Q2's RELATIVE VELOCITY

2014(a) Unusual .

2014(b) type(a) find d ships are ≤ 10km apart (junction)

2013 (a) type(a) but unusual → they are shortest dist apart

2013(b) Very quick if you notice

?

2012 (a) Rain - different

2012 (b) type (a) junction

2011 (a) type (a) → shortest dist before B reaches jn

2011 (b) type (d)(river)

2010 (a) show they will collide

2010 (b) wind rel to man type(b)

2009 (a) type (a) but 'how far from junction?'

2009 (b) algebra type (c) airplane

2008 (a) from jn type (a)

(!) 2008 (b) wind rel to man but not ususal 45° type (b)

(!) 2007 (a) type (a) but with a bit of a difference

2007 (b) type (d) river

2006 (a) type (c) airplane

2006 (b) unusual

2005 (a) type (d) river algebra (!)

2005 (b) type (a) but $V_A = P$.

2004 (a)

2004 (b) type (a) with unusual angles!!! (must draw)

2003 (a) wind rel type (b)

2003 (b)	type (a) dist from junction
2002 (a)	type (a) $\leq$ 9 km apart
(!) 2002 (b)	type (e) !! V_{PW}, V_{QW} find V_{PQ}
2001 (a)	
2001 (b)	type (c) airplane algebra
2000 (a)	unusual, algebra (!!) (!)

type (a)	Junction, how long apart, how far from jn
type (b)	wind rel to man in one direction, other direction
type (c)	airplane in air
type (d)	Boat in water - downstream distance
type (e)	$V_{AB} = V_{AW} - V_{BW}$
unusual	

1:
2014:

ⓘ

2 (a) X and Z : at 15:00 $S_x = 6i + 12j$ $\begin{pmatrix} 2i + 4i \\ 7j + 5j \end{pmatrix}$

$S_z = 12i + 9j$

(i)

$S_{zx} = 6i - 3j$

$V_x = 4i + 5j$
$V_z = 2i + 6j$ $V_{xz} = 2i - j$

Since $kV_{xz} = S_{zx}$ or $3(V_{xz}) = S_{zx}$

they <u>will</u> collide in 3 hours from 15:00

they will collide at 18:00

(ii) At 18:00 Y changes course

Collision is at displacement $18i + 27j$ $\begin{pmatrix} S_z = 12i + 9j \\ + 3(2i + 6j) \end{pmatrix}$

At 18:00 y is at displacement:

$6i + 9j + 3\tfrac{1}{2}(3i + 4j)$

$6i + 10{\cdot}5i + 9j + 14j = 16{\cdot}5i + 23j$

$V_{CY} = 18i + 27j - (16{\cdot}5i + 23j) = 1{\cdot}5i + 4j$

$|CY| = \sqrt{1{\cdot}5^2 + 4^2} = 4{\cdot}272$ km

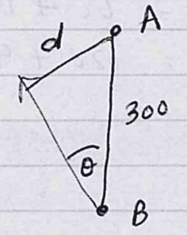
Speed of y is $\sqrt{3^2 + 4^2} = 5$ km/hr

so time for y to get to collision $= \frac{4{\cdot}272}{5} = 0{\cdot}8544$ hrs
$= 51{\cdot}26$ mins

Ans: 18:00 + 51·26 = 18:51 (to nearest min)

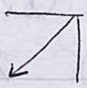
2014

2(b)



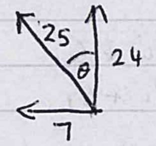
$V_A = -24i - 24j$

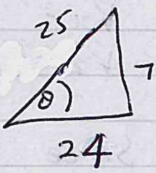
$V_B = -31i + 0j$



$V_{BA} = -7i + 24j$

$\sqrt{7^2+24^2} = 25$



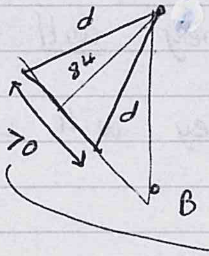


$\sin\theta = \frac{7}{25}$

$\cos\theta = \frac{24}{25}$

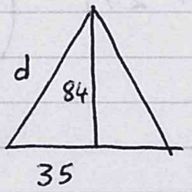
$d = 300 \sin\theta$

$d = 300 \cdot \frac{7}{25} = 84m$



$V_{BA} = 25$ km/hr

in 2·8 hours this is a distance of 70 km

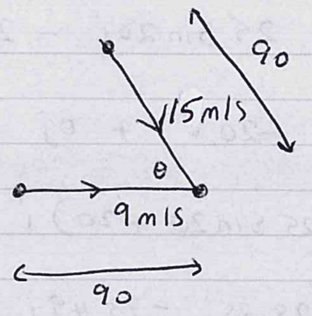


$d = \sqrt{84^2 + 35^2}$

$d = 91$ km

2013

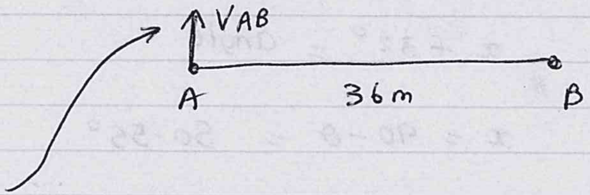
Q2 (a)



B reaches intersection in $\frac{90}{15} = 6$ secs

A travels 54m in 6s

So B is at jn and A is $90 - 54 = 36$m from it



If 36m = Shortest distance then V_{AB} must be ⊥ to

Since 36 = ⊥ distance from path of A rel to B

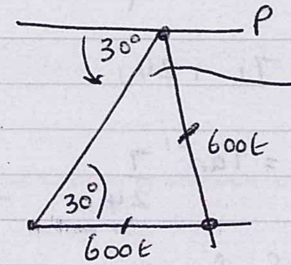
So $V_{AB} = 0i + xj$

$V_A = 9i + 0j$

$V_B = 15\cos\theta i - 15\sin\theta j \Rightarrow V_{AB} = (9 - 15\cos\theta)i - 15\sin\theta j$

But, $9 - 15\cos\theta = 0i$ So $\cos\theta = \frac{9}{15}$

2(b) in time t,

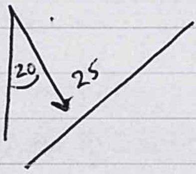


must be 30° due to isosceles Δ

So Ans: 60° South of West

2012

(a)

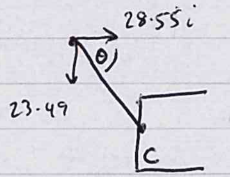


$V_r = 25 \sin 20 i - 25 \cos 20 j$

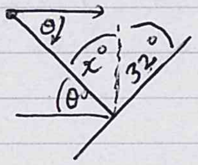
$V_c = -20 i + 0 j$

$V_{rc} = (25 \sin 20 + 20) i - 25 \cos 20 j$

$= 28.55 i - 23.49 j$



$\theta = \tan^{-1} \frac{23.49}{28.55} = 39.45°$

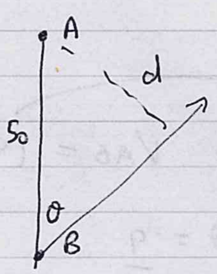


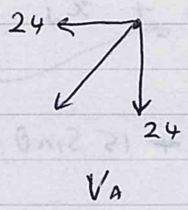
$x + 32° = \text{angle}$

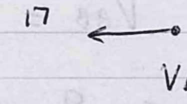
$x = 90 - \theta = 50.55°$

So Ans: $50.55 + 32 = 82.55°$

(b.)





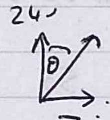


$V_A = -24i - 24j$

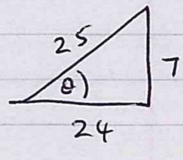
$V_B = -17i$

$V_{BA} = -17i - (-24i - 24j)$

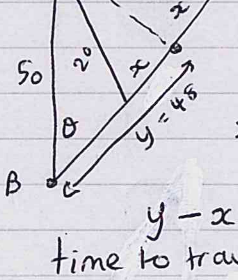
$= 7i + 24j$



$\theta = \tan^{-1} \frac{7}{24}$



$\sin\theta = \frac{7}{25}$, $\cos\theta = \frac{24}{25}$



$d = 50 \sin\theta = \frac{50.7}{25} = 14m$

$x = \sqrt{20^2 - 14^2}$

$x = 14.28$

$y = 50 \cos\theta = \frac{50.24}{25} = 48km$

$y - x = 33.72$

time to travel 33.72m at 25m/s

$= 33.72/25 = 1.35hrs = 1hr\ 21mins$ So 12:00 + 1:21 = 13:21

now time to go distance of 2x = $2x = 28.56$

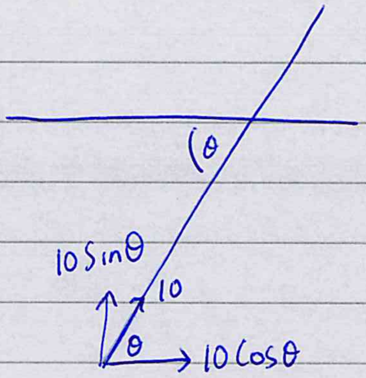
$\frac{28.56}{25} = 1.14hrs = 1hr\ 8.54mins$

So 12:00 + 1hr 8.54mins = 13:09

Very straight forward (But note shortest dist before B reached jn)

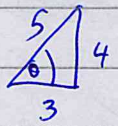
2

2011 (a)



$V_A = 5i + 0j$

$V_B = 10\cos\theta i + 10\sin\theta j$



$V_{AB} = 6i + 8j$

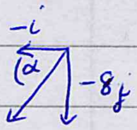
(i) $\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$V_{AB} = -i - 8j$

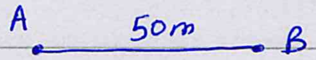
$|V_{AB}| = \sqrt{(1)^2 + (-8)^2} = \sqrt{65}$

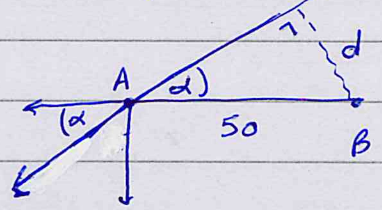
direction $\tan^{-1} 8$ S of W

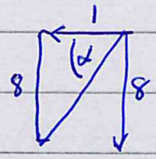


(ii) Bring B to junction → 10secs

In that 10s, A travels 50m so



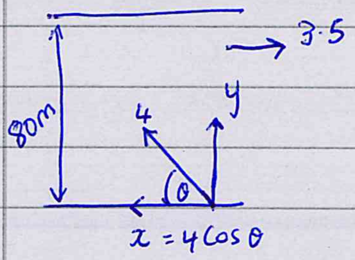




$\sin\alpha = 8/\sqrt{65}$, $\cos\alpha = 1/\sqrt{65}$

$d = 50 \sin\alpha = 50\left(\frac{8}{\sqrt{65}}\right) = 49.614$ m

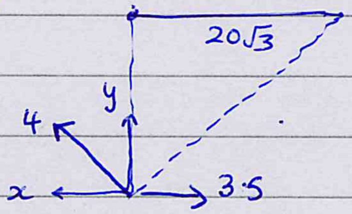
(b.)



To cross to B : $x = 3.5 = 4\cos\theta$

$\cos^{-1}\frac{3.5}{4} = \theta = 28.955^\circ$

To cross to C :



time for her to travel $20\sqrt{3}$ in horiz $\left(20\sqrt{3}/3.5-x\right)$

= time for her to travel 80m vertically $(80/y)$

$\frac{20\sqrt{3}}{3.5-x} = \frac{80}{y}$, also $x^2+y^2=4 \leftarrow y = \frac{280-80x}{20\sqrt{3}}$

$x = 2$, $y = 2\sqrt{3}$

$\tan\theta = \sqrt{3}$ $\theta = 60^\circ$

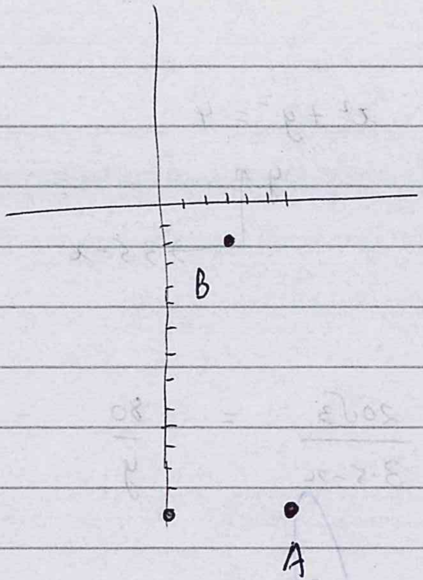
↖ ANS

$x = \frac{79}{19}$, $y = -1.52$

$\tan\theta = -0.3656$

$\theta = 340^\circ, \theta = 160^\circ$

2010: 2 (a)



$V_{BA} = 5i - 7j - (4i - 3j)$

$V_{BA} = i - 4j$

$S_{AB} = 6i - 14j - (3i - 2j)$

$= 3i - 12j$

$S_{AB} = 3(V_{BA})$

So they will collide

(b.) Appears means V_W rel to MC. from $WM = 0i + 12.5j$

$V_W = xi + yj$

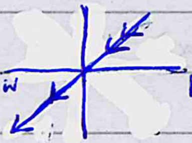
$V_{WM} = xi + (y - 12.5)j$

x comp = y comp

$x = +y \quad 12.5$

$$\boxed{x - y = -12.5}$$

V_{WM}



$-x$ comp $= -y$ comp,

x comp $= y$ comp

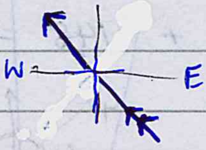
$V_M = 0i - 12.5j$

$V_W = xi + yj$

$V_{WM} = xi + (y + 12.5)j$

$x = -(y + 12.5)$

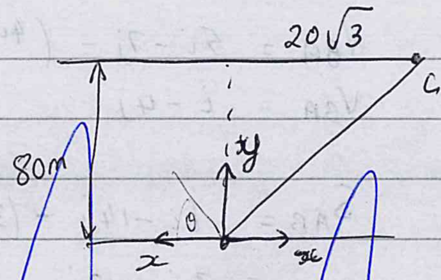
$$\boxed{x + y = -12.5}$$



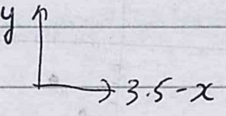
x comp $= -(y$ comp$)$

$y = 0, \; x = -12.5$

2011
2
(a)



$x^2 + y^2 = 4$



time to reach C = t

$D = S \times t$

$\frac{D}{S} = t$

$t = \frac{\cancel{3.5 - x}}{20\sqrt{3}}$ = ~ 8 $\frac{20\sqrt{3}}{3.5 - x} = \frac{80}{y}$.

$y = \frac{(3.5x)80}{20\sqrt{3}}$ $y = \frac{280 - 80x}{20\sqrt{3}}$

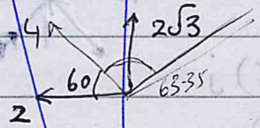
$x^2 + y^2 = 4^2$ $x^2 + \frac{(280 - 80x)(280 - 80x)}{400(3)} = 16$

$6400x^2 + 1200x^2 - 44800x + 78400 = \cancel{100}\ 19200$

$760\cancel{0}\cancel{0}x^2 - 448\cancel{0}\cancel{0}x + 592\cancel{0}\cancel{0} = 0$

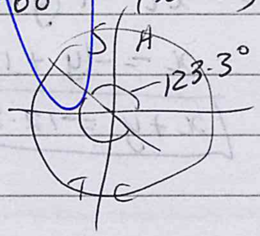
$19x^2 - 112x + 148 = 0$

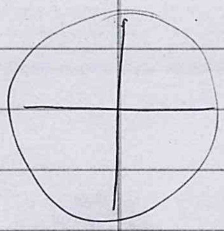
$x = \frac{74}{19} > 2$ $x =$



$\theta = 60°$ $(x = 2)$

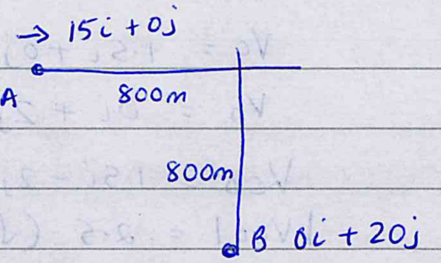
$x = \cancel{4}\ \frac{79}{19}$, $y = -1.52$





2009:

2(a)



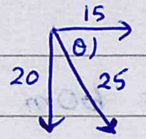
Bring B to junction

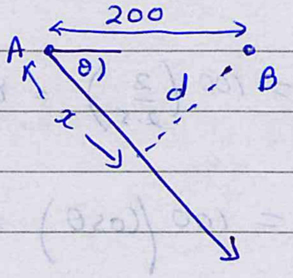
$\frac{800}{20} = 40$ secs

In 40secs A will travel 600m

$V_{AB} = 15i + 0j - (0i + 20j)$

$= 15i - 20j$





(i) $d = 200 \sin\theta = 200\left(\frac{4}{5}\right) = 160$ m

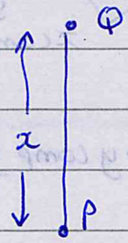
(ii) $x = 200 \cos\theta = 200 \cdot \frac{3}{5} = 120$m

120m @ 25m/s = 4.8 secs

In 4.8 secs B would travel 4.8 x 20 = 96m so B is 96m from junction

In 4.8 secs A would travel 72m so A would be 200 − 72 = 128m from junction

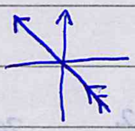
2(b)

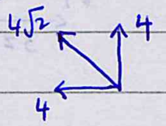


no wind then

$T = \frac{\text{dist}}{\text{speed}} = \frac{x}{u} \Rightarrow x = uT$

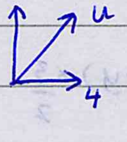
with a wind

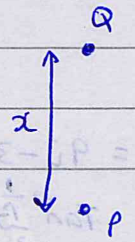




$\sqrt{u^2-4^2} = \sqrt{u^2-16}$

V_{plane}

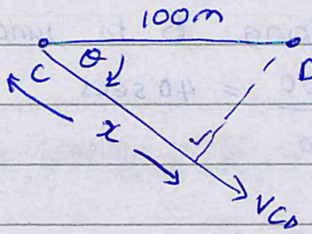




$\text{Time} = \frac{\text{dist}}{\text{speed}} = \frac{x}{\sqrt{u^2-16}+4} = \frac{uT}{\sqrt{u^2-16}+4}$

2008

2(a)

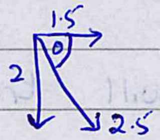


$V_C = 1{\cdot}5i + 0j$

$V_D = 0i + 2j$

$V_{CD} = 1{\cdot}5i - 2j$

$|V_{CD}| = 2{\cdot}5 \quad (\sqrt{1{\cdot}5^2 - 2^2})$



Nearest distance $d = 100 \sin\theta$

$\tan\theta = \frac{2}{1{\cdot}5}$; $\sin\theta = \frac{2}{2{\cdot}5}$

$\cos\theta = 1{\cdot}5/2{\cdot}5$

$d = 100\left(\frac{2}{2{\cdot}5}\right) = 80m$

$x = 100(\cos\theta) = 100\left(\frac{1{\cdot}5}{2{\cdot}5}\right) = 60m$

time to travel $x = \frac{60}{2{\cdot}5} = 24$ secs

In 24s;

C will travel $1{\cdot}5 \times 24 = 36m$ So C will be $100-36m$ from junction $= 64m$

(ii) In 24s; B will travel $2 \times 24 = 48m$ So B will be 48m from jn.

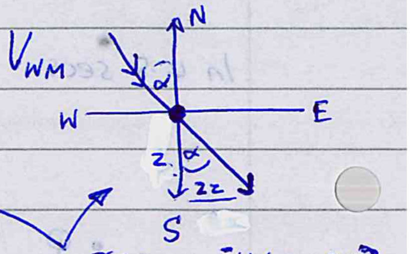
(b.) $V_W = xi - 3j$ $\quad V_m = yi + 0j$

$V_W = xi - 3j$

$V_{WM} = (x-y)i - 3j$

$(x-y) = -3(-2)$

$\boxed{x - y = +6}$

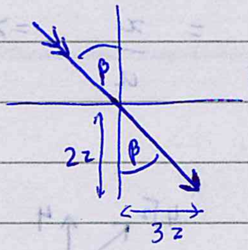


$x\text{comp} = {}^-y\text{ comp} \times 2$

$V_m = 0i + yj$

$V_W = xi - 3j$

$V_{WM} = xi - (3+y)j$



$x\text{comp} = -y\text{ comp} \times \frac{3}{2}$

$x = +(3+y) \times \frac{3}{2}$

$\boxed{2x = 9 + 3y}$

$-2x + 2y = -12$

$2x - 3y = 9$

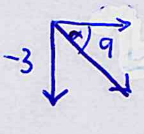
$-y = -3$

$y = 3$

$x = 3 + 6$

$x = 9$

Ans: $V_W = 9i - 3j$

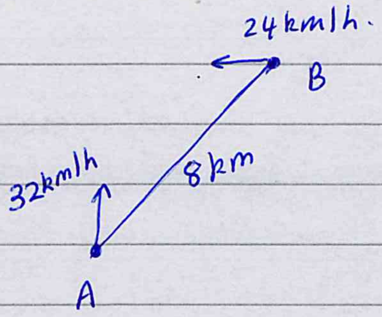


$\tan^{-1}\frac{1}{3} = \alpha$

2007:

(a)

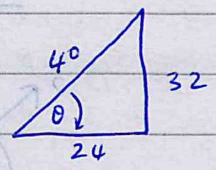
(i)



$V_A = 0i + 32j$

$V_B = -24i + 0j$

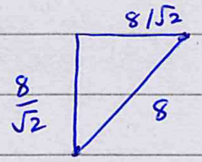
$V_{AB} = 24i + 32j$



$\sin\theta = \frac{32}{40} = \frac{4}{5}$

$\cos\theta = 3/5$

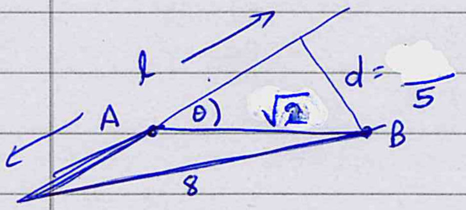
(ii) Bring A horizontal with B :



$\frac{8/\sqrt{2}}{32} = \frac{\sqrt{2}}{8}$ hrs

in $\frac{\sqrt{2}}{8}$ hrs, B will travel $24 \times \frac{\sqrt{2}}{8} = 3\sqrt{2}$ km

$\frac{8}{\sqrt{2}} - 3\sqrt{2} = \sqrt{2}$



$\sqrt{2}\sin\theta = d$ $= \sqrt{2} \cdot \frac{4}{5} = \frac{4\sqrt{2}}{5}$

$l = \sqrt{64 - \left(\frac{4\sqrt{2}}{5}\right)^2} = 7.92$ km

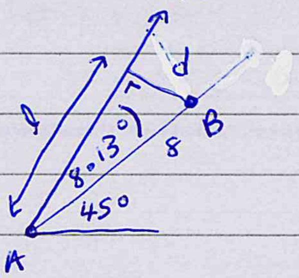
$2l = 15.84$

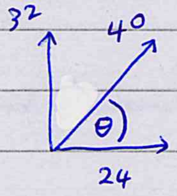
time to travel $2l$ @ 40km/h $= \frac{15.84}{40} = 0.396$ hrs

$= 23.76$ mins

OR

(a)(ii)





$\theta = \tan^{-1}\frac{32}{24}$

$\theta = 53.13°$

$53.13 - 45 = 8.13°$

$d = 8\sin 8.13°$

$d = 1.13$

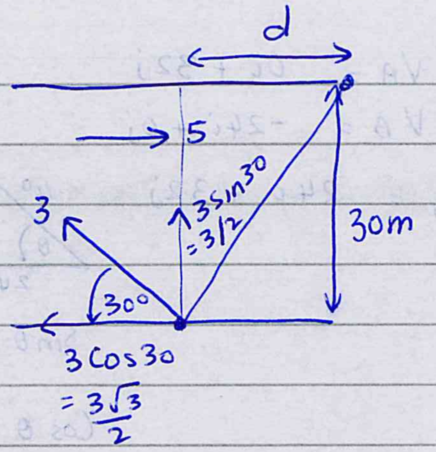
$l = 8\cos 8.13$

$l = 7.92$km $\quad 2l = 15.84$

∴ time to travel $2l$ @ 40km/h $= \frac{15.84}{40} = 0.396$ hrs

$= 23.76$ mins

2007 2 (b.)



$$\text{time to cross} = \frac{30}{3/2} = \frac{d}{-\frac{3\sqrt{3}}{2}+5} \quad \begin{matrix}\times 2\\ \times 2\end{matrix}$$

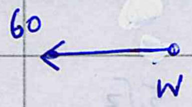
$$20 = \frac{2d}{-3\sqrt{3}+10}$$

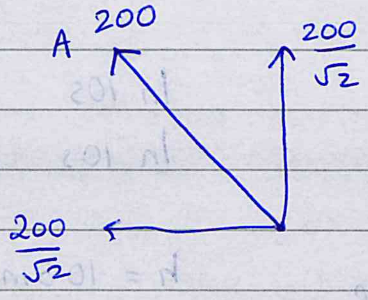
$$10(10-3\sqrt{3}) = d$$

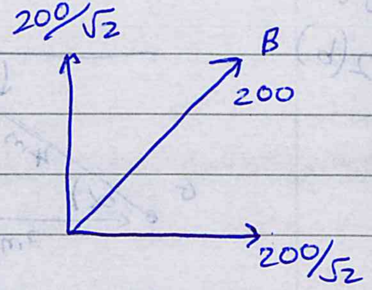
$$d = 100 - 30\sqrt{3} = 48.04\,m$$

2006

2(a)

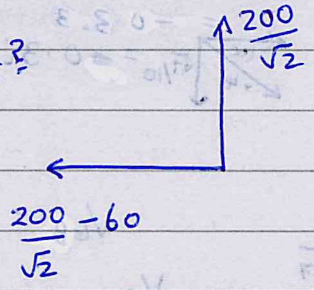






In still air

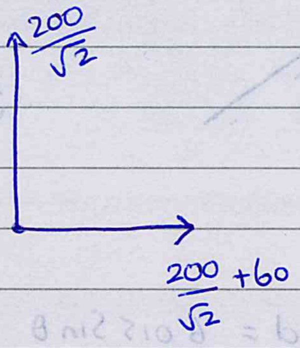
A in still air?



so that when you 'add' the wind you get 200 = speed of plane NW

speed of A in still air = $\sqrt{\left(\frac{200}{\sqrt{2}} - 60\right)^2 + \left(\frac{200}{\sqrt{2}}\right)^2}$ = 163·185

B in still air?



So that when you 'add' wind you get 200 = speed NE

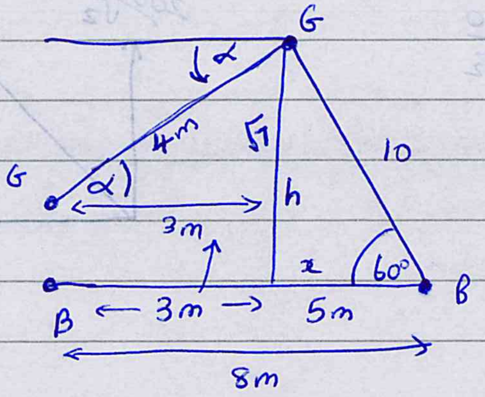
$$V_{AB} = V_A - V_B = \left(60 - \frac{200}{\sqrt{2}}\right)i + \left(\frac{200}{\sqrt{2}}\right)j - \left[\left(\frac{200}{\sqrt{2}} + 60\right)i + \left(\frac{200}{\sqrt{2}}\right)j\right]$$

$$= \left(-\frac{200}{\sqrt{2}} - \frac{200}{\sqrt{2}}\right)i + 0j$$

$$= -\frac{400}{\sqrt{2}}\,i + 0j$$ = 282·84 km/h West

2006

2(b)



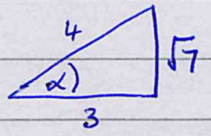
In 10s $S_B = 0.8 \times 10 = 8m$

In 10s $S_G = 0.4 \times 10 = 4m$

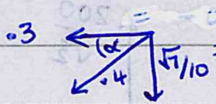
$h = 10 \sin 60 = 5\sqrt{3}$

$x = 10 \cos 60 = 5m$

$\cos \alpha = \frac{3}{4}$

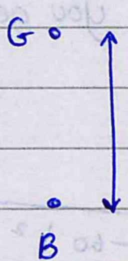


so $V_G = -0.4 \cos \alpha - 0.4 \sin \alpha$



$= -0.3i - \frac{\sqrt{7}}{10} j$

After 10s



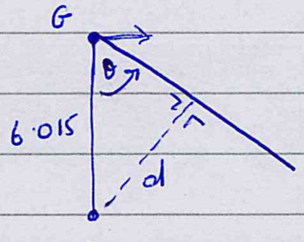
$h - \sqrt{7} = 5\sqrt{3} - \sqrt{7}$

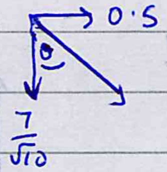
$= 6.015$

$V_G = -0.3i - \frac{\sqrt{7}}{10} j$

$V_B = -0.8i + 0j$

$V_{GB} = 0.5i - \sqrt{7}/10\, j$





$\tan \theta = \frac{0.5}{\frac{\sqrt{7}}{10}} = 5$

$\theta = 62.114°$

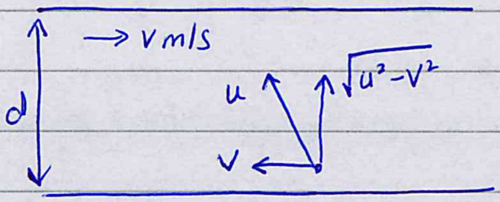
shortest distance $d = 6.015 \sin \theta$

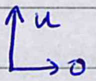
$= 5.32m$

(!)(!)

2005:

2(a)



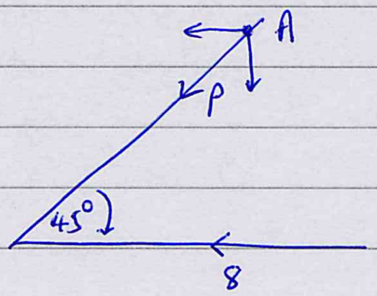
crossing in shortest time means 

so time = $\frac{d}{u} = 10$

$\therefore d = 10u$

speed ↑ = $\sqrt{u^2-v^2}$ time = $\frac{d}{\sqrt{u^2-v^2}} = \frac{10u}{\sqrt{u^2-v^2}}$

2(b)



$V_A = -P\cos 45_i - P\sin 45_j$

$= -P/\sqrt{2} - P/\sqrt{2}$

$V_B = -8i$

$V_{AB} = \left(\frac{-P}{\sqrt{2}} + 8\right)_i - \left(\frac{P}{\sqrt{2}}\right)_j = -2i - 10j$

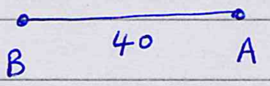
$\frac{P}{\sqrt{2}} = 10$ $P = 10\sqrt{2}$

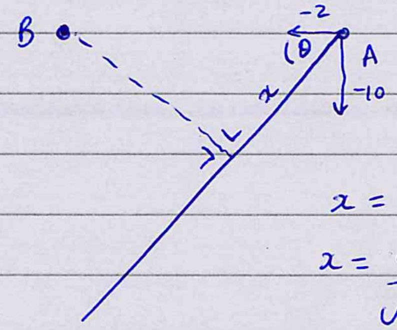
Bring A to intersection:

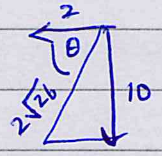
$\frac{220\sqrt{2}}{10\sqrt{2}}$ = time = 22 secs, in that time B travels 8×22

= 176 m

so B passes jn.

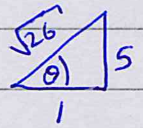
So situation now is 





$\theta = Tan^{-1}\frac{10}{2}$

$\theta = Tan^{-1} 5$



$\sin\theta = \frac{5}{\sqrt{26}}$

$\cos\theta = \frac{1}{\sqrt{26}}$

$x = d\cos\theta$

$x = \frac{40}{\sqrt{26}} = 7.84$ m

7.84 m @ $2\sqrt{26}$ m/s ⇒ time = 0.769 secs

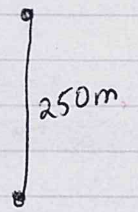
How far is A from junction after 0.769 secs?

$10\sqrt{2} \times 0.769 = 10.87$m or 11m.

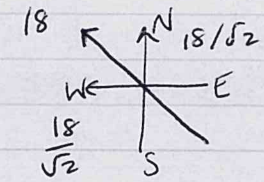
!!!

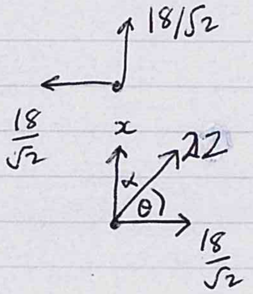
2004

(a)

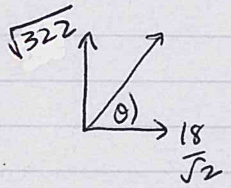


Wind ~~from~~ SE





$x = \sqrt{22^2 - \left(\frac{18}{\sqrt{2}}\right)^2} = \sqrt{322}$



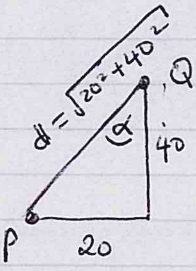
$\tan\theta = \frac{\sqrt{322}}{18/\sqrt{2}}$

$\theta = 54.65$

$\alpha = 35.35$

$\text{Time} = \frac{\text{Dist}}{\text{speed}} = \frac{250}{\sqrt{322} + \frac{18}{\sqrt{2}}} = 8.15 \text{ secs}$

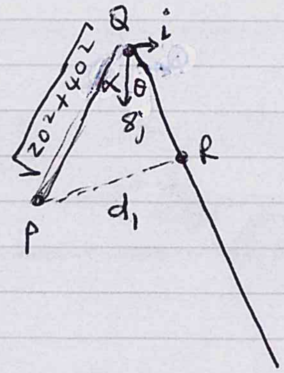
(b.)

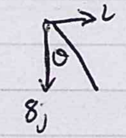


$V_{QP} = 4i - 3j - (3i + 5j)$
$= i - 8j$

$\alpha = \tan^{-1} \frac{20}{40}$

$\alpha = 26.57°$





$\theta = \tan^{-1} \frac{1}{8}$

$\theta = 7.125$

$d_1 = \sqrt{20^2 + 40^2} \sin(\alpha + \theta)$

$= \sqrt{20^2 + 40^2} \sin(26.57 + 7.125)$

$= \sqrt{20^2 + 40^2} \sin(33.7)$

$= 24.81$

(iii) $|QR| = \sqrt{20^2 + 40^2} \cos(33.7)$
$= 37.21 \text{ m}$

$\text{speed} = \sqrt{1^2 + (-8)^2} = \sqrt{65}$

$\text{time} = 37.21 / \sqrt{65} = 4.61 \text{ secs}$

2003:

2(a) $V_{WdW} = xi + yj - (0i + 10j)$

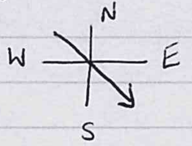
$= xi + (y - 10)j$ appears to blow $\longrightarrow$

$\Rightarrow j$ comp $= 0$

$\Rightarrow y = 10.$

$V_{WdW} = xi + yj - (0i + 30j)$

$= xi + (y - 30)j$ appears to blow from NW



$x = -(y - 30)$

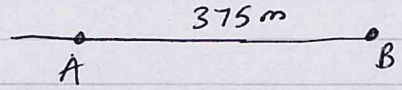
$x = -y + 30$

$x = -10 + 30$

$x = 20$

$V_W = 20i + 10j$

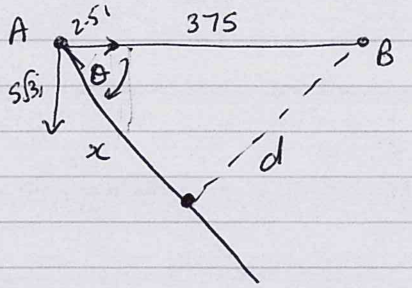
2(b)



$V_A = 7{\cdot}5i + 0j$

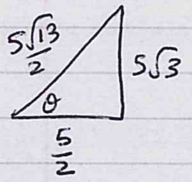
$V_B = 10\cos 60i + 10\sin 60j$

$V_B = 5i + 5\sqrt{3}j$



$V_{AB} = 2{\cdot}5i - 5\sqrt{3}j$

$\theta = \tan^{-1} \frac{5\sqrt{3}}{2{\cdot}5} =$



$\sin\theta = \frac{\cancel{5}\sqrt{3}}{\frac{\cancel{5}\sqrt{13}}{2}} = \frac{2\sqrt{3}}{\sqrt{13}}$

$375\sin\theta = d = 375\left(\frac{2\sqrt{3}}{\sqrt{\sqrt{3}}}\right)$

$= 360{\cdot}288$

$375\cos\theta = x = 104m$

speed of $V_{AB} = \sqrt{2{\cdot}5^2 + (5\sqrt{3})^2} = \frac{5\sqrt{13}}{2}$

$\frac{d}{s} = t = \frac{104}{5\sqrt{13}/2} = 11{\cdot}54s$

in 11·54s A travels $11{\cdot}54 \times 7{\cdot}5 = 85{\cdot}5$

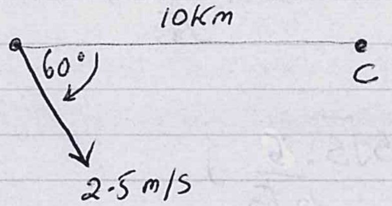
so A is $375 - 85{\cdot}5 = 288m$

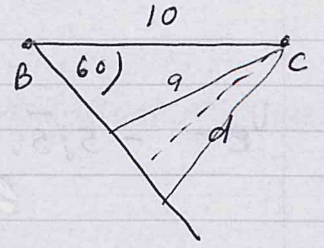
in 11·54s B travels 115·4m so

B is 115·4m from jn

2002

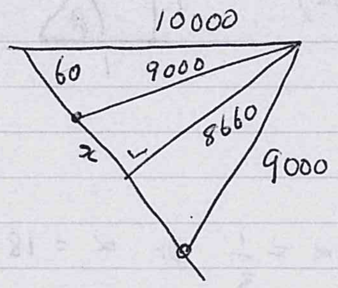
2 (a)





$d = 10 \sin 60 = \frac{10\sqrt{3}}{2} = 5\sqrt{3}$ Km

$= 8660$m



$x = \sqrt{9000^2 - 8660^2}$

$x = 2450.39$

$2x = 4900.8$

!!

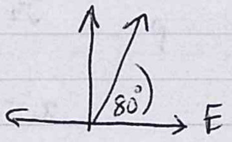
time $= \frac{2x}{|V_{AB}|} = \frac{4900.8}{2.5}$

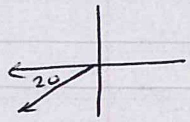
$= 1960.35$

2 (b)

$V_{PW} = 20\cos 80i + 20\sin 80j$

$V_{QW} = -10\cos 20i - 10\sin 20j$





$V_P = V_{PW} + V_W$

$V_Q = V_{QW} + V_W$

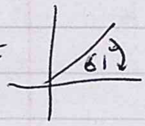
$V_{PQ} = V_P - V_Q$

$V_{PQ} = V_{PW} - V_{QW}$

$V_{PQ} = (20\cos 80 + 10\cos 20)\,i + (20\sin 80 + 10\sin 20)\,j$

$= 12.9i + 23.1j$

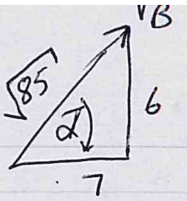
$= \sqrt{12.9^2 + 23.1^2}$ $\quad \tan^{-1}\left(\frac{23.1}{12.9}\right) = \alpha = 61°$ N of E



$= 26.457 = 26$ km/h

2001
2 (a)

$$V_B = 5\sqrt{34}\cdot\frac{7}{\sqrt{85}}\,i + 5\sqrt{34}\cdot\frac{6}{\sqrt{85}}\,j$$

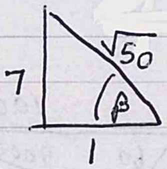


$\sin\alpha = \frac{6}{\sqrt{85}}$

$\cos\alpha = \frac{7}{\sqrt{85}}$

$$V_C = -5\sqrt{5}\cdot\frac{1}{5\sqrt{2}}\,i + 5\sqrt{5}\cdot\frac{7}{5\sqrt{2}}\,j$$

$$V_{BC} = \left(\frac{35\sqrt{34}}{\sqrt{85}} + \frac{\sqrt{5}}{\sqrt{2}}\right)i + \left(\frac{30\sqrt{34}}{\sqrt{85}} - \frac{7\sqrt{5}}{\sqrt{2}}\right)j$$



$\sin\beta = \frac{7}{5\sqrt{2}}$

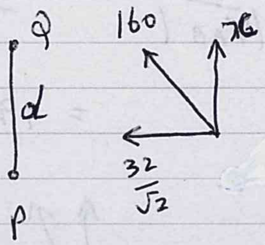
$\cos\beta = \frac{1}{5\sqrt{2}}$

$$7{\cdot}5\sqrt{10}\;i + 2{\cdot}5\sqrt{10}\,j$$

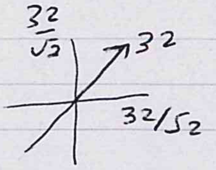
$$= 23{\cdot}72\,i + 7{\cdot}9\,j$$

$|v_{BC}| = 25$ km/h $\quad \tan\alpha = \frac{1}{3}$ or $\alpha = 18{\cdot}4°$ N of E

2 (b)



$V_W =$



$$x = \sqrt{160^2 - \left(\frac{32}{\sqrt{2}}\right)^2} = 158{\cdot}39$$

$$x + \frac{32}{\sqrt{2}} = 181{\cdot}019$$

$$\text{so time up} = \frac{d}{181{\cdot}019}$$

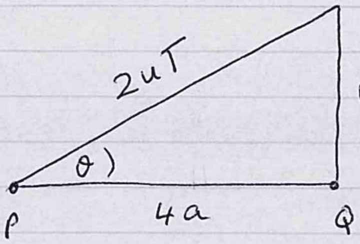
Back Down: $x - \frac{32}{\sqrt{2}} = 135{\cdot}765 \quad$ time down $= \frac{d}{135{\cdot}765}$

$$\frac{d}{181{\cdot}019} + \frac{d}{135{\cdot}765} = 5$$

$$d = \frac{5}{\frac{1}{181{\cdot}019} + \frac{1}{135{\cdot}765}} = 387{\cdot}899$$

$$= 387{\cdot}9 \;/\; 388 \text{ km}$$

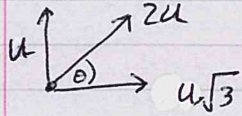
2000a 2(a)



$\sin\theta = \frac{uT}{2uT} = \frac{1}{2}$

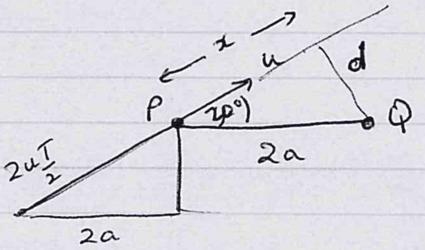
$\theta = 30°$

Time T : time for Horizontal comp of P to travel 4a



$T = \frac{4a}{u\sqrt{3}} = \frac{\text{dist}}{\text{speed}} = T$

At $\frac{T}{2}$



$V_P = u\cos 30 i + u\sin 30 j$

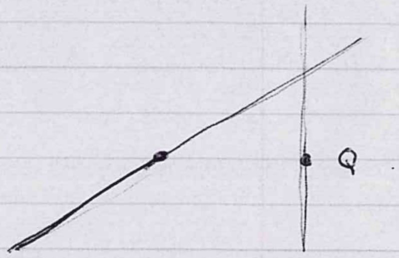
$V_P = \frac{u\sqrt{3}}{2} i + \frac{u}{2} j$

$V_Q = 0i + uj$

$d = 2a\sin 30$

$d = a$

$x = 2a\cos 30 = a\sqrt{3}$ So p has moved :



$x + \frac{2uT}{2}$ $(T = 4a/u\sqrt{3})$

$a\sqrt{3} + \frac{4a}{\sqrt{3}} = \frac{3a + 4a}{\sqrt{3}} = \frac{7a}{\sqrt{3}}$

Q has moved $\frac{uT}{2} + u$(time it takes P to travel x) $\nwarrow \frac{x}{u}$

$= \frac{2a}{\sqrt{3}} + x$

$= \frac{2a}{\sqrt{3}} + a\sqrt{3} = \frac{5a}{\sqrt{3}}$