Question 1

Question 5 **(25 marks)**

The function f is such that $f(x) = 2x^3 + 5x^2 - 4x - 3$, where $x \in \mathbb{R}$.

(a) Show that $x = -3$ is a root of $f(x)$ **and** find the other two roots.

(b) Find the co-ordinates of the local maximum point **and** the local minimum point of the function f.

Local maximum point: Local minimum point:

(c) $f(x) + a$, where a is a constant, has only one real root.
Find the range of possible values of a.

Question 2

(a) **(i)** Air is pumped into a spherical exercise ball at the rate of 250 cm^3 per second.
Find the rate at which the radius is increasing when the radius of the ball is 20 cm.
Give your answer in terms of π.

(ii) Find the rate at which the surface area of the ball is increasing when the radius of the ball is 20 cm.

(b) The inflated ball is kicked into the air from a point O on the ground. Taking O as the origin, $(x,\ f(x))$ approximately describes the path followed by the ball in the air, where

$$f(x) = -x^2 + 10x$$

and both x and $f(x)$ are measured in metres.

(i) Find the values of x when the ball is on the ground.

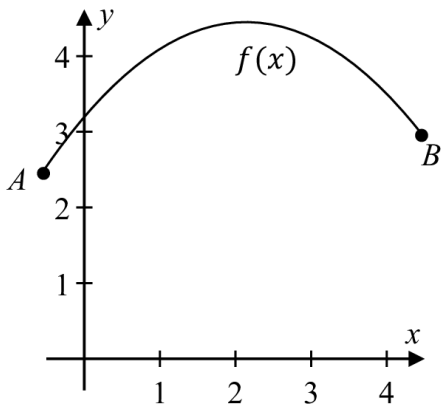
(ii) Find the average height of the ball above the ground, during the interval from when it is kicked until it hits the ground again.

Question 3

(a) The diagram shows Sarah's first throw at the basket in a basketball game. The ball left her hands at A and entered the basket at B. Using the co-ordinate plane with $A(-0{\cdot}5, 2{\cdot}565)$ and $B(4{\cdot}5, 3{\cdot}05)$, the equation of the path of the centre of the ball is

$$f(x) = -0{\cdot}274x^2 + 1{\cdot}193x + 3{\cdot}23,$$

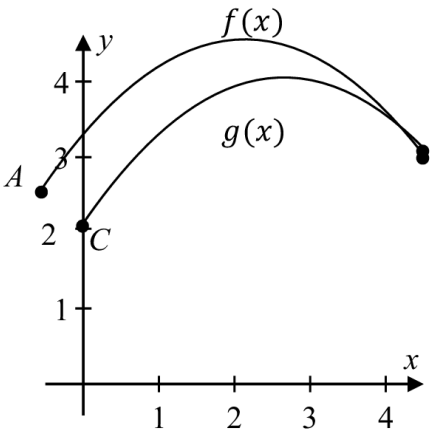
where both x and $f(x)$ are measured in metres.



(i) Find the maximum height reached by the centre of the ball, correct to three decimal places.

(ii) Find the acute angle to the horizontal at which the ball entered the basket. Give your answer correct to the nearest degree.

(iii) Sarah took a second throw. This throw followed the path of the parabola $g(x)$ as shown.
The ball left Sarah's hands at the point $C(0, 2)$.
The graph $y = g(x)$ is the image of the graph $y = f(x)$ under the translation which maps A onto C.
Using your result from part **a(i)**, show that the centre of this ball reached its maximum height at the point $(2{\cdot}677, 3{\cdot}964)$, correct to three decimal places.

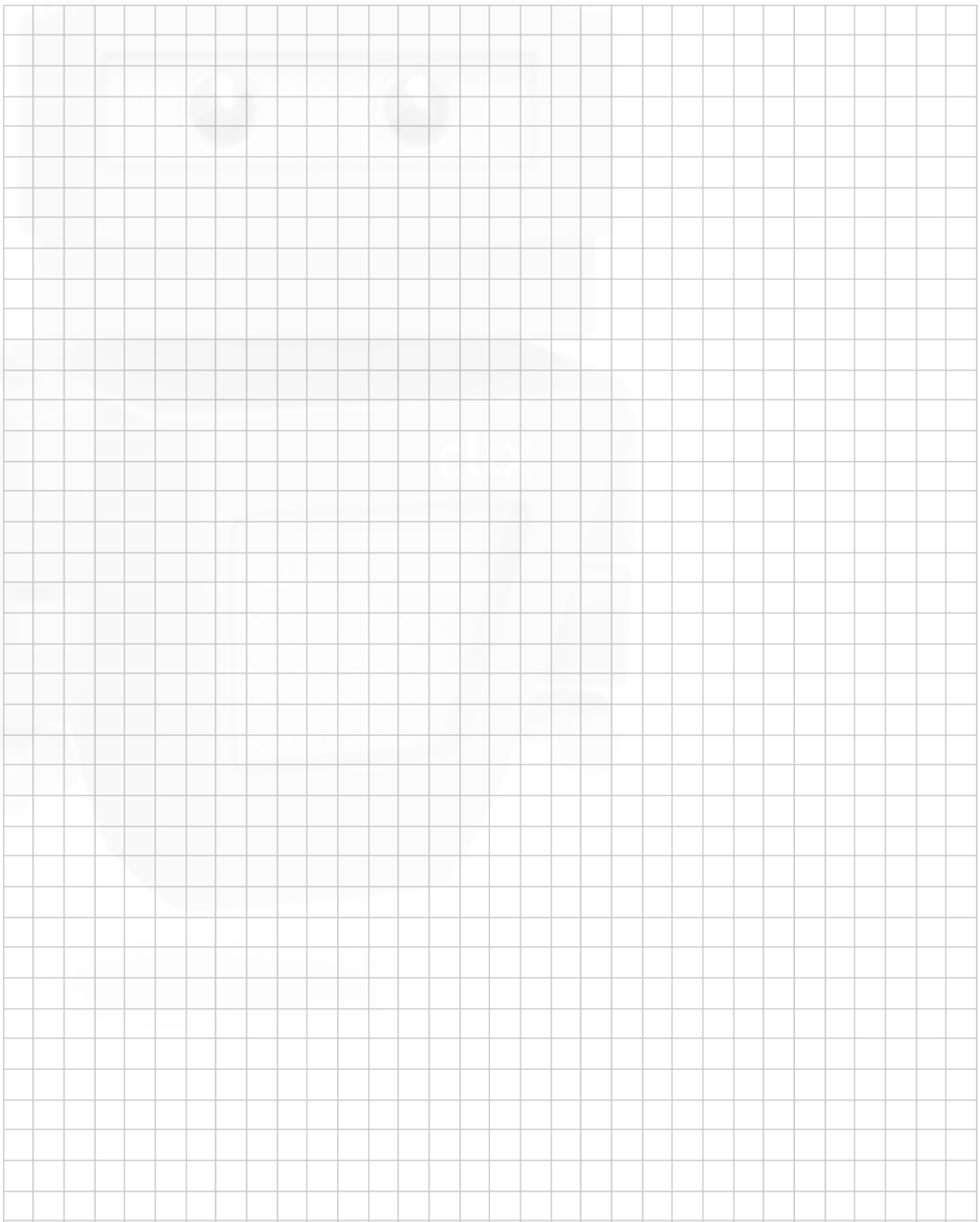


(iv) Hence, or otherwise, find the equation of the parabola $g(x)$.

Question 2 **(25 marks)**

Solve the equation $x^3 - 3x^2 - 9x + 11 = 0$.

Write any irrational solution in the form $a + b\sqrt{c}$, where $a, b, c \in \mathbb{Z}$.



Question 5

(a) Solve the equation $x = \sqrt{x+6},\ x \in \mathbb{R}$.

Question 3 **(25 marks)**

A cubic function f is defined for $x \in \mathbb{R}$ as

$$f : x \mapsto x^3 + (1-k^2)x + k, \quad \text{where } k \text{ is a constant.}$$

(a) Show that $-k$ is a root of f.

(b) Find, in terms of k, the other two roots of f.

(c) Find the set of values of k for which f has exactly one real root.