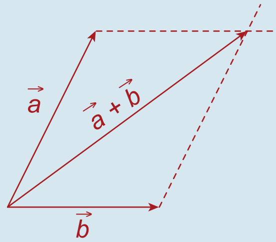
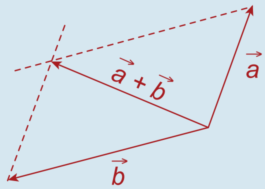
Chapter 1 Exercise 1A

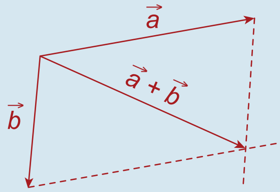
Q. 1. (i)



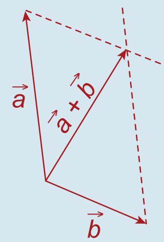
(ii)



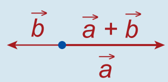
(iii)



(iv)

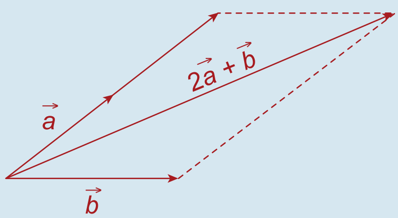


(v)

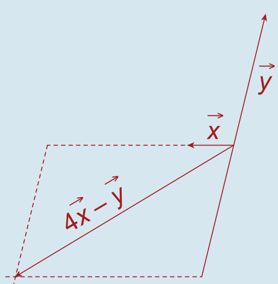


(vi) $\vec{0}$, The null vector

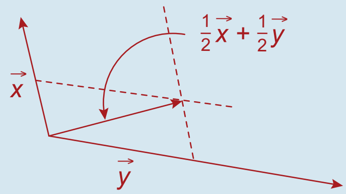
Q. 2.



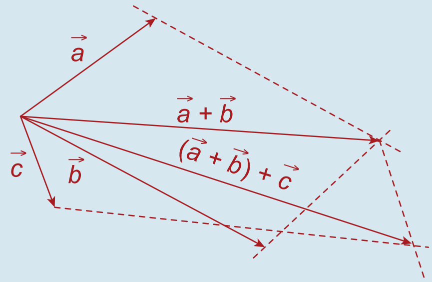
Q. 3.



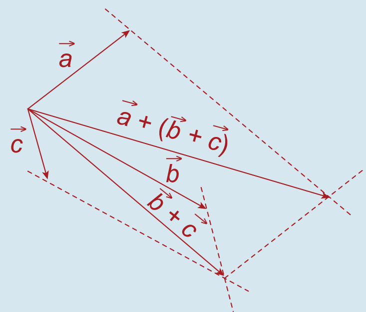
Q. 4.



Q. 5.

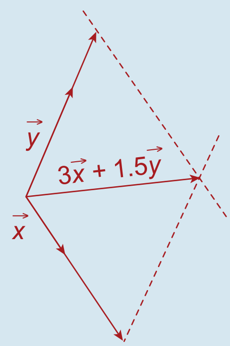


Q. 6.

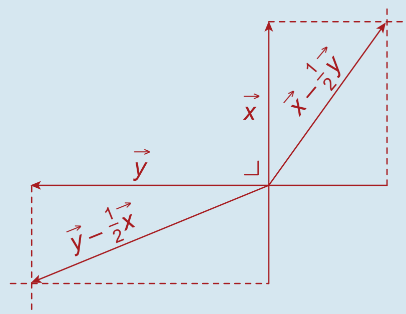


Yes; $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$.

Q. 7.



Q. 8.



Q. 9. 5 cm; E 53° N.

Q. 10. Approximately 7 cm due East.

Q. 11. 13 cm.

Exercise 1B

Q. 1. (i) $\sqrt{29}$, E 21° 48′ N.

(ii) $\sqrt{8}$, NE.

(iii) 5, E 36° 52′ S.

(iv) 13, W 67° 23′ S.

(v) $\sqrt{20}$, W 26° 34′ N.

(vi) $\sqrt{2}$, NE.

(vii) $\frac{1}{\sqrt{2}}$, SE.

(viii) 1, W 53° 8′ S.

(ix) 2, W 30° N.

(x) $\sqrt{12}$, E 30° N.

(xi) 4, due West.

Q. 2. (i) $\vec{a} + \vec{b} = (3\vec{i} - \vec{j}) + (2\vec{i} - 3\vec{j})$
$= 5\vec{i} - 4\vec{j}$

(ii) $\vec{a} - \vec{b} = (3\vec{i} - \vec{j}) - (2\vec{i} - 3\vec{j})$
$= \vec{i} + 2\vec{j}$

(iii) $\vec{b} - \vec{a} = (2\vec{i} - 3\vec{j}) - (3\vec{i} - \vec{j})$
$= -\vec{i} - 2\vec{j}$

(iv) $2\vec{a} - 3\vec{b} = 2(3\vec{i} - \vec{j}) - 3(2\vec{i} - 3\vec{j})$
$= 7\vec{j}$

Q. 3. $\vec{x} + \vec{y} = (2\vec{i} + 3\vec{j}) + (10\vec{i} + 2\vec{j})$
$= 12\vec{i} + 5\vec{j}$

(i) $\sqrt{13}$

(ii) $\sqrt{104}$

(iii) $|\vec{x} + \vec{y}| = \sqrt{12^2 + 5^2} = 13$

(iv) $|\vec{x}| + |\vec{y}| = \sqrt{13} + \sqrt{104}$
$= 3.606 + 10.20$
$= 13.806$

$\therefore |\vec{x} + \vec{y}| < |\vec{x}| + |\vec{y}|$
(since $13 < 13.806$)

Q. 4. (i) $4\vec{i} + 8\vec{j}$

(ii) $\sqrt{16 + 64} = \sqrt{80} = 8.944$

(iii) $\sqrt{80} < \sqrt{10} + \sqrt{50}$
since $8.944 < 10.233$

Q. 5. $\sqrt{5} \geqslant \sqrt{20} - \sqrt{5}$
since $2.236 \geqslant 4.472 - 2.236$

Q. 6. (i) Magnitude $= \sqrt{3^2 + 4^2} = 5$

$\therefore$ Unit vector $= \frac{1}{5}(3\vec{i} + 4\vec{j})$

(ii) $\frac{1}{\sqrt{5}}(\vec{i} + 2\vec{j})$

(iii) $\frac{1}{\sqrt{2}}(\vec{i} - \vec{j})$

(iv) $\frac{1}{\sqrt{10}}(-3\vec{i} - \vec{j})$

(v) $\frac{1}{2}(\sqrt{3}\,\vec{i} + \vec{j})$

Q. 7. $k(2\vec{i} - \vec{j}) + l(4\vec{i} + 3\vec{j}) = 2\vec{i} - 11\vec{j}$

$\therefore 2k + 4l = 2$ and $-k + 3l = -11$

Solving gives $l = -2$, $k = 5$

Q. 8. $4\vec{i} - 2\vec{j} + t(7\vec{i} + 5\vec{j}) = k\vec{i} + 0\vec{j}$

$\therefore 4 + 7t = k$ and $-2 + 5t = 0$

$\therefore t = 0.4$

Q. 9. $\sqrt{65} = \sqrt{49 + k^2}$

$\therefore k = \pm 4$

Q. 10. $\sqrt{50} = \sqrt{2p^2}$

$\therefore p = \pm 5$

Q. 11. $\sqrt{121 + k^2} = \sqrt{125}$

$\therefore k = \pm 2$

Q. 12. $\sqrt{2k^2} = \sqrt{50}$

$\therefore k = \pm 5$

Q. 13. $\frac{3}{4} \times \frac{-8}{6} = \frac{-24}{24} = -1$

$\therefore \perp$

Q. 14. $\frac{-2}{5} \times \frac{20}{8} = \frac{-40}{40} = -1$

$\therefore \perp$

Q. 15. $\frac{3}{1} \times \frac{-2}{6} = \frac{-6}{6} = -1$

$\therefore \perp$

Q. 16. $\frac{-t}{9} \times \frac{6}{2} = -1$

$\therefore t = 3$

Q. 17. $\frac{p}{4} \times \frac{-2}{p + 1} = -1$

$\therefore 4p + 4 = 2p$

$\therefore p = -2$

Exercise 1C

Q. 1. (i) $|AB| = H\cos\theta;\ |BC| = H\sin\theta$

(ii) $|AB| = H\sin\theta;\ |BC| = H\cos\theta$

(iii) $|AC| = H\cos\theta;\ |BC| = H\sin\theta$

(iv) $|AC| = H\cos\theta;\ |AB| = H\sin\theta$

Q. 2. (i) $\cos A = \frac{12}{13},\ \sin\theta = \frac{5}{13}$

(ii) $\cos A = \frac{35}{37},\ \sin\theta = \frac{12}{37}$

(iii) $\sin A = \frac{\sqrt{7}}{4},\ \tan A = \frac{\sqrt{7}}{3}$

(iv) $\cos = \frac{40}{41}$

Q. 3. (i) $|AB| = 4\sqrt{3}$ cm; $|BC| = 4$ cm

(ii) $|XY| = 2$ m; $|YZ| = 2$ m

(iii) $|AB| = 10\sin 40° = 6.428$ m

$|BC| = 10\cos 40° = 7.66$ m

(iv) $|XY| = 20\cos 35° = 16.38$ cm

$|XZ| = 20\sin 35° = 11.47$ cm

(v) $|PQ| = 40\cos 20° = 37.59$ m

$|QR| = 40\sin 20° = 13.68$ m

(vi) $|PQ| = 12\cos 60° = 6$ m

$|RQ| = 12\sin 60° = 6\sqrt{3}$ m

(vii) $|AB| = 15\cos\theta = 12$ cm

$|BC| = 15\sin\theta = 9$ cm

(viii) $|RQ| = 78\cos\alpha = 30$ m

$|PQ| = 78\sin\alpha = 72$ m

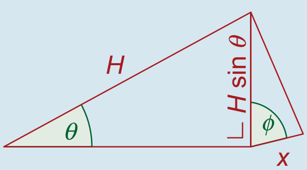
(ix) $|XY| = \sqrt{13}\cos\theta = 3$

$|YZ| = \sqrt{13}\sin\theta = 2$

(x) $|AB| = \sqrt{20}\cos\alpha = 4$

$|BC| = \sqrt{20}\sin\alpha = 2$

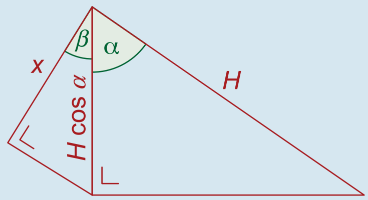
Q. 4.



$x = \text{ADJ} = H\sin\theta\cos\phi$

$\therefore x = (13)\left(\frac{3}{5}\right)\left(\frac{5}{13}\right) = 3$

Q. 5.



$x = \text{ADJ} = H\cos\alpha\cos\beta$

$\therefore x = H\left(\frac{7}{\sqrt{50}}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{7}{10}H$

$\therefore H : x = 10 : 7$

Exercise 1D

Q. 1. (i) $2\cos 60°\vec{i} + 2\sin 60°\vec{j} = \vec{i} + \sqrt{3}\vec{j}$

(ii) $10\cos 18°\vec{i} + 10\sin 18°\vec{j}$

$= 9.511\vec{i} + 3.09\vec{j}$

(iii) $8\cos 45°\vec{i} - 8\sin 45°\vec{j}$

$= 4\sqrt{2}\vec{i} - 4\sqrt{2}\vec{j}$

(iv) $-20\cos 20°\vec{i} + 20\sin 20°\vec{j}$

$= -18.794\vec{i} + 6.84\vec{j}$

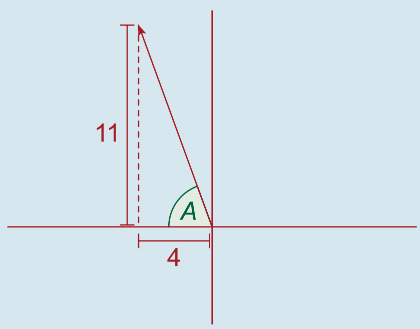
(v) $-\sqrt{50}\cos 45°\vec{i} - \sqrt{50}\sin 45°\vec{j}$

$= -5\vec{i} - 5\vec{j}$

(vi) $12\cos 39°\vec{i} - 12\sin 39°\vec{j}$

$= 9.3252\vec{i} - 7.5516\vec{j}$

Q. 2.



$\vec{u} = -10\cos\alpha\vec{i} - 10\sin\alpha\vec{j}$

$= -8\vec{i} - 6\vec{j}$

$\vec{v} = 13\cos\beta\vec{i} - 13\sin\beta\vec{j}$

$= 12\vec{i} - 5\vec{j}$

$\therefore \vec{u} + \vec{v} = 4\vec{i} - 11\vec{j}$

$\vec{w} = -(\vec{u} + \vec{v})$

$= -4\vec{i} + 11\vec{j}$

$|\vec{w}| = \sqrt{(-4)^2 + (11)^2}$

$= \sqrt{137}$

$= 11.7$

$\text{Tan } A = \frac{11}{4}$

$\Rightarrow A = 70°$

Direction is W 70° N.

Q. 3. $\vec{p} = \sqrt{8}\cos 45°\vec{i} + \sqrt{8}\sin 45°\vec{j}$

$= 2\vec{i} + 2\vec{j}$

$\vec{q} = 4\cos 30°\,\vec{i} - 4\sin 30°\,\vec{j}$

$= 2\sqrt{3}\,\vec{i} - 2\vec{j}$

$\therefore \vec{p} + \vec{q} = (2 + 2\sqrt{3})\vec{i} + 0\vec{j}$

Q. 4. $\vec{r} = -10\cos 40°\vec{i} - 10\sin 40°\vec{j}$

$= -7.66\vec{i} - 6.428\vec{j}$

$\vec{s} = -10\cos 58°\vec{i} + 10\sin 58°\vec{j}$

$= -5.299\vec{i} + 8.48\vec{j}$

$\vec{t} = 11\cos 20°\,\vec{i} + 11\sin 20°\vec{j}$

$= 10.3367\vec{i} + 3.762\vec{j}$

$\therefore \vec{r} + \vec{s} + \vec{t} = -2.6\vec{i} + 5.8\vec{j}$

Q. 5. (i) $\vec{a} = 12\vec{i}$

$\vec{b} = -13\cos\alpha\vec{i} + 13\sin\alpha\vec{j}$

$= -12\vec{i} + 5\vec{j}$

(ii) $\therefore \vec{a} + \vec{b} = 12\vec{i} - 12\vec{i} + 5\vec{j}$

$= 5\vec{j}$, along the $\vec{j}$-axis

(iii) $|\vec{a} + \vec{b}| = 5$ units

Q. 6. (i) $\vec{x} = -25\cos A\vec{i} + 25\sin A\vec{j}$

$= -20\vec{i} + 15\vec{j}$

$y = 17\cos B\vec{i} - 17\sin B\vec{j}$

$= 8i - 15\vec{j}$

(ii) $\therefore \vec{x} + \vec{y} = -20\vec{i} + 15\vec{j} + 8\vec{i} - 15\vec{j}$

$= -12\vec{i}$, which has no $\vec{j}$-component

(iii) $|\vec{x} + \vec{y}| = 12$ units

Exercise 1E

Q. 1. $\vec{a} = 10\vec{i}$

$\vec{b} = -26\cos\alpha\,\vec{i} + 26\sin\alpha\vec{j}$

$\therefore \vec{a} + \vec{b} = (10 - 26\cos\alpha)\vec{i} + 26\sin\alpha\vec{j}$

No $\vec{i}$-component

$\Rightarrow 10 - 26\cos\alpha = 0$

$\Rightarrow \cos\alpha = \frac{5}{13}$

$\Rightarrow \sin\alpha = \frac{12}{13}$

$\therefore \vec{a} + \vec{b} = 0\vec{i} + 26\left(\frac{12}{13}\right)\vec{j}$

$= 24\vec{j}$

$\therefore |\vec{a} + \vec{b}| = 24$ units

Q. 2. $\vec{a} = \sqrt{32}\left(\frac{1}{\sqrt{2}}\right)\vec{i} - \sqrt{32}\left(\frac{1}{\sqrt{2}}\right)\vec{j}$

$= 4\vec{i} - 4\vec{j}$

$\vec{b} = -5\cos\alpha\vec{i} + 5\sin\alpha\vec{j}$

$\therefore \vec{a} + \vec{b} = (4 - 5\cos\alpha)\vec{i} + (-4 + 5\sin\alpha)\vec{j}$

No $\vec{i}$-component

$\Rightarrow 4 - 5\cos\alpha = 0$

$\Rightarrow \cos\alpha = \frac{4}{5}$

$\Rightarrow \sin\alpha = \frac{3}{5}$

$\therefore \vec{a} + \vec{b} = 0\vec{i} + \left(-4 + 5\left(\frac{3}{5}\right)\right)\vec{j}$

$= 0\vec{i} - \vec{j}$

Q. 3. Let $|\vec{b}| = x$

$\vec{a} = -4\vec{i}$

$\vec{b} = \frac{1}{2}x\vec{i} + \frac{\sqrt{3}}{2}x\vec{j}$

$\therefore \vec{a} + \vec{b} = (-4 + \frac{1}{2}x)\vec{i} + \frac{\sqrt{3}}{2}x\vec{j}$

No $\vec{i}$-component $\Rightarrow -4 + \frac{1}{2}x = 0$

$\Rightarrow x = 8$

i.e. $|\vec{b}| = 8$

$\therefore \vec{a} + \vec{b} = 0\vec{i} + 4\sqrt{3}\vec{j}$

$|\vec{a} + \vec{b}| = 4\sqrt{3}$ units

Q. 4. $\vec{i} = -10\cos 70°\,\vec{i} + 10\sin 70°\,\vec{j}$

$= -3.420\vec{i} + 9.397\vec{j}$

Let $|\vec{s}| = x \Rightarrow \vec{s} = x\vec{i}$

$\therefore \vec{r} + \vec{s} = (-3.420 + x)\vec{i} + 9.397\vec{j}$

No $\vec{i}$-component $\Rightarrow x = 3.420$

i.e. $|\vec{s}| = 3.420$

Q. 5. $\vec{a} = 10\cos\theta\,\vec{i} + 10\sin\theta\,\vec{j}$

$= 10\left(\frac{4}{5}\right)\vec{i} + 10\left(\frac{3}{5}\right)\vec{j}$

$= 8\vec{i} + 6\vec{j}$

Let $|\vec{b}| = x \Rightarrow \vec{b} = -x\vec{j}$

$\therefore \vec{a} + \vec{b} = 8\vec{i} + (6 - x)\vec{j}$

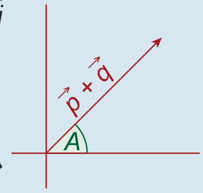
$= k\vec{i} + 0\vec{j}$

$\therefore x = 6$ and $k = 8$

Q. 6. (i) $\vec{p} = 35\cos\alpha\vec{i} + 35\sin\alpha\vec{j}$

$= 35\left(\frac{3}{5}\right)\vec{i} + 35\left(\frac{4}{5}\right)\vec{j}$

$= 21i + 28\vec{j}$



$\vec{q} = 13\cos\beta\vec{i} + 13\sin\beta\vec{j}$

$= 13\left(\frac{12}{13}\right)\vec{i} + 13\left(\frac{5}{13}\right)\vec{j}$

$= 12\vec{i} + 5\vec{j}$

(ii) $\therefore \vec{p} + \vec{q} = 21\vec{i} + 28\vec{j} + 12\vec{i} + 5\vec{j}$

$= 33\vec{i} + 33\vec{j}$

$\therefore \tan A = \frac{33}{33} = 1$

$\Rightarrow A = 45°$

Q. 7. $\vec{r} = -3.42\vec{i} + 9.397\vec{j}$

Let $|\vec{s}| = x$

$\therefore \vec{s} = x\cos 10°\vec{i} + x\sin 10°\vec{j}$

$= 0.9848x\vec{i} + 0.1736x\vec{j}$

$\therefore \vec{r} + \vec{s} = (-3.42 + 0.9848x)\vec{i} + (9.397 + 0.1736x)\vec{j}$

Since $(\vec{r} + \vec{s})$ is in a NE direction, the $\vec{i}$ and $\vec{j}$ components must be equal.

$\therefore -3.42 + 0.9848x = 9.397 + 0.1736x$

$\Rightarrow x = 15.8$

Q. 8. (i) $|\vec{p}| = \sqrt{8^2 + 1^2} = \sqrt{65}$

$|\vec{q}| = \sqrt{(-7)^2 + 4^2} = \sqrt{65}$

$\therefore |\vec{p}| = |\vec{q}|$

(ii) "Slope" of $p = \frac{\vec{j}\text{-bit}}{\vec{i}\text{-bit}} = \frac{1}{8} = m_1$

"Slope" of $q = \frac{4}{-7} = -\frac{4}{7} = m_2$

$m_1 \times m_2 = -\frac{1}{14} \neq -1$

$\therefore$ Not perpendicular

Q. 9. $|-7\vec{i} + \vec{j}| = \sqrt{49 + 1} = \sqrt{50}$

$|p(\vec{i} + \vec{j})| = |p\vec{i} + p\vec{j}|$

$= \sqrt{p^2 + p^2} = \sqrt{2p^2}$

$\therefore 2p^2 = 50$

$\Rightarrow p = 5$ (since $p > 0$)

Q. 10. $\vec{a} = 10\cos 80°\vec{i} + 10\sin 80°\vec{j}$

$= 1.736\vec{i} + 9.848\vec{j}$

Let $|\vec{b}| = x$

Therefore, $\vec{b} = x\vec{i}$

$\therefore \vec{a} + \vec{b} = (1.736 + x)\vec{i} + 9.848\vec{j}$

Since the direction of $(\vec{a} + \vec{b})$ is 21° 48′,

the slope of $(\vec{a} + \vec{b})$ must equal tan 21° 48′.

$\therefore \frac{9.848}{1.736 + x} = 0.4$

$\Rightarrow x = 22.88$