}{3} = 0.3333$

$\therefore \theta = 18^\circ\ 26'$ (Since θ is acute)

Q. 5. (i) $u_1 = 5\cos\theta\vec{i} + 5\sin\theta\vec{j}$

$= 5\left(\frac{4}{5}\right)\vec{i} + 5\left(\frac{3}{5}\right)\vec{j}$

$= 4\vec{i} + 3\vec{j}$

$u_2 = -4\sqrt{2}\cos 45^\circ\vec{i} + 4\sqrt{2}\sin 45^\circ\vec{j}$

$= -4\vec{i} + 4\vec{j}$

Before	(Mass)	After
$4\vec{i} + 3\vec{j}$	2	$p\vec{i} + 3\vec{j}$
$-4\vec{i} + 4\vec{j}$	3	$q\vec{i} + 4\vec{j}$

$2(4) + 3(-4) = 2p + 3q$

$\Rightarrow 2p + 3q = -4$

$\frac{p-q}{4+4} = \frac{-7}{8}$

$\Rightarrow p - q = -7$

Solving these gives $p = -5,\ q = 2$.

(ii) $-5\vec{i} + 3\vec{j},\ 2\vec{i} + 4\vec{j}$ m/s

(iii) $K.E._{before} = \frac{1}{2}(2)(4^2 + 3^2)$

$+ \frac{1}{2}(3)((-4)^2 + 4^2)$

$= 73$ J

$K.E._{after} = \frac{1}{2}(2)((-5)^2 + 3^2)$

$+ \frac{1}{2}(3)(2^2 + 4^2)$

$= 64$ J

Loss $= 73 - 64 = 9$ J

Q. 6.

Before	(Mass)	(After)
$5\vec{i} + 4\vec{j}$	5	$p\vec{i} + 4\vec{j}$
$-2\vec{i} - 3\vec{j}$	10	$q\vec{i} - 3\vec{j}$

$5(5) + 10(-2) = 5p + 10q$

$\Rightarrow p + 2q = 1$

$\frac{p-q}{5+2} = \frac{1}{7}$

$\Rightarrow p - q = -1$

Solving these gives: $p = -\frac{1}{3},\ q = \frac{2}{3}$.

(i) $-\frac{1}{3}\vec{i} + 4\vec{j}$

(ii) $\frac{2}{3}\vec{i} - 3\vec{j}$

$K.E._{before} =$

$\frac{1}{2}(5)(25 + 16) + \frac{1}{2}(10)(4 + q)$

$= 102.5 + 65$

$= 167.5$ J

$K.E._{after} =$

$\frac{1}{2}(5)\left(\frac{1}{9} + 16\right) + \frac{1}{2}(10)\left(\frac{4}{9} + 9\right)$

$= \frac{725}{18}$

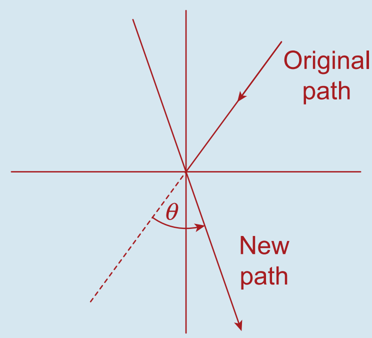
$= \frac{850}{18}$

$= \frac{1,575}{18}$

$= 87.5$ J

Loss $= 167.5 - 87.5 = 80$ J **QED**

(iii)



$M_1 = \frac{-3}{-2} = \frac{3}{2};\quad M_2 = \frac{-3}{\frac{2}{5}} = \frac{-9}{2}$

$\tan\theta = \pm\frac{\frac{3}{2} + \frac{9}{2}}{1 - \frac{27}{4}} = \pm\frac{24}{23}$

$\tan\theta = \frac{24}{23}$, since θ is acute.

Q. 7.

Before	(Mass)	(After)
$p\vec{i} + q\vec{j}$	4	$0\vec{i} + q\vec{j}$
$0\vec{i} + 0\vec{j}$	m	$r\vec{i} + 0\vec{j}$

Momentum in the $\vec{i}$ – direction is conserved

$\Rightarrow 4p = mr$

N.E.L.

$\frac{0 - r}{p - 0} = -\frac{4}{7}$

$\Rightarrow 4p = 7r$

$\Rightarrow m = 7$

Q. 8.

Before	Mass	After
$8\vec{i} + 4\vec{j}$	m	$x\vec{i} + 4\vec{j}$
$0\vec{i} + 0\vec{j}$	$2m$	$y\vec{i} + 0\vec{j}$

(i) $8\vec{i} + 4\vec{j} \perp x\vec{i} + 4\vec{j}$

$\therefore \frac{4}{8} \cdot \frac{4}{x} = -1$

$\Rightarrow x = -2$

$\therefore$ New velocity $= -2\vec{i} + 4\vec{j}$ m/s

(ii) $m(8) + 2m(0) = m(x) + 2my$

$8m = -2m + 2my$

$\therefore y = 5$

$\therefore$ Its velocity $= 5\vec{i}$ m/s

(iii) $\frac{x - y}{8 - 0} = -e$

$\therefore \frac{-2 - 5}{8} = -e$

$\therefore e = \frac{7}{8}$

Q. 9. (i)

Before	(Mass)	After
$p\vec{i} + q\vec{j}$	4	$r\vec{i} + q\vec{j}$
$x\vec{i} + y\vec{j}$	2	$0\vec{i} + 0\vec{j}$

Momentum in the $\vec{i}$ – direction is conserved

$\Rightarrow 4p + 2x = 4r$

$\Rightarrow 2p + x = 2r$ … **Equation 1**

N.E.L.

$\frac{r - 0}{p - x} = -\frac{1}{2}$

$\Rightarrow p - x = -2r$ … **Equation 2**

Adding equations **1** and **2** we get

$3p = 0$

$\Rightarrow p = 0$

$\vec{j}$ – velocity of 2nd sphere is unchanged

$\Rightarrow y = 0$

$\Rightarrow$ Velocities before impact were $q\vec{j}$ and $x\vec{i}$ … these are $\perp$ to each other.

(ii) **4 kg mass:**

$\text{K.E.}_{\text{before}} = \frac{1}{2}(4)q^2 = 2q^2$

$\text{K.E.}_{\text{after}} = \frac{1}{2}(4)(r^2 + q^2)$

$= 2(r^2 + q^2)$

Gain $= 2(r^2 + q^2) - 2q^2$

$= 2r^2$

2 kg mass:

$\text{K.E.}_{\text{before}} = \frac{1}{2}(2)x^2 = x^2$

… but $x = 2r$ (from **Equation 1**)

$\Rightarrow \text{K.E.}_{\text{before}} = (2r)^2 = 4r^2$

$\text{K.E.}_{\text{after}} = 0$

$\Rightarrow$ Loss $= 4r^2$

$\Rightarrow$ Gain in K.E. of 4 kg mass $= \frac{1}{2}$ (Loss in K.E. of 2 kg mass)

Q. 10. (i) First collision between P and Q

	Before	(Mass)	After
P:	$12u$	4	p
Q:	0	8	q

$4(12u) + 8(0) = 4(p) + 8(q)$

$\Rightarrow p + 2q = 12u$ … **Equation 1**

$\frac{p - q}{12u - 0} = -\frac{1}{4}$

$\Rightarrow -p + q = 3u$ … **Equation 2**

Adding equations **1** and **2** we get

$3q = 15u$

$\Rightarrow q = 5u$ … speed of Q after 1st collision

$p = q - 3u$ … from **Equation 2**

$\Rightarrow p = 5u - 3u$

$\Rightarrow p = 2u$ … speed of P after 1st collision

Collision of Q with the wall

$\frac{\text{NEW}}{\text{OLD}} = -e$

$\Rightarrow \frac{v}{5u} = -e$

$\Rightarrow v = -5eu$

Second collision between P and Q

	Before	**(Mass)**	**After**
P:	$2u$	4	r
Q:	$-5eu$	8	0

$$4(2u) + 8[-5eu] = 4r$$

$$\Rightarrow r = 2u - 10eu$$

$$\frac{r-0}{2u+5eu} = -\frac{1}{4}$$

$$\Rightarrow 4r = -2u - 5eu$$

$$\Rightarrow 4(2u - 10eu) = -2u - 5eu$$

$$\Rightarrow 8 - 40e = -2 - 5e$$

$$\Rightarrow 35e = 10$$

$$\Rightarrow e = \frac{2}{7}$$

(ii) $r = 2u - 10eu$

$$\Rightarrow r = 2u - 10\left(\frac{2}{7}\right)u$$

$$\Rightarrow r = 2u - \frac{20}{7}u$$

$$\Rightarrow r = -\frac{6}{7}u \quad \text{... final speed of } P$$

(iii) $\text{K.E.}_{\text{before}} = \frac{1}{2}(4)(12u)^2$

$$= 288u^2 \text{ J}$$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}(4)\left(\frac{6}{7}u\right)^2$$

$$= \frac{72}{49}u^2 \text{ J}$$

$$\text{Loss} = 288u^2 - \frac{72}{49}u^2$$

$$= 286.53u^2 \text{ J}$$

Exercise 7D

Q. 1.

Before	**(Mass)**	**After**
$u\cos A\vec{i} + u\sin A\vec{j}$	M	$v\cos B\vec{i} + v\sin B\vec{j}$
$0\vec{i} + 0\vec{j}$	M	$q\vec{i} + 0\vec{j}$

$u\sin A = V\sin B$ ... **Equation 1**

$$M(u\cos A) + M(0) = M(v\cos B) + Mq$$

$$\Rightarrow q = u\cos A - v\cos B \quad \text{... } \textbf{Equation 2}$$

$$\frac{v\cos B - q}{u\cos A - 0} = -\frac{1}{4}$$

$\Rightarrow 4v\cos B - 4q = -u\cos A$ ... **Equation 3**

But $q = u\cos A - v\cos B$

$\therefore$ **3** reads:

$$4v\cos B - 4u\cos A + 4v\cos B = -u\cos A$$

$\Rightarrow 3u\cos A = 8v\cos B$ ... **Equation 4**

Dividing equation **1** by equation **4**, we get:

$$\frac{u\sin A}{3u\cos A} = \frac{v\sin B}{8v\cos B}$$

$\Rightarrow 8\tan A = 3\tan B$ **QED**

Q. 2. **Before** **(Mass)** **After**

$2u\cos A\vec{i} + 2u\sin A\vec{j}$ $2M$ $p\vec{i} + 2u\sin A\vec{j}$

$-u\vec{i} + 0\vec{j}$ M $0\vec{i} + 0\vec{j}$

(i) $2M(2u\cos A) + M(-u) = 2M(p) + M(0)$

$$\Rightarrow p = 2u\cos A - \frac{1}{2}u$$

$$\frac{p-0}{2u\cos A + u} = \frac{-5}{118}$$

$$\Rightarrow 118p = -10u\cos A - 5u$$

But $p = 2u\cos A - \frac{1}{2}u$

$$\therefore 118\left(2u\cos A - \frac{1}{2}u\right)$$

$$= -10u\cos A - 5u$$

$$\Rightarrow 236\cos A - 59 = -10\cos A - 5$$

$$\Rightarrow \cos A = \frac{9}{41}$$

$$\Rightarrow \sin A = \frac{40}{41}$$

(ii) $p = 2u\cos A - \frac{1}{2}u$

$$= 2u\left(\frac{9}{41}\right) - \frac{1}{2}u = \frac{-5u}{82}$$

$$2u\sin A = 2u\left(\frac{40}{41}\right) = \frac{80u}{41} = \frac{160u}{82}$$

$\therefore$ Velocity after impact

$$= \frac{u}{82}(-5\vec{i} + 160\vec{j})$$

$$= \frac{5u}{82}(-\vec{i} + 32\vec{j}) \text{ m/s}$$

Q. 3. (i) **Before** **(Mass)** **After**

$4u$	m	p
$2u$	m	q

$m(4u) + m(2u) = m(p) + m(q)$

… divide by m

$\Rightarrow p + q = 6u$ **Equation 1**

$$\frac{p-q}{4u-2u} = -e$$

$\Rightarrow p - q = -2eu$ **Equation 2**

Adding equations **1** and **2** we get

$$2p = 2u(3-e)$$

$$\Rightarrow p = u(3-e)$$

$q = 6u - p$ from **Equation 1**

$$\Rightarrow q = 6u - u(3-e)$$

$$\Rightarrow q = 3u + eu$$

$$\Rightarrow q = u(3+e)$$

$$\text{K.E.}_{\text{before}} = \frac{1}{2}(m)(4u)^2 + \frac{1}{2}(m)(2u)^2$$

$$= 10mu^2$$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}(m)[u(3-e)^2]$$

$$+ \frac{1}{2}(m)[u(3+e)]^2$$

$$= \frac{1}{2}(m)[u^2(9 - 6e + e^2 + 9$$

$$+ 6e + e^2)]$$

$$= \frac{1}{2}(m)[u^2(18 + 2e^2)]$$

$$= mu^2(9 + e^2)$$

$$\text{Loss} = 10mu^2 - mu^2(9+e^2)$$

$$= mu^2 - mu^2e^2$$

$$= mu^2(1-e^2)$$

(ii) Firstly, calculate velocity of first sphere before impact:

$$4u\cos 30°\vec{i} + 4u\sin 30°\vec{j}$$

$$= 4u\left(\frac{\sqrt{3}}{2}\right)\vec{i} + 4u\left(\frac{1}{2}\right)\vec{j}$$

$$= 2u\sqrt{3}i + 2u\vec{j}$$

Before	**(Mass)**	**After**
$2u\sqrt{3}\vec{i} + 2u\vec{j}$	m	$p\vec{i} + 2u\vec{j}$
$0\vec{i} + 0\vec{j}$	m	$q\vec{i} + 0\vec{j}$

Momentum in the $\vec{i}$ – direction is conserved

$$\Rightarrow m(2u\sqrt{3}) + m(0) = m(p) + m(q)$$

… divide by m

$\Rightarrow p + q = 2u\sqrt{3}$ **Equation 3**

N.E.L.

$$\frac{p-q}{2u\sqrt{3} - 0} = -e$$

$\Rightarrow p - q = -2eu\sqrt{3}$ **Equation 4**

Adding equations **3** and **4** we get

$$2p = 2u\sqrt{3}(1-e)$$

$$\Rightarrow p = u\sqrt{3}(1-e)$$

$\Rightarrow$ velocity of 1st sphere after impact

$$= u\sqrt{3}(1-e)\vec{i} + 2u\vec{j}$$

$q = 2u\sqrt{3} - p$ from **Equation 3**

$\Rightarrow q = 2u\sqrt{3} - u\sqrt{3}(1 - e)$

$= u\sqrt{3} + eu\sqrt{3}$

$\Rightarrow q = u\sqrt{3}(1 + e)$

$\Rightarrow$ velocity of 2nd sphere after impact $= u\sqrt{3}(1 + e)\vec{i}$

$$\text{K.E.}_{\text{before}} = \frac{1}{2}(m)(4u)^2$$

$$= 8mu^2$$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}(m)[\{u\sqrt{3}(1 - e)\}^2 + \{2u\}^2] + \frac{1}{2}(m)\,[u\sqrt{3}(1 + e)]^2$$

$$= \frac{m}{2}[3u^2(1 - 2e + e^2) + 4u^2 + 3u^2(1 + 2e + e^2)]$$

$$= \frac{m}{2}[10u^2 + 6e^2u^2]$$

$$= mu^2(5 + 3e^2)$$

$$\text{Loss} = 8mu^2 - mu^2(5 + 3e^2)$$

$$= 3mu^2 - 3mu^2e^2$$

$$= 3mu^2(1 - e^2)$$

Q. 4. (i)

	Before	**(Mass)**	**After**
A:	$\frac{u}{2}\vec{i} + \frac{u\sqrt{3}}{2}\vec{j}$	m	$p\vec{i} + \frac{u\sqrt{3}}{2}\vec{j}$
B:	$0\vec{i} + 0\vec{j}$	m	$q\vec{i} + 0\vec{j}$

$$\tan\theta = \frac{\frac{u\sqrt{3}}{2}}{p} = \frac{u\sqrt{3}}{2p}$$

Momentum in the $\vec{i}$ −direction is conserved

$\Rightarrow m\left(\frac{u}{2}\right) + m(0) = m(p) + m(q)$ … divide by m

$$\Rightarrow p + q = \frac{u}{2}$$

$\Rightarrow 2p + 2q = u$ **Equation 1**

$$\frac{p - q}{\frac{u}{2} - 0} = -e$$

$$\Rightarrow p - q = -\frac{eu}{2}$$

$2p - 2q = -eu$ **Equation 2**

Adding equations **1** and **2** we get

$$4p = u(1 - e)$$

$$\Rightarrow p = \frac{u}{4}(1 - e)$$

$$\tan\theta = \frac{u\sqrt{3}}{2p}$$

$$\Rightarrow p = \frac{u\sqrt{3}}{2\tan\theta}$$

$q = \frac{u}{2} - p$ … from **Equation 1**

$$\Rightarrow q = \frac{u}{2} - \frac{u}{4}(1 - e) = \frac{u}{4} + \frac{ue}{4} = \frac{u}{4}(1 + e)$$

$\Rightarrow \frac{u\sqrt{3}}{2\tan\theta} = \frac{u}{4}(1 - e)$ … multiply by $\frac{2}{u}$

$$\Rightarrow \frac{\sqrt{3}}{\tan\theta} = \frac{1}{2}(1 - e)$$

$$\Rightarrow (1 - e)\tan\theta = 2\sqrt{3}$$

$$\Rightarrow \tan\theta = \frac{2\sqrt{3}}{1 - e}$$

(ii) $\text{K.E.}_{\text{before}} = \frac{1}{2}mu^2 \text{ J}$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}m\left[p^2 + \left(\frac{u\sqrt{3}}{2}\right)^2\right] + \frac{1}{2}mq^2$$

$$= \frac{1}{2}m\left[\frac{u^2}{16}(1-e)^2 + \frac{3u^2}{4}\right]$$

$$+ \frac{1}{2}m\left[\frac{u^2}{16}(1+e)^2\right] \text{ ...let } e = 0$$

$$= \left[\frac{1}{2}m\frac{u^2}{16} + \frac{3u^2}{4} + \frac{u^2}{16}\right]$$

$$= \frac{1}{2}m\left(\frac{7u^2}{8}\right)$$

$$= \frac{7mu^2}{16} \text{ J}$$

$$\text{Loss} = \frac{mu^2}{2} - \frac{7mu^2}{16} = \frac{mu^2}{16} \text{ J}$$

$$\Rightarrow \% \text{ loss} = \frac{\frac{1}{16}mu^2}{\frac{1}{2}mu^2} \times 100 = 12\tfrac{1}{2}\%$$

(iii) $\text{K.E.}_{A(\text{after})} = \frac{1}{2}m\left[p^2 + \left(\frac{u\sqrt{3}}{2}\right)^2\right]$

$$= \frac{1}{2}m\left[\frac{u^2}{16}(1-e)^2 + \frac{3u^2}{4}\right]$$

$$= \frac{1}{2}m\left[\frac{u^2}{16}(1 - 2e + e^2) + \frac{3u^2}{4}\right]$$

$$= \frac{1}{2}m\left[\frac{13u^2}{16} - \frac{2eu^2}{16} + \frac{e^2u^2}{16}\right]$$

$$= \frac{mu^2}{32}[13 - 2e + e^2]$$

$$\text{K.E.}_{B(\text{after})} = \frac{1}{2}mq^2 = \frac{1}{2}m\left[\frac{u^2}{16}(1+e)^2\right]$$

$$= \frac{mu^2}{32}(1 + 2e + e^2)$$

$$\frac{\text{K.E.}_{A(\text{after})}}{\text{K.E.}_{B(\text{after})}} = \frac{7}{1}$$

$$\Rightarrow \frac{13 - 2e + e^2}{1 + 2e + e^2} = 7$$

$$\Rightarrow 13 - 2e + e^2 = 7 + 14e + 7e^2$$

$$\Rightarrow 6e^2 + 16e - 6 = 0$$

$$\Rightarrow 3e^2 + 8e - 3 = 0$$

$$\Rightarrow (3e - 1)(e + 3) = 0$$

$$\Rightarrow e = \frac{1}{3} \quad 0 \leqslant e \leqslant 1$$

Q. 5. (i)

Before	**(Mass)**	**After**
$A: p\vec{i} + q\vec{j}$	m	$r\vec{i} + q\vec{j}$
$B: 0\vec{i} + 0\vec{j}$	$2m$	$t\vec{i} + 0\vec{j}$

$$\tan\theta = \frac{q}{p}$$

$$\frac{q}{p} \times \frac{q}{r} = -1 \quad \text{... new path at right angles to old path}$$

$$\Rightarrow \frac{q^2}{pr} = -1$$

$$\Rightarrow r = -\frac{q^2}{p}$$

Also, $\sqrt{r^2 + q^2} = 0.2$

$$\Rightarrow r^2 + q^2 = 0.04$$

$$\Rightarrow \frac{q^4}{p^2} + q^2 = 0.04$$

$$\Rightarrow \frac{q^4 + p^2q^2}{p^2} = 0.04$$

$$\Rightarrow \frac{q^2(p^2 + q^2)}{p^2} = 0.04 \text{ ... } \sqrt{p^2 + q^2} = 0.06$$

so, $p^2 + q^2 = 0.36$

$$\Rightarrow \frac{0.36q^2}{p^2} = 0.04 \quad \text{... divide by 0.04}$$

$$\Rightarrow \frac{9q^2}{p^2} = 1$$

$$\Rightarrow \frac{q^2}{p^2} = \frac{1}{9}$$

$$\Rightarrow \frac{q}{p} = \frac{1}{3} = \tan\theta$$

(ii) Momentum in the $\vec{i}$ −direction is conserved

$$m(p) + 2m(0) = m(r) + 2m(t)$$

...divide by m

$$\Rightarrow r + 2t = p$$

$$\Rightarrow t = \frac{p - r}{2} \quad \textbf{Equation 1}$$

N.E.L.

$$\frac{r - t}{p - 0} = -e$$

$$\Rightarrow r - t = -pe \quad \textbf{Equation 2}$$

$$\Rightarrow r - \frac{p - r}{2} = -pe$$

$$\Rightarrow 2r - p + r = -2pe$$

$$\Rightarrow 3r - p = -2pe$$

$$\Rightarrow e = \frac{p - 3r}{2p}$$

$$= \frac{p + \frac{3q^2}{p}}{2p}$$

$$= \frac{p^2 + 3q^2}{2p^2}$$

$$= \frac{1}{2} + \frac{3}{2}\left(\frac{q^2}{p^2}\right)$$

$$= \frac{1}{2} + \frac{3}{2}\left(\frac{1}{9}\right)$$

$$= \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$$

(iii) $\text{K.E.}_{\text{before}} = \frac{1}{2}m(0.6)^2 = 0.18m$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}m(0.2)^2 + \frac{1}{2}(2m)t^2$$

$$= 0.02m + mt^2$$

$$p = 0.6 \cos\theta = 0.6\left(\frac{3}{\sqrt{10}}\right) = \frac{1.8}{\sqrt{10}}$$

$r + 2t = \frac{1.8}{\sqrt{10}}$ ... from **Equation 1**

$r - t = -\frac{2p}{3}$ ... from **Equation 2**

$$\Rightarrow -r + t = \frac{3.6}{3\sqrt{10}}$$

$$= \frac{1.2}{\sqrt{10}}$$

$$\text{Add } 3t = \frac{3}{\sqrt{10}}$$

$$\Rightarrow t = \frac{1}{\sqrt{10}}$$

$$\Rightarrow \text{K.E.}_{\text{after}} = 0.02m + 0.1m = 0.12m$$

$$\text{Loss} = 0.18m - 0.12m = 0.06m$$

Q. 6. (a) $\frac{b}{a} \times \frac{d}{c} = -1$

$$\Rightarrow \frac{b}{a} = -\frac{c}{d}$$

$$\Rightarrow ac = -bd$$

(b) $\cos\theta = \frac{3}{7}$

$$\Rightarrow \sin\theta = \frac{\sqrt{40}}{7}$$

Before	**(Mass)**	**After**
$\frac{3v}{7}\vec{i} + \frac{v\sqrt{40}}{7}\vec{j}$	$2m$	$p\vec{i} + \frac{v\sqrt{40}}{7}\vec{j}$
$-u\vec{i} + 0\vec{j}$	m	$q\vec{i} + 0\vec{j}$

$\frac{3v}{7}(p) = -\frac{v\sqrt{40}}{7}\left(\frac{v\sqrt{40}}{7}\right)$ ... from part (a)

$$\Rightarrow 3vp = -\frac{40v^2}{7}$$

$$\Rightarrow p = -\frac{40v}{21}$$

Momentum in the $\vec{i}$ – direction is conserved

$$\Rightarrow 2m\left(\frac{3v}{7}\right) + m(-u) = 2m\left(\frac{-40v}{21}\right) + m(q)$$

... multiply by $\frac{21}{m}$

$$\Rightarrow 18v - 21u = -80v + 21q$$

$$\Rightarrow 98v - 21u = 21q$$

$\Rightarrow 14v - 3q = 3u$ ... **Equation 1**

N.E.L.

$$\frac{\frac{-40v}{21} - q}{\frac{3v}{7} + u} = -\frac{3}{4}$$

$$\Rightarrow \frac{-40v - 21q}{9v + 21u} = -\frac{3}{4}$$

$$\Rightarrow 160v + 84q = 27v + 63u$$

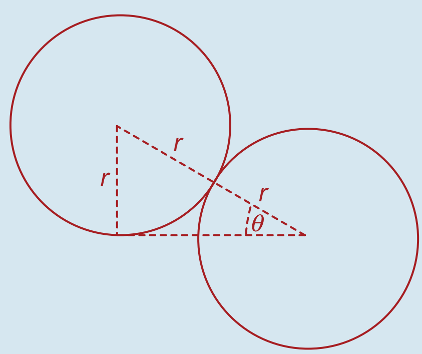
$\Rightarrow 133v + 84q = 63u$ ... **Equation 2**

$392v - 84q = 84u$ ... from **Equation 1**

$$525v = 147u$$

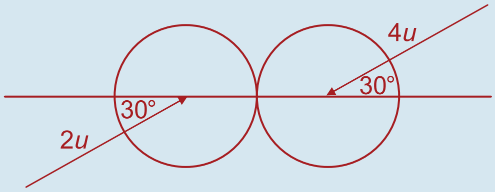
$$v = \frac{147u}{525} = \frac{7u}{25}$$

Q. 7. (i)



$$\sin\theta = \frac{r}{2r} = \frac{1}{2} \Rightarrow \theta = 30°$$

(ii) Rotate diagram as shown:



Before	(Mass)	After
$u\sqrt{3}\vec{i} + u\vec{j}$	$2m$	$p\vec{i} + u\vec{j}$
$-2u\sqrt{3}\vec{i} - 2u\vec{j}$	m	$q\vec{i} - 2u\vec{j}$

Momentum in the $\vec{i}$–direction is conserved

$\Rightarrow 2m(u\sqrt{3}) + m(-2u\sqrt{3}) = 2m(p) + m(q)$

... divide by m

$\Rightarrow 2p + q = 0$ **Equation 1**

N.E.L.

$$\frac{p - q}{u\sqrt{3} + 2u\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

$\Rightarrow p - q = -3u$ **Equation 2**

Adding equations **1** and **2** we get

$3p = -3u$

$\Rightarrow p = -u$

Speed of 1st sphere after impact

$= \sqrt{p^2 + u^2}$

$= \sqrt{(-u)^2 + u^2}$

$= \sqrt{2u^2}$

$= u\sqrt{2}$

$q = -2p$ ... from **Equation 1**

$\Rightarrow q = 2u$

Speed of 2nd sphere after impact

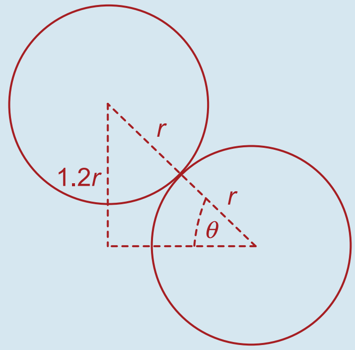
$= \sqrt{q^2 + (-2u)^2}$

$= \sqrt{(2u)^2 + (-2u)^2}$

$= \sqrt{8u^2}$

$= 2u\sqrt{2}$

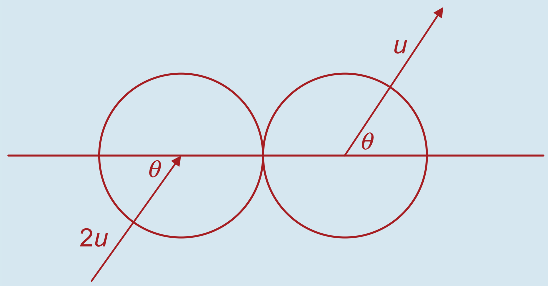
Q. 8. (i) Point of collision:



$\sin\theta = \frac{1.2r}{2r} = \frac{3}{5}$

$\Rightarrow \cos\theta = \frac{4}{5}$

Rotate diagram as shown in the diagram below:



Before	(Mass)	After
$\frac{8u}{5}\vec{i} + \frac{6u}{5}\vec{j}$	m	$p\vec{i} + \frac{6u}{5}\vec{j}$
$\frac{4u}{5}\vec{i} + \frac{3u}{5}\vec{j}$	m	$q\vec{i} + \frac{3u}{5}\vec{j}$

Momentum in the $\vec{i}$–direction is conserved

$\Rightarrow m\left(\frac{8u}{5}\right) + m\left(\frac{4u}{5}\right) = m(p) + m(q)$

... multiply by $\frac{5}{m}$

$\Rightarrow 5p + 5q = 12u$ **Equation 1**

N.E.L.

$$\frac{p - q}{\frac{8u}{5} - \frac{4u}{5}} = -\frac{1}{2}$$

$$\Rightarrow \frac{5p - 5q}{4u} = -\frac{1}{2}$$

$\Rightarrow 5p - 5q = -2u$ **Equation 2**

Adding equations **1** and **2** we get

$10p = 10u$

$\Rightarrow p = u$

$\Rightarrow$ Velocity of 1st sphere after impact

$= u\vec{i} + \frac{6u}{5}\vec{j}$

$\Rightarrow$ Speed of 1st sphere after impact

$$= \sqrt{u^2 + \left(\frac{6u}{5}\right)^2}$$

$$= \sqrt{u^2 + \frac{36u^2}{25}}$$

$$= \sqrt{\frac{25u^2 + 36u^2}{25}}$$

$$= \sqrt{\frac{61u^2}{25}}$$

$$= \frac{u}{5}\sqrt{61}$$

$$= \frac{\sqrt{61}}{5}u$$

$q = \frac{1}{5}(5p + 2u)$ ... from **Equation 2**

$\Rightarrow q = \frac{1}{5}(5u + 2u)$

$\Rightarrow q = \frac{7u}{5}$

$\Rightarrow$ Velocity of 2nd sphere after impact

$\frac{7u}{5}\vec{i} + \frac{3u}{5}\vec{j}$

$\Rightarrow$ Speed of 2nd sphere after impact

$$\sqrt{\left(\frac{7u}{5}\right)^2 + \left(\frac{3u}{5}\right)^2} = \sqrt{\frac{49u^2 + 9u^2}{25}}$$

$$= \sqrt{\frac{58u^2}{25}} = \frac{u}{5}\sqrt{58} = \frac{\sqrt{58}}{5}u$$

(ii) Velocities after impact are $\vec{v_1} = u\vec{i} + \frac{6u}{5}\vec{j}$ and $\vec{v_2} = \frac{7u}{5}\vec{i} + \frac{3u}{5}\vec{j}$

The slope of $\vec{v_1}$ is given by $m_1 = \frac{\frac{6u}{5}}{u}$

$$= \frac{6}{5}$$

The slope of $\vec{v_2}$ is given by $m_2 = \frac{\frac{3u}{5}}{\frac{7u}{5}} = \frac{3}{7}$

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$$

$$\Rightarrow \tan\theta = \pm\frac{\frac{6}{5} - \frac{3}{7}}{1 + \frac{18}{35}}$$

$$= \pm\frac{42 - 15}{35 + 18}$$

$$= \pm\frac{27}{53}$$

Take the plus case to find the acute angle

$\Rightarrow \theta = \tan^{-1}\frac{27}{53} = 27°$