Chapter 3 Exercise 3A

Q. 1. (i) $s_y = 28(3) + \frac{1}{2}(-9.8)(3)^2$

$= 39.9$ m

(ii) Let H = maximum height

$u_y = 28, \quad a_y = -9.8,$

$v_y = 0, \quad s = H$

$v^2 = u^2 + 2as$

$\therefore 0 = (28)^2 + 2(-9.8)H$

$\therefore 0 = 784 - 19.6H$

$\therefore H = 40$

(iii) $s_y = 0$

$\Rightarrow 28t - 4.9t^2 = 0$

$\Rightarrow t = 0$ **OR** $t = \frac{40}{7}$

At $t = \frac{40}{7}$, $s_x = 21t$

$= 21\left(\frac{40}{7}\right)$

$= 120$ m

Q. 2. $v_x = 56$

$v_y = 56 - gt$

$s_x = 56t$

$s_y = 56t - \frac{1}{2}gt^2$

(i) Let H = maximum height

$u_y = 56, \quad a_y = -9.8,$

$v_y = 0, \quad s_y = H$

$v^2 = u^2 + 2as$

$\therefore 0 = (56)^2 + 2(-9.8)H$

$\therefore 0 = 3{,}136 - 19.6H$

$\therefore H = 160$ m

(ii) Range: s_x when $s_y = 0$

$s_y = 0$

$\Rightarrow 56t - \frac{1}{2}gt^2 = 0$

$\Rightarrow 112t - gt^2 = 0$

$\Rightarrow t(112 - gt) = 0$

$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \qquad \underbrace{t = \frac{112}{g}}_{\text{Time of Flight}}$

$\Rightarrow \text{Range} = 56\left[\frac{112}{g}\right]$

$= 640$ m

(iii) Velocity after 4 seconds: Find v_x and v_y when $t = 4$

$v_x = 56$

$v_y = 56 - g(4)$

$= 16.8$

$\Rightarrow \vec{v} = 56\vec{i} + 16.8\vec{j}$ m/s

Q. 3. $v_x = 70$

$v_y = 105 - gt$

$s_x = 70t$

$s_y = 105t - \frac{1}{2}gt^2$

(i) Need v_x and v_y when $t = 10$

$v_x = 70$

$v_y = 105 - g(10) = 7$

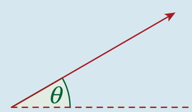
$\Rightarrow \vec{v} = 70\vec{i} + 7\vec{j}$

Magnitude

$|\vec{v}| = \sqrt{70^2 + 7^2}$

$= 70.35$ m/s

Direction



$\tan\theta = \frac{7}{70} = \frac{1}{10}$

$\Rightarrow \theta = \tan^{-1}\left[\frac{1}{10}\right] = 5.71°$

(ii) Range: s_x when $s_y = 0$

$s_y = 0$

$\Rightarrow 105t - \frac{1}{2}gt^2 = 0$

$\Rightarrow 210t - gt^2 = 0$

$\Rightarrow t(210 - gt) = 0$

$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \qquad \underbrace{t = \frac{210}{g}}_{\text{Time of Flight}}$

$\Rightarrow \text{Range} = 70\left[\frac{210}{g}\right]$

$= 1{,}500$ m

Q. 4. (i) $s_y = 21t - 4.9t^2$

$= 22.4$

$\Rightarrow 7t^2 - 30t + 32 = 0$

$\Rightarrow (7t - 16)(t - 2) = 0$

$t = \frac{16}{7}$ **OR** $t = 2$

(ii) When it is at its greatest height, $v_y = 0$ and $v_x = 14$, as always.

$\therefore \vec{v} = 14\vec{i} + 0\vec{j}$

Q. 5. (i) $u_y = 49, \quad a_y = -9.8,$

$v_y = 0, \quad s_y = H$

$v^2 = u^2 + 2as$

$\therefore 0 = (49)^2 + 2(-9.8)H$

$\therefore H = 122.5$ m

(ii) $s_y = 0$

$\Rightarrow 105t - 4.9t^2 = 0$

$t = 0$ **OR** $t = \frac{105}{4.9} = \frac{150}{7}$

At $t = \frac{150}{7}$, $s_x = 70t$

$= 70\left(\frac{150}{7}\right)$

$= 1{,}500$ m

(iii) $v_x = 10$

$v_y = 49 - (9.8)(6)$

$= -9.8$

$\therefore \vec{v} = 10\vec{i} - 9.8\vec{j}$ m/s

Q. 6. $v_x = 70$

$v_y = 140 - gt$

$s_x = 70t$

$s_y = 140t - \frac{1}{2}gt^2$

(i) Need to find v_x and v_y when $t = 5$

$v_x = 70$

$v_y = 140 - g(5) = 91$

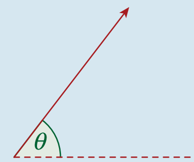
$\Rightarrow \vec{v} = 70\vec{i} + 91\vec{j}$

Magnitude

$|\vec{v}| = \sqrt{70^2 + 91^2}$

$= 114.8$ m/s

Direction



$\tan\theta = \frac{91}{70}$

$= \frac{13}{10}$

$\Rightarrow \theta = \tan^{-1}\left[\frac{13}{10}\right]$

$= 52.43°$

(ii) Range: s_x when $s_y = 0$

$s_y = 0$

$\Rightarrow 140t - \frac{1}{2}gt^2 = 0$

$\Rightarrow 280t - gt^2 = 0$

$\Rightarrow t(280 - gt) = 0$

$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \qquad \underbrace{t = \frac{280}{g}}_{\text{Time of Flight}}$

Range $= 70\left[\frac{280}{g}\right]$

$= 2{,}000$ m

Q. 7. $v_x = 20$

$v_y = 28 - gt$

$s_x = 20t$

$s_y = 28t - \frac{1}{2}gt^2$

(i) Need s_y when $t = 3$

$= 28(3) - \frac{1}{2}g(9)$

$= 39.9$ m

(ii) Need v_x and v_y when $t = 4$

$v_x = 20$

$v_y = 28 - g(4)$

$= -11.2$

$\Rightarrow \vec{v} = 20\vec{i} - 11.2\vec{j}$

$\Rightarrow$ Speed $= \sqrt{20^2 + (-11.2)^2}$

$= 22.92$ m/s

Q. 8. (i) $v_y = 0$

$\Rightarrow 35 - 9.8t = 0$

$\Rightarrow t = \frac{35}{9.8}$

$= \frac{25}{7}$

(ii) $v_y = 10.5$

$\Rightarrow 35 - 9.8t = 10.5$

$\Rightarrow t = \frac{24.5}{9.8}$

$= \frac{5}{2}$

(iii) $v_y = -10.5$

$\Rightarrow 35 - 9.8t = -10.5$

$\Rightarrow t = \frac{45.5}{9.8}$

$= \frac{65}{14}$

(iv) Midway between (ii) and (iii) is

$\frac{\frac{5}{2} + \frac{65}{14}}{2} = \frac{25}{7} = \text{(i)}$ **QED**

Q. 9. $s_y = -490$

$\Rightarrow -4.9t^2 = -490$

$\Rightarrow t = 10$

At $t = 10$, $s_x = 200t$

$= 200(10)$

$= 2{,}000$ m

Q. 10. (i) $v_y = 0$

$\Rightarrow 98 - 9.8t = 0$

$\Rightarrow t = 10$

(ii) At $t = 10$, $s_x = 10t$

$= 10(10)$

$= 100$ m

$s_y = 98t - 4.9t^2$

$= 98(10) - 4.9(100)$

$= 490$ m

$\therefore \vec{r} = 100\vec{i} + 490\vec{j}$ m

Q. 11. $v_x = 100$

$v_y = 98 - gt$

$s_x = 100t$

$s_y = 98t - \frac{1}{2}gt^2$

(i) Need to find t when $s_y = 470.4$

$\Rightarrow 98t - \frac{1}{2}gt^2 = 470.4$

$\Rightarrow 98t - 4.9t^2 = 470.4$

$\Rightarrow 4.9t^2 - 98t + 470.4 = 0$

$\Rightarrow t^2 - 20t + 96 = 0$

$\Rightarrow (t - 8)(t - 12) = 0$

$\Rightarrow t = 8, t = 12$

(ii) Find v_x and v_y when $t = 8$

$v_x = 100$

$v_y = 98 - g(8)$

$= 19.6$

$\Rightarrow \vec{v} = 100\vec{i} + 19.6\vec{j}$

$\Rightarrow \text{Speed} = \sqrt{100^2 + 19.6^2}$

$= 102$ m/s

Find v_x and v_y when $t = 12$

$v_x = 100$

$v_y = 98 - g(12)$

$= -19.6$

$\Rightarrow \vec{v} = 100\vec{i} - 19.6\vec{j}$

$\Rightarrow \text{Speed} = \sqrt{100^2 + (-19.6)^2}$

$= 102$ m/s

Q. 12. $v_x = 8$

$v_y = 28 - gt$

$\sqrt{8^2 + (28 - gt)^2} = 10$

$\Rightarrow 64 + (784 - 56gt + g^2t^2) = 100$

$\Rightarrow g^2t^2 - 56gt + 748 = 0$

$\Rightarrow \frac{2{,}401}{25}t^2 - \frac{2{,}744}{5}t + 748 = 0$

$\Rightarrow 2{,}401t^2 - 13{,}720t + 18{,}700 = 0$

$$t = \frac{13{,}720 \pm \sqrt{13{,}720^2 - 4(2{,}401)(18{,}700)}}{2(2{,}401)}$$

$t_1 = \frac{110}{49}$ s $\qquad t_2 = \frac{170}{49}$ s

Exercise 3B

Q. 1. (i) $\cos\alpha = \frac{2}{\sqrt{5}}, \sin\alpha = \frac{1}{\sqrt{5}}$

$\vec{u} = 7\sqrt{5}\cos\alpha\vec{i} + 7\sqrt{5}\sin\alpha\vec{j}$

$= 14\vec{i} + 7\vec{j}$

(ii) $s_y = 0$

$\Rightarrow 7t - 4.9t^2 = 0$

$\Rightarrow t = 0$ **OR** $t = \frac{7}{4.9}$

$= \left(\frac{10}{7}\right)$

At $t = \frac{10}{7}, s_x = 14t$

$= 14\left(\frac{10}{7}\right)$

$= 20$ m

Q. 2. $\vec{u} = 35\cos\theta\vec{i} + 35\sin\theta\vec{j}$

$= 35\left(\frac{4}{5}\right)\vec{i} + 35\left(\frac{3}{5}\right)\vec{j}$

$= 28\vec{i} + 21\vec{j}$

$s_y = 10$

$\Rightarrow 21t - 4.9t^2 = 10$

$\Rightarrow 49t^2 - 210t + 100 = 0$

$\Rightarrow t = 0.546$ (**OR** 3.740)

At $t = 0.546$,

$v_x = 28$ and $v_y = 21 - 9.8t$

$= 21 - 9.8(0.546)$

$= 15.65$

$\therefore \vec{v} = 28\vec{i} + 15.65\vec{j}$

$\Rightarrow |\vec{v}| = \sqrt{28^2 + (15.65)^2}$

$= 32.08$ m/s

Q. 3. $s_y = 0$

$\Rightarrow 7t - 4.9t^2 = 0$

$\Rightarrow t = 0$ **OR** $t = \frac{7}{4.9} = \frac{10}{7}$

At $t = \frac{10}{7}, s_x = 10t$

$= 10\left(\frac{10}{7}\right)$

$= \frac{100}{7}$ m

$= R$, the range

$\frac{3}{4}R = \frac{75}{7}$ m

$s_x = \frac{75}{7}$

$\Rightarrow t = \frac{75}{70}$

$= \frac{15}{14}$

At $t = \frac{15}{14}, s_y = 7\left(\frac{15}{14}\right) - 4.9\left(\frac{225}{196}\right)$

$= 7.5 - 5.625$

$= 1.875$ m

Q. 4. $s_y = 0$

$\Rightarrow 4t - 4.9t^2 = 0$

$\Rightarrow t = 0$ **OR** $t = \frac{4}{4.9} = \frac{40}{49}$

At $t = \frac{40}{49}, s_x = 3t$

$= 3\left(\frac{40}{49}\right)$

$= \frac{120}{49} = R$, the range

$\therefore \frac{1}{5}R = \frac{24}{49}$ m

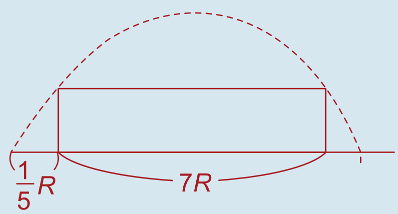
When is $s_x = \frac{24}{49}$? When $3t = \frac{24}{49}$

$\Rightarrow t = \frac{8}{49}$

At $t = \frac{8}{49}, s_y = 4\left(\frac{8}{49}\right) - 4.9\left(\frac{64}{2{,}401}\right)$

$= \frac{32}{49} - \frac{64}{490}$

$= \frac{128}{245}$



By symmetry, it will reach the same height when $s_x = \frac{4}{5}R$. The time will be $\frac{4}{5}$ of the time of flight $= \frac{4}{5}\left(\frac{40}{49}\right) = \frac{32}{49}$ s

Q. 5. (a) $(7t + 50)(t - 10)$

(b) $s_y = -350$

$\Rightarrow 14t - 4.9t^2 = -350$

$\Rightarrow 7t^2 - 20t - 50 = 0$

$\Rightarrow (7t + 50)(t - 10) = 0$

$\Rightarrow t = 10$ $\left(t = -\frac{50}{7}\text{ is rejected}\right)$

At $t = 10$, $s_x = 10t$
$= 10(10)$
$= 100$ m

Q. 6. $s_y = 49t - \frac{1}{2}(9.8)t^2 = 0$
$\therefore 49t - 4.9t^2 = 0$
$\therefore t(49 - 4.9t) = 0$
$\therefore t = 0$ **OR** $t = 10$

at $t = 10$, $s_x = 50t$
$= 50(10)$
$= 500$ m

Q. 7. (i) $u_y = 14, \quad a_y = -9.8,$
$s_y = H, \quad v_y = 0$
$v^2 = u^2 + 2as$
$\therefore 0 = (14)^2 + 2(-9.8)H$
$\therefore H = 10$ m

(ii) $\frac{3}{4}H = \frac{3}{4}H(10)$
$= 7.5$
$s_y = 7.5$
$\Rightarrow 14t - 4.9t^2 = 7.5$
$\Rightarrow 49t^2 - 140t + 75 = 0$
$\Rightarrow (7t - 5)(7t - 15) = 0$
$\Rightarrow t = \frac{5}{7}$ **OR** $\frac{15}{7}$

Q. 8. $s_y = -82.5$
$\Rightarrow 8t - 4.9t^2 = -82.5$
$\Rightarrow 49t^2 - 80t - 825 = 0$
$\Rightarrow t = 5$ **OR** $-\frac{330}{98}$
At $t = 5$, $s_x = 12t$
$= 12(5)$
$= 60$ m

Q. 9. (i) $s_y = -78.4$
$\Rightarrow -4.9t^2 = -78.4$
$\Rightarrow t^2 = 16$
$\Rightarrow t = 4$

(ii) $\vec{u} = 98\cos 30°\vec{i} + 98\sin 30°\vec{j}$
$= 84.868\,\vec{i} + 49\vec{j}$
$s_y = -78.4$
$\Rightarrow 49t - 4.9t^2 = -78.4$
$\Rightarrow 49t^2 - 490t - 784 = 0$
$\Rightarrow t^2 - 10t - 16 = 0$
$\Rightarrow t = 11.4 \quad (-1.4$ is rejected$)$

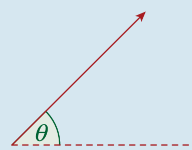
Q. 10. $u_x = 50\cos\theta$
$= 50\left(\frac{3}{5}\right)$
$= 30$
$u_y = 50\sin\theta$
$= 50\left(\frac{4}{5}\right)$
$= 40$
$v_x = 30$
$v_y = 40 - gt$
$s_x = 30t$
$s_y = 40t - \frac{1}{2}gt^2$

(i) $\vec{r} = 30t\vec{i} + \left[40t - \frac{1}{2}gt^2\right]\vec{j}$
$= 30t\vec{i} + (40 - 4.9t^2)\vec{j}$ m

(ii) $v_x = 30$
$v_y = 40 - g(1) = 30.2$
$\Rightarrow \vec{v} = 30\vec{i} + 30.2\vec{j}$ m/s

Magnitude
$|\vec{v}| = \sqrt{30^2 + 30.2^2}$
$= 43$ m/s

Direction
$\tan\theta = \frac{30.2}{30}$
$\Rightarrow \theta = \tan^{-1}\frac{30.2}{30}$
$= 45°$



(iii) Range: s_x when $s_y = 0$
$s_y = 0$
$\Rightarrow 40t - \frac{1}{2}gt^2 = 0$
$\Rightarrow 80t - gt^2 = 0$
$\Rightarrow t(80 - gt) = 0$
$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \qquad \underbrace{t = \frac{80}{g}}_{\text{Time of Flight}}$

Range $= 30\left(\frac{80}{g}\right)$
$= 245$ m

Q. 11. $s_x = 12t$

$s_y = kt - \frac{1}{2}gt^2$

$s_x = 30$ when $s_y = 9.375$

$\Rightarrow 12t = 30$

$\Rightarrow t = \frac{5}{2}$

when $kt - 4.9t^2 = 9.375$

$\Rightarrow k\left(\frac{5}{2}\right) - 4.9\left(\frac{5}{2}\right)^2 = 9.375$

$\Rightarrow \frac{5k}{2} - \frac{245}{8} = \frac{75}{8}$

$\Rightarrow 20k - 245 = 75$

$\Rightarrow 20k = 320 \Rightarrow k = 16$

Q. 12. $u_x = u\cos\alpha$ $\qquad u_y = u\sin\alpha$

$v_x = u\cos\alpha$

$v_y = u\sin\alpha - gt$

$s_x = ut\cos\alpha$

$s_y = ut\sin\alpha - \frac{1}{2}gt^2$

(i) Greatest height: s_y when $v_y = 0$

$v_y = 0$

$\Rightarrow u\sin\alpha - gt = 0$

$\Rightarrow t = \frac{u\sin\alpha}{g}$

$\Rightarrow$ Greatest height $= u\left(\frac{u\sin\alpha}{g}\right)\sin\alpha - \frac{1}{2}g\left(\frac{u\sin\alpha}{g}\right)^2$

$\Rightarrow$ Greatest height $= \frac{u^2\sin^2\alpha}{g} - \frac{u^2\sin^2\alpha}{2g}$

$\Rightarrow$ Greatest height $= \frac{2u^2\sin^2\alpha - u^2\sin^2\alpha}{2g}$

$\Rightarrow$ Greatest height $= \frac{u^2\sin^2\alpha}{2g}$

OR

$v_y{}^2 = u_y{}^2 + 2a_y s_y$

$0 = (u\sin\alpha)^2 + 2(-g)H$

$\therefore H = \frac{u^2\sin^2\alpha}{2g}$

(ii) Range: s_x when $s_y = 0$

$s_y = 0$

$\Rightarrow ut\sin\alpha - \frac{1}{2}gt^2 = 0$

$\Rightarrow 2ut\sin\alpha - gt^2 = 0$

$\Rightarrow t(2u\sin\alpha - gt) = 0$

$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \qquad \underbrace{t = \frac{2u\sin\alpha}{g}}_{\text{Time of Flight}}$

$\Rightarrow$ Range $= u\left(\frac{2u\sin\alpha}{g}\right)\cos\alpha$

$\Rightarrow$ Range $= \frac{2u^2\sin\alpha\cos\alpha}{g}$

(iii) $u = 70$ and Greatest height $= 125$

$\Rightarrow \frac{4{,}900\sin^2\alpha}{2g} = 125$

$\Rightarrow 250\sin^2\alpha = 125$

$\Rightarrow \sin^2\alpha = \frac{1}{2}$

$\Rightarrow \sin\alpha = \frac{1}{\sqrt{2}}$

$\Rightarrow \alpha = 45°$

$\Rightarrow$ Range $= \frac{9{,}800\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)}{9.8}$

$= 500$ m

Q. 13. $\dfrac{u^2\sin^2\alpha}{2g} = \dfrac{2u^2\sin\alpha\cos\alpha}{g}$

$\Rightarrow \tan\alpha = 4$

$\Rightarrow \alpha = 76°$

Q. 14. Let the speed of projection $= u$ and the angle of projection $= \alpha$

$u_x = u\cos\alpha \qquad u_y = u\sin\alpha$

$v_x = u\cos\alpha$

$v_y = u\sin\alpha - gt$

$s_x = ut\cos\alpha$

$s_y = ut\sin\alpha - \frac{1}{2}gt^2$

Maximum height: s_y when $v_y = 0$

$v_y = 0$

$\Rightarrow u\sin\alpha - gt = 0$

$\Rightarrow t = \dfrac{u\sin\alpha}{g}$

$\Rightarrow$ Maximum height $= u\left(\dfrac{u\sin\alpha}{g}\right)\sin\alpha - \dfrac{1}{2}g\left(\dfrac{u\sin\alpha}{g}\right)^2$

$\Rightarrow$ Maximum height $= \dfrac{u^2\sin^2\alpha}{g} - \dfrac{u^2\sin^2\alpha}{2g}$

$\Rightarrow$ Maximum height $= \dfrac{2u^2\sin^2\alpha - u^2\sin^2\alpha}{2g}$

$\Rightarrow$ Maximum height $= \dfrac{u^2\sin^2\alpha}{2g}$

OR

$v_y{}^2 = u_y{}^2 + 2a_ys_y$

$\therefore 0 = (u\sin\alpha)^2 + 2(-g)H$

$\therefore H = \dfrac{u^2\sin^2\alpha}{2g}$

Range: s_x when $s_y = 0$

$s_y = 0$

$\Rightarrow ut\sin\alpha - \frac{1}{2}gt^2 = 0$

$\Rightarrow 2ut\sin\alpha - gt^2 = 0$

$\Rightarrow t(2u\sin\alpha - gt) = 0$

$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \qquad \underbrace{t = \dfrac{2u\sin\alpha}{g}}_{\text{Time of Flight}}$

$\Rightarrow$ Range $= u\left(\dfrac{2u\sin\alpha}{g}\right)\cos\alpha$

$\Rightarrow$ Range $= \dfrac{2u^2\sin\alpha\cos\alpha}{g}$

Maximum Height = 2(Range)

$\Rightarrow \dfrac{\cancel{u^2}\sin^2\alpha}{2\cancel{g}} = 2\left(\dfrac{2\cancel{u^2}\sin\alpha\cos\alpha}{\cancel{g}}\right)$

$\Rightarrow \sin^2\alpha = 8\sin\alpha\cos\alpha$

$\Rightarrow \sin\alpha(\sin\alpha - 8\cos\alpha) = 0$

$\Rightarrow \cancel{\sin\alpha = 0} \qquad \sin\alpha - 8\cos\alpha = 0$ … divide by $\cos\alpha$

$\Rightarrow \tan\alpha - 8 = 0$

$\Rightarrow \tan\alpha = 8$

$\Rightarrow \alpha = \tan^{-1} 8$

$= 83°$

Q. 15. (i) $\frac{u^2 \sin^2 \alpha}{2g} = 3.6 \ldots.$ ①

$\frac{2u^2 \sin\alpha \cos\alpha}{g} = 19.2 \ldots.$ ②

Dividing ① by ②

$\Rightarrow \frac{\tan \alpha}{4} = \frac{3}{16}$

$\Rightarrow \tan\alpha = \frac{3}{4}$

$\Rightarrow \sin\alpha = \frac{3}{5}$

Putting this result into ① gives

$\frac{u^2\left(\frac{9}{25}\right)}{19.6} = 3.6$

$\Rightarrow u^2 = 196$

$\Rightarrow u = 14$ m/s

(ii) $R_{max} = \frac{u^2}{g}$

$= \frac{196}{9.8} = 20$ m

Q. 16. (i) 30°, 150°

(ii) $R = \frac{u^2 \sin 2\alpha}{g}$

$= 40$

$\Rightarrow \frac{784 \sin 2\alpha}{9.8} = 40$

$\Rightarrow \sin 2\alpha = \frac{1}{2}$

$\Rightarrow 2\alpha = 30°$ **OR** 150°

$\Rightarrow \alpha = 15°$ **OR** 75°

Q. 17. $H = \frac{u^2 \sin^2 \alpha}{2g}$

$\Rightarrow 2.5 = \frac{100 \sin^2\alpha}{19.6}$

$\Rightarrow \sin^2\alpha = 0.49$

$\Rightarrow \sin\alpha = 0.7$

Q. 18. As in Q. 18(iii), $\alpha = 45°$

$H = \frac{u^2 \sin^2 45°}{2g}$

$= \frac{u^2\left(\frac{1}{2}\right)}{2g}$

$= \frac{u^2}{4g}$

$R = \frac{u^2 \sin 2(45°)}{g}$

$= \frac{u^2}{g}$

Therefore, $H : R = 1 : 4$

Q. 19. (i) Let T = time of flight

$\therefore PT = 60 \quad \ldots.$ (I) and $QT - \frac{1}{2}(9.8)T^2 = 0 \quad \ldots.$ (II)

When it is at greatest height

$v_y^{\,2} = u_y^{\,2} + 2a_y s_y$

$0 = Q^2 + 2(-9.8)(5.625)$

$\therefore 110.25 = Q^2$

$\therefore Q = 10.5$

Put this into (II): $10.5T - 4.9T^2 = 0$

$\therefore T = \frac{15}{7}$

Put this into (I): $P\left(\frac{15}{7}\right) = 60$

$\therefore P = 28$

(ii) The greatest distance $= 7 \times \frac{15}{7}$

$= 15$ m

Q. 20. Let the angle of projection $= \alpha$

$u_x = u\cos\alpha \qquad u_y = u\sin\alpha$

$v_x = u\cos\alpha$

$v_y = u\sin\alpha - gt$

$s_x = ut\cos\alpha$

$s_y = ut\sin\alpha - \frac{1}{2}gt^2$

Greatest height: s_y when $v_y = 0$

$v_y = 0$

$\Rightarrow u\sin\alpha - gt = 0$

$\Rightarrow t = \frac{u\sin\alpha}{g}$

$\Rightarrow$ Greatest height $= u\left(\frac{u\sin\alpha}{g}\right)\sin\alpha - \frac{1}{2}g\left(\frac{u\sin\alpha}{g}\right)^2$

$\Rightarrow$ Greatest height $= \frac{u^2\sin^2\alpha}{g} - \frac{u^2\sin^2\alpha}{2g}$

$\Rightarrow$ Greatest height $= \frac{2u^2\sin^2\alpha - u^2\sin^2\alpha}{2g}$

$\Rightarrow$ Greatest height $= \frac{u^2\sin^2\alpha}{2g}$

Range: s_x when $s_y = 0$

$s_y = 0$

$\Rightarrow ut\sin\alpha - \frac{1}{2}gt^2 = 0$

$\Rightarrow 2ut\sin\alpha - gt^2 = 0$

$\Rightarrow t(2u\sin\alpha - gt) = 0$

$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \qquad \underbrace{t = \frac{2u\sin\alpha}{g}}_{\text{Time of Flight}}$

$\Rightarrow$ Range $= u\left(\frac{2u\sin\alpha}{g}\right)\cos\alpha$

$\Rightarrow$ Range $= \frac{2u^2\sin\alpha\cos\alpha}{g}$

$\frac{\text{Greatest height}}{\text{Range}} = \frac{2}{7}$

$\Rightarrow 2(\text{Range}) = 7(\text{Greatest height})$

$\Rightarrow 2\left(\frac{2u^2\sin\alpha\cos\alpha}{g}\right) = 7\left(\frac{u^2\sin^2\alpha}{2g}\right)$ … multiply by $\frac{2g}{u^2}$

$\Rightarrow 8\sin\alpha\cos\alpha = 7\sin^2\alpha$

$\Rightarrow \sin\alpha(8\cos\alpha - 7\sin\alpha) = 0$

$\Rightarrow$ ~~$\sin\alpha = 0$~~ $\qquad 8\cos\alpha - 7\sin\alpha = 0$ … divide by $\cos\alpha$

$\Rightarrow 8 - 7\tan\alpha = 0$

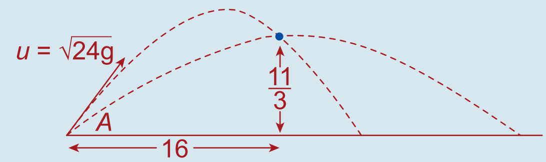
$\Rightarrow \tan\alpha = \frac{8}{7}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{8}{7}\right)$

$= 49°$

Exercise 3C

Q. 1. $u_x = \sqrt{24g}\cos A \qquad u_y = \sqrt{24g}\sin A$



$v_x = \sqrt{24g}\cos A$

$v_y = \sqrt{24g}\sin A - gt$

$s_x = \sqrt{24g}\, t\cos A$

$s_y = \sqrt{24g}\, t\sin A - \frac{1}{2}gt^2$

$s_x = 16$ when $\vec{s_y} = \frac{11}{3}$

$\Rightarrow \sqrt{24g}\, t\cos A = 16 \qquad$ when $\qquad \sqrt{24g}\, t\sin A - \frac{1}{2}gt^2 = \frac{11}{3}$

$\Rightarrow t = \frac{16}{\sqrt{24g}\cos A}$

$\boxed{\frac{\sin A}{\cos A} = \tan A} \qquad \boxed{\frac{1}{\cos^2 A} = 1 + \tan^2 A}$

$$\Rightarrow \sqrt{24g}\left[\frac{16}{\sqrt{24g}\cos A}\right]\sin A - \frac{1}{2}g\left[\frac{256}{24g\cos^2 A}\right] = \frac{11}{3}$$

$$\Rightarrow 16\tan A - \frac{16}{3}(1 + \tan^2 A) = \frac{11}{3}$$

$$\Rightarrow 48\tan A - 16 - 16\tan^2 A = 11$$

$$\Rightarrow 16\tan^2 A - 48\tan A + 27 = 0 \quad \ldots \text{ let } x = \tan A$$

$$\Rightarrow 16x^2 - 48x + 27 = 0$$

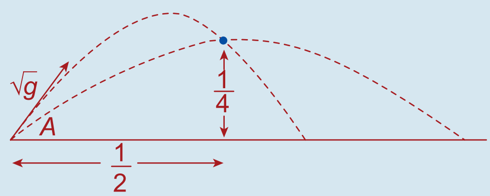
$$\Rightarrow (4x - 9)(4x - 3) = 0$$

$$\Rightarrow x = \frac{9}{4} \quad \textbf{OR} \quad x = \frac{3}{4}$$

$$\Rightarrow \tan A = \frac{9}{4} \quad \textbf{OR} \quad \tan A = \frac{3}{4}$$

$$\Rightarrow A = 66^\circ \quad \textbf{OR} \quad A = 37^\circ$$

Q. 2. $u_x = \sqrt{g}\cos A \qquad u_y = \sqrt{g}\sin A$



$v_x = \sqrt{g}\cos A$

$v_y = \sqrt{g}\sin A - gt$

$s_x = t\sqrt{g}\cos A$

$s_y = t\sqrt{g}\sin A - \frac{1}{2}gt^2$

$s_x = \frac{1}{2} \qquad$ when $\qquad s_y = \frac{1}{4}$

$\Rightarrow t\sqrt{g}\cos A = \frac{1}{2} \qquad$ when $\qquad t\sqrt{g}\sin A - \frac{1}{2}gt^2 = \frac{1}{4}$

$\Rightarrow t = \frac{1}{2\sqrt{g}\cos A}$

$\boxed{\frac{\sin A}{\cos A} = \tan A} \qquad \boxed{\frac{1}{\cos^2 A} = 1 + \tan^2 A}$

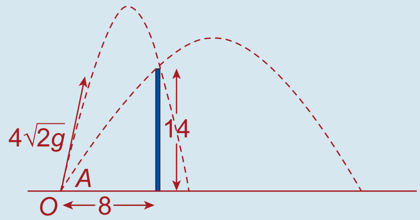
$$\Rightarrow \left[\frac{1}{2\sqrt{g}\cos A}\right]\sqrt{g}\sin A - \frac{1}{2}g\left[\frac{1}{4g\cos^2 A}\right] = \frac{1}{4}$$

$$\Rightarrow \frac{1}{2}\tan A - \frac{1}{8}(1 + \tan^2 A) = \frac{1}{4} \quad \ldots \text{ multiply by 8.}$$

$$\Rightarrow 4\tan A - 1 - \tan^2 A = 2$$

$$\Rightarrow \tan^2 A - 4\tan A + 3 = 0 \quad \ldots \text{ let } x = \tan A$$
$$\Rightarrow x^2 - 4x + 3 = 0$$
$$\Rightarrow (x-3)(x-1) = 0$$
$$\Rightarrow x = 3 \quad \textbf{OR} \quad x = 1$$
$$\Rightarrow \tan A = 3 \quad \textbf{OR} \quad \tan A = 1$$
$$\Rightarrow A = 72^\circ \quad \textbf{OR} \quad A = 45^\circ$$

Q. 3. $u_x = 4\sqrt{2g}\cos A \qquad u_y = 4\sqrt{2g}\sin A$



$v_x = 4\sqrt{2g}\cos A$

$v_y = 4\sqrt{2g}\sin A - gt$

$s_x = 4t\sqrt{2g}\cos A$

$s_y = 4t\sqrt{2g}\sin A - \frac{1}{2}gt^2$

$\boxed{\frac{\sin A}{\cos A} = \tan A} \qquad \boxed{\frac{1}{\cos^2 A} = 1 + \tan^2 A}$

$s_x = 8 \qquad$ when $\quad s_y = 14$

$\Rightarrow 4t\sqrt{2g}\cos A = 8 \qquad$ when $\quad 4t\sqrt{2g}\sin A - \frac{1}{2}gt^2 = 14$

$\Rightarrow t\sqrt{2g}\cos A = 2$

$\Rightarrow t = \dfrac{2}{\sqrt{2g}\cos A}$

$$\Rightarrow \quad 4\left[\frac{2}{\sqrt{2g}\cos A}\right]\sqrt{2g}\sin A - \frac{1}{2}g\left[\frac{2}{g\cos^2 A}\right] = 14$$
$$\Rightarrow 8\tan A - (1 + \tan^2 A) = 14$$
$$\Rightarrow \tan^2 A - 8\tan A + 15 = 0 \quad \ldots \text{ let } x = \tan A$$
$$\Rightarrow x^2 - 8x + 15 = 0$$
$$\Rightarrow (x-5)(x-3) = 0$$
$$\Rightarrow x = 5 \quad \textbf{OR} \quad x = 3$$
$$\Rightarrow \tan A = 5 \quad \textbf{OR} \quad \tan A = 3$$
$$\Rightarrow A = 79^\circ \quad \textbf{OR} \quad A = 72^\circ$$

We get the time of flight by letting $s_x = 8$ (Flight ends when it hits the target)

Firstly, look at the particle fired at an angle A where $\tan A = 5 \Rightarrow \cos A = \dfrac{1}{\sqrt{26}}$

$\Rightarrow 4t\sqrt{2g}\cos A = 8$

$\Rightarrow t\sqrt{2g}\cos A = 2$

$$\Rightarrow t = \frac{2}{\sqrt{2g}\cos A}$$
$$= \sqrt{\frac{2}{g}}\left[\frac{1}{\cos A}\right]$$
$$= \sqrt{\frac{2}{g}}\sqrt{26}$$
$$= \sqrt{\frac{52}{g}} = 2.3 \text{ s}$$

Secondly, look at the particle fired at an angle A where $\tan A = 3 \Rightarrow \cos A = \dfrac{1}{\sqrt{10}}$

$$t = \sqrt{\frac{2}{g}}\left[\frac{1}{\cos A}\right]$$
$$= \sqrt{\frac{2}{g}}\sqrt{10} = \sqrt{\frac{20}{g}} = 1.4 \text{ s}$$

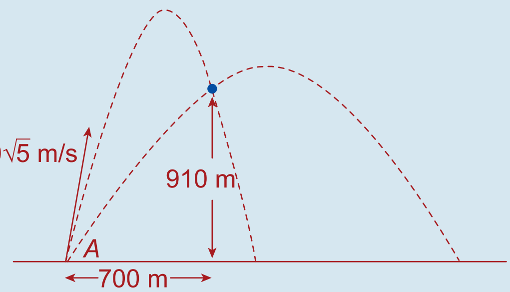
Q. 4. $u_x = 70\sqrt{5}\cos A$ $\quad u_y = 70\sqrt{5}\sin A$

$v_x = 70\sqrt{5}\cos A$

$v_y = 70\sqrt{5}\sin A - gt$

$s_x = 70t\sqrt{5}\cos A$

$s_y = 70t\sqrt{5}\sin A - \frac{1}{2}gt^2$



(i) $s_x = 700$ when $s_y = 910$

$70t\sqrt{5}\cos A = 700$ when $70t\sqrt{5}\sin A - \frac{1}{2}gt^2 = 910$

$\Rightarrow t = \frac{700}{70\sqrt{5}\cos A}$

$\Rightarrow t = \frac{10}{\sqrt{5}\cos A}$

$\frac{\sin A}{\cos A} = \tan A$ $\quad \frac{1}{\cos^2 A} = 1 + \tan^2 A$

$$\Rightarrow 70\left[\frac{10}{\sqrt{5}\cos A}\right]\sqrt{5}\sin A - \frac{1}{2}g\left[\frac{100}{5\cos^2 A}\right] = 910$$

$$\Rightarrow 700\tan A - 10g(1 + \tan^2 A) = 910$$

$$\Rightarrow 700\tan A - 98 - 98\tan^2 A = 910$$

$$\Rightarrow 98\tan^2 A - 700\tan A + 1{,}008 = 0 \quad \text{... divide by 14}$$

$$\Rightarrow 7\tan^2 A - 50\tan A + 72 = 0 \quad \text{... let } x = \tan A$$

$$\Rightarrow 7x^2 - 50x + 72 = 0$$

$$\Rightarrow (7x - 36)(x - 2)$$

$\Rightarrow x = \frac{36}{7}$ **OR** $x = 2$

$\Rightarrow \tan A = \frac{36}{7}$ **OR** $\tan A = 2$

$\Rightarrow x = 79^\circ$ **OR** $x = 63^\circ$

(ii) First particle is fired at an angle A where $\tan A = \frac{36}{7} \Rightarrow \cos A = \frac{7}{\sqrt{1{,}345}}$

Time of Flight: Let $s_x = 700$ (flight ends when it hits the target)

$\Rightarrow 70t\sqrt{5}\cos A = 700$

$\Rightarrow t\sqrt{5}\cos A = 10$

$$\Rightarrow t = \frac{10}{\sqrt{5}\cos A}$$

$$= \frac{2\sqrt{5}}{\cos A}$$

$$= 2\sqrt{5}\left[\frac{\sqrt{1{,}345}}{7}\right] = 23.43 \text{ s}$$

Second particle is fired at an angle A where $\tan A = 2 \Rightarrow \cos A = \frac{1}{\sqrt{5}}$

Time of Flight: Let $s_x = 700$

$\Rightarrow 70t\sqrt{5}\cos A = 700$

$\Rightarrow t = \frac{2\sqrt{5}}{\cos A}$

$= 2\sqrt{5}\left[\frac{\sqrt{5}}{1}\right] = 10 \text{ s}$

(iii) $23.43 - 10 = 13.43$

$\Rightarrow$ 13.5 seconds elapses between 1st and 2nd hit to the nearest half second.

Q. 5. (a) $\sin^2 A + \cos^2 A = 1\ [\div \cos^2 A]$

$\Rightarrow \tan^2 A + 1 = \sec^2 A$

(b) $\vec{u} = 35\sqrt{5}\cos\alpha\vec{i} + 35\sqrt{5}\sin\alpha\vec{j}$

$s_x = 35\sqrt{5}\cos\alpha t = 350$

$\Rightarrow t = \dfrac{350}{35\sqrt{5}\cos\alpha}$

$= \dfrac{10}{\sqrt{5}\cos\alpha}$

At $t = \dfrac{10}{\sqrt{5}\cos\alpha}$

$s_y = 35\sqrt{5}\sin\alpha t - 4.9t^2$

$= 210$

$\Rightarrow 35\sqrt{5}\sin\alpha\left(\dfrac{10}{\sqrt{5}\cos\alpha}\right) - 4.9\left(\dfrac{100}{5\cos^2\alpha}\right) = 210$

$\Rightarrow 350\tan\alpha - 98\sec^2\alpha = 210$

$\Rightarrow 25\tan\alpha - 7\sec^2\alpha = 15$

$\Rightarrow 25\tan\alpha - 7(\tan^2\alpha + 1) = 15$

$\Rightarrow 25T - 7T^2 - 7 = 15$ (where $T = \tan\alpha$)

$\Rightarrow 7T^2 - 25T + 22 = 0$

$\Rightarrow (7T - 11)(T - 2) = 0$

$\Rightarrow \tan\alpha = \dfrac{11}{7}$ **OR** 2

$\left(\Rightarrow \cos\alpha = \dfrac{7}{\sqrt{170}}\right.$ **OR** $\left.\dfrac{1}{\sqrt{5}}\right)$

Now $t = \dfrac{10}{\sqrt{5}\cos\alpha}$

$= \dfrac{10}{7}\sqrt{34}$ s **OR** 10 s

Q. 6. Bullet

Angle of projection is α where $\tan\alpha = \dfrac{1}{2}$

$\Rightarrow \cos\alpha = \dfrac{2}{\sqrt{5}}$ and $\sin\alpha = \dfrac{1}{\sqrt{5}}$

$u_x = 70\sqrt{5}\cos\alpha$

$= 70\sqrt{5}\left(\dfrac{2}{\sqrt{5}}\right)$

$= 140$

$u_y = 70\sqrt{5}\sin\alpha$

$= 70\sqrt{5}\left(\dfrac{1}{\sqrt{5}}\right)$

$= 70$

$v_x = 140$

$v_y = 70 - gt$

$s_x = 140t$

$s_y = 70t - \dfrac{1}{2}gt^2$

Bullet will strike fighter when bullet has travelled 210 m vertically, i.e. when $s_y = 210$.

$70t - \dfrac{1}{2}gt^2 = 210$

$\Rightarrow 4.9t^2 - 70t + 210 = 0$ … divide by 0.7

$\Rightarrow 7t^2 - 100t + 300 = 0$

$\Rightarrow (7t - 30)(t - 10) = 0$

$\Rightarrow t = \dfrac{30}{7}, t = 10$ … bullet will strike fighter on the way up

$\Rightarrow t = \dfrac{30}{7}$ s

Q. 7. **Bullet**

(i) Let the angle of projection be α.

$u_x = v_x = 35\cos\alpha = 28$ … must match the speed of the bird

$\Rightarrow \cos\alpha = \frac{4}{5}$

$\Rightarrow \tan\alpha = \frac{3}{4}$

$\Rightarrow \alpha = \tan^{-1}\frac{3}{4}$

$u_y = 35\sin\alpha$

$= 35\left(\frac{3}{5}\right)$

$= 21$

$\Rightarrow s_y = 21t - \frac{1}{2}gt^2$

Bullet will reach bird when $s_y = 5.6$

$\Rightarrow 21t - 4.9t^2 = 5.6$

$\Rightarrow 4.9t^2 - 21t + 5.6 = 0$ … divide by 0.7

$\Rightarrow 7t^2 - 30t + 8 = 0$

$\Rightarrow (7t - 2)(t - 4) = 0$

$\Rightarrow t = \frac{2}{7}, t = 4$ … bullet will strike bird on the way up

$\Rightarrow t = \frac{2}{7}$ s

Q. 8. (i) $u_x = 35\cos A$

$u_y = 35\sin A$

$x = s_x = 35t\cos A$

$y = s_y = 35t\sin A - \frac{1}{2}gt^2$

$250y = 250(\tan A)x - (1 + \tan^2 A)x^2$

$\Leftrightarrow 250\left(35t\sin A - \frac{1}{2}gt^2\right) = 250(\tan A)(35t\cos A) - (1 + \tan^2 A)(35t\cos A)^2$

$\Leftrightarrow 8{,}750t\sin A - 1{,}225t^2 = 8{,}750\tan A\cos A - (1 + \tan^2 A)(1{,}225t^2\cos^2 A)$

$\Leftrightarrow 8{,}750\sin A - 1{,}225t^2 = 8{,}750\sin A - \left(\frac{1}{\cancel{\cos^2 A}}\right)(1{,}225t^2\cancel{\cos^2 A})$

$\Leftrightarrow 8{,}750\sin A - 1{,}225t^2 = 8{,}750\sin A - 1225t^2$ **QED**

(ii) $250y = 250(\tan A)x - (1 + \tan^2 A)x^2$ … let $x = 40$ and $y = 20$

$\Rightarrow 5{,}000 = 10{,}000(\tan A) - (1 + \tan^2 A)(1{,}600)$

$\Rightarrow 1{,}600\tan^2 A - 10{,}000\tan A + 6{,}600 = 0$ … divide by 200

$\Rightarrow 8\tan^2 A - 50\tan A + 33 = 0$

$\Rightarrow (4\tan A - 3)(2\tan A - 11) = 0$

$\Rightarrow \tan A = \frac{3}{4}, \tan A = \frac{11}{2}$

Will reach target when $x = 40$

$\tan A = \frac{3}{4}$

$\Rightarrow \cos A = \frac{4}{5}$

$\Rightarrow x = 35t\left(\frac{4}{5}\right)$

$\Rightarrow = 40$

$\Rightarrow 28t = 40$

$\Rightarrow t = \frac{10}{7}$ s

$\tan A = \frac{11}{2}$

$\Rightarrow \cos A = \frac{2}{\sqrt{125}} = \frac{2}{5\sqrt{5}}$

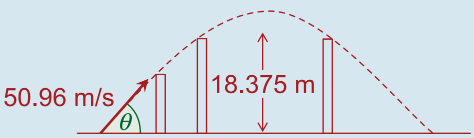
$\Rightarrow x = 35t\frac{2}{5\sqrt{5}}$

$= 40$

$\Rightarrow \frac{14t}{\sqrt{5}} = 40$

$\Rightarrow t = \frac{20\sqrt{5}}{7}$ s

Q. 9.



$\tan\theta = \frac{5}{12} \Rightarrow \cos\theta = \frac{12}{13}$ and $\sin\theta = \frac{5}{13}$

$u_x = 50.96\cos\theta = 50.96\left(\frac{12}{13}\right) = 47.04$

$u_y = 50.96\sin\theta = 50.96\left(\frac{5}{13}\right) = 19.6$

$vc_x = 47.04$

$v_y = 19.6 - 9.8t$

$s_x = 47.04t$

$s_y = 19.6t - 4.9t^2$

Passes over a wall 14.7 m high on its upward path.

Find s_x when $s_y = 14.7$ to find the horizontal displacement of the first wall.

$s_y = 14.7$

$\Rightarrow 19.6t - 4.9t^2 = 14.7$ ... divide by 4.9

$\Rightarrow 4t - t^2 = 3$

$\Rightarrow t^2 - 4t + 3 = 0$

$\Rightarrow (t - 1)(t - 3) = 0$

$\Rightarrow t = 1$ **OR** $t = 3$

$\Rightarrow$ Passes over wall after 1 second.

$s_x = 47 - 04t$ ... let $t = 1$

$\Rightarrow s_x = 47.04$ m

Now find s_x when $s_y = 18.375$ to find horizontal displacements of the two possible walls of height 18.375 m.

$s_y = 18.375$

$\Rightarrow 19.6t - 4.9t^2 = 18.375$ ... divide by 4.9

$\Rightarrow 4t - t^2 = 3.75$

$\Rightarrow t^2 - 4t + 3.75 = 0$ ... multiply by 4

$\Rightarrow 4t^2 - 16t + 15 = 0$

$\Rightarrow (2t - 3)(2t - 5) = 0$

$\Rightarrow t = \frac{3}{2}$ **OR** $t = \frac{5}{2}$

$s_x = 47.04t$ ... let $t = \frac{3}{2}$

$\Rightarrow s_x = 70.56$

$70.56 - 47.04 = 23.52$

$\Rightarrow$ Second wall must not be less than 23.52 m from first wall.

$s_x = 47.04t$ ... let $t = \frac{5}{2}$

$s_x = 117.6$

$117.6 - 47.04 = 70.56$

$\Rightarrow$ Second wall must not be more than 70.56 m from first wall.

Q. 10. (i) Let initial velocity = u and let the angle of projection be θ

$u_x = u\cos\theta \qquad u_y = u\sin\theta$

$s_x = ut\cos\theta$

$s_y = ut\sin\theta - \frac{1}{2}gt^2$

$s_x = 27$ and $s_y = 0$ when $t = 3$

$\Rightarrow 3u\cos\theta = 27$ and $3u\sin\theta - 44.1 = 0$

$\Rightarrow u\cos\theta = 9$ **Equation 1** $\Rightarrow u\sin\theta = 14.7$ **Equation 2**

$\Rightarrow$ Horizontal component of initial velocity = 9

Vertical component of initial velocity = 14.7

(ii) Need to find s_y when $s_x = 5.4$

$\Rightarrow 9t = 5.4$

$\Rightarrow t = 0.6$

$s_y = 14.7t - 4.9t^2$ ... let $t = 0.6$

$\Rightarrow s_y = 14.7(0.6) - 4.9(0.6)^2$

$= 7.056$ m ... height of wall

(iii) Need to find v_x and v_y when $t = 0.6$

$v_x = 9$

$v_y = 14.7 - gt$

$= 14.7 - 9.8(0.6)$

$= 8.82$

$\vec{v} = 9\vec{i} + 8.82\vec{j}$

Speed = $|\vec{v}|$

$= \sqrt{9^2 + 8.82^2}$

$= 12.6$ m/s

Q. 11. (i) $s_x = ut$

$s_y = -\frac{1}{2}gt^2$

$s_y = -0.1$ when $s_x = 2$

$ut = 2$

$\Rightarrow t = \frac{2}{u}$

$-\frac{1}{2}gt^2 = -0.1$ ... let $t = \frac{2}{u}$

$\Rightarrow \frac{1}{2}g\left(\frac{4}{u^2}\right) = 0.1$

$\Rightarrow \frac{2g}{u^2} = 0.1$

$\Rightarrow u^2 = 20g$

$\Rightarrow u = \sqrt{20g}$ m/s

$t = \frac{2}{u}$

$= \frac{2}{\sqrt{20g}}$

$= \frac{1}{7}$ s

(ii) $u = \sqrt{20g}$

$= 14$ m/s

(iii) $v_x = 14$

$v_y = -gt$

$= -9.8\left(\frac{1}{7}\right)$

$= -1.4$

$\vec{v} = 14\vec{i} - 1.4\vec{j}$

Speed = $|\vec{v}| = \sqrt{14^2 + (-1.4)^2}$

$= 14.07$ m/s

Q. 12. (i) $v_x = u\cos\alpha$

$v_y = u\sin\alpha - gt$

$s_x = ut\cos\alpha$

$s_y = ut\sin\alpha - \frac{1}{2}gt^2$

Range: s_x when $s_y = 0$

$ut\sin\alpha - \frac{1}{2}gt^2 = 0$

$\Rightarrow 2ut\sin\alpha - gt^2 = 0$

$\Rightarrow t(2u\sin\alpha - gt) = 0$

$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \qquad \underbrace{t = \frac{2u\sin\alpha}{g}}_{\text{Time of Flight}}$

$\Rightarrow \text{Range} = u\left[\frac{2u\sin\alpha}{g}\right]\cos\alpha = \frac{2u^2\sin\alpha\,\cos\alpha}{g}$

(ii) Passes through $3\vec{i} + \vec{j}$

$\Rightarrow s_x = 3$ when $s_y = 1$

$\Rightarrow ut\cos\alpha = 3$ when $ut\sin\alpha - \frac{1}{2}gt^2 = 1$

$\Rightarrow t = \frac{3}{u\cos\alpha} \quad \Rightarrow \quad u\left[\frac{3}{u\cos\alpha}\right]\sin\alpha - \frac{1}{2}g\left[\frac{9}{u^2\cos^2\alpha}\right] = 1$

$\Rightarrow 3\tan\alpha - \frac{9g}{2u^2}(1 + \tan^2\alpha) = 1$

$\Rightarrow 6u^2\tan\alpha - 9g(1 + \tan^2\alpha) = 2u^2$

$\Rightarrow 2u^2(3\tan\alpha - 1) = 9g(1 + \tan^2\alpha)$

$\Rightarrow 2u^2 = \frac{9g(1 + \tan^2\alpha)}{3\tan\alpha - 1}$

Passes through $\vec{i} + 3\vec{j} \quad \Rightarrow \quad s_x = 1$ when $s_y = 3$

$\Rightarrow ut\cos\alpha = 1$ when $ut\sin\alpha - \frac{1}{2}gt^2 = 3$

$\Rightarrow t = \frac{1}{u\cos\alpha} \quad \Rightarrow \quad u\left[\frac{1}{u\cos\alpha}\right]\sin\alpha - \frac{1}{2}g\left[\frac{1}{u^2\cos^2\alpha}\right] = 3$

$\Rightarrow \tan\alpha - \frac{g}{2u^2}(1 + \tan^2\alpha) = 3 \quad \ldots\ 2u^2 = \frac{9g(1 + \tan^2\alpha)}{3\tan\alpha - 1}$

$\Rightarrow \tan\alpha - g\left[\frac{3\tan\alpha - 1}{9g(1 + \tan^2\alpha)}\right](1 + \tan^2\alpha) = 3$

$\Rightarrow \tan\alpha - \frac{3\tan\alpha - 1}{9} = 3 \quad$ ... multiply by 3

$\Rightarrow 9\tan\alpha - 3\tan\alpha + 1 = 27$

$\Rightarrow 6\tan\alpha = 26$

$\Rightarrow \tan\alpha = \frac{13}{3} \quad$ ... as required

$$\Rightarrow \cos\alpha = \frac{3}{\sqrt{178}} \quad \text{and} \quad \sin\alpha = \frac{13}{\sqrt{178}}$$

$$2u^2 = \frac{9g(1 + \tan^2\alpha)}{3\tan\alpha - 1} = \frac{9g\left(1 + \frac{169}{9}\right)}{13 - 1} = \frac{9g + 169g}{12} = \frac{178g}{12}$$

$$\text{Range} = \frac{2u^2 \sin\alpha\cos\alpha}{g} = \frac{\left(\frac{178g}{12}\right)\left(\frac{13}{\sqrt{178}}\right)\left(\frac{3}{\sqrt{178}}\right)}{g} = \frac{13}{4}\text{ m}$$

Q. 13. $\sin\theta = \frac{3}{5} \quad \Rightarrow \quad \cos\theta = \frac{4}{5}$

$u_x = u\cos\theta = \frac{4u}{5} \qquad u_y = u\sin\theta = \frac{3u}{5}$

$v_x = \frac{4u}{5}$

$v_y = \frac{3u}{5} - gt$

$s_x = \frac{4u}{5}t$

$s_y = \frac{3u}{5}t - \frac{1}{2}gt^2$

$s_x = 240 \quad \text{when} \quad s_y = 90$

$\frac{4u}{5}t = 240$

$\Rightarrow t = \frac{300}{u}$

$\Rightarrow \frac{3u}{5}\left[\frac{300}{u}\right] - \frac{1}{2}g\left[\frac{9,0000}{u^2}\right] = 90$

$\Rightarrow 180 - \frac{441,000}{u^2} = 90$

$\Rightarrow \frac{441,000}{u^2} = 90$

$\Rightarrow u^2 = 4,900$

$\Rightarrow u = 70$ m/s

Q. 14. (i) **Bullet**

$v_x = 70\cos\theta$

$v_y = 70\sin\theta - gt$

$s_x = 70t\cos\theta$

$s_y = 70t\sin\theta - \frac{1}{2}gt^2$

Target

$v_x = 42\sqrt{2}\cos 45° = 42$

$v_y = 42\sqrt{2}\sin 45° = 42$

$s_x = 42t$

$s_y = 42t + 10$

If bullet is to hit target, then x-velocities must match

$\Rightarrow 70\cos\theta = 42 \quad \Rightarrow \quad \cos\theta = \frac{42}{70} = \frac{3}{5} \quad \Rightarrow \quad \tan\theta = \frac{4}{3}$

(ii) Hits target when y-displacements match

$\Rightarrow 70t\sin\theta - \frac{1}{2}gt^2 = 42t + 10 \quad \ldots \sin\theta = \frac{4}{5}$

$\Rightarrow 70t\left(\frac{4}{5}\right) - \frac{1}{2}gt^2 = 42t + 10$

$\Rightarrow 56t - 4.9t^2 = 42t + 10$

$\Rightarrow 4.9t^2 - 14t + 10 = 0$

$\Rightarrow 49t^2 - 140t + 100 = 0$

$\Rightarrow (7t - 10)(7t - 10) = 0$

$\Rightarrow t = \frac{10}{7}$ seconds

Horizontal distance $= s_x = 42t = 42\left(\frac{10}{7}\right) = 60$ m