Chapter 2 Exercise 2A

Q. 1. (i) $u = 0, \quad v = 10, \quad t = 5, \quad a = ?$

$$v = u + at$$
$$10 = 0 + 5a$$
$$a = 2 \text{ m/s}^2$$

(ii) $u = 0, \quad a = 2, \quad t = 5, \quad s = ?$

$$s = ut + \frac{1}{2}at^2$$
$$s = (0)(5) + \frac{1}{2}(2)(25)$$
$$= 25 \text{ m}$$

Q. 2. (i) $u = 0, \quad v = 24, \quad a = 3, \quad t = ?$

$$v = u + at$$
$$24 = 0 + 3t$$
$$t = 8 \text{ s}$$

(ii) $u = 0, \quad a = 3, \quad t = 8, \quad s = ?$

$$s = ut + \frac{1}{2}at^2$$
$$s = (0)(8) + \frac{1}{2}(3)(64) = 96 \text{ m}$$

Q. 3. $u = 0, \quad a = 3, \quad s = 6, \quad v = ?$

$$v^2 = u^2 + 2as$$
$$v^2 = 0 + 2(3)(6)$$
$$v = 6 \text{ m/s}$$

Q. 4. $u = 50, \quad v = 70, \quad s = 300, \quad a = ?$

$$v^2 = u^2 + 2as$$
$$4{,}900 = 2{,}500 + 2(a)(300)$$
$$a = 4 \text{ m/s}^2$$

$u = 50, \quad v = 70, \quad a = 4, \quad t = ?$

$$v = u + at$$
$$70 = 50 + 4t$$
$$t = 5 \text{ s}$$

Q. 5. $a = 0.5, \quad s = 600, \quad t = 40, \quad u = ?$

$$s = ut + \frac{1}{2}at^2$$
$$600 = u(40) + \frac{1}{2}(0.5)(1{,}600)$$
$$600 = 40u + 400$$
$$200 = 40u$$
$$u = 5 \text{ m/s}$$

Q. 6. $u = 3, \quad v = 11, \quad t = 6, \quad s = ?$

$$s = \left(\frac{u + v}{2}\right)t$$
$$s = \left(\frac{3 + 11}{2}\right)(6)$$
$$= 42 \text{ m}$$

Q. 7. $u = 3, \quad v = 0, \quad s = 6, \quad a = ?$

$$v^2 = u^2 + 2as$$
$$0 = 9 + 2a(6)$$
$$a = -\frac{3}{4} \text{ m/s}^2$$

$u = 3, \quad v = 0, \quad a = -\frac{3}{4}, \quad t = ?$

$$v = u + at$$
$$0 = 3 + -\left(\frac{3}{4}\right)t$$
$$t = 4 \text{ s}$$

Q. 8. (i) $u = 70, \quad v = 50, \quad t = 8, \quad a = ?$

$$v = u + at$$
$$70 = 50 + a(8)$$
$$a = -2\frac{1}{2} \text{ m/s}^2$$

$u = 70, \quad t = 8, \quad a = -2\frac{1}{2}, \quad s = ?$

$$s = ut + \frac{1}{2}at^2$$
$$s = 70(8) + \frac{1}{2}\left(-2\frac{1}{2}\right)(64)$$
$$= 560 - 80 = 480 \text{ s}$$

(ii) $u = 50, \quad v = 0, \quad a = -2\frac{1}{2}, \quad s = ?$

$$v^2 = u^2 + 2as$$
$$0 = 2{,}500 - 5s$$
$$s = 500 \text{ m}$$

Q. 9. (i) $u = 24, \quad v = 0, \quad a = -8, \quad s = ?$

$$v^2 = u^2 + 2as$$
$$0 = 576 + 2(-8)s$$
$$s = 36 \text{ m}$$

(ii) $v^2 = u^2 + 2as$

$$0 = 2{,}304 + 2(-8)s$$
$$s = 144 \text{ m}$$

Q. 10. (i) $72 \text{ km/hr} = \dfrac{72{,}000 \text{ m}}{3{,}600 \text{ s}}$

$$= 20 \text{ m/s}$$

(ii) $48 \text{ km/hr} = \dfrac{48{,}000 \text{ m}}{3{,}600 \text{ s}} = \dfrac{40}{3} \text{ m/s}$

$$v^2 = u^2 + 2as$$
$$\frac{1{,}600}{9} = 400 + 2a(500)$$
$$a = -\frac{2}{9} \text{ m/s}^2$$
$$v = u + at$$
$$\frac{40}{3} = 20 + \left(-\frac{2}{9}\right)t$$
$$t = 30 \text{ s}$$

(iii) $v^2 = u^2 + 2as$

$0 = \frac{1,600}{9} + 2\left(-\frac{2}{9}\right)s$

$s = 400$ m

Q. 11. (i) $1 \text{ km/hr} = \frac{1,000 \text{ m}}{3,600 \text{ s}} = \frac{5}{18}$ m/s

$\therefore$ 72 km/hr = 20 m/s and

54 km/hr = 15 m/s

$v^2 = u^2 + 2as$

$225 = 400 + 2(a)(35)$

$a = -2\frac{1}{2}$ m/s^2

(ii) $v^2 = u^2 + 2as$

$0 = 225 + 2\left(-2\frac{1}{2}\right)s$

$s = 45$ m

Q. 12. Let t be the time of meeting.

$s_1 = 5t + \frac{1}{2}(3)t^2 = 5t + \frac{3}{2}t^2$

$s_2 = 7t + \frac{1}{2}(2)t^2 = 7t + t^2$

$s_1 + s_2 = 162$

$12t + \frac{5}{2}t^2 = 162$

$t = 6 \left(t = -\frac{54}{5} \text{ rejected}\right)$

At $t = 6$,

$v_1 = u + at = 5 + (3)(6)$

$= 23$ m/s

$v_2 = 7 + (2)(6)$

$= 19$ m/s

Q. 13. (i) They will meet when $s_1 + s_2 = 400$.

$\therefore 3t + \frac{1}{2}(4)t^2 + 7t + \frac{1}{2}(2)t^2 = 400$

$\therefore 10t + 3t^2 = 400$

$\therefore 3t^2 + 10t - 400 = 0$

$\therefore (3t + 40)(t - 10) = 0$

$\therefore t = -\frac{40}{3}$ **OR** $t = 10$

$\therefore t = 10$ s

($t = -\frac{40}{3}$ is rejected as impossible)

(ii) $s_1 = 3(10) + \frac{1}{2}(4)(10)^2 = 230$ m

$s_2 = 7(10) + \frac{1}{2}(2)(10)^2 = 170$ m

Q. 14. (i) $30 \text{ km/hr} = \frac{30,000 \text{ m}}{3,600 \text{ s}} = \frac{25}{3}$ m/s

$v = u + at$

$\frac{25}{3} = 0 + 2t$

$t = 4\frac{1}{6}$ s

(ii) $v^2 = u^2 + 2as$

$\left(\frac{50}{3}\right)^2 = 0^2 + 2(2)s$

$s = \frac{625}{9}$ m

(iii) $v^2 = u^2 + 2as$

$0^2 = \left(\frac{50}{3}\right)^2 + 2(a)(2)$

$a = -\frac{625}{9}$ m/s^2

Exercise 2B

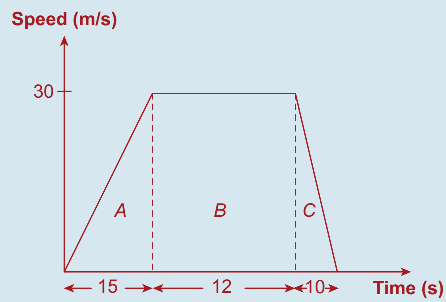
Q. 1. (i) $u = 0, \quad a = 2, \quad v = 30$

$v = u + at$

$30 = 0 + 2t$

$t = 15$ s

(ii)



Total distance covered = Area under graph

$= A + B + C$

$= \frac{1}{2}(15)(30) + (12)(30) + \frac{1}{2}(10)(30)$

$= 225 + 360 + 150$

$= 735$ m

(iii) $u = 30, \quad v = 0, \quad t = 10$

$v = u + at$

$0 = 30 + 10a$

$10a = -30$

$a = -3$ m/s^2

Magnitude of deceleration is 3 m/s^2

Q. 2. (i) $v = u + at$

$20 = 0 + a(5)$

$a = 4$ m/s^2

(ii) $s = ut + \frac{1}{2}at^2$

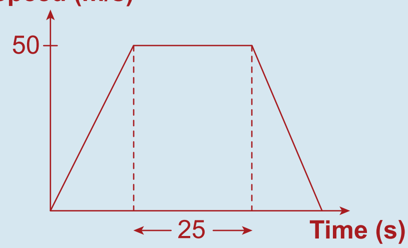
$s = 0(5) + \frac{1}{2}(4)(25) = 50$ m

(iii) Remaining distance = 200 m.

200 m at 20 m/s takes 10 seconds.

The total time taken = 5 + 10 = 15 s

Q. 3. (i)



During acceleration

$u = 0, \quad v = 50, \quad a = 5$

$v = u + at$

$50 = 0 + 5t$

$t = 10$ s

During deceleration

$u = 50, \quad v = 0, \quad a = -10$

$v = u + at$

$0 = 50 - 10t$

$10t = 50$

$t = 5$ s

Total distance travelled = Area under graph

$= \frac{1}{2}(10)(50) + (25)(50) + \frac{1}{2}(5)(50)$

$= 250 + 1{,}250 + 125$

$= 1{,}625$ m

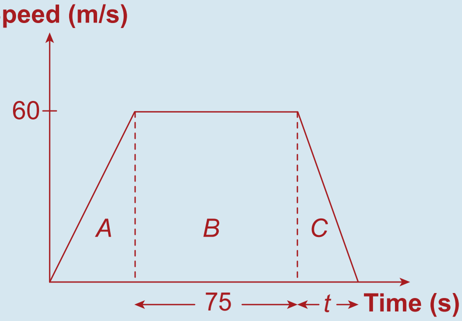
(ii) Average speed $= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{1{,}625}{10 + 25 + 5}$

$= \frac{1{,}625}{40}$

$= 40.625$ m/s

Q. 4.



(i) **During deceleration**

Area under graph = 150

Area C = 150

$\frac{1}{2}(t)(60) = 150$

$30t = 150$

$t = 5$ s

(ii) **During acceleration**

$u = 0, \quad v = 60, \quad a = 3$

$v = u + at$

$60 = 0 + 3t$

$t = 20$ s

Total distance covered = Area under graph

$= \frac{1}{2}(20)(60) + (75)(60) + 150$

$= 600 + 4{,}500 + 150$

$= 5{,}250$ m

(iii) Average speed $= \frac{5{,}250}{100}$

$= 52.5$ m/s

Q. 5. (i) $v^2 = u^2 + 2as$

$(40)^2 = 0^2 + 2(2)s$

$s = 400$ m

$v = u + at$

$40 = 0 + 2t$

$t = 20$ s

(ii) $v^2 = u^2 + 2as$

$0 = (40)^2 + 2(-5)s$

$s = 160$ m

$v = u + at$

$0 = 40 + (-5)t$

$t = 8$ s

(iii) Remainder = 1,000 − 400 − 160
= 440 m

440 m at 40 m/s takes 11 seconds.

∴ Total time = 20 + 11 + 8 = 39 s

Q. 6. (i) $v = u + at$

$27 = 0 + a(9)$

$a = 3$ m/s^2

(ii) $v^2 = u^2 + 2as$

$0 = (27)^2 + 2(a)(54)$

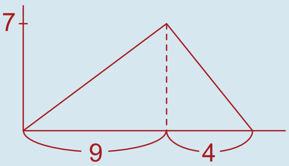
$a = -6\frac{3}{4}$ m/s^2

(iii) 2nd part:

$v = u + at$

$0 = 27 + \left(-6\frac{3}{4}\right)t$

$t = 4$ s



Distance = Area under the curve

$= \frac{1}{2}(13)(27)$

$= 175\frac{1}{2}$ m

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{175\frac{1}{2}}{13}$$

$$= 13\frac{1}{2} \text{ m/s}$$

(iv) First part:

$v = u + at$

$15 = 0 + 3t$

$t = 5$ s

2nd part:

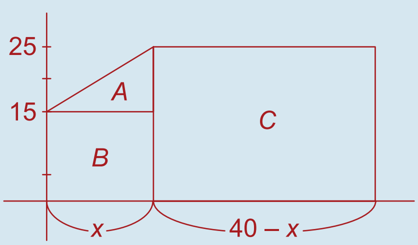
$v = u + at$

$15 = 27 + \left(-6\frac{3}{4}\right)t$

$t = 1\frac{7}{9}$ s

Answer: After 5 seconds and after $\left(9 + 1\frac{7}{9}\right) = 10\frac{7}{9}$ s

Q. 7.



Area under the curve = 980

$\frac{1}{2}(x)(10) + (15)(x) + (40 - x)(25) = 980$

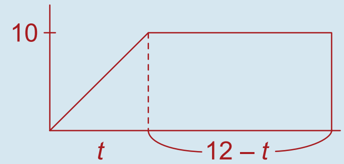
$x = 4$ s

$v = u + at$

$25 = 15 + (a)(4)$

$a = 2\frac{1}{2}$ m/s^2

Q. 8.



Let t = time spent accelerating

$\frac{1}{2}(t)(10) + (12 - t)(10) = 100$

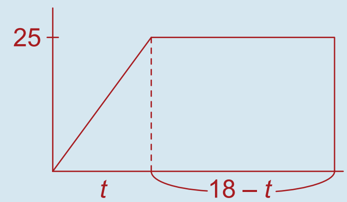
$5t + 120 - 10t = 100$

$20 = 5t$

$t = 4$

$\therefore a = \frac{10}{4} = 2.5$ m/s^2

Q. 9.



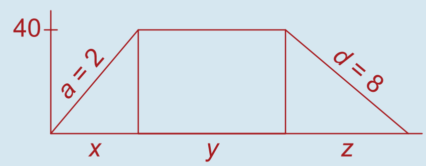
$\frac{1}{2}(t)(25) + (18 - t)(25) = 350$

$12.5t + 450 - 25t = 350$

$\therefore 100 = 12.5t$

$\therefore t = 8$ s

Q. 10.



$x = \frac{40}{2} = 20$

$z = \frac{40}{8} = 5$

Area = 700

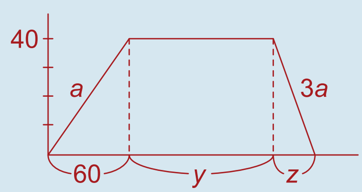
$\therefore \frac{1}{2}(20)(40) + 40y + \frac{1}{2}(5)(40) = 700$

$\therefore 400 + 40y + 100 = 700$

$\therefore y = 5$

$\therefore$ Time = 20 + 5 + 5 = 30 s

Q. 11. (i)



$a = \frac{40}{60} = \frac{2}{3}$ m/s^2

Since $60a = 40 = 3az$

$\therefore z = \frac{60a}{3a} = \frac{60}{3} = 20$ s

Area = 8,800

$\therefore \frac{1}{2}(60)(40) + 40y + \frac{1}{2}(20)(40) = 8{,}800$

$\therefore 1{,}200 + 40y + 400 = 8{,}800$

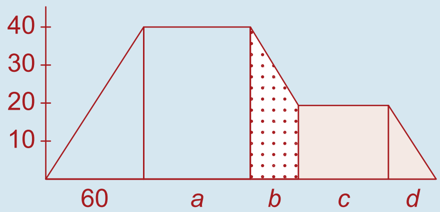
$\therefore 40y = 7{,}200$

$\therefore y = 180$

$\therefore$ Distances are 1,200, 7,200, 400 metres

Total time = 60 + 180 + 20 = 260 s

(ii)



Shaded region = 1 km = 1,000 m

The deceleration $= 3a = 3\left(\frac{2}{3}\right)$

$= 2$ m/s^2 (from (i))

$\therefore d = \frac{20}{2} = 10$ s

Shaded region = 1,000

$\therefore 20c + \frac{1}{2}(d)(20) = 1{,}000$

$\therefore 20c + \frac{1}{2}(10)(20) = 1{,}000$

$\therefore 20c + 100 = 1{,}000$

$\therefore 20c = 900$

$\therefore c = 45$

Dotted region: $u = 40$, $v = 20$, $a = -2$, $t = b$

$v = u + at$

$\therefore 20 = 40 - 2b$

$\therefore b = 10$

Area $= 10(20) + \frac{1}{2}(10)(20) = 300$ m

Total area = 8,800

$\therefore \frac{1}{2}(60)(40) + 40a + 300 + 900 + 100$

$= 8{,}800$

$\therefore 40a = 6{,}300$

$\therefore a = 157.5$

Total time = 60 + 157.5 + 10 + 45 + 10

= 282.5

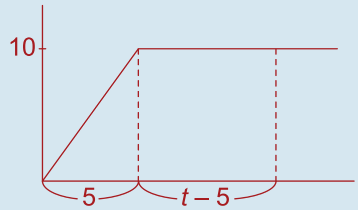
Extra time = 282.5 − 260

= 22.5 seconds more than the first time

QED

Exercise 2C

Q. 1.



$v = u + at$

$10 = 0 + 2t$

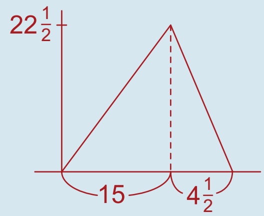
$t = 5$ s

Area under the curve = 100

$\frac{1}{2}(5)(10) + (t - 5)(10) = 100$

$t = 12.5$ s

Q. 2. (i)



$v = u + at$

$v = 0 + \left(1\frac{1}{2}\right)(15) = 22\frac{1}{2}$ m/s

$v = u + at$

$0 = 22\frac{1}{2} + (-5)t$

$t = 4\frac{1}{2}$ s

(ii) Distance = Area under the curve

$= \frac{1}{2}\left(19\frac{1}{2}\right)\left(22\frac{1}{2}\right) = 219\frac{3}{8}$ m

Q. 3. (i) $v = u + at$

$24 = 0 + 2t$

$t = 12$ s

$s = \left(\frac{u + v}{2}\right)t$

$48 = \left(\frac{24 + 0}{2}\right)t$

$t = 4$ s

(ii) First part: $a = ut + \frac{1}{2}at^2$

$= 0(12) + \frac{1}{2}(2)(12)^2$

$= 144$ m

$\therefore$ Total distance $= 144 + 48 = 192$ m

Total time $= 12 + 4 = 16$ s

$\therefore$ Average speed $= \frac{192}{16} = 12$ m/s

Q. 4. $s_1 = ut + \frac{1}{2}at^2 = 0(t) + \frac{1}{2}(4)t^2 = 2t^2$

$s_2 = 20t$

$s_1 = s_2$

$2t^2 = 20t$

$t = 10$ s

$s = 200$ m

Q. 5. (i) $v_1 = 10 + 3t$; $v_2 = 20 + 2t$

(ii) $s_1 = 10t + \frac{3}{2}t^2$; $s_2 = 20t + t^2$

(iii) $v_1 = v_2$

$10 + 3t = 20 + 2t$

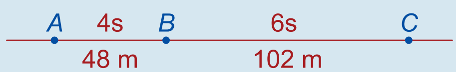
$t = 10$ s

(iv) $s_1 = s_2$

$10t + \frac{3}{2}t^2 = 20t + t^2$

$t = 0$ **OR** $t = 20$ s

Q. 6.



A to B: $u = u$, $t = 4$, $s = 48$, $a = a$

$s = ut + \frac{1}{2}at^2$

$48 = 4u + 8a$

$u + 2a = 12$

A to C: $u = u$, $t = 10$, $s = 150$, $a = a$

$s = ut + \frac{1}{2}at^2$

$150 = 10u + 50a$

$u + 5a = 15$

Solving gives $a = 1$, $u = 10$

Answer: 1 m/s^2

Q. 7. (i) $s = ut + \frac{1}{2}at^2$

$18 = u(2) + \frac{1}{2}(a)(4)$

$u + a = 9$ (1st part)

$s = ut + \frac{1}{2}at^2$

$48 = u(4) + \frac{1}{2}(a)(16)$

$u + 2a = 12$ (1st and 2nd parts)

Solving these gives (i) $a = 3$ m/s^2

(ii) $u = 6$ m/s

(iii) First 6 seconds:

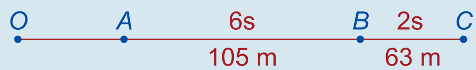
$s = ut + \frac{1}{2}at^2$

$s = (6)(6) + \frac{1}{2}(3)(36) = 90$ m

The distance travelled $= 90 - 48$

$= 42$ m

Q. 8.



(i) A to B: $u = u$, $s = 105$, $t = 6$, $a = a$

$s = ut + \frac{1}{2}at^2$

$105 = 6u + 18a$

A to C: $u = u$, $s = 168$, $t = 8$, $a = a$

$s = ut + \frac{1}{2}at^2$

$168 = 8a + 32a$

Solving gives $a = 3.5$, $u = 7$

Answer: $a = 3.5$ m/s

(ii) O to A: $u = 0$, $v = 7$, $a = 3.5$, $s = s$

$v^2 = u^2 + 2as$

$49 = 0 + 7s$

$s = 7$ m

Q. 9. 1st part: $s = ut + \frac{1}{2}at^2$

$39 = u(1) + \frac{1}{2}a(1)^2$

$2u + a = 78$

1st and 2nd parts: $76 = u(2) + \frac{1}{2}a(2)^2$

$2u + 2a = 76$

First three parts: $111 = u(3) + \frac{1}{2}a(3)^2$

$6u + 9a = 222$

$u + 3a = 74$

Solving the first two equations gives $a = -2$, $u = 40$

Solving the last two equations gives $a = -2$, $u = 40$

$\therefore$ They are consistent.

Stopping: $v^2 = u^2 + 2as$

$0 = (40)^2 + 2(-2)s$

$s = 400$ m

$\therefore$ It will travel a further $400 - 111 = 289$ m

Q. 10. a to b: $s = ut + \frac{1}{2}at^2$

$20 = u(5) + \frac{1}{2}(a)(5)^2$

$2u + 5a = 8$

a to c: $40 = u(8) + \frac{1}{2}a(8)^2$

$2u + 8a = 10$

Solving these gives $u = \frac{7}{3}$ m/s, $a = \frac{2}{3}$ m/s^2

a to d: $a = ut + \frac{1}{2}at^2$

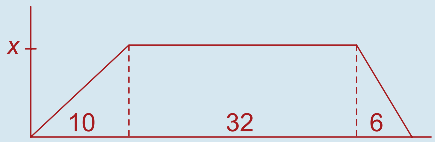
$60 = \frac{7}{3}t + \frac{1}{2}\left(\frac{2}{3}\right)t^2$

$t^2 + 7t - 180 = 0$ (use formula)

$t = 10.4$ s (-17.4 is rejected).

The time taken $= 10.4 - 8 = 2.4$ s

Q. 11.



Area $= \frac{1}{2}(10)x + 32x + \frac{1}{2}(6)x = 1{,}000$

$x = 25$ m/s

Distances are: $\frac{1}{2}(10)(25) = 125$;

$32(25) = 800$;

$\frac{1}{2}(6)(25) = 75$ m

Q. 12. (i) $t_1 : t_2 = 3 : 1 = \frac{3}{4} : \frac{1}{4}$

$\therefore t_1 = \frac{3}{4}(20) = 15$ s

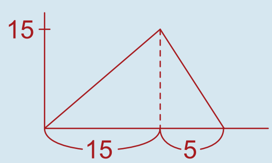
$\therefore t_2 = \frac{1}{4}(20) = 5$ s

(ii) $v = u + at$

$v = 0 + (1)(15)$

$= 15$ m/s

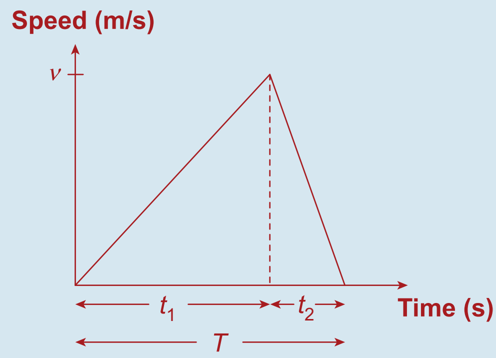
(iii)



$s =$ Area under curve

$= \frac{1}{2}(20)(15) = 150$ m

Q. 13.



Let the top speed $= v$

$t_1 = \frac{v}{2}$

$t_2 = \frac{v}{7}$

The time taken $= 90$ seconds $= T$

$T = \frac{v}{2} + \frac{v}{7} = \frac{7v + 2v}{14}$

$= \frac{9v}{14}$

$\frac{9v}{14} = 90$

$9v = 1{,}260$

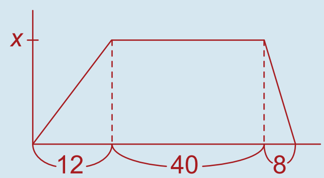
$v = 140$ m/s

Distance $= \frac{1}{2}(90)(140)$

$= 6{,}300$ m

$= 6.3$ km

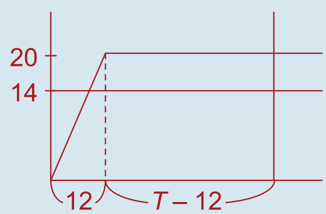
Q. 14. (i)



Area $= \frac{1}{2}(12)x + 40x + \frac{1}{2}(8)x = 1{,}000$

$x = 20$ m/s

(ii)



$\text{Area}_1 = \text{Area}_2$

$14T = \frac{1}{2}(12)(20) + 20(T - 12)$

$T = 20$ s

$s = 14T$

$= 280$ m

Q. 15. 1st part: $s = ut + \frac{1}{2}at^2$

$$24 = u(2) + \frac{1}{2}a(2)^2$$

$$u + a = 12$$

1st and 2nd parts: $48 = u(3) + \frac{1}{2}a(3)^2$

$$2u + 3a = 32$$

Solving these gives $a = 8$, $u = 4$

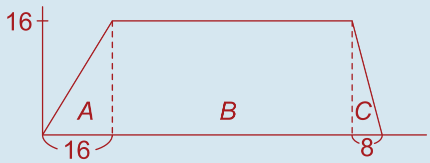
First three parts: $72 = 4t + \frac{1}{2}(8)t^2$

$$t^2 + t - 18 = 0$$

$t = 3.772$ ($t = -4.772$ is rejected)

Time taken $= 3.772 - 3$
$= 0.772$ s

Q. 16. (i)



Distance in $A = \frac{1}{2}(16)(16) = 128$ m

Distance in $C = \frac{1}{2}(8)(16) = 64$ m

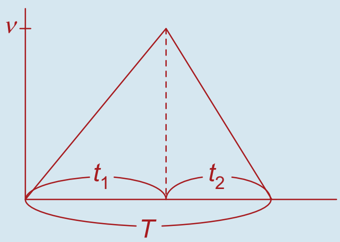
Total distance travelled $= 192$ m

Remainder for $B = 300 - 192$
$= 108$ m

Time taken $= \frac{108}{16} = 6.75$ s

Total time $= 16 + 6.75 + 8$
$= 30.75$ s

(ii)



Let T = the time taken

$t_1 : t_2 = d : a = 2 : 1 = \frac{2}{3} : \frac{1}{3}$

$\therefore t_1 = \frac{2}{3}T$ and $t_2 = \frac{1}{3}T$

$$v = u + at$$

$$v = 0 + (1)\left(\frac{2}{3}T\right) = \frac{2}{3}T$$

Area $= 300$

$$\frac{1}{2}(T)\left(\frac{2}{3}T\right) = 300$$

$T = 30$ s

Q. 17. (i) $s_1 = 0 + \frac{1}{2}\left(\frac{1}{2}\right)t^2 = \frac{1}{4}t^2$

$$s_2 = 0 + \frac{1}{2}(1)t^2 = \frac{1}{2}t^2$$

Distance apart, $s = s_1 + s_2$

$$= \frac{3}{4}t^2$$

When $t = 10$, $s = \frac{3}{4}(100) = 75$ m

(ii) When $s = 108$

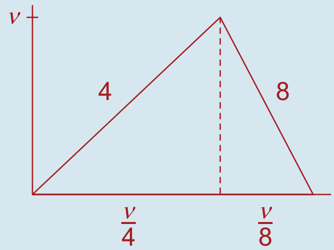
$$\frac{3}{4}t^2 = 108$$

$$t^2 = 144$$

$$t = 12 \text{ s}$$

This is $12 - 10 = 2$ seconds later

Q. 18. Let v = top speed



$$\frac{1}{2}\left(\frac{v}{4} + \frac{v}{8}\right)v = 1{,}200$$

$$\frac{1}{2}\left(\frac{3v}{8}\right)v = 1{,}200$$

$$\frac{3v^2}{16} = 1{,}200$$

$$v^2 = 6{,}400$$

$$v = 80 \text{ m/s}$$

$\therefore$ Time $= \frac{v}{4} + \frac{v}{8} = \frac{80}{4} + \frac{80}{8} = 30$ s

Q. 19. Let t = time after cyclist passes P

Greatest gap $v_1 = v_2$

$$12 = 0 + 1(t - 5)$$

$$t = 17$$

at $t = 17$, $s_1 = 12(17) = 204$ m

$s_2 = \frac{1}{2}(1)(17 - 5)^2 = 72$ m

Gap $= 204 - 72 = 132$ m

Q. 20. Greatest gap $\Rightarrow$ $v_1 = v_2$

$$8 + 4t = 30 + 3(t - 2)$$

$$8 + 4t = 30 + 3t - 6$$

$$t = 16$$

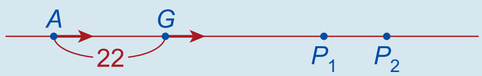
at $t = 16$, $s_1 = 8(16) + \frac{1}{2}(4)(16)^2 = 640$

at $t = 16 - 2 = 14$,

$s_2 = 30\,(14) + \frac{1}{2}(3)(14)^2 = 714$

Gap $= 714 - 640 = 74$ m **QED**

Q. 21.



After t seconds, Alberto has travelled

$s_1 = 12t + \frac{1}{2}t^2$

After t seconds, Gustav has travelled $s_2 = t^2$

At both P_1 and P_2,

$$s_1 = s_2 + 22$$

$$12t + \frac{1}{2}t^2 = t^2 + 22$$

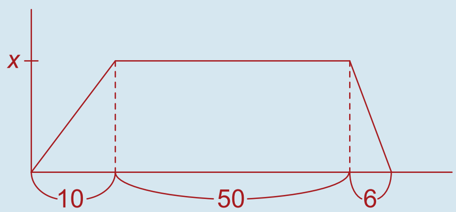
$$t^2 - 24t + 44 = 0$$

$$t = 2, 22$$

Answer: (i) After 2 seconds

(ii) After 20 seconds more

Q. 22.



Area under the curve $= 696$

$$\frac{1}{2}(10)x + 50x + \frac{1}{2}(6)x = 696$$

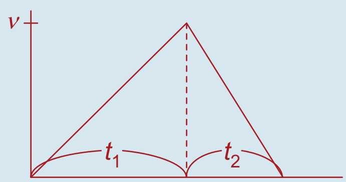
$$x = 12$$

$$v = u + at$$

$$12 = 0 + a_1(10)$$

$$a_1 = 1.2 \text{ m/s}^2$$

Similarly, $a_2 = 2$ m/s^2



$$t_1 : t_2 = 2 : 1.2 = 5 : 3 = \frac{5}{8} : \frac{3}{8}$$

$$\therefore t_1 = \frac{5}{8}T, \quad t_2 = \frac{3}{8}T$$

$$v = u + at$$

$$v = 0 + (1.2)\left(\frac{5}{8}T\right) = \frac{3}{4}T$$

Area $= 696$

$$\frac{1}{2}(T)\left(\frac{3}{4}T\right) = 696$$

$$T^2 = 1{,}856$$

$$T = 8\sqrt{29}$$

Q. 23. Take speeds, accelerations, distances relative to the goods train.

Let p = the passenger train and g = the goods train.

The initial relative speed,

$$u_{pg} = u_p - u_g$$

$$= 80 - 30$$

$$= 50 \text{ m/s}$$

The relative distance $= 1{,}500$ m

The final relative speed is zero, since the two trains must eventually be travelling at the same speed to avoid a crash.

$$v^2 = u^2 + 2as$$

$$0 = (50)^2 + 2a(1{,}500)$$

$$a = -\frac{5}{6} \text{ m/s}^2$$

The relative deceleration is, therefore, $\frac{5}{6}$ m/s^2

The actual deceleration of the passenger train is $\frac{5}{6}$ m/s^2, since the goods train does not decelerate at all.

Q. 24. (i) Initial relative speed,

$u = 20 - 8 = 12$ m/s

Relative distance $= 120$ m

Final relative speed $= 0$ m/s

$$v^2 = u^2 + 2as$$

$$0 = (12)^2 + 2a(120)$$

$$a = -\frac{3}{5} \text{ m/s}^2$$

(ii) (i) $u = 12$

$a = -1$

$s = 120 - 66 = 54$

$t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$54 = 12t + \frac{1}{2}(-1)t^2$$

$$t^2 - 24t + 108 = 0$$

$$(t - 6)(t - 18) = 0$$

$$t = 6, 18$$

Answer: After 6 seconds

(ii) $u = 12$

$a = -1$

$s = s$

$t = t$

$s = ut + \frac{1}{2}at^2$

$s = 12t - \frac{1}{2}t^2$

$\frac{ds}{dt} = 12 - t$

$= 0$ (Since s is a minimum)

$t = 12$

At $t = 12$, $s = 12(12) - \frac{1}{2}(12)^2 = 72$ m

This means that they have travelled a distance of 72 m towards each other, and so the distance between then will be $120 - 72 = 48$ m.

Exercise 2D

Q. 1. (i) $s = 35t - 4.9t^2 = 0$

$\therefore t(35 - 4.9\,t) = 0$

$\therefore t = 0$ **OR** $t = \frac{350}{49} = \frac{50}{7}$ s

(ii) $v = 0$

$35 - 9.8t = 0$

$\therefore t = \frac{350}{98} = \frac{50}{14} = \frac{25}{7}$

$s = 35\left(\frac{25}{7}\right) - 4.9\left(\frac{25}{7}\right)^2$

$= 125 - 62.5 = 62.5$ m

Q. 2. (i) $u(1) - 4.9(1)^2 = 16.1$

$\therefore u - 4.9 = 16.1$

$\therefore u = 21$ m/s

(ii) $v = 0$

$21 - 9.8t = 0$

$t = \frac{21}{9.8} = \frac{210}{98} = \frac{15}{7}$

at $t = \frac{15}{7}$,

$s_y = 21\left(\frac{15}{7}\right) - 4.9\left(\frac{15}{7}\right)^2$

$= 45 - 22.5 = 22.5$ m

(iii) $s_y = 0$

$21t - 4.9t^2 = 0$

$\therefore t = 0$ **OR** $t = \frac{21}{4.9}$

$= \frac{210}{49} = \frac{30}{7}$ s

Q. 3. (i) $s_1 = s_2$

$\therefore 0(t) + 4.9t^2$

$= 14.7\,(t - 1) + 4.9\,(t - 1)^2$

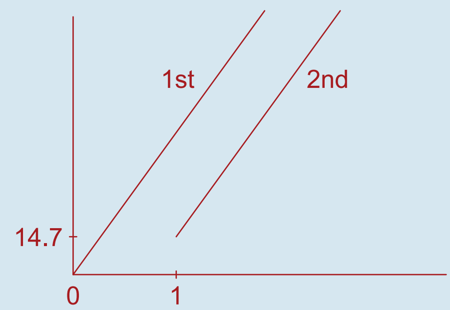
$\therefore t^2 = 3(t - 1) + (t - 1)^2$

$\therefore t^2 = 3t - 3 + t^2 - 2t + 1$

$\therefore t = 2$

(ii) $4.9(2)^2 = 19.6$ m

(iii)



Q. 4. (i) Let t = time Q is in motion.

$\therefore t + 2$ = time P is in motion.

$s_P = s_Q$

$47(t + 2) - 4.9(t + 2)^2 = 64.6t - 4.9t^2$

$47t + 94 - 4.9t^2 - 19.6t - 19.6$
$= 64.6t - 4.9t^2$

$74.4 = 37.2t$

$\therefore t = 2$ s

(ii) $64.6(2) - 4.9(2)^2 = 109.6$ m

Q. 5. First t seconds

$u = u$, $s = 70$, $a = -9.8$, $t = t$

$\therefore 70 = ut - 4.9t^2$ … **Equation 1**

First $2t$ seconds

$u = u$, $s = 70 + 50 = 120$,
$a = -9.8$, $t = 2t$

$120 = 2ut - 4.9\,(2t)^2$

$\therefore 120 = 2ut - 19.6t^2$ … **Equation 2**

Eq 2: $120 = 2ut - 19.6t^2$

$-2 \times$ **Eq. 1:** $\underline{-140 = -2ut + 9.8t^2}$

$-20 = -9.8t^2$

$200 = 98t^2$

$\therefore t^2 = \frac{100}{49}$

$\therefore t = \frac{10}{7}$ s

$70 = u\left(\frac{10}{7}\right) - 4.9\frac{100}{49}$

$\therefore 70 = \frac{10u}{7} - 10$

$\therefore u = 56 \text{ m/s}$

Q. 6. (i) $u = u, \quad a = -9.8, \quad s = -30, \quad t = 5$

$s = ut + \frac{1}{2}at^2$

$\therefore -30 = 5u + \frac{1}{2}(-9.8)(25)$

$\therefore 92.5 = 5u$

$\therefore u = 18.5 \text{ m/s}$

(ii) $v = u + at$

$= 18.5 + (-9.8)(5)$

$= -30.5$

$\therefore$ Speed $= 30.5$ m/s

Q. 7. $49t - 4.9t^2 = 78.4$

$\therefore 10t - t^2 = 16$

$\therefore t^2 - 10t + 16 = 0$

$\therefore (t - 2)(t - 8) = 0$

$\therefore t = 2, 8$

$\therefore t_1 = 2, t_2 = 8$

$\therefore t_1 + t_2 = 10$ **QED**

OR

$t_1 + t_2 = -\frac{b}{a}$

$\therefore t_1 + t_2 = -\frac{(-10)}{1}$

$\therefore t_1 + t_2 = 10$

Q. 8. $70t - 4.9t^2 = 210$

$\therefore 700t - 49t^2 = 2{,}100$

$\therefore 7t^2 - 100t + 300 = 0$

Product of roots $= \frac{c}{a}$

$\therefore t_1 t_2 = \frac{300}{7}$

$\therefore 7\, t_1 t_2 = 300$ **QED**

Exercise 2E

Q. 1. $s = d, \quad t = n, \quad u = 0, \quad a = a$

$s = ut + \frac{1}{2}at^2$

$d = \frac{1}{2}an^2 \ldots$ **Equation A**

$a = d + k, \quad t = 2n, \quad u = 0, \quad a = a$

$s = ut + \frac{1}{2}at^2$

$d + k = \frac{1}{2}a(2n)^2$

$d + k = 2an^2 \ldots$ **Equation B**

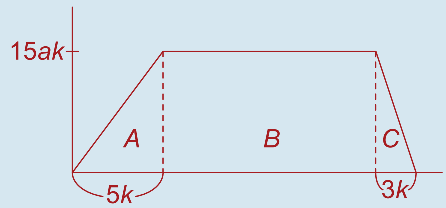
$4 \times$ **Equation A** $\Rightarrow$ $2an^2 = 4d$

But $2an^2 = d + k$

$\therefore d + k = 4d$

$k = 3d$ **QED**

Q. 2. (i)



Distance in $A = \frac{1}{2}(5k)(15ak) = 37\frac{1}{2}ak^2$

Distance in $C = \frac{1}{2}(3k)(15ak) = 22\frac{1}{2}ak^2$

Remainder $= 90ak^2 - 37\frac{1}{2}ak^2 - 22\frac{1}{2}ak^2$

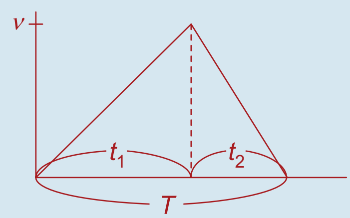
$= 30ak^2$

Time for $B = \frac{30ak^2}{15ak} = 2k$ s

Total time $= 5k + 2k + 3k$

$= 10k$ s

(ii)



Let T = the total time

$t_1 : t_2 = d : a = 5a : 3a = \frac{5}{8} : \frac{3}{8}$

$\therefore t_1 = \frac{5}{8}T$ and $t_2 = \frac{3}{8}T$

$v = u + at$

$v = 0 + (3a)\left(\frac{5}{8}T\right) = \frac{15}{8}aT$

Area $= 90ak^2$

$\frac{1}{2}(T)\left(\frac{15}{8}aT\right) = 90ak^2$

$T^2 = 96k^2$

$T = 4\sqrt{6}\,k$

Q. 3. $h = ut + \frac{1}{2}(-g)t^2$

$2h = 2ut - gt^2$

$gt^2 - 2ut + 2h = 0$

Product of roots $= \frac{c}{a}$

$\therefore t_1 t_2 = \frac{2h}{g}$ **QED**

Q. 4. Let $|PQ| = |QR| = x$

(i) The journey $P \to R$: $u = u, \quad v = 7u,$
$s = 2x, \quad a = a$

$\therefore v^2 = u^2 + 2as$

$\therefore 49u^2 = u^2 + 2(a)(2x)$

$\therefore 48u^2 = 4ax$

$\therefore 12u^2 = ax$

$\therefore a = \frac{12u^2}{x}$

The journey $P \to Q$: $u = u, \quad v = v,$

$s = x, \quad a = \frac{12u^2}{x}$

$v^2 = u^2 + 2as$

$\therefore v^2 = u^2 + 2\left(\frac{12u^2}{x}\right)(x)$

$\therefore v^2 = u^2 + 24u^2$

$\therefore v^2 = 25u^2$

$\therefore v = 5u$

(ii) P to Q: $u = u, \quad v = 5u, \quad s = x,$
$t = t_1, \quad a = a$

$v = u + at$

$\therefore 5u = u + at_1$

$\therefore t_1 = \frac{4u}{a}$

Q to R: $u = 5u, \quad v = 7u, \quad t = t_2,$
$a = a$

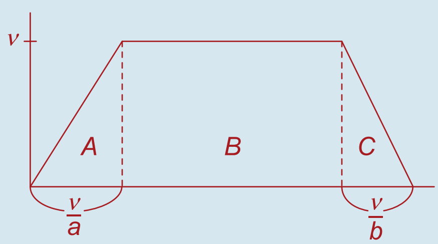
$v = u + at$

$\therefore 7u = 5u + at_2$

$\therefore t_2 = \frac{2u}{a}$

$\therefore t_1 = 2t_2$ **QED**

Q. 5. (i)



Distance $A = \frac{1}{2}\left(\frac{v}{a}\right)v = \frac{v^2}{2a}$

Distance $C = \frac{1}{2}\left(\frac{v}{b}\right)v = \frac{v^2}{2b}$

Remainder $= s - \frac{v^2}{2a} - \frac{v^2}{2b}$

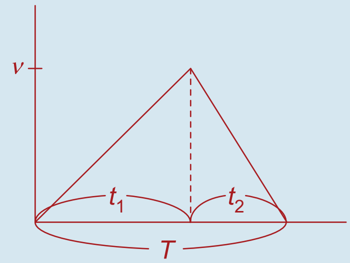
Time taken in $B = \left(s - \frac{v^2}{2a} - \frac{v^2}{2b}\right) \div v$

$= \frac{s}{v} - \frac{v}{2a} - \frac{v}{2b}$

Total time taken $= \frac{v}{a} + \left(\frac{s}{v} - \frac{v}{2a} - \frac{v}{2b}\right) + \frac{v}{b}$

$= \frac{v}{2a} + \frac{v}{2b} + \frac{s}{v}$ **QED**

(ii)



Let T = the total time

$t_1 : t_2 = b : a = \frac{b}{a+b} : \frac{a}{a+b}$

$\therefore t_1 = \frac{bT}{a+b}, \quad t_2 = \frac{aT}{a+b}$

$v = u + at$

$v = 0 + (a)\left(\frac{bT}{a+b}\right) = \frac{abT}{a+b}$

Area $= s$

$\frac{1}{2}(T)\left(\frac{abT}{a+b}\right) = s$

$T = \sqrt{2s\left(\frac{a+b}{ab}\right)}$ **QED**

Q. 6. (a) $t = n - 1, \quad u = u, \quad a = a, \quad s = s_1$

$s = ut + \frac{1}{2}at^2$

$s_1 = u(n-1) + \frac{1}{2}a(n-1)^2$

$= un - u + \frac{1}{2}an^2 - an + \frac{1}{2}a$

$t = n, \quad u = u, \quad a = a, \quad s = s_2$

$s = ut + \frac{1}{2}at^2$

$s_2 = un + \frac{1}{2}an^2$

Distance travelled, $s = s_2 - s_1$

$= u + an - \frac{1}{2}a$ **QED**

(b) When $n = 2$, $s = 17$

$17 = u + 2a - \frac{1}{2}a$

$u + 1\frac{1}{2}a = 17$

When $n = 7$, $s = 47$

$47 = u + 7a - \frac{1}{2}a$

$u + 6\frac{1}{2}a = 47$

Solving these gives $a = 6$, $u = 8$

(i) $n = 10$

$s = u + an - \frac{1}{2}a$

$= 8 + (6)(10) - \frac{1}{2}(6)$

$= 65$ m

(ii) $n = n$

$s = 8 + 6n - \frac{1}{2}(6)$

$= (6n + 5)$ m

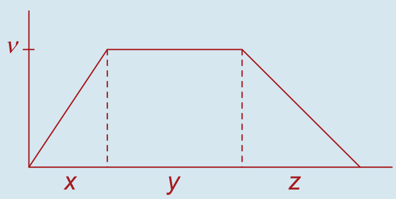
(c) $s_n + s_{n+1} = 256$

$6n + 5 + 6(n + 1) + 5 = 256$

$n = 20$

Answer: In the 20th and 21st seconds

Q. 7.



Average speed $= \dfrac{\text{total distance}}{\text{total time}}$

$$= \frac{\frac{1}{2}xv + yv + \frac{1}{2}zv}{x + y + z}$$

$$= \frac{5v}{6}$$

$\therefore 5xv + 5yv + 5zv = 3xv + 6yv + 3zv$

$\therefore 5x + 5y + 5z = 3x + 6y + 3z$

$\therefore 2x + 2z = y$

$\therefore x + z = \frac{1}{2}y$

Fraction of distance travelled at constant speed $= \dfrac{yv}{\frac{1}{2}xv + yv + \frac{1}{2}zv}$

$$= \frac{y}{\frac{1}{2}x + y + \frac{1}{2}z}$$

$$= \frac{2y}{x + 2y + z}$$

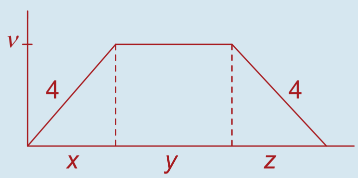
$$= \frac{2y}{(x + z) + 2y}$$

$$= \frac{2y}{\frac{1}{2}y + 2y}$$

$$= \frac{2y}{2\frac{1}{2}y}$$

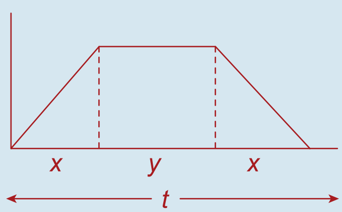
$= \frac{4}{5}$ **QED**

Q. 8. (i)



(ii) $v = 4x = 4z$

$\therefore x = z$ and $v = 4x$



$2x + y = t$

$\therefore x = \dfrac{t - y}{2}$

$\therefore v = 4x = 2(t - y)$

Area under curve $= d$

$$\frac{1}{2}\left(\frac{t - y}{2}\right)(2(t - y)) + y(2)(t - y) + \frac{1}{2}\left(\frac{t - y}{2}\right)(2(t - y)) = d$$

$\therefore \frac{1}{2}(t - y)^2 + 2yt - 2y^2 + \frac{1}{2}(t - y)^2 = d$

$\therefore (t - y)^2 + 2yt - 2y^2 = d$

$\therefore t^2 - 2yt + y^2 + 2yt - 2y^2 = d$

$\therefore t^2 - y^2 = d$

$\therefore y^2 = t^2 - d$

$\therefore y = \sqrt{t^2 - d}$ **QED**