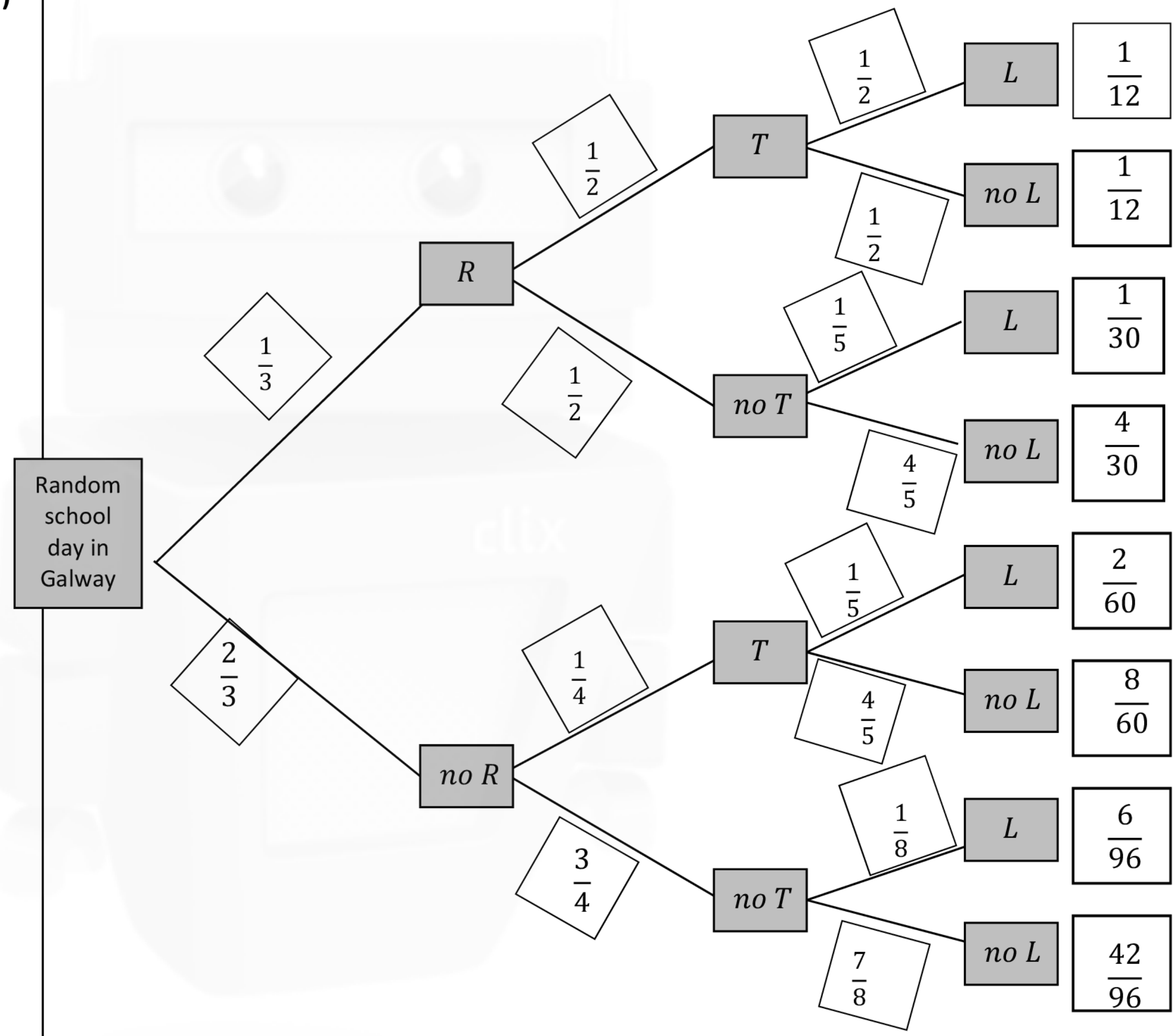
Question 1 (2017)

(a)	$\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}=\frac{256}{78125}$ **or** $=0{\cdot}0032768$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • $\frac{4}{5}$ • $(\frac{1}{5})^3$ *High Partial Credit:* • $\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}$ in any order
(b)	$\binom{6}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)$ $=\frac{1280}{78125}$ **or** $\frac{256}{15625}$ **or** $0{\cdot}016384$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • $\binom{6}{3}$ or $\left(\frac{1}{5}\right)^3$ or $\left(\frac{4}{5}\right)^3$ • $\frac{1}{5}$ for last day *Mid Partial Credit:* • $\binom{6}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^3$ and stops or continues • $\binom{7}{4}\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^3$ and continues *High Partial Credit:* • $\binom{6}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)$
(c)	$1-\left(\frac{4}{5}\right)^n$	**Scale 5B (0, 3, 5)** *Partial Credit:* • 1 or $\left(\frac{4}{5}\right)^n$ • any correct term from the expansion
(d)	$1-\left(\frac{4}{5}\right)^n>0{\cdot}99$ $\left(\frac{4}{5}\right)^n<0{\cdot}01$ $\left(\frac{4}{5}\right)^{20{\cdot}6377}\approx 0{\cdot}01000000517$ $n=21$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • Ans **(c)** $>0{\cdot}99$ *High Partial Credit:* • viable solution to inequality • n = 20·6377 and stops

(b)
(i)



(b)
(i)

Scale 15C (0, 6, 9, 15)

Low Partial Credit:

- 3 boxes filled correctly

High Partial Credit:

- 10 boxes filled correctly

(b) (ii)	$\frac{1}{12}+\frac{1}{30}+\frac{2}{60}+\frac{6}{96}=\frac{17}{80}$ **or** $0{\cdot}2125$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • 2 relevant fractions transferred *High Partial Credit:* • 4 relevant fractions identified but fails to complete
(b) (iii)	$P(R\|L)=\frac{P(R\cap L)}{P(L)}=\frac{\frac{1}{12}+\frac{1}{30}}{\frac{17}{80}}$ $=\frac{28}{51}$ **or** $0{\cdot}5490$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • $P(L)$ • $P(R\cap L)$ *High Partial Credit:* • Formula fully substituted

Question 3 (2016)

Q5	**Model Solution – 25 Marks**	**Marking Notes**
(a) (i)	John ✓ ✓ × ✓ David ✓ × ✓ ✓ Mike × ✓ ✓ ✓	Scale 5B (0, 2, 5) *Partial Credit* • 1 correct column
(a) (ii)	$P(win)=\left(\frac{1}{5}\times\frac{1}{6}\times\frac{3}{4}\right)+\left(\frac{1}{5}\times\frac{5}{6}\times\frac{1}{4}\right)$ $+\left(\frac{4}{5}\times\frac{1}{6}\times\frac{1}{4}\right)+\left(\frac{1}{5}\times\frac{1}{6}\times\frac{1}{4}\right)$ $=\frac{13}{120}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • one correct triple (numerical or descriptive) • probability of any one *Miss* *High Partial Credit* • 4 correct triples (numerical)

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$P(M,3,3)=\frac{1}{26}\times\frac{1}{10}\times\frac{1}{10}=\frac{1}{2600}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • any correct relevant probability *High Partial credit* • correct probabilities but not expressed as single fraction or equivalent **Note:** Accept correct answer without supporting work
(b)	See tables below	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • 1 correct entry to table *High Partial Credit* • all entries correct but fails to finish or finishes incorrectly • no conclusion

Event	Payout	Prob (P(x))	x.P(x)
Win	1000	$\frac{1}{2600}$	$\frac{1000}{2600}$
letter 1 No.	50	$\frac{9}{2600}$	$\frac{450}{2600}$
letter 2nd No	50	$\frac{9}{2600}$	$\frac{450}{2600}$
letter only	50	$\frac{81}{2600}$	$\frac{4050}{2600}$
Fail to win	0		0

$$\sum x.P(x)=\frac{5950}{2600}=2{\cdot}29$$

Club loses 29 cent per play

Or

Event	Pay out	Prob (P(x)	x.P(x)
Win	-998	${}^{1}/_{2600}$	${}^{-998}/_{2600}$
letter + 1st No.	-48	${}^{9}/_{2600}$	${}^{-432}/_{2600}$
Letter + 2nd No	-48	${}^{9}/_{2600}$	${}^{-432}/_{2600}$
letter only	-48	${}^{81}/_{2600}$	${}^{-3888}/_{2600}$
Fail to Win	$+2$	${}^{2500}/_{2600}$	${}^{5000}/_{2600}$

$$\sum x.P(x)=-\frac{750}{2600}=-29 \text{ cent}$$

(c)	Profit = Revenue – Pay-out $600 = 845(x - 2{\cdot}29)$ $$x = \frac{600 + 845(2{\cdot}29)}{845}$$ $$x = 3$$ **or** $$\frac{600}{845} = 0{\cdot}71$$ $$0{\cdot}71 + 2{\cdot}29 = 3$$	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • links profit, revenue and payout *High partial Credit* • formula fully substituted

(a) Complete the table below to show all possible outcomes of the experiment.

		Die 2					
		1	2	3	4	5	6
Die 1	1	L	L	L	L	L	L
	2	L	L	L	L	L	L
	3	L	L	L	L	L	W
	4	L	L	L	L	W	W
	5	L	L	L	W	W	W
	6	L	L	W	W	W	W

(b) **(i)** Find the probability of a win on one throw of the two dice.

$$P(W) = \frac{10}{36} = \frac{5}{18}$$

(ii) Find the probability that each of 3 successive throws of the two dice results in a loss. Give your answer correct to four decimal places.

$$P(L, L, L) = \left(\frac{13}{18}\right)^3 = 0 \cdot 3767$$

(c) The experiment is repeated until a total of 3 wins occur. Find the probability that the third win occurs on the tenth throw of the two dice. Give your answer correct to four decimal places.

$$P(2 \text{ wins in } 9) = \binom{9}{2}\left(\frac{5}{18}\right)^2\left(\frac{13}{18}\right)^7$$

$$P(3 \text{ wins, 3rd on 10th throw}) = \binom{9}{2}\left(\frac{5}{18}\right)^2\left(\frac{13}{18}\right)^7\left(\frac{5}{18}\right) = 0 \cdot 0791$$

(a) Find the probability that Michael is successful (S) with all three of his first three free throws in a game.

$$P(S, S, S) = 0 \cdot 7 \times 0 \cdot 8 \times 0 \cdot 8 = 0 \cdot 448$$

(b) Find the probability that Michael is unsuccessful (U) with his first two free throws and successful with the third.

$$P(U, U, S) = 0 \cdot 3 \times 0 \cdot 4 \times 0 \cdot 6 = 0 \cdot 072$$

(c) List all the ways that Michael could be successful with his third free throw in a game and hence find the probability that Michael is successful with his third free throw.

S, S, S U, U, S S, U, S U, S, S

$$P(S, S, S) = 0 \cdot 7 \times 0 \cdot 8 \times 0 \cdot 8 = 0 \cdot 448$$
$$P(U, U, S) = 0 \cdot 3 \times 0 \cdot 4 \times 0 \cdot 6 = 0 \cdot 072$$
$$P(S, U, S) = 0 \cdot 7 \times 0 \cdot 2 \times 0 \cdot 6 = 0 \cdot 084$$
$$P(U, S, S) = 0 \cdot 3 \times 0 \cdot 6 \times 0 \cdot 8 = 0 \cdot 144$$

$$P = 0 \cdot 448 + 0 \cdot 072 + 0 \cdot 084 + 0 \cdot 144 = 0 \cdot 748$$

(d) **(i)** Let p_n be the probability that Michael is successful with his n^{th} free throw in the game (and hence $(1-p_n)$ is the probability that Michael is unsuccessful with his n^{th} free throw). Show that $p_{n+1} = 0 \cdot 6 + 0 \cdot 2p_n$.

$$\begin{aligned} p_{n+1} &= \mathrm{P}(\mathrm{S,S}) + \mathrm{P}(\mathrm{U,S}) \\ &= p_n \times 0 \cdot 8 + (1 - p_n)0 \cdot 6 \\ &= 0 \cdot 6 + 0 \cdot 2p_n \end{aligned}$$

(ii) Assume that p is Michael's success rate in the long run; that is, for large values of n, we have $p_{n+1} \approx p_n \approx p$.
Using the result from part **(d) (i)** above, or otherwise, show that $p = 0 \cdot 75$.

$$p \approx p_n \approx p_{n+1} = 0 \cdot 6 + 0 \cdot 2p_n$$
$$\Rightarrow 0 \cdot 8p_n = 0 \cdot 6$$
$$\Rightarrow p_n = \frac{0 \cdot 6}{0 \cdot 8} = 0 \cdot 75 = p$$

(e) For all positive integers n, let $a_n = p - p_n$, where $p = 0 \cdot 75$ as above.

(i) Use the ratio $\frac{a_{n+1}}{a_n}$ to show that a_n is a geometric sequence with common ratio $\frac{1}{5}$.

$$\begin{aligned} \frac{a_{n+1}}{a_n} &= \frac{p - p_{n+1}}{p - p_n} \\ &= \frac{0 \cdot 75 - (0 \cdot 6 + 0 \cdot 2p_n)}{0 \cdot 75 - p_n} \\ &= \frac{0 \cdot 15 - 0 \cdot 2p_n}{5(0 \cdot 15 - 0 \cdot 2p_n)} = \frac{1}{5} \end{aligned}$$

(ii) Find the smallest value of n for which $p - p_n < 0{\cdot}00001$.

$$a_n = p - p_n$$
$$a_1 = p - p_1 = 0 \cdot 75 - 0 \cdot 7 = 0 \cdot 05$$

$$ar^{n-1} = 0 \cdot 05(0 \cdot 2)^{n-1} < 0 \cdot 00001$$
$$(n-1)\ln 0 \cdot 2 < \ln 0 \cdot 0002$$
$$\Rightarrow n - 1 > \frac{\ln 0 \cdot 0002}{\ln 0 \cdot 2} = 5 \cdot 29$$
$$\Rightarrow n > 6 \cdot 29$$

$$n = 7$$

(f) You arrive at a game in which Michael is playing. You know that he has already taken many free throws, but you do not know what pattern of success he has had.

(i) Based on this knowledge, what is your estimate of the probability that Michael will be successful with his next free throw in the game?

Answer: $0{\cdot}75$ **or** p

(ii) Why would it **not** be appropriate to consider Michael's subsequent free throws in the game as a sequence of Bernoulli trials?

Events not independent

(a) John played Game A four times and tells us that he has won a total of €8. In how many different ways could he have done this?

5, 3, 0, 0;	3, 5, 0, 0;	0, 5, 3, 0;	0, 3, 5, 0;	
5, 0, 3, 0;	3, 0, 5, 0;	0, 5, 0, 3;	0, 3, 0, 5;	12 ways
5, 0, 0, 3;	3, 0, 0, 5;	0, 0, 5, 3;	0, 0, 3, 5.	

(b) To spin either arrow once, the player pays €3. Which game of chance would you expect to be more successful in raising funds for the charity? Give a reason for your answer.

Expected outcome $E(X)=\sum x.P(x)$

Game A: $E(X)=0\left(\frac{2}{5}\right)+3\left(\frac{1}{5}\right)+5\left(\frac{1}{5}\right)+6\left(\frac{1}{5}\right)=2\frac{4}{5}$

Game B: $E(X)=0\left(\frac{1}{6}\right)+1\left(\frac{1}{6}\right)+2\left(\frac{1}{6}\right)+3\left(\frac{1}{6}\right)+4\left(\frac{1}{6}\right)+5\left(\frac{1}{6}\right)=2\frac{3}{6}=2\frac{1}{2}$

Game B - it pays out less money.

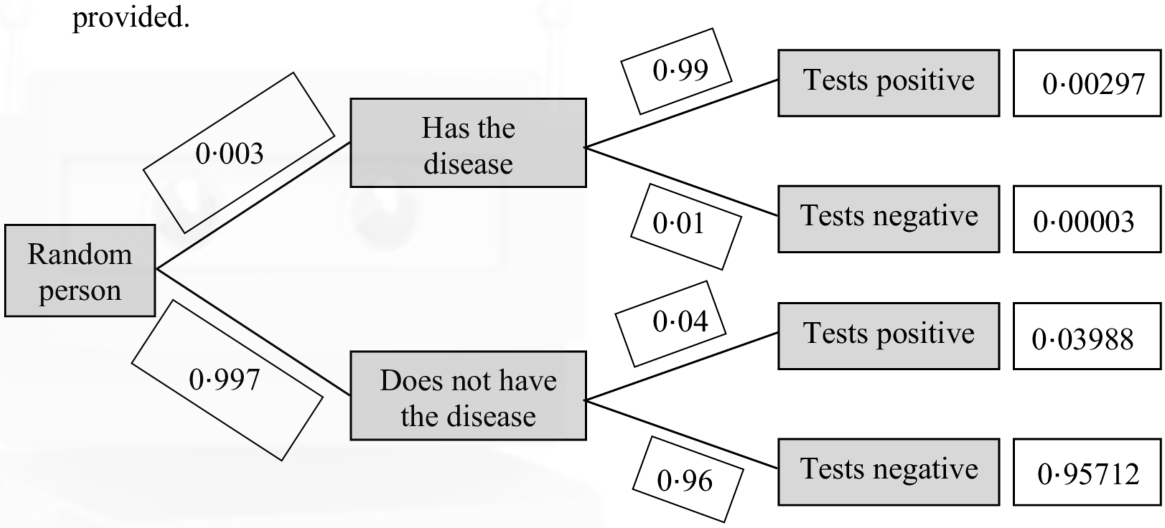
Or

On average, over the long term:
Game A pays out €14 for every €15 taken in.
Game B pays out €15 for every €18 taken in.
Game B – it pays out a smaller proportion on the money taken in.

(c) Mary plays Game B six times. Find the probability that the arrow stops in the €4 sector exactly twice.

P(stops in €4 sector exactly twice) $=\binom{6}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4=0\cdot 2$

(a) **(i)** Write the probability associated with each branch of the tree diagram in the blank boxes provided.



(ii) Hence, or otherwise, calculate the probability that a person selected at random from the population tests positive for the disease.

$$\text{P(Positive test)} = 0 \cdot 00297 + 0 \cdot 03988 = 0 \cdot 04285$$

(iii) A person tests positive for the disease. What is the probability that the person actually has the disease. Give your answer correct to three significant figures.

$$\text{P(Has disease}|\text{positive test)} = \frac{0 \cdot 00297}{0 \cdot 04285} = 0 \cdot 0693$$

(iv) The health authority is considering using a test on the general population with a view to treatment of the disease. Based on your results, do you think that the above test would be an effective way to do this? Give a reason for your answer.

Test is not very useful.
A person who tests positive has the disease only 7% of the time.

(i) Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to justify the company's claim. State the null hypothesis and state your conclusion clearly.

H_0: The new drug is not more successful than the generic drug.
$p = 0\cdot 51$

95% margin of error $= \dfrac{1}{\sqrt{500}} = 0\cdot 045$

The success rate for the new drug is $\dfrac{296}{500} = 0\cdot 592$.
This is outside the interval $[0\cdot 51 - 0\cdot 045, 0\cdot 51 + 0\cdot 045] = [0\cdot 465, 0\cdot 555]$
Result is significant, reject the null hypothesis.
There is evidence to conclude that the new drug is more successful than the generic.

Or

H_0: The new drug is not more successful than the generic drug.
H_1: The new drug is more successful than the generic drug.
$p = 0\cdot 51$

95% margin of error $= \dfrac{1}{\sqrt{500}} = 0\cdot 045$

The success rate for the new drug is $\dfrac{296}{500} = 0\cdot 592$.
The 95% confidence interval for the population is
$0\cdot 592 - 0\cdot 045 < p < 0\cdot 592 + 0\cdot 045 = 0\cdot 547 < p < 0\cdot 637$
$p = 0\cdot 51$ is outside this interval.
Result is significant, reject the null hypothesis.
There is evidence to conclude that the new drug is more successful than the generic

(ii) The null hypothesis was accepted for Drug *B*. Estimate the greatest number of patients in that trial who could have been successfully treated with Drug *B*.

The result must lie in the interval $[0\cdot 465, 0\cdot 555]$

Thus, $\dfrac{n}{500} < 0\cdot 555 \Rightarrow n < 277\cdot 5$
Hence, 277 patients.

Or

$k - 0\cdot 045 < 0\cdot 51 < k + 0\cdot 045$

$\Rightarrow k - 0\cdot 045 < 0\cdot 51$

$\Rightarrow k < 0\cdot 555$

Number of patients $< 0\cdot 555 \times 500 = 277\cdot 5$
Hence, 277 patients.

(a) Complete the probability table below and hence calculate $E(X)$, the expected value of X.

x	13	14	15	16
$P(X=x)$	0.383	0.575	**0.038**	0.004

Note that the sum of the probabilities must be 1. Therefore $P(15) = 1 - 0.383 - 0.575 - 0.004 = 0.038$.
$E(X) = 13(0.383) + 14(0.575) + 15(0.038) + 16(0.004) = 13.663$.

(b) If X is the age, in complete years, on 1 January 2013 of a student selected at random from among all second-year students in Irish schools, explain what $E(X)$ represents.

$E(X)$ represents the mean of the ages of all second-year students in Irish schools on 1 January 2013.

(c) If ten students are selected at random from this population, find the probability that exactly six of them were 14 years old on 1 January 2013. Give your answer correct to three significant figures.

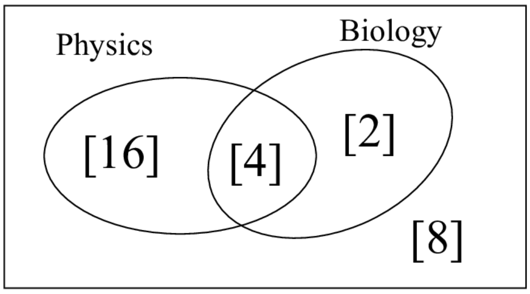
We have 10 Bernoulli trials with $p = P(X = 14) = 0.575$. So the probability of exactly six successes is given by

$$\begin{aligned} \binom{10}{6} p^6(1-p)^4 &= \frac{10!}{6!4!}(0.575)^6(1-0.575)^4 \\ &= 210(0.575)^6(0.425)^4 \\ &= 0.248 \end{aligned}$$

correct to three significant places.

(b) In a class of 30 students, 20 study Physics, 6 study Biology and 4 study both Physics and Biology.

(i) Represent the information on the Venn Diagram.



A student is selected at random from this class.
The events E and F are:

E: The student studies Physics

F: The student studies Biology.

(ii) By calculating probabilities, investigate if the events E and F are independent.

$P(E \cap F) = \frac{4}{30}$

$P(E) \times P(F) = \frac{20}{30} \times \frac{6}{30} = \frac{4}{30}$

$P(E \cap F) = P(E) \times P(F) \Rightarrow$ E and F are independent events

Trials are independent of each other.
Probability of success is the same each time.

[Only two outcomes …. (Given)]
[Finite number of throws…… (Given)]

(b) Based on such assumption(s), find, correct to three decimal places, the probability that:

(i) she scores on exactly four of the six shots

$$P(X=4) = {}^6C_4(0{\cdot}6)^4(0{\cdot}4)^2 = 0{\cdot}31104$$
$$= 0{\cdot}311 \text{ to three decimal places.}$$

(ii) she scores for the second time on the fifth shot.

Exactly one success among first four throws, followed by success on fifth:

$$\left({}^4C_1(0{\cdot}6)(0{\cdot}4)^3\right)(0{\cdot}6) = 0{\cdot}09216$$
$$= 0{\cdot}092 \text{ to three decimal places.}$$

Question 12 (2012)

Question 1 **(25 marks)**

The events A and B are such that $P(A) = 0.7, P(B) = 0.5$ and $P(A \cap B) = 0.3$.

(a) Find $P(A \cup B)$.

$$\begin{aligned} P(A\cup B) &= P(A)+P(B)-P(A\cap B) \\ &= 0.7+0.5-0.3 \\ &= 0.9 \end{aligned}$$

(b) Find $P(A|B)$.

$$\begin{aligned} P(A|B) &= \frac{P(A\cap B)}{P(B)} \\ &= \frac{0.3}{0.5} \\ &= 0.6 \end{aligned}$$

(c) State whether A and B are independent events and justify your answer

If A and B are independent events then $P(A\cap B) = P(A)P(B)$.
Here, $P(A\cap B) = 0.3$ but $P(A)P(B) = (0.7)(0.9) = 0.63$.
So A and B are NOT independent events.

Question 13 (2011)

Let x = number of boys, who study French.

$\therefore 12 - x$ = number of girls who study French.

$$\frac{12-x}{16} = 1{\cdot}5\left(\frac{x}{8}\right)$$

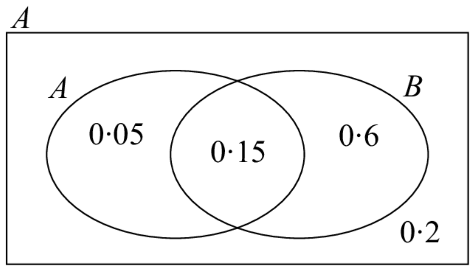
$$96 - 8x = 24x$$

$$32x = 96$$

$$x = 3$$

Three boys study French.

(a) Complete this Venn diagram.



(b) Find the probability that neither A nor B happens.

$$0{\cdot}2.$$

or

$$P(A \cup B)' = 1 - P(A \cup B) = 1 - (0{\cdot}05 + 0{\cdot}15 + 0{\cdot}6) = 0{\cdot}2$$

(c) Find the conditional probability $P(A \mid B)$.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{0{\cdot}15}{0{\cdot}75} = 0{\cdot}2.$$

(d) State whether A and B are independent events and justify your answer.

A and B are independent events as, $P(A \mid B) = P(A) = 0{\cdot}2$.

or

A and B are independent events as, $P(A)P(B) = (0{\cdot}2)(0{\cdot}75) = 0{\cdot}15 = P(A \cap B)$.

(a) From the diagram, estimate the correlation coefficient.

Answer: – 0·75

(b) Circle the *outlier* on the diagram and write down the person's age and maximum heart rate.

Age = 47 years

Max. heart rate = 137 bpm

(c) The line of best fit is shown on the diagram. Use the line of best fit to estimate the maximum heart rate of a 44-year-old person.

Answer: 176 bpm

(d) By taking suitable readings from the diagram, calculate the slope of the line of best fit.

Possible Readings

$(10, 200)$ and $(90, 144)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{144 - 200}{90 - 10} = -\frac{-56}{80} = -\frac{7}{10} \text{ or } m = -0{\cdot}7.$$

(e) Find the equation of the line of best fit and write it in the form: $MHR = a - b \times (\text{age})$, where MHR is the maximum heart rate.

$$y - y_1 = m(x - x_1)$$
$$y - 200 = -0{\cdot}7(x - 10)$$
$$y = -0{\cdot}7x + 207$$
$$MHR = 207 - 0{\cdot}7 \times (age)$$

(f) The researchers compared their new rule for estimating maximum heart rate to an older rule. The older rule is: $MHR = 220 - \text{age}$. The two rules can give different estimates of a person's maximum heart rate. Describe how the level of agreement between the two rules varies according to the age of the person. Illustrate your answer with two examples.

For young adults the old rule gives a greater *MHR* than the new rule.

<u>*Adult aged 20*</u>

$MHR = 220 - 20 = 200$ bpm (Old rule)

$MHR = 207 - 0{\cdot}7(20) = 193$ bpm (New Rule)

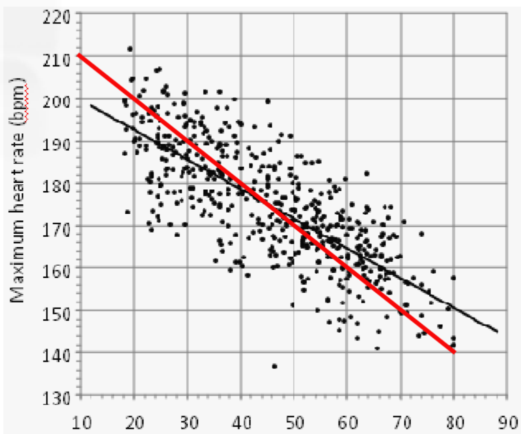
Towards middle age there is a greater agreement between the rules.

For older people the new rule gives a greater *MHR* than the old rule.

<u>*Adult aged 70*</u>

$MHR = 220 - 70 = 150$ bpm

$MHR = 207 - 0{\cdot}7(70) = 158$ bpm



(g) A particular exercise programme is based on the idea that a person will get most benefit by exercising at 75% of their estimated *MHR*. A 65-year-old man has been following this programme, using the old rule for estimating *MHR*. If he learns about the researchers' new rule for estimating *MHR*, how should he change what he is doing?

He should exercise a bit more intensely.

Using the old rule he exercises to 75% of $(220 - 65) = 116$ bpm.

Using the new rule he can exercise to 75% of $(207 - 0{\cdot}7 \times 65) = 121$ bpm.

(a) What is the probability that a randomly selected rod will be less than 39·7 mm in length?

$$P(X < 39{\cdot}7) = P\left(Z < \frac{39{\cdot}7 - 40}{0{\cdot}2}\right) = P(Z < -1{\cdot}5)$$
$$= P(z > 1{\cdot}5)$$
$$= 1 - P(Z \le 1{\cdot}5)$$
$$= 1 - 0{\cdot}9332$$
$$= 0{\cdot}0668$$

(b) Five rods are selected at random. What is the probability that at least two of them are less than 39·7 mm in length?

Binomial distribution with $n = 5$, $p = 0{\cdot}0668$, $q = 0{\cdot}9332$.

$$P(X \ge 2) = 1 - P(X < 2) = 1 - \left[P(X=1) + P(X=0)\right]$$
$$= 1 - \left[\binom{5}{1}(0{\cdot}0668)(0{\cdot}9332)^4 + \binom{5}{0}(0{\cdot}9332)^5\right]$$
$$= 0{\cdot}03895.$$

Or

$$P(X \ge 2) = P(X=2) + P(X=3) + P(X=4) + P(X=5)$$
$$= \binom{5}{2}(0{\cdot}0668)^2(0{\cdot}9332)^3 + \binom{5}{3}(0{\cdot}0668)^3(0{\cdot}9332)^2 + \binom{5}{4}(0{\cdot}0668)^4(.9332) + \binom{5}{5}(0{\cdot}0668)^5$$
$$= 0{\cdot}03895$$

$H_0 : \mu = 40$ mm (null hypothesis)
$H_1 : \mu \ne 40$ mm (alternative hypothesis)

$$\sigma_{\bar{x}} = \frac{0{\cdot}2}{\sqrt{10}} = 0{\cdot}0632456$$

Observed value of $\bar{x} = 39{\cdot}87$

$$\therefore \text{ Observed } z = \frac{39{\cdot}87 - 40}{0{\cdot}0632456} = -2{\cdot}055$$

The critical values for the test are $\pm 1{\cdot}96$

As $-2{\cdot}055 < -1{\cdot}96$, we reject the null hypothesis at the 5% level of significance and we conclude that the machine setting has become inaccurate.