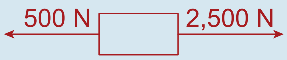
Chapter 5 Exercise 5A

Q. 1. (i)



$F = 2{,}500 - 500$
$= 2{,}000$

$F = ma$
$2{,}000 = 1{,}000a$
$\Rightarrow a = 2 \text{ m/s}^2$

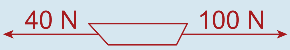
(ii) $a = 2, \quad u = 0, \quad t = 20, \quad s = ?$

$s = ut + \frac{1}{2}at^2$

$= 0(20) + \frac{1}{2}(2)(400)$

$= 400 \text{ m}$

Q. 2.



$F = 100 - 40 = 60$

$F = ma$
$\Rightarrow 60 = 120a$
$\Rightarrow a = \frac{1}{2} \text{ m/s}^2$

$F = ma$
$\Rightarrow 60 = 180a$
$\Rightarrow a = \frac{1}{3} \text{ m/s}^2$

Q. 3. $F = ma$

$\Rightarrow (t - 40) = 150\left(\frac{1}{2}\right)$

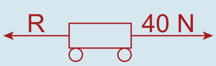
$\Rightarrow T = 115 \text{ N}$

$F = ma$

$\Rightarrow (t - 40) = 240\left(\frac{1}{2}\right)$

$\Rightarrow T = 160 \text{ N}$

Q. 4.



$F = ma$

$\Rightarrow (40 - R) = 120\left(\frac{1}{8}\right)$

$\Rightarrow R = 25 \text{ N}$

Q. 5. $F = ma$
$-900 = (0.060)a$
$\Rightarrow a = -15{,}000 \text{ m/s}^2$

$u = 150, \quad v = 0, \quad a = -15{,}000, \quad s = ?$

$v^2 = u^2 + 2as$

$0 = 22{,}500 + 2(-15{,}000)s$

$\Rightarrow s = 0.75 \text{ m} = 75 \text{ cm}$

Q. 6. $u = 0, \quad s = 8, \quad t = 8, \quad a = ?$

$s = ut + at^2$

$\Rightarrow 8 = (0)8 + \frac{1}{2}(a)(64)$

$\Rightarrow a = \frac{1}{4} \text{ m/s}^2$

$F = ma$

$\Rightarrow (T - 20) = 80\left(\frac{1}{4}\right)$

$T = 40 \text{ N}$

Q. 7. (i) $u = 0, \quad v = 10, \quad s = 50, \quad a = ?$

$v^2 = u^2 + 2as$

$\Rightarrow 100 = 0 + 2(a)(50)$

$\Rightarrow a = 1 \text{ m/s}^2$

(ii) $F = ma$

$\Rightarrow T - 350 = (800)(1)$

$\Rightarrow T = 1{,}150 \text{ N}$

Q. 8. $u = 200, \quad v = 0, \quad s = 1, \quad a = ?$

$v^2 = u^2 + 2as$
$\Rightarrow 0 = 40{,}000 + 2(a)(1)$
$\Rightarrow a = -20{,}000$

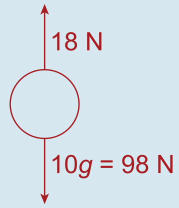
$F = ma$
$\Rightarrow R = (0.050)(-20{,}000)$
$= -1{,}000 \text{ N}$

$u = 400, \quad v = 0, \quad a = -20{,}000,$
$s = ?$

$v^2 = u^2 + 2as$
$\Rightarrow 0 = 160{,}000 + 2(-20{,}000)s$
$\Rightarrow s = 4 \text{ m}$

Q. 9. (a) $F = 98 - 18$

$= 80$ N



$F = ma$

$\Rightarrow 80 = 10(a)$

$\Rightarrow a = 8$ m/s^2

(b) (i) $u = 0, \quad a = 8, \quad t = 10, \quad s = ?$

$s = ut + \frac{1}{2}at^2$

$\Rightarrow s = 0(10) + \frac{1}{2}(8)(100)$

$= 400$ m

(ii) $u = 0, \quad a = 8, \quad t = 20, \quad s = ?$

$s = ut + \frac{1}{2}at^2$

$\Rightarrow s = (0)(10) + \frac{1}{2}(8)(400)$

$= 1{,}600$ m

$\therefore$ Distance $= 1{,}600 - 400$

$= 1{,}200$ m

Q. 10. $u = 300, \quad v = 200, \quad s = 0.1, \quad a = ?$

$v^2 = u^2 + 2as$

$\Rightarrow 40{,}000 = 90{,}000 + 2(a)(0.1)$

$\Rightarrow a = -250{,}000$

$F = ma$

$\Rightarrow -25{,}000 = m(-250{,}000)$

$\Rightarrow m = 0.1$ kg

$= 100$ grammes

$u = 300, \quad v = 0, \quad a = -250{,}000,$
$s = ?$

$v^2 = u^2 + 2as$

$0 = 90{,}000 + 2(-250{,}000)s$

$\Rightarrow s = 0.18$ m

$= 18$ cm

Q. 11. $u = u, \quad v = 0, \quad s = s, \quad a = ?$

$v^2 = u^2 + 2as$

$\Rightarrow 0 = u^2 + 2as$

$\Rightarrow a = \frac{-u^2}{2s}$

$F = ma$

$\Rightarrow R = m\left(\frac{-u^2}{2s}\right)$

$= \frac{-mu^2}{2s}$

$v = u + at$

$\Rightarrow 3u = 2u - \frac{u^2}{2s}t$

$\Rightarrow \quad t = \frac{2s}{u}$

$u = 3u, \quad s = 5s, \quad a = \frac{-u^2}{2s}, \quad v = ?$

$v^2 = u^2 + 2as$

$\Rightarrow v^2 = 9u^2 + 2\left(\frac{-u^2}{2s}\right)(5s)$

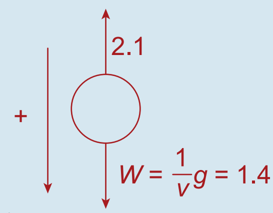
$\Rightarrow v^2 = 9u^2 - 5u^2 = 4u^2$

$\Rightarrow v = 2u$ m/s

$v = u + at \Rightarrow 3u = 2u - \frac{u^2}{2s}t \Rightarrow t = \frac{2s}{u}$

Q. 12. $F = 1.4 - 2.1$

$= -0.7$ N



$F = ma$

$\Rightarrow -0.7 = \left(\frac{1}{7}\right)a$

$\Rightarrow a = -4.9$ m/s

$u = 1.4, \quad a = -4.9, \quad v = 0, \quad s = ?$

$v^2 = u^2 + 2as$

$\Rightarrow 0 = 1.96 + 2(-4.9)s$

$\Rightarrow s = 0.2$ m

$= 20$ cm

Q. 13. **In Air**

$u = 0, \quad a = 9.8, \quad s = 2.5, \quad v = ?$

$v^2 = u^2 + 2as$

$\Rightarrow v^2 = 0 + 2(9.8)(2.5)$

$= 49$

$\Rightarrow v = 7$ m/s

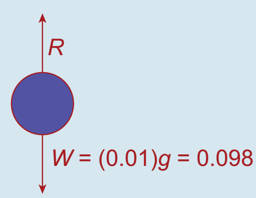
In Material

$u = 7, \quad v = 0, \quad s = 0.35, \quad a = ?$

$v^2 = u^2 + 2as$

$\Rightarrow 0 = 49 + 2(a)(0.35)$

$\Rightarrow a = -70$ m/s^2



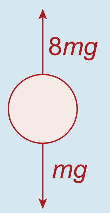
$F = ma$
$\Rightarrow (0.098 - R) = (0.01)(-70)$
$\Rightarrow R = 0.798$ N

Q. 14. (i) **In Air**

$u = 0, \quad s = 4h, \quad a = g, \quad v = ?$

$v^2 = u^2 + 2as$
$\Rightarrow v^2 = 0 + 2(g)(4h)$
$\Rightarrow v = \sqrt{8gh}$

In Marsh



$F = mg - 8mg$
$= -7mg$

$F = ma$
$\Rightarrow -7mg = ma$
$\Rightarrow a = -7g$

$u = \sqrt{8gh}, \quad v = 0, \quad a = -7g,$
$s = ?$

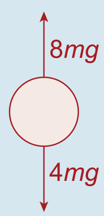
$v^2 = u^2 + 2as$
$0 = 8gh + 2(-7g)s$
$\Rightarrow s = \frac{4}{7}h$

In Air

$u = 0, \quad s = h, \quad a = g, \quad v = ?$

$v^2 = u^2 + 2as$
$\Rightarrow v^2 = 0 + 2gh$
$\Rightarrow v = \sqrt{2gh}$

In Marsh



$F = 4mg - 8mg$
$= -4mg$

$F = ma$
$\Rightarrow -4mg = -(4m)a$
$\Rightarrow a = -g$

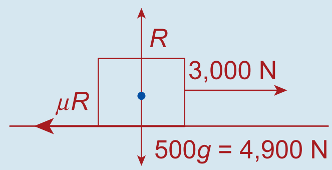
$u = \sqrt{2gh}, \quad a = -g, \quad v = 0, \quad s = ?$

$v^2 = u^2 + 2as$
$0 = 2gh + 2(-g)s$
$\Rightarrow s = h$

Answer: No

Exercise 5B

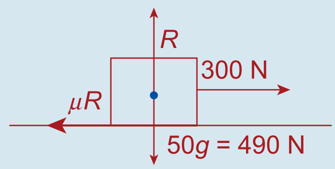
Q. 1.



$R = 4{,}900$
$\Rightarrow \mu R = (0.4)(4{,}900)$
$= 1{,}960$

$F = ma$
$\Rightarrow (3{,}000 - 1{,}960) = 500a$
$\Rightarrow a = 2.08$ m/s^2

Q. 2.

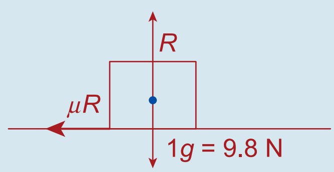


$R = 490$

$\Rightarrow \mu R = 0.6(490)$
$= 294$ N

$F = ma$
$\Rightarrow (300 - 294) = (50)a$
$\Rightarrow a = 0.12$ m/s^2

Q. 3. (i)



$R = 9.8$

$\Rightarrow$ Friction $= \mu R$

$= \left(\frac{1}{7}\right)(9.8)$

$= 1.4$ N

(ii) $F = ma$

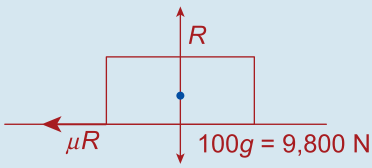
$(-1.4) = (1)a$
$\Rightarrow a = -1.4$ m/s^2

The deceleration is 1.4 m/s^2

(iii) $u = 3.5, \quad v = 0, \quad a = -1.4, \quad s = ?$

$v^2 = u^2 + 2as$
$\Rightarrow 0 = 12.25 + 2(-1.4)s$
$\Rightarrow s = 4.375$ m

Q. 4. (i)



$R = 9{,}800$ N

$\Rightarrow \mu R = \frac{1}{4}(9{,}800)$

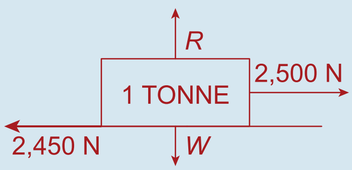
$= 2{,}450$ N

$\Rightarrow$ Limiting friction $= 2{,}450$ N

(ii) $\frac{2{,}450}{250} = 9.8$

$\therefore$ 10 slaves needed.

(iii)

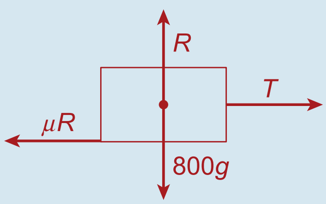


$F = ma$

$\Rightarrow (2{,}500 - 2{,}450) = (1{,}000)a$

$\Rightarrow a = 0.05$ m/s^2

Q. 5. (i)



$R = 800g$

$= 7{,}840$

$\Rightarrow \mu R = \frac{1}{8}(7{,}840)$

$= 980$ N ... limiting friction

(ii) $\frac{980}{200} = 4.9 \Rightarrow 5$ dogs required.

(iii) $T = 5(200) = 1{,}000$ N

$F = ma$

$\Rightarrow 1000 - 980 = 800a$

$\Rightarrow 800a = 20$

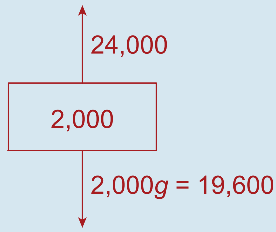
$\Rightarrow a = 0.025$ m/s^2

Q. 6. (a) Momentum is the product of mass and velocity.

(b) A newton is the force required to accelerate one kilogram at one metre per second squared.

(c) The change in momentum per unit time is proportional to the applied force and takes place along the straight line in which the force acts.

(d) (i) $1{,}440 + 8(70) = 2{,}000$

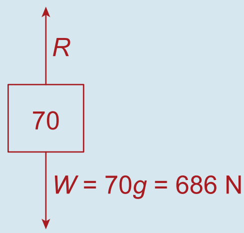


$F = ma$

$\Rightarrow (24{,}000 - 19{,}600) = 2{,}000a$

$\Rightarrow a = 2.2$ m/s^2

(ii)

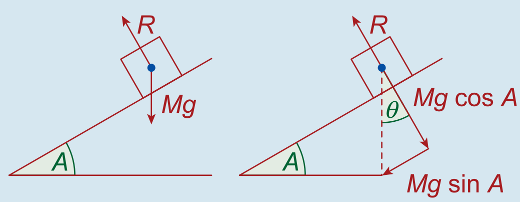


$F = ma$

$\Rightarrow (R - 686) = (70)(2.2)$

$\Rightarrow R = 840$ N

Q. 7. **Forces** **Resolved**



$F = ma$

$\Rightarrow Mg \sin A = Ma$

$\Rightarrow a = \frac{1}{7}g$

$= 1.4$ m/s^2

$v = u + at$

$\Rightarrow 7 = 0 + (1.4)t$

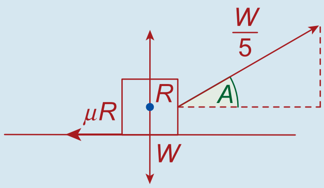
$\Rightarrow t = 5$ s

$s = ut + \frac{1}{2}at^2$

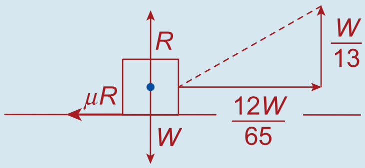
$= 0(5) + \frac{1}{2}(1.4)(25)$

$= 17.5$ m

Q. 8. **Forces**



Resolved

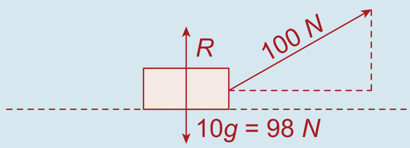


① $R + \frac{W}{13} = W \Rightarrow R = \frac{12W}{13}$

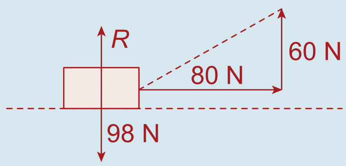
② $\mu R = \frac{12W}{65}$

Dividing ② by ① gives $\mu = \frac{1}{5}$

Q. 9. (i) **Forces**



Resolved



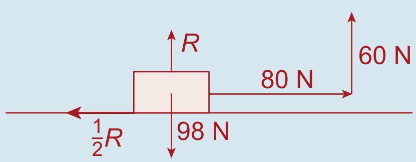
① $R + 60 = 98 \Rightarrow R = 38$ N

② $F = ma$

$\Rightarrow 80 = 10a$

$\Rightarrow a = 8$ m/s^2

(ii) **Forces (Resolved)**



① $R + 60 = 98 \Rightarrow R = 38$

② $\therefore \frac{1}{2}R = 19$ N

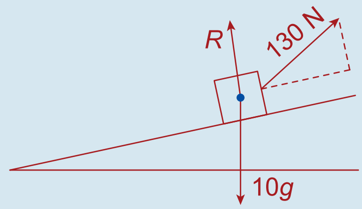
= The friction force

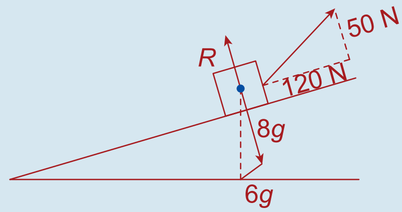
$F = ma$

$\Rightarrow (80 - 19) = 10a$

$\Rightarrow a = 6.1$ m/s^2

Q. 10. (i)



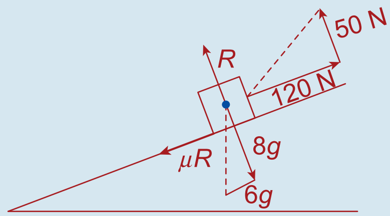


$F = ma$

$\Rightarrow 120 - 6g = 10a$

$\Rightarrow a = 6.12$ m/s^2

(ii)



$R + 50 = 8g$

$\Rightarrow R = 28.4$

$\Rightarrow \mu R = \frac{1}{4}(28.4)$

$= 7.1$

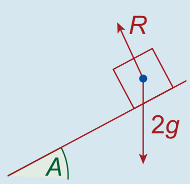
$F = ma$

$\Rightarrow 120 - 6g - 7.1 = 10a$

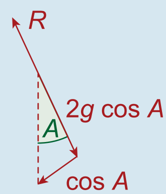
$\Rightarrow a = 5.41$ m/s^2

Q. 11. 4. Since $\sin A = \frac{1}{5}$, $\cos A = \frac{\sqrt{24}}{5}$.

Forces



Resolved



1. $R = 2g \cos A$

$= 2g\left(\frac{\sqrt{24}}{5}\right)$

$= \frac{4\sqrt{6}}{5}g$ **QED**

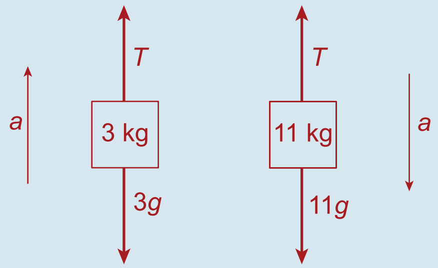
2. $F = ma$

$\Rightarrow 2g \sin A = 2a$

$\Rightarrow a = \frac{1}{5}g$ **QED**

Exercise 5C

Q. 1. (i)



$T - 3g = 3a$ $\qquad$ $11g - T = 11a$

Add equations:

$14a = 8g$

$\Rightarrow a = \frac{4g}{7}$

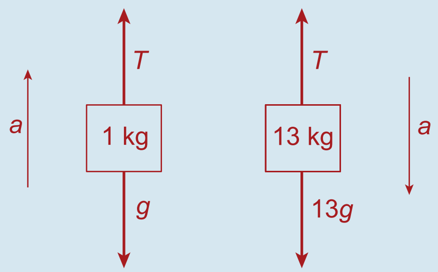
$= 5.6 \text{ m/s}^2$

(ii) $T - 3g = 3a$

$\Rightarrow T - 29.4 = 16.8$

$\Rightarrow T = 46{\cdot}2 \text{ N}$

Q. 2. (i)



$T - g = a$ $\qquad$ $13g - T = 13a$

Add equations:

$14a = 12g$

$\Rightarrow a = \frac{6g}{7}$

$= 8.4 \text{ m/s}^2$

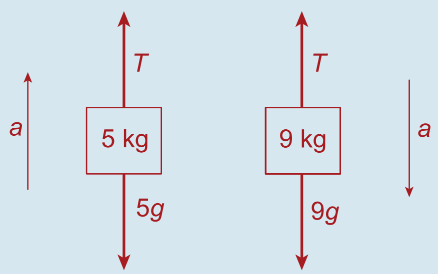
(ii) $u = 0, \quad a = 8.4, \quad t = 3$

$v = u + at$

$\Rightarrow v = 0 + (8.4)(3)$

$\Rightarrow v = 25.2 \text{ m/s}$

Q. 3. (i)



$T - 5g = 5a$ $\qquad$ $9g - T = 9a$

Add equations:

$14a = 4g$

$\Rightarrow a = \frac{2g}{7}$

$= 2.8 \text{ m/s}^2$

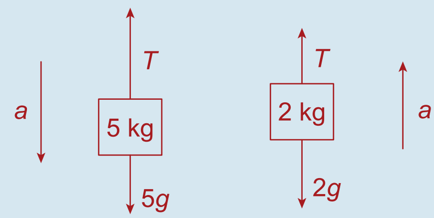
(ii) $u = 0, \quad a = 2.8, \quad t = 3$

$s = ut + \frac{1}{3}at^2$

$\Rightarrow s = (0)(3) + \frac{1}{2}(2.8)(3)^2$

$\Rightarrow s = 12.6 \text{ m}$

Q. 4. (i)



$5g - T = 5a$ $\qquad$ $T - 2g = 2a$

Add equations:

$7a = 3g$

$a = \frac{3g}{7}$

$= 4.2 \text{ m/s}^2$

(ii) $u = 0, \quad s = 2, \quad a = 4.2, \quad v = ?$

$v^2 = u^2 + 2as$

$\Rightarrow v^2 = 0 + 2(4.2)(2)$

$\Rightarrow v = \sqrt{16.8} \text{ m/s}$

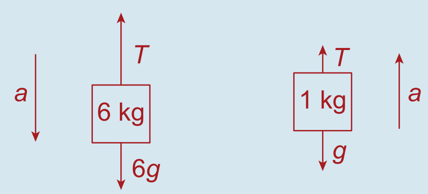
(iii) $u = \sqrt{16.8}, \quad v = 0, \quad a = -9.8, \quad s = ?$

$v^2 = u^2 + 2as$

$0 = 16.8 + 2(-9.8)(s)$

$\Rightarrow s = \frac{6}{7} \text{ m}$

Q. 5. (i)



$6g - T = 6a$ $\qquad$ $T - g = 1a$

Add equations:

$7a = 5g$

$a = \frac{5g}{7}$

$= 7 \text{ m/s}^2$

(ii) $v = u + at$

$v = 0 + 7(1)$

$= 7$ m/s

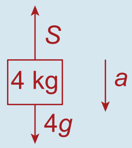
(iii) $u = 7, \quad v = 0, \quad a = -9.8, \quad s = ?$

$v^2 = u^2 + 2as$

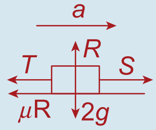
$\Rightarrow 0 = 49 + 2(-9.8)s$

$\Rightarrow s = 2.5$ m

Q. 6. (i)



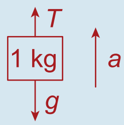
$4g - S = 4a$ **Equation 1**



$R = 2g$

$\Rightarrow \mu R = \frac{1}{2}R = g$

$S - T - g = 2a$ **Equation 2**



$T - g = a$ **Equation 3**

Add equations:

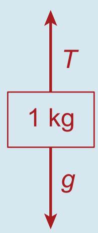
$7a = 2g$

$a = \frac{2g}{7}$

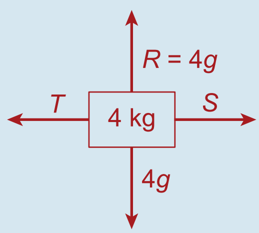
$= 2.8$ m/s

(ii) $\therefore T = 12.6$ N and $S = 28$ N

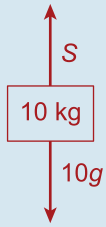
Q. 7. (i)



$T - g = a$ **Equation 1**



$S - T = 4a$ **Equation 2**



$10g - S = 10a$ **Equation 3**

Add 3 equations:

$15a = 9g$

$\Rightarrow a = \frac{3g}{5}$ m/s^2

(ii) Additional force of $\mu R = \frac{1}{2}(4g) = 2g$ opposing the motion of the 4 kg mass, i.e. to the left.

Equation 2 becomes: $S - T - 2g = 4a$

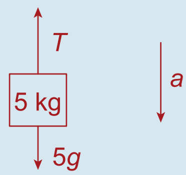
Equation 1: $T - g = a$

Equation 2: $10g - S = 10a$

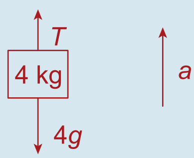
$15a = 7g$

$\Rightarrow a = \frac{7g}{15}$ m/s^2

Q. 8.



$5g - T = 5a$ **Equation 1**



$T - 4g = 4a$ **Equation 2**

Add equations:

(i) $9a = g$

$a = \frac{1}{9}g$ m/s^2

(ii) $T = \frac{40}{9}g$ N

$v = u + at$

$\Rightarrow v = 0 + \left(\frac{1}{9}g\right)(2)$

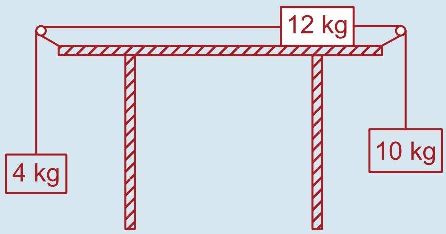
$= \frac{2}{9}g$ m/s

$v^2 = u^2 + 2as$

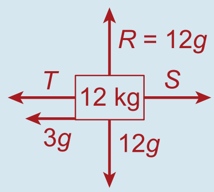
$\Rightarrow 0 = \left(\frac{2}{9}g\right)^2 + 2(-g)s$

$\Rightarrow s = \frac{2}{81}g$ metres

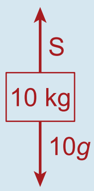
Q. 9.



$T - 4g = 4a$ **Equation 1**



$S - T - 3g = 12a$ **Equation 2**



$10g - S = 10a$ **Equation 3**

Add 3 equations:

$26a = 3g$

$\Rightarrow a = \frac{3}{26} g \text{ m/s}^2$

Now, find least value of μ for which the particles will not move:

Equation 1: $T - 4g = 4a$

Equation 2: $S - T - 12\mu g = 12a$

Equation 3: $10g - S = 10a$ … add

$6g - 12\mu g = 26a$

$\Rightarrow 3g - 6\mu g = 13a$

$\Rightarrow a = \frac{3g(1 - 2\mu)}{13}$

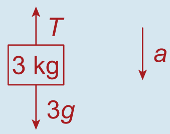
Let $a = 0$

$\Rightarrow 3g(1 - 2\mu) = 0$

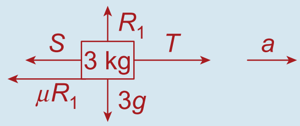
$\Rightarrow 1 - 2\mu = 0$

$\Rightarrow \mu = \frac{1}{2}$ … least value of μ for which the particles will not move.

Q. 10. (i)



$3g - T = 3a$ **Equation 1**

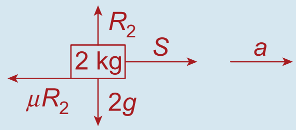


$R_1 = 3g$

$\Rightarrow \mu R_1 = \frac{1}{4}(3g)$

$= \frac{3}{4}g$

$T - S - \frac{3}{4}g = 3a$ **Equation 2**



$R_2 = 2g$

$\Rightarrow \mu R_2 = \frac{1}{4}(2g)$

$= \frac{1}{2}g$

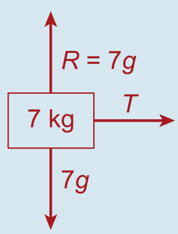
$S - \frac{1}{2}g = 2a$ **Equation 3**

Solving 3 equations:

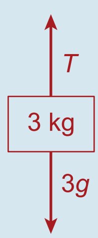
$a = \frac{7}{32}g \text{ m/s}^2$

(ii) $\therefore S = \frac{15}{16}g$ N and $T = \frac{75}{32}g$ N

Q. 11. (i)



$T = 7a$ **Equation 1**



$3g - T = 3a$ **Equation 2**

Add 2 equations:

$10a = 3g$

$\Rightarrow a = \frac{3g}{10}$

$= 2.94 \text{ m/s}^2$

$u = 0, \quad a = 2.94, \quad s = 3.3075$

$v^2 = u^2 + 2as$

$\Rightarrow v^2 = 2(2.94)(3.3075)$

$\Rightarrow v = 4.41 \text{ m/s}$

$t = \frac{v - u}{a}$

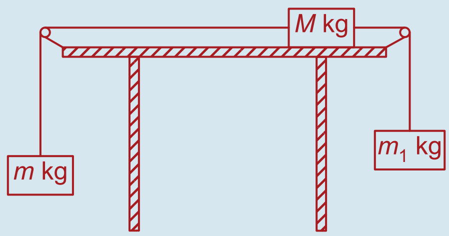
$= \frac{4.41 - 0}{2.94}$

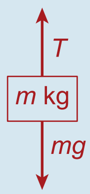
$= 1.5 \text{ s}$

(ii) 7 kg mass now 3.3075 m from the edge of the table and moves at a constant speed of 4.41 m/s.

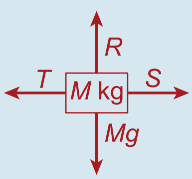
$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{3.3075}{4.41} = 0.75 \text{ s}$

Q. 12.

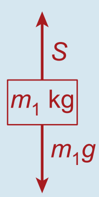




$T - mg = ma$ **Equation 1**



$S - T = Ma$ **Equation 2**



$m_1 g - S = m_1 a$ **Equation 3**

Add 3 equations:

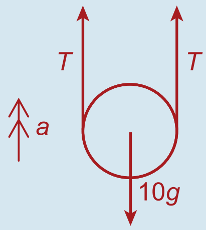
$m_1 a + Ma + ma = m_1 g - mg$

$\Rightarrow a(m_1 + M + m) = (m_1 - m)g$

$\Rightarrow a = \left(\frac{m_1 - m}{m_1 + m + M}\right)g$

Exercise 5D

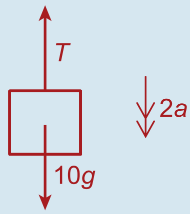
Q. 1. (i) **Pulley *A*:**



$2T - 10g = 10a$

$\Rightarrow T - 5g = 5a$ **Equation 1**

Particle *B*:



$10g - T = 10(2a)$

$\Rightarrow 10g - T = 20a$ **Equation 2**

Adding equations **1** and **2** gives

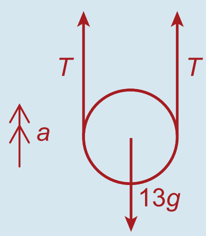
$5g = 25a$

$\Rightarrow a = \frac{g}{5} \text{ m/s}^2$

(ii) $T - 5g = 5a$

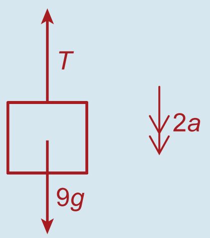
$\Rightarrow T = 5\left(\frac{g}{5}\right) + 5g = 6g \text{ N}$

Q. 2. (i) **Pulley *A*:**



$2T - 13g = 13a$ **Equation 1**

Particle B:



$9g - T = 9(2a)$

$\Rightarrow 18g - 2T = 36a$ **Equation 2**

Adding equations **1** and **2** gives

$5g = 49a$

$\Rightarrow a = \frac{5g}{49}$

$= \frac{49}{49}$

$= 1 \text{ m/s}^2$

(ii) Acceleration of $B = 2a$

$= 2(1)$

$= 2 \text{ m/s}^2$

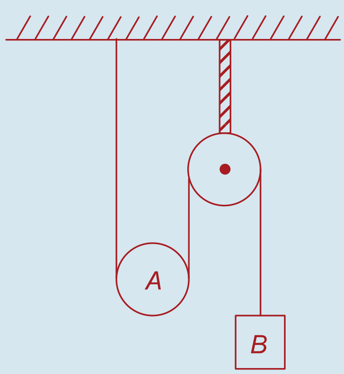
(iii) $2T - 13g = 13a$

$\Rightarrow 2T = 13(1) + 13g$

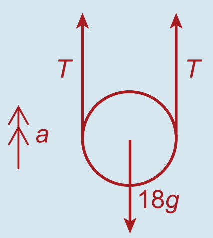
$\Rightarrow 2T = 140.4$

$\Rightarrow T = 70.2 \text{ N}$

Q. 3.



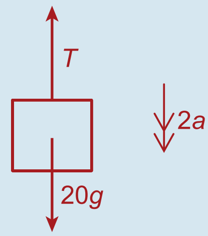
Pulley A:



$2T - 18g = 18a$

$\Rightarrow T - 9g = 9a$ **Equation 1**

Particle B:



$20g - T = 20(2a)$

$\Rightarrow 20g - T = 40a$ **Equation 2**

Adding equations **1** and **2** gives

$11g = 49a$

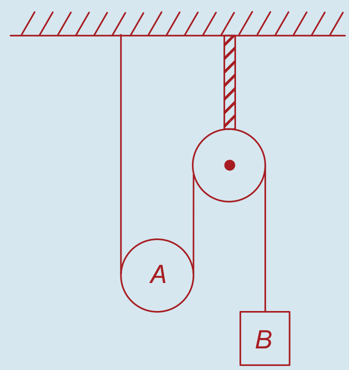
$\Rightarrow a = \frac{11g}{49} \quad \Rightarrow \quad a = 2.2 \text{ m/s}^2$

$T - 9g = 9a$

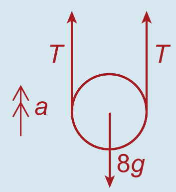
$\Rightarrow T = 9(2.2) + 9g$

$\Rightarrow T = 108 \text{ N}$

Q. 4.



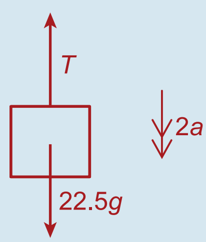
(i) **Pulley A:**



$2T - 8g = 8a$

$\Rightarrow T - 4g = 4a$ **Equation 1**

Particle B:



$22.5g - T = 22.5(2a)$

$\Rightarrow 22.5g - T = 45a$ **Equation 2**

Adding equations **1** and **2** gives

$18.5g = 49a$

$\Rightarrow a = \dfrac{18.5g}{49}$

$= 3.7 \text{ m/s}^2$

(ii) Acceleration of $B = 2a$

$= 2(3.7)$

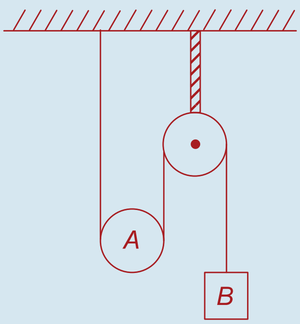
$= 7.4 \text{ m/s}^2$

(iii) $T - 4g = 4a$

$\Rightarrow T = 4(3.7) + 4g$

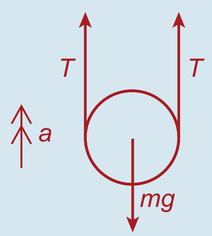
$\Rightarrow T = 54 \text{ N}$

Q. 5.



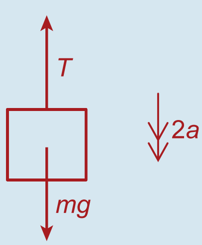
If A moves up 1 metre, B will move down 2 metres, since there will be 2 metres more string available. Hence, if A moves up x metres while B moves down y metres, it follows that $y = 2x$. Hence, the velocity and acceleration of B will be twice those of A. Therefore, if the acceleration of A is a, then the acceleration of B will be $2a$.

(i) **Pulley A:**



$2T - mg = ma$ **Equation 1**

Particle B:



$mg - 2T = m(2a)$

$\Rightarrow 2mg - 2T = 4ma$ **Equation 2**

Adding equations **1** and **2** gives

$mg = 5ma$

$\Rightarrow a = \frac{1}{5}g$

(ii) $2T - mg = ma$

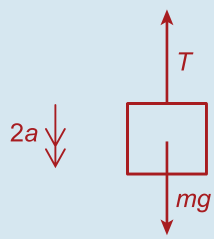
$\Rightarrow 2T - mg = m\left(\frac{g}{5}\right)$

$\Rightarrow 10T - 5mg = mg$

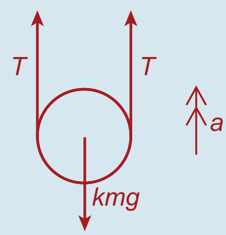
$\Rightarrow 10T = 6mg$

$\Rightarrow T = \frac{3}{5}mg$

Q. 6. (i) **Particle E:**



Pulley C:



(ii) $mg - T = m(2a)$

$\Rightarrow 2mg - 2T = 4ma$ **Equation 1**

$2T - kmg = kma$ **Equation 2**

Adding equations **1** and **2** gives

$2mg - kmg = 4ma + kma$

$\Rightarrow 2g - kg = 4a + ka$

$\Rightarrow (2 - k)g = a(4 + k)$

$\Rightarrow a = \dfrac{(2 - k)g}{4 + k}$

(iii) Let $k = 0.5$

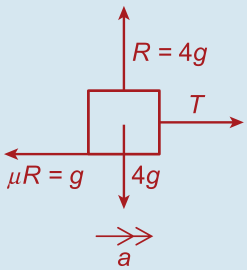
$\Rightarrow a = \dfrac{1.5g}{4.5} = \dfrac{g}{3}$

$\Rightarrow T = mg - 2ma$ from **Equation 1**

$\Rightarrow T = mg - 2m\left(\frac{g}{3}\right)$

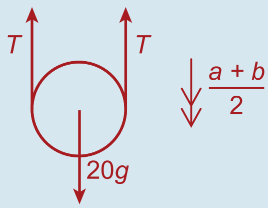
$\Rightarrow T = mg - \frac{2}{3}mg = \frac{1}{3}mg$

Q. 7. **4 kg Mass**



$T - g = 4a$ **Equation 1**

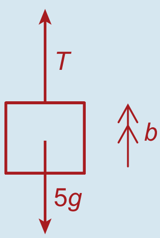
20 kg Pulley



$20g - 2T = 20\left(\frac{a+b}{2}\right)$

$\Rightarrow 10g - T = 5(a + b)$ **Equation 2**

5 kg Mass



$T - 5g = 5b$ **Equation 3**

$T - g = 4a$ **Equation 1**

$\Rightarrow a = \frac{T-g}{4}$

$T - 5g = 5b$ **Equation 3**

$\Rightarrow b = \frac{T-5g}{5}$

$10g - T = 5(a + b)$ **Equation 2**

$\Rightarrow 10g - T = 5a + 5b$

$\Rightarrow 10g - T = 5\left(\frac{T-g}{4}\right) + 5\left(\frac{T-5g}{5}\right)$

$\Rightarrow 10g - T = \frac{5}{4}T - \frac{5}{4}g + T - 5g$

… multiply by 4

$\Rightarrow 40g - 4T = 5T - 5g + 4T - 20g$

$\Rightarrow 13T = 65g$

$\Rightarrow T = 5g$

Acceleration of 20 kg

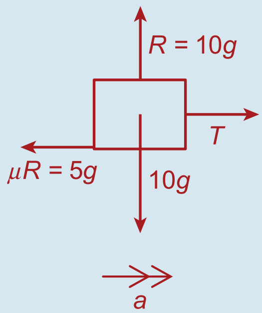
pulley $= \frac{1}{2}(a + b)$

$= \frac{1}{2}\left(\frac{T-g}{4} + \frac{T-5g}{5}\right)$ … $T = 5g$

$= \frac{1}{2}(g + 0)$

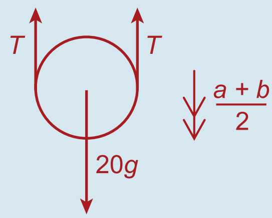
$= \frac{g}{2}$

Q. 8. **10 kg Mass**



$T - 5g = 10a$ **Equation 1**

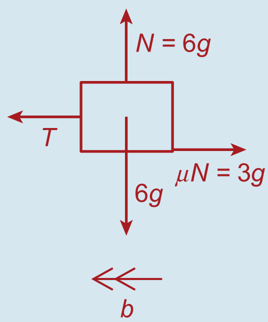
20 kg Pulley



$20g - 2T = 20\left(\frac{a+b}{2}\right)$

$\Rightarrow 10g - T = 5a + 5b$ **Equation 2**

6 kg Mass



$T - 3g = 6b$ **Equation 3**

$T - 5g = 10a$ **Equation 1**

$\Rightarrow a = \frac{T-5g}{10}$

$T - 3g = 6b$ **Equation 3**

$\Rightarrow b = \dfrac{T - 3g}{6}$

$10g - T = 5a + 5b$ **Equation 2**

$\Rightarrow 10g - T = 5\left(\dfrac{T - 5g}{10}\right) + 5\left(\dfrac{T - 3g}{6}\right)$

… multiply by 6

$\Rightarrow 60g - 6T = 3T - 15g + 5T - 15g$

$\Rightarrow 14T = 90g$

$\Rightarrow T = \dfrac{45g}{7} = 63$ N

$a = \dfrac{T - 5g}{10}$

$= \dfrac{63 - 49}{10}$

$= 1.4$ m/s^2 … acceleration of 10 kg particle.

$b = \dfrac{T - 3g}{6}$

$= \dfrac{63 - 29.4}{6}$

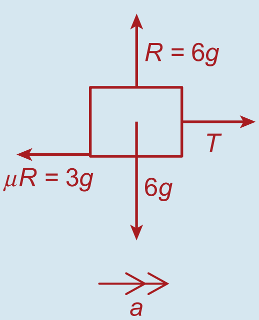
$= 5.6$ m/s^2 … acceleration of 6 kg particle

$\dfrac{a + b}{2} = \dfrac{1.4 + 5.6}{2}$

$= 3.5$ m/s^2 … acceleration of 20 kg pulley

Q. 9. Note firstly that, as M increases, the first particle to move will be the 6 kg mass. Initially, therefore, the only moving particles will be the 6 kg mass and the pulley. The system forces, just as the 6 kg mass starts to move, will look like this:

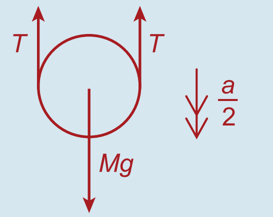
6 kg Mass



$T - 3g = 6a$

$\Rightarrow 2T - 6g = 12a$ **Equation 1**

Pulley



$Mg - 2T = M\left(\frac{a}{2}\right)$ **Equation 2**

Adding equations **1** and **2** gives

$Mg - 6g = \dfrac{Ma}{2} + 12a$

$\Rightarrow g(M - 6) = a\left(\dfrac{M}{2} + 12\right)$

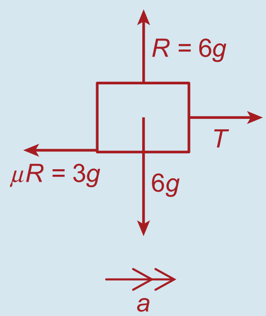
$\Rightarrow a = \dfrac{g(M - 6)}{\frac{M}{2} + 12} = \dfrac{2g(M - 6)}{M + 24}$

$a = 0$

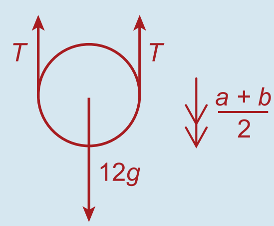
$\Rightarrow M - 6 = 0$

$\Rightarrow M = 6$ … this is the value of M at which the friction between the 6 kg mass and the table is just overcome. If the value of M is below this, there will be no movement.

Now, let $M = 12$

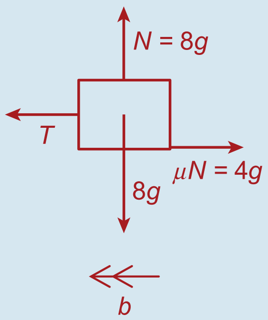


$T - 3g = 6a$ **Equation 1**



$12g - 2T = 12\left(\dfrac{a + b}{2}\right)$

$\Rightarrow 6g - T = 3a + 3b$ **Equation 2**



$T - 4g = 8b$ **Equation 3**

$T - 3g = 6a$ **Equation 1**

$$\Rightarrow a = \frac{T - 3g}{6}$$

$T - 4g = 8b$ **Equation 3**

$$\Rightarrow b = \frac{T - 4g}{8}$$

$6g - T = 3a + 3b$ **Equation 2**

$$\Rightarrow 6g - T = 3\left(\frac{T - 3g}{6}\right) + 3\left(\frac{T - 4g}{8}\right)$$

$$\Rightarrow 6g - T = \left(\frac{T - 3g}{2}\right) + 3\left(\frac{T - 4g}{8}\right)$$

$$\Rightarrow 48g - 8T = 4T - 12g + 3T - 12g$$

$$\Rightarrow 15T = 72g$$

$$\Rightarrow T = \frac{24g}{5}\text{ N}$$

$$a = \frac{T - 3g}{6}$$

$$= \frac{\frac{24g}{5} - 3g}{6}$$

$$= \frac{24g - 15g}{30}$$

$$= \frac{9g}{30}$$

$$= \frac{3g}{10}\text{ m/s}^2$$ … acceleration of 6 kg mass

$$b = \frac{T - 4g}{8}$$

$$= \frac{24g}{5} - \frac{4g}{8}$$

$$= \frac{24g - 20g}{40}$$

$$= \frac{4g}{40}$$

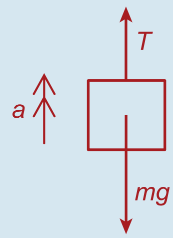
$$= \frac{g}{10}\text{ m/s}^2$$ … acceleration of 8 kg mass

$$\frac{a + b}{2} = \frac{\frac{3g}{10} + \frac{g}{10}}{2}$$

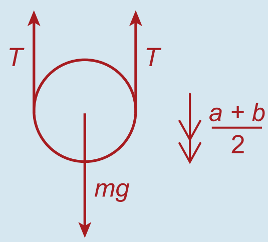
$$= \frac{3g + g}{20} = \frac{4g}{20}$$

$$= \frac{g}{5}\text{ m/s}^2$$ … acceleration of pulley

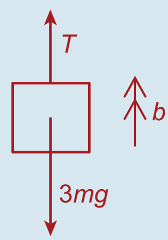
Q. 10. (i) **Mass *m***



Pulley *B*



Mass 3*m*



(ii) Mass m: $T - mg = ma$ **Equation 1**

Mass $3m$: $T - 3mg = 3mb$ **Equation 2**

Pulley *B*:

$Mg - 2T = M\left(\frac{a + b}{2}\right)$ **Equation 3**

$a = \frac{T - mg}{m}$ from **Equation 1**

$b = \frac{T - 3mg}{3m}$ from **Equation 2**

$Mg - 2T = M\left(\frac{a + b}{2}\right)$ **Equation 3**

$$\Rightarrow 2Mg - 4T = Ma + Mb$$

$$\Rightarrow 2Mg - 4T = M\left(\frac{T - mg}{m}\right) + M\left(\frac{T - 3mg}{3m}\right)$$

… multiply by $3m$

$\Rightarrow 6Mmg - 12mT = 3MT - 3Mmg + MT - 3Mmg$

$\Rightarrow 12Mmg = 12mT + 4MT$ … divide by 4

$\Rightarrow 3Mmg = 3mT + MT$

$\Rightarrow 3Mmg = T(3m + M)$

$\Rightarrow g = T\left(\frac{3m + M}{3Mm}\right)$

$\Rightarrow g = T\left(\frac{1}{M} + \frac{1}{3m}\right)$ … as required

(iii) Let $M = 3m$

$\Rightarrow g = T\left(\frac{1}{3m} + \frac{1}{3m}\right)$

$\Rightarrow g = T\left(\frac{2}{3m}\right)$

$\Rightarrow T = \frac{3mg}{2}$

$a = \frac{T - mg}{m}$

$= \frac{\frac{3mg}{2} - mg}{m}$

$= \frac{3mg - 2mg}{2m}$

$= \frac{mg}{2m}$

$= \frac{g}{2}$ m/s^2 … m mass will move.

$b = \frac{T - 3mg}{3m}$

$= \frac{\frac{3mg}{2} - 3mg}{3m}$

$= \frac{3mg - 6mg}{6m}$

$= \frac{-3mg}{6m}$

$= -\frac{g}{2}$ m/s^2 … $3m$ mass will move.

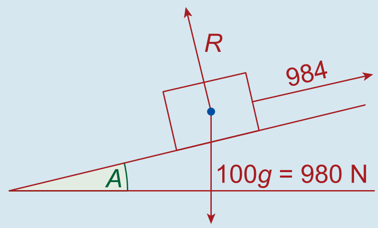
$\frac{a + b}{2} = \frac{\frac{g}{2} - \frac{g}{2}}{2}$

$= 0$ m/s^2 … pulley B will not move.

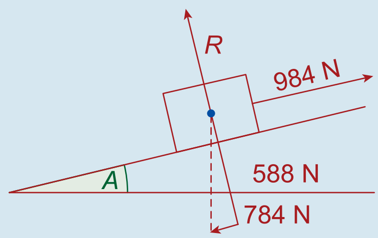
Exercise 5E

Q. 1. (a) $\tan A = \frac{4}{3} \Rightarrow \sin A = \frac{4}{5}, \quad \cos A = \frac{3}{5}$

Forces



Resolved

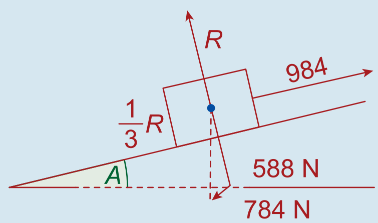


$F = ma$

$\Rightarrow (984 - 784) = 100a$

$\Rightarrow a = 2$ m/s^2

(b) **Forces (Resolved)**



$R = 588 \Rightarrow$ Friction $= \frac{1}{3}R = 196$ N

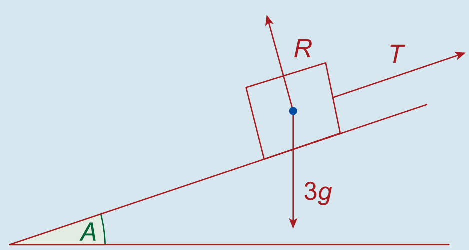
$F = ma$

$\Rightarrow (984 - 784 - 196) = 100a$

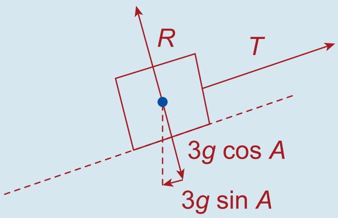
$\Rightarrow a = 0.04$ m/s^2

Q. 2. **3 kg**

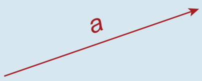
Forces



Resolved



Acceleration



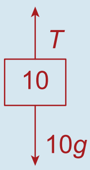
$F = ma$

$\Rightarrow T - 3g \sin A = 3a$

$\Rightarrow T - g = 3a$ **Equation 1**

10 kg

Forces **Accelerations**



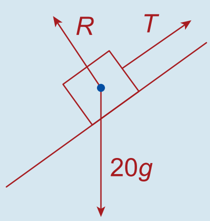
$10g - T = 10a$ **Equation 2**

Solving these gives $a = \frac{9}{13}g$, $T = \frac{40g}{13}$

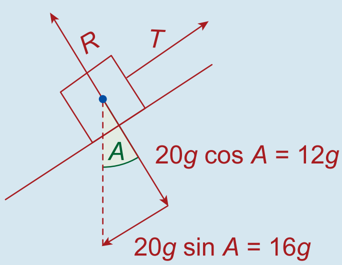
Q. 3. (i) $\tan A = \frac{4}{3} \Rightarrow \sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$.

20 kg's

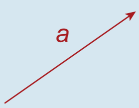
Forces



Resolved



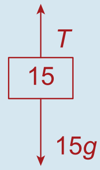
Acceleration



$T - 16g = 20a$ **Equation 1**

15 kg's

Forces **Acceleration**



$15g - T = 15a$ **Equation 2**

Solving these gives

$a = -\frac{1}{35}g$

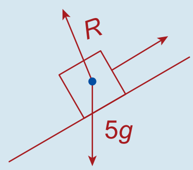
$\Rightarrow$ acceleration $= \frac{9.8}{35}$

$= 0.28 \text{ m/s}^2$

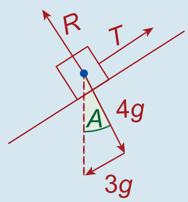
(ii) The 15 kg rises (since the downward "a" was negative – i.e. it should be upward).

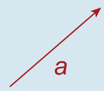
Q. 4. (i) **5 kg's**

Forces



Resolved **Acceleration**

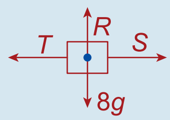




$T - 3g = 5a$ **Equation 1**

8 kg's

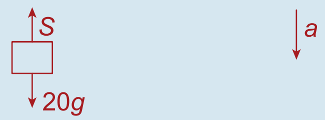
Forces **Acceleration**



$S - T = 8a$ **Equation 2**

20 kg's

Forces **Acceleration**



$20g - S = 20a$ **Equation 3**

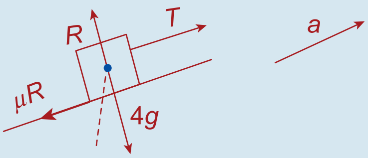
Adding these gives

$17g = 33a$

$\Rightarrow a = \frac{17}{33}g$

(ii) **5 kg's**

Resolved Forces **Acceleration**



$R = 4g$

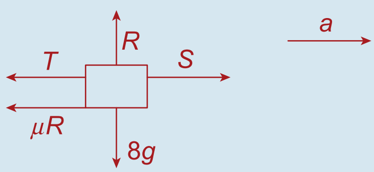
$\Rightarrow \mu R = \frac{1}{4}(4g) = g$

$\therefore T - g - 3g = 5a$

$\Rightarrow T - 4g = 5a$ **Equation 1**

8 kg's

Forces **Acceleration**



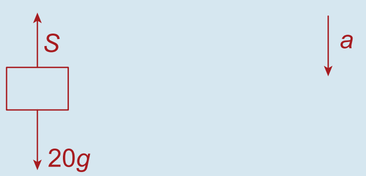
$R = 8g$

$\Rightarrow \mu R = \frac{1}{4}(8g) = 2g$

$S - 2g - T = 8a$ **Equation 2**

20 kg's

Forces **Acceleration**



$20g - S = 20a$ **Equation 3**

Adding gives

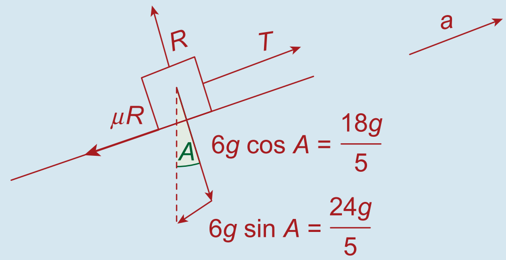
$14g = 33a$

$\Rightarrow a = \frac{14g}{33}$

Q. 5. Since $\tan A = \frac{4}{3}$, $\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$

18 kg's

Forces **Acceleration**



$R = \frac{18g}{5}$

$\Rightarrow \mu R = \frac{1}{6}\left(\frac{18g}{5}\right)$

$= \frac{3g}{5}$

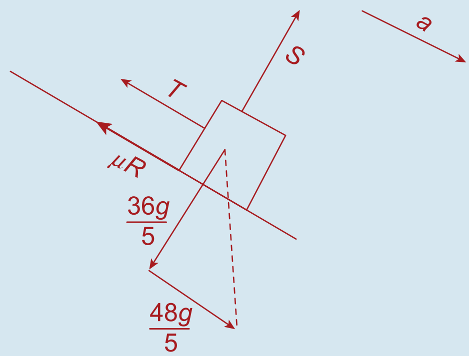
$F = ma$

$\Rightarrow T - \frac{3g}{5} - \frac{24g}{5} = 6a$

$\Rightarrow T - \frac{27g}{5} = 6a$ **Equation 1**

2 kg's

Forces **Acceleration**



$= \frac{36g}{5} \Rightarrow \mu S = \frac{1}{6}\left(\frac{36g}{5}\right) = \frac{6g}{5}$

$= Ma \Rightarrow \frac{48g}{5} - \frac{6g}{5} - T = 12a$

$\Rightarrow \frac{42g}{5} - T = 12a$ **Equation 2**

Adding these gives:

$3g = 18a$

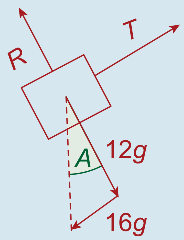
$\Rightarrow a = \frac{1}{6}g$

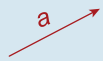
$\Rightarrow T = \frac{32g}{5}$

Q. 6. Since $\tan A = \frac{4}{3}$, $\sin A = \left(\frac{4}{5}\right)$, $\cos A = \frac{3}{5}$

(i) **20 kg's**

Forces **Acceleration**



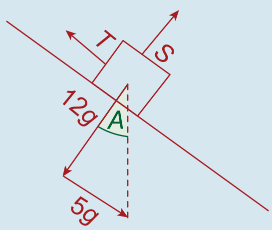


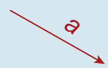
$T - 16g = 20a$ **Equation 1**

Since $\tan B = \frac{5}{12}$, $\sin B = \frac{5}{13}$, $\cos B = \frac{12}{13}$

13 kg's

Forces **Acceleration**





$5g - T = 13a$ **Equation 2**

Adding these gives

$-11g = 33a$

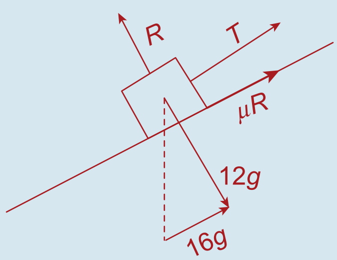
$\Rightarrow a = -\frac{1}{3}g$ (i.e. they go the other way)

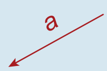
The acceleration of the masses is

$\frac{1}{3}g$ m/s^2 and $T = \frac{28}{3}g$ N

(ii) **20 kg's**

Forces **Acceleration**





$R = 12g$

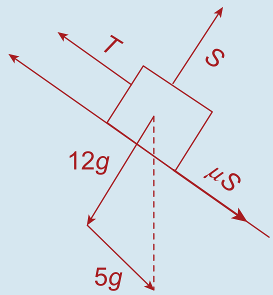
$\Rightarrow \mu R = \frac{1}{4}(12g) = 3g$

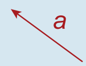
$16g - 3g - T = 20a$

$13g - T = 20a$ **Equation 1**

(ii) **12 kg's**

Forces **Acceleration**





Similarly,

$S = 12g \Rightarrow \mu S = \frac{1}{4}(12g) = 3g$

$T - 3g - 5g = 13a$

$\Rightarrow T - 8g = 13a$ **Equation 2**

Adding these gives:

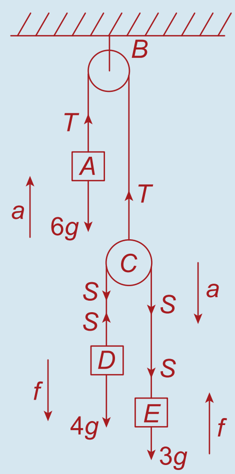
$5g = 33a$

$\Rightarrow a = \frac{5}{33}g$ m/s^2

$\Rightarrow T = \frac{329}{330}g$ N

Exercise 5F

Q. 1.



A: $T - 6g = 6a$

C: $2S - T = 0$

$\Rightarrow T = 2s$

D: $4g - S = 4(a + f)$

E: $S - 3g = 3(f - a)$

A becomes $2S - 6g = 6a$

$S = 3a + 3g$

$\therefore D$ becomes $4g - 3a - 3g = 4a + 4f$

$\Rightarrow 7a + 4f = g$

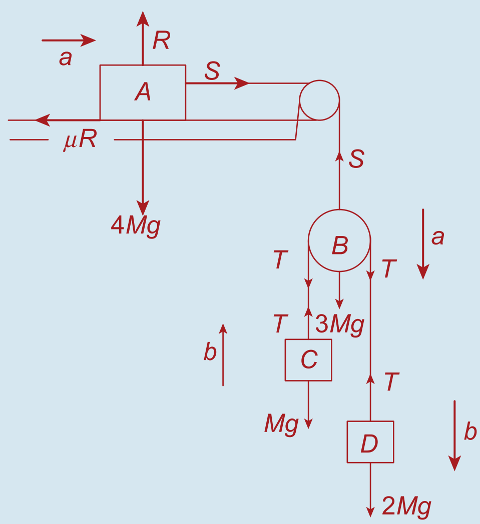
E becomes $3a + 3g - 3g = 3f - 3a$

$$\Rightarrow f - 2a = 0$$

Solving these gives $a = \frac{1}{15}g$ m/s^2,

$f = \frac{2}{15}g$ m/s^2

Q. 2.



Since $R = 4Mg$, $\mu R = \frac{1}{2}(4Mg) = 2Mg$

A: $S - 2Mg = 4Ma$

B: $3Mg + 2T - S = 3Ma$

C: $T - Mg = M(b - a)$

D: $2Mg - T = 2M(b + a)$

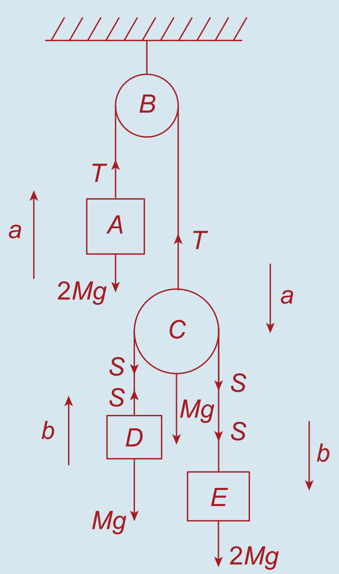
Adding C and $D \Rightarrow 3b + a = g$

$A + B - C + D \Rightarrow b + 10a = 4g$

Solving these gives $a = \frac{11}{29}g$ m/s^2,

$b = \frac{6}{29}g$ m/s^2

Q. 3.



A: $T - 2Mg = 2Ma$

C: $Mg + 2S - T = Ma$

D: $S - Mg = M(b - a)$

E: $2Mg - S = 2m(b + a)$

$D + E \Rightarrow Mg = 3Mb + Ma$

$\Rightarrow 3b + a = g$

$A + C - D + E \Rightarrow 2Mg = Mb + 6Ma$

$$\Rightarrow b + 6a = 2g$$

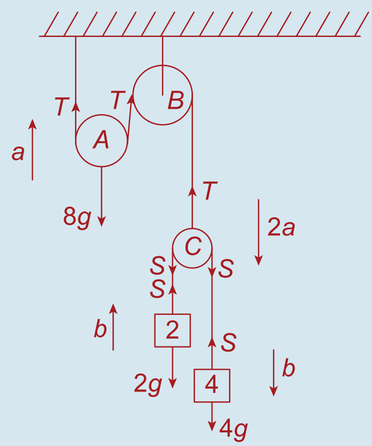
Solving these gives $a = \frac{5}{17}g$, $b = \frac{4}{17}g$

Acceleration of $D = b - a = \frac{-g}{17}$

i.e. $\frac{8}{17}$ downward.

Acceleration of $E = b + a = \frac{9}{17}g$ downward.

Q. 4. (i)



A: $2T - 8g = 8a$

C: $2s - T = 0(2a)$

$$\Rightarrow T = 2s$$

D: (2 kg): $S - 2g = 2(b - 2a)$

E: (4 kg): $4g - S = 4(b + 2a)$

C: $T = 2S$

$\Rightarrow A$ becomes $4S - 8g = 8a$

$$\Rightarrow S = 2a + 2g$$

$\therefore D$ becomes $2a + 2g - 2g = 2(b - 2a)$

$$\Rightarrow 6a - 2b = 0$$

E becomes $4g - 2a - 2g = 4(b + 2a)$

$$\Rightarrow 5a + 2b = g$$

Solving these gives $a = \frac{g}{11}$, $b = \frac{3g}{11}$

(ii) Pulley A: Acceleration: $a = \frac{g}{11}$ m/s^2

Pulley C: Acceleration: $2a = \frac{2g}{11}$ m/s^2

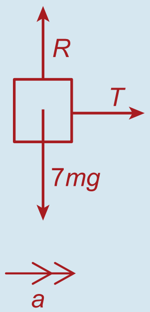
2 kg particle:

Acceleration: $b - 2a = \frac{g}{11}$ m/s^2

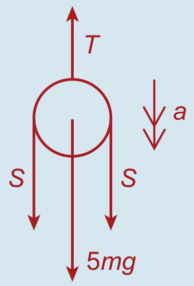
4 kg particle:

Acceleration: $b + 2a = \frac{8g}{11}$ m/s^2

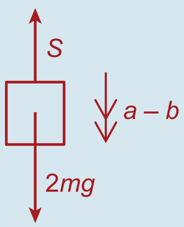
Q. 5. ***7m* mass**



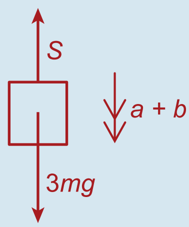
***5m* pulley**



***2m* mass**



***3m* mass**



Equations of Motion

A: $T = 7ma$

B: $2S + 5mg - T = 5ma$

$\Rightarrow 2S + 5mg - 7ma = 5ma$

$\Rightarrow S = \frac{m(12a - 5g)}{2}$

C: $2mg - S = 2m(a - b)$

D: $3mg - S = 3m(a + b)$

C becomes: $2mg - \frac{m(12a - 5g)}{2} = 2m(a - b)$ ... multiply by $\frac{2}{m}$

$\Rightarrow 4g - 12a + 5g = 4a - 4b$

$\Rightarrow 16a - 4b = 9g$ ... **Equation E**

D becomes: $3mg - \frac{m(12a - 5g)}{2} = 3m(a + b)$ ... multiply by $\frac{2}{m}$

$\Rightarrow 6g - 12a + 5g = 6a + 6b$

$\Rightarrow 18a + 6b = 11g$ **Equation F**

Equation E ($\times 3$): $48a - 12b = 27g$

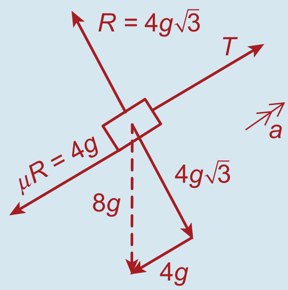
Equation F ($\times 2$): $36a + 12b = 22g$ ... add

$84a = 49g$

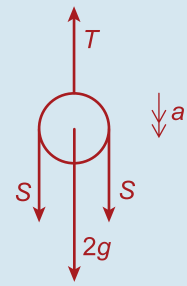
$a = \frac{49g}{84}$

$\Rightarrow a = \frac{7}{12}g$ m/s^2 ... acceleration of $7m$ mass

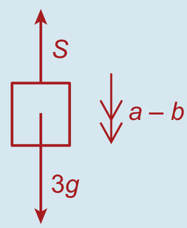
Q. 6. **8 kg mass**



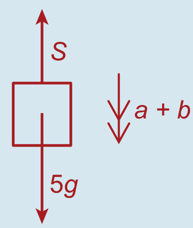
2 kg pulley



3 kg mass



5 kg mass



8 kg mass **A:** $T - 8g = 8a$

$T = 8g + 8a$

2 kg pulley **B:** $2g + 2S - T = 2a$

$\Rightarrow 2g + 2S - 8g - 8a = 2a$

$\Rightarrow S = 5a + 3g$

3 kg mass **C:** $3g - S = 3(a - b)$

5 kg mass **D:** $5g - S = 5(a + b)$

Equation C becomes: $3g - 5a - 3g = 3a - 3b$

$\Rightarrow 8a - 3b = 0$ **Equation E**

Equation D becomes:

$5g - 5a - 3g = 5a + 5b$

$10a + 5b = 2g$ **Equation F**

Equation E $(\times 5)$: $40a - 15b = 0$

Equation F $(\times 3)$: $\underline{30a + 15b = 6g}$ ...add

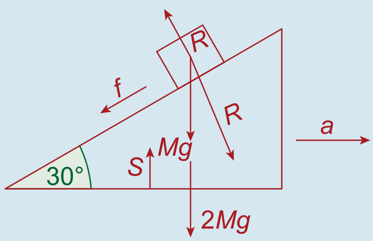
$70a = 6g$

$a = \dfrac{6g}{70}$

$\Rightarrow a = \dfrac{3}{35} g \text{ m/s}^2$ ... acceleration of 8 kg mass

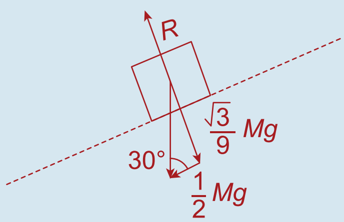
Exercise 5G

Q. 1.

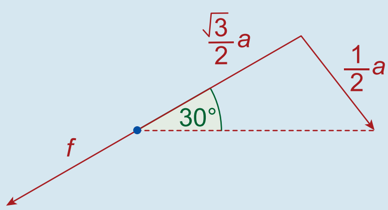


(i) **The Particle:**

Forces



Accelerations



Along the slope : $F = ma$

$\Rightarrow \frac{1}{2} Mg = M(f - \frac{\sqrt{3}}{2a})$

$\Rightarrow g = 2f - \sqrt{3}a$ .... ①

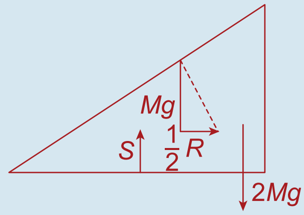
Perpendicular to the slope: $F = ma$

$\Rightarrow \frac{\sqrt{3}}{2} Mg - R = M\left(\frac{1}{2}a\right)$

$\Rightarrow \sqrt{3}\, Mg - 2R = Ma$ .... ②

(ii) **The Wedge:**

Forces



Acceleration

$F = ma$

$\Rightarrow \frac{1}{2} R = 2Ma \quad R = 4Ma$ .... ③

Putting this result into equation ② gives:

$\sqrt{3} Mg - 8Ma = Ma$

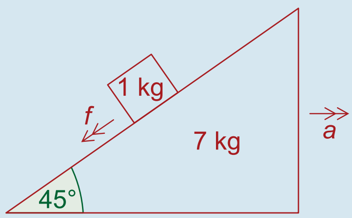
$\Rightarrow a = \frac{\sqrt{3}}{9} g \text{ m/s}^2$

Putting this result into equation ① gives:

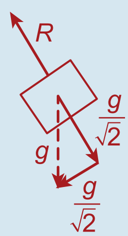
$g = 2f - \sqrt{3}\left(\frac{\sqrt{3}}{9} g\right)$

$\Rightarrow f = \frac{2}{3} g \text{ m/s}^2$

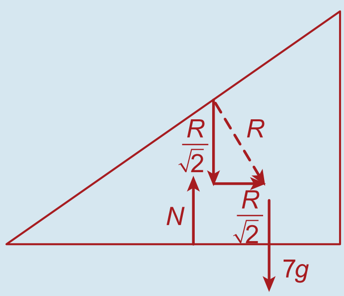
Q. 2.



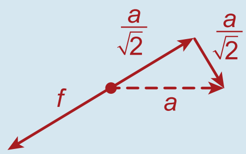
Forces on 1 kg mass



Forces on wedge



Acceleration of 1 kg mass



Acceleration of wedge

1 kg mass along slope: $F = ma$

$$\Rightarrow \frac{g}{\sqrt{2}} = 1\left(f - \frac{a}{\sqrt{2}}\right)$$

$$\Rightarrow g = f\sqrt{2} - a \quad \ldots ①$$

1 kg mass perpendicular to slope:

$$F = ma$$

$$\Rightarrow \frac{g}{\sqrt{2}} - R = \frac{a}{\sqrt{2}}$$

$$\Rightarrow g - R\sqrt{2} = a \quad \ldots ②$$

Wedge horizontal: $F = ma$

$$\Rightarrow \frac{R}{\sqrt{2}} = 7a$$

$$\Rightarrow R = 7a\sqrt{2} \quad \ldots ③$$

Putting this result into equation ② gives:

$$g - 14a = a$$

$$\Rightarrow 15a = g$$

$$\Rightarrow a = \frac{1}{15} g \text{ m/s}^2$$ … acceleration of wedge

Putting this result into equation ① gives

$$g = f\sqrt{2} - \frac{1}{15}g$$

$$\Rightarrow 15g = 15f\sqrt{2} - g$$

$$\Rightarrow 15f\sqrt{2} = 16g$$

$$\Rightarrow f = \frac{16}{15\sqrt{2}}g$$

$$= \frac{8\sqrt{2}}{15}g \text{ m/s}^2$$ … acceleration of particle relative to the wedge

$$u = 0, \quad v = \sqrt{2}, \quad f = \frac{8\sqrt{2}}{15}g$$

$$t = \frac{v - u}{f}$$

$$= \sqrt{2}\left(\frac{15}{8g\sqrt{2}}\right)$$

$$= \frac{15}{8g} \text{ seconds}$$

Now, find speed of wedge when $t = \frac{15}{8g}$

$$u = 0, \quad a = \frac{1}{15}g, \quad t = \frac{15}{8g}$$

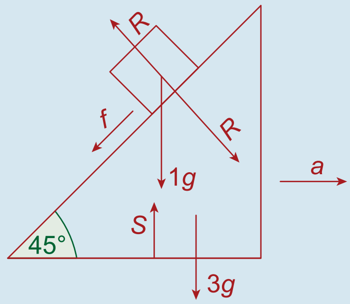
$$v = u + at$$

$$= 0 + \left(\frac{g}{15}\right)\left(\frac{15}{8g}\right)$$

$$= \frac{1}{8}$$

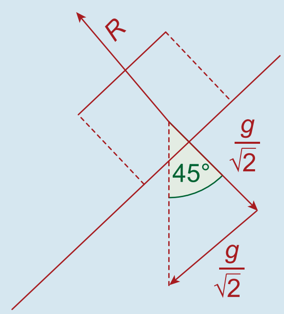
$$= 0.125 \text{ m/s}$$

Q. 3.

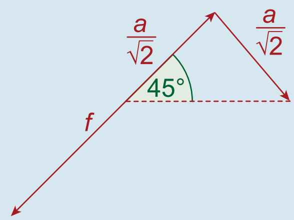


The Particle:

Forces



Acceleration



Parallel to the slope:

$$F = ma$$

$$\Rightarrow \frac{g}{\sqrt{2}} = 1\left(f - \frac{a}{\sqrt{2}}\right)$$

$$\Rightarrow g = \sqrt{2}f - a \quad \ldots. ①$$

Perpendicular to the slope:

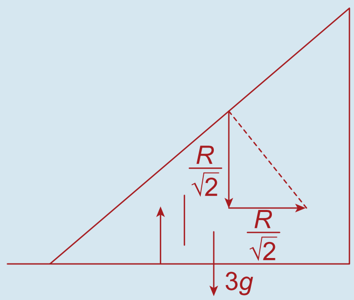
$$F = ma$$

$$\Rightarrow \frac{g}{\sqrt{2}} - R = 1\left(\frac{a}{\sqrt{2}}\right)$$

$$\Rightarrow g - \sqrt{2}R = a \quad \ldots ②$$

The Wedge:

Forces



Acceleration

$F = ma$

$\Rightarrow \dfrac{R}{\sqrt{2}} = 3a$

$\Rightarrow R = 3\sqrt{2}a \quad .... ③$

Putting this result into equation ② gives:

$g - \sqrt{2}(3\sqrt{2}a) = a$

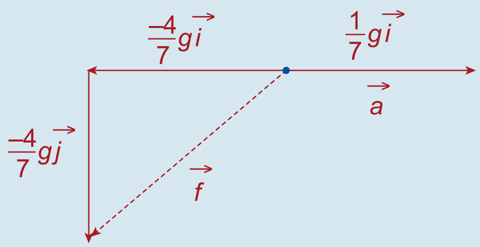
$\Rightarrow a = \frac{1}{7}g \text{ m/s}^2$

Putting this result into equation ① gives:

$g = \sqrt{2}f - \dfrac{1}{7g}$

$\Rightarrow f = \dfrac{4\sqrt{2}g}{7} \text{ m/s}^2$

Here are $\vec{f}$ and $\vec{a}$ resolved:



$\therefore \vec{f} + \vec{a} = \left(-\frac{4}{7}g\vec{i} - \frac{4}{7}g\vec{j}\right) + \frac{1}{7}g\vec{i}$

$= -\frac{3}{7}g\vec{i} - \frac{4}{7}g\vec{j}$

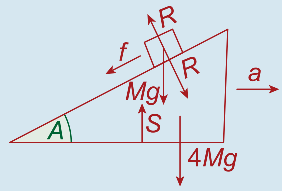
$\therefore |\vec{f} + \vec{a}| = \sqrt{\frac{9}{49}g^2 + \frac{16}{49}g^2}$

$= \sqrt{\frac{25}{49}g^2}$

$= \frac{5}{7}g$

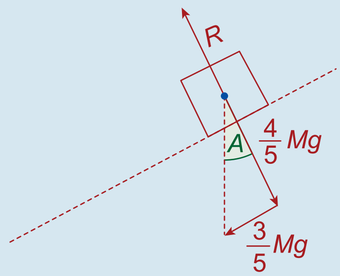
This is the magnitude of the actual acceleration of the particle.

Q. 4. Since $\tan A = \frac{3}{4}$, $\sin A = \frac{3}{5}$, $\cos A = \frac{4}{5}$

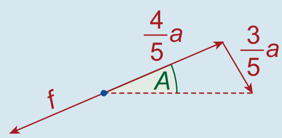


The Particle:

Forces



Accelerations



Parallel to the slope:

$F = ma$

$\Rightarrow \frac{3}{5}Mg = M\left(f - \frac{4}{5}a\right)$

$\Rightarrow 3g = 5f - 4a \quad ... ①$

Perpendicular to the slope:

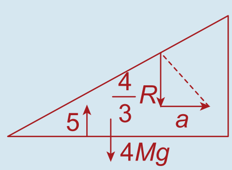
$F = ma$

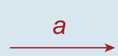
$\Rightarrow \frac{4}{5}Mg - R = M\left(\frac{3}{5}a\right)$

$\Rightarrow 4Mg - 5R = 3Ma \quad ... ②$

The Wedge:

Forces **Acceleration**





$F = ma$

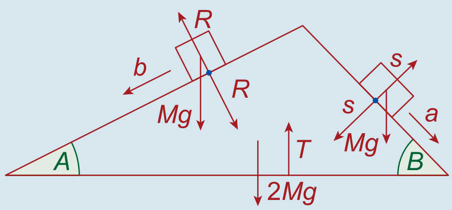
$\Rightarrow \frac{3}{5}R = 4Ma$

$\Rightarrow R = \frac{20}{3}Ma \quad ... ③$

Putting this result into equation ② gives:

$4Mg - \dfrac{100}{3}Ma = 3Ma \Rightarrow a = \dfrac{12g}{109} \text{ m/s}^2$

Q. 5.

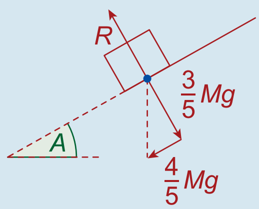


Since $\tan a = \frac{4}{3}$, $\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$

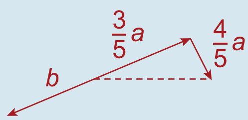
Since $\tan B = \frac{3}{4}$, $\sin B = \frac{3}{5}$, $\cos B = \frac{4}{5}$

First Particle:

Forces



Accelerations



Along plane:

$F = ma$

$\Rightarrow \frac{4}{5}Mg = M\left(b - \frac{3}{5}a\right)$

$\Rightarrow 4g = 5b - 3a \quad \ldots ①$

Perpendicular to the plane:

$F = ma$

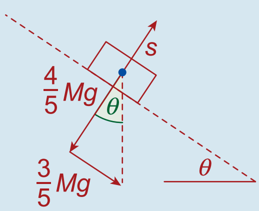
$\Rightarrow \frac{3}{5}Mg - R = M\left(\frac{4}{5}a\right)$

$\Rightarrow 3\ Mg - 5R = 4Ma$

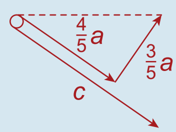
$\Rightarrow R = \dfrac{3Mg - 4Ma}{5} \quad \ldots ②$

Other Particle:

Forces



Accelerations



Along the plane: $\frac{3}{5}Mg = M\left(\frac{4}{5}a + c\right)$

$\Rightarrow 3g = 4a + 5c \quad \ldots.. ③$

Perpendicular to the plane:

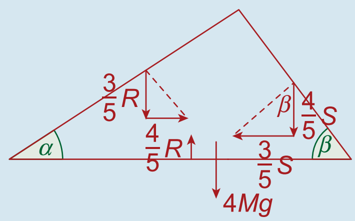
$S - \frac{4}{5}Mg = M\left(\frac{3}{5}a\right)$

$\Rightarrow 5S - 4Mg = 3Ma$

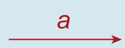
$\Rightarrow S = \dfrac{3Ma + 4Mg}{5} \quad \ldots. ④$

The Wedge:

Forces



Acceleration



$\frac{4}{5}R - \frac{3}{5}S = 2Ma$

$\Rightarrow \frac{4}{5}\left(\frac{3Mg - 4Ma}{5}\right) - \frac{3}{5}\left(\frac{3Ma + 4Mg}{5}\right) = 2Ma$

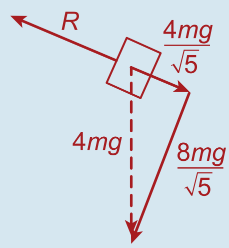
(from ② and ④)

$\Rightarrow \frac{1}{25}(12Mg - 16Ma - 9Ma - 12Mg) = 2Ma$

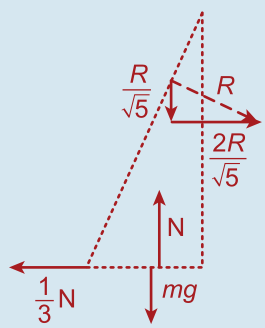
$\Rightarrow a = 0$

$\therefore$ it remains at rest. **QED**

Q. 6. (i) **4*m* mass**

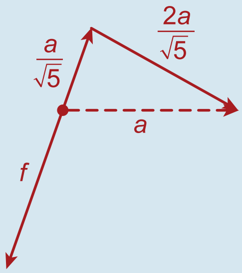


Wedge



Accelerations

4*m* mass



Wedge

(ii) $4m$ particle along the plane:

$F = ma$

$\Rightarrow \frac{8mg}{\sqrt{5}} = 4m\left(f - \frac{a}{\sqrt{5}}\right)$

... multiply by $\frac{\sqrt{5}}{4m}$

$\Rightarrow 2g = f\sqrt{5} - a \quad \ldots ①$

$4m$ particle perpendicular to the plane:
$F = ma$

$\Rightarrow \frac{4mg}{\sqrt{5}} - R = 4m\left(\frac{2a}{\sqrt{5}}\right)$

... multiply by $\frac{\sqrt{5}}{4m}$

$\Rightarrow g - \frac{\sqrt{5}}{4m}R = 2a \quad \ldots ②$

Wedge horizontal: $F = ma$

$\Rightarrow \frac{2R}{\sqrt{5}} - \frac{1}{3}N = ma \quad \ldots N = mg + \frac{R}{\sqrt{5}}$

$\Rightarrow \frac{2R}{\sqrt{5}} - \frac{1}{3}\left(mg + \frac{R}{\sqrt{5}}\right) = ma$

... multiply by $3\sqrt{5}$

$\Rightarrow 6R - mg\sqrt{5} - R = 3ma\sqrt{5}$

$\Rightarrow 5R = m\sqrt{5}(3a + g)$

$\Rightarrow R = \frac{m}{\sqrt{5}}(3a + g) \ldots ③$

Putting this result into equation ② gives:

$g - \frac{\sqrt{5}}{4m}\left(\frac{m}{\sqrt{5}}(3a + g)\right) = 2a$

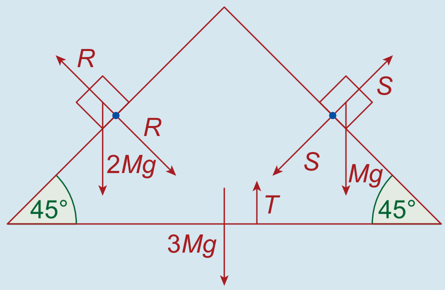
... multiply by 4

$\Rightarrow 4g - (3a + g) = 8a$

$\Rightarrow 3g = 11a$

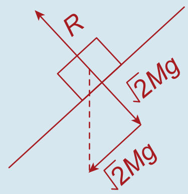
$\Rightarrow a = \frac{3g}{11}$ ... acceleration of the wedge

Q. 7.

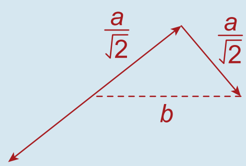


The 2*M* mass:

Forces



Acceleration



Along the plane: $\sqrt{2}Mg = 2M\left(b - \frac{a}{\sqrt{2}}\right)$

$\Rightarrow g = \sqrt{2}b - a \quad \ldots ①$

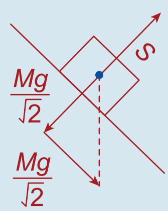
Perpendicular to the plane:

$\sqrt{2}Mg - R = 2M\left(\frac{a}{\sqrt{2}}\right)$

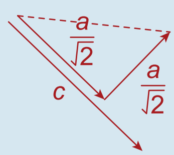
$\Rightarrow R = \sqrt{2}Mg - \sqrt{2}\,Ma \quad \ldots ②$

The *M* mass:

Forces



Accelerations



Along the plane: $\frac{Mg}{\sqrt{2}} = M\left(c + \frac{a}{\sqrt{2}}\right)$

$\Rightarrow g = \sqrt{2}\,c + a \quad \ldots ③$

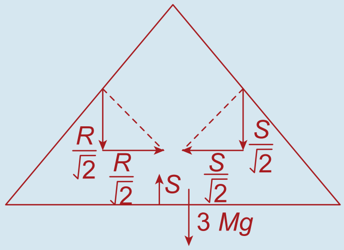
Perpendicular to the plane:

$S - \frac{Mg}{\sqrt{2}} = M\left(\frac{a}{\sqrt{2}}\right)$

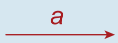
$\Rightarrow S = \frac{Ma}{\sqrt{2}} + \frac{Mg}{\sqrt{2}} \quad \ldots ④$

The Wedge:

Forces



Acceleration



Along the horizontal: $\frac{R}{\sqrt{2}} - \frac{S}{\sqrt{2}} = 3Ma$

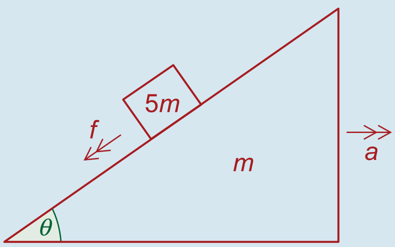
$\Rightarrow R - S = 3\sqrt{2}\,Ma$

$\Rightarrow \sqrt{2}Mg - \sqrt{2}Ma - \frac{Ma}{\sqrt{2}} - \frac{Mg}{\sqrt{2}} = 3\sqrt{2}Ma$ (from ② and ④)

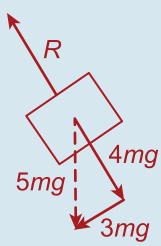
$\Rightarrow 2Mg - 2Ma - Ma - Mg = 6Ma$

$\Rightarrow a = \frac{1}{9}g \text{ m/s}^2$

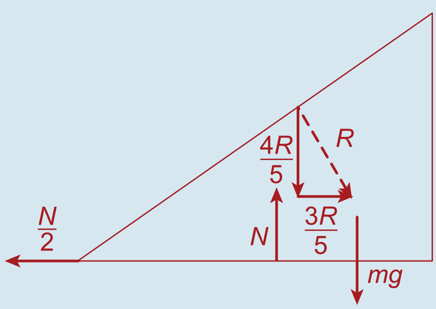
Q. 8. (i) $\tan\theta = \frac{3}{4}$, $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$



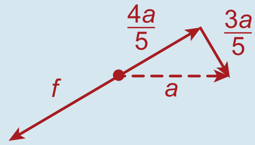
Forces on 5*m* mass



Forces on wedge



Acceleration of 1 kg mass



Acceleration of wedge

5 kg mass along slope:

$F = ma$

$\Rightarrow 3mg = 5m\left(f - \frac{4a}{5}\right)$

$\Rightarrow 3g = 5\left(f - \frac{4a}{5}\right)$ … ①

5 kg mass perpendicular to slope:

$F = ma$

$\Rightarrow 4mg - R = 5m\left(\frac{3a}{5}\right)$

$\Rightarrow 4mg - R = 3ma$ … ②

Wedge horizontal:

$F = ma$

$\Rightarrow \frac{3R}{5} - \frac{N}{2} = ma$ … $N = mg + \frac{4R}{5}$

$\Rightarrow \frac{3R}{5} - \frac{1}{2}\left(mg + \frac{4R}{5}\right) = ma$

$\Rightarrow 6R - 5mg - 4R = 10ma$

$\Rightarrow 2R = 5m(2a + g)$

$\Rightarrow R = \frac{5m}{2}(2a + g)$ … ③

Putting this result into equation ② gives:

$4mg - \frac{5m}{2}(2a + g) = 3ma$

… multiply by $\frac{2}{m}$

$\Rightarrow 8g - 10a - 5g = 6a$

$\Rightarrow 16a = 3g$

$\Rightarrow a = \frac{3}{16}g$ … acceleration of wedge

Putting this result into equation ① gives

$3g = 5\left(f - \frac{4}{5}\left(\frac{3g}{16}\right)\right)$

$\Rightarrow 3g = 5f - \frac{3g}{4}$

$\Rightarrow 12g = 20f - 3g$

$\Rightarrow 20f = 15g$

$\Rightarrow f = \frac{3}{4}g$ … acceleration of the particle relative to the wedge

(ii) Motion of wedge:

$u = 0, \quad s = 1, \quad a = \frac{3g}{16},$

$\Rightarrow s = ut + \frac{1}{2}at^2$

$\Rightarrow 1 = \frac{1}{2}\left(\frac{3g}{16}\right)t^2$

$\Rightarrow \frac{3gt^2}{32} = 1$

$t = \sqrt{\frac{32}{3g}}$ s

Motion of particle relative to wedge:

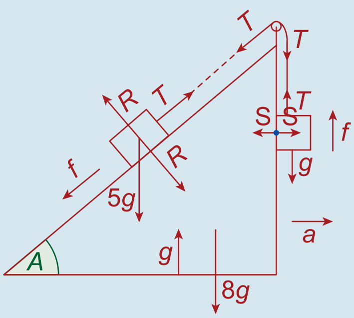
$u = 0, \quad t = \sqrt{\frac{32}{3g}}, \quad f = \frac{3g}{4}$

$s = ut + \frac{1}{2}ft^2$

$= \frac{1}{2}\left(\frac{3g}{4}\right)\left(\frac{32}{3g}\right)$

$= 4$ m

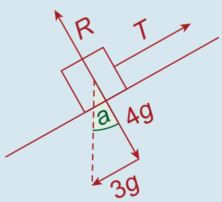
Q. 9.



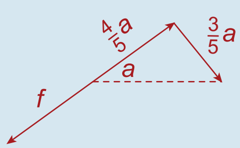
Since $\tan A = \frac{3}{4}$, $\sin A = \frac{3}{5}$, $\cos A = \frac{4}{5}$

The 5kg Mass:

Forces



Acceleration



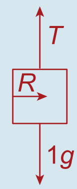
(i) Along the plane: $3g - T = 5\left(f - \frac{4}{5}a\right)$

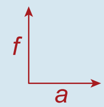
(ii) Perpendicular to the plane

$4g - R = 5\left(\frac{3}{5}a\right)$

The 1kg Mass:

Forces **Acceleration:**



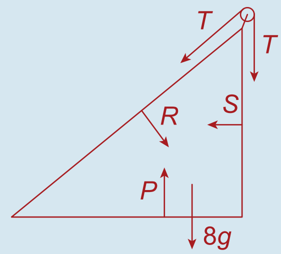


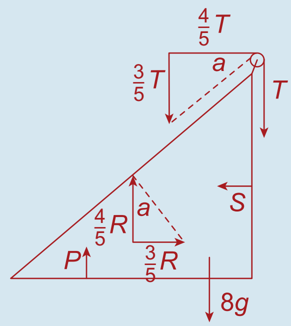
(iii) Along the vertical : $T - g = f$

(iv) Along the horizontal: $S = a$

The Wedge:

Forces

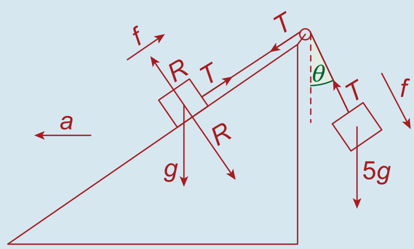




Acceleration

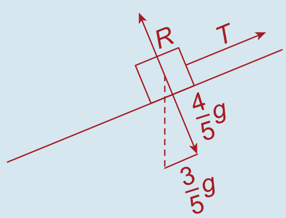
(v) $\frac{3}{5}R - \frac{4}{5}T - S = 8a$

Q. 10.

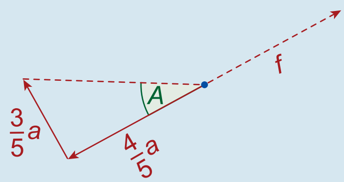


The 1kg Mass:

Forces



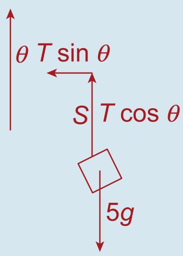
Accelerations



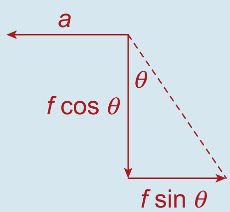
1. $T - \frac{3}{5}g = 1\left(f - \frac{4}{5}a\right)$
2. $R - \frac{4}{5}g = 1\left(\frac{3}{5}a\right)$

The 5kg Mass:

Forces



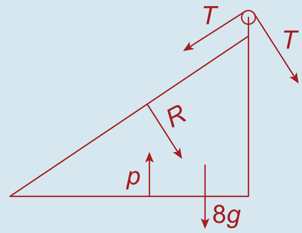
Accelerations



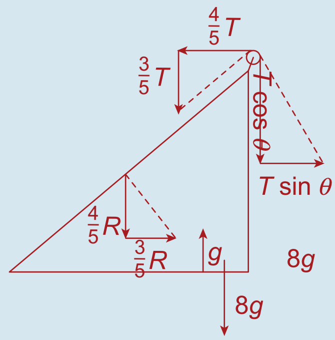
3. $5g - T\cos\theta = 5(f\cos\theta)$
4. $T\sin\theta = 5(a - f\sin\theta)$

The Wedge:

Forces



Resolved



Acceleration

5. $\frac{4}{5}T - \frac{3}{5}R - T\sin\theta = 8a$