Question 1

Question 1 **(25 marks)**

When Conor rings Ciara's house, the probability that Ciara answers the phone is $\frac{1}{5}$.

(a) Conor rings Ciara's house once every day for 7 consecutive days. Find the probability that she will answer the phone on the 2nd, 4th, and 6th days but not on the other days.

(b) Find the probability that she will answer the phone for the 4th time on the 7th day.

(c) Conor rings her house once every day for *n* days. Write, in terms of *n*, the probability that Ciara will answer the phone at least once.

(d) Find the minimum value of *n* for which the probability that Ciara will answer the phone at least once is greater than 99%.

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(b) In Galway, rain falls in the morning on $\frac{1}{3}$ of the school days in the year.

When it is raining the probability of heavy traffic is $\frac{1}{2}$.

When it is not raining the probability of heavy traffic is $\frac{1}{4}$.

When it is raining and there is heavy traffic, the probability of being late for school is $\frac{1}{2}$.

When it is not raining and there is no heavy traffic, the probability of being late for school is $\frac{1}{8}$.

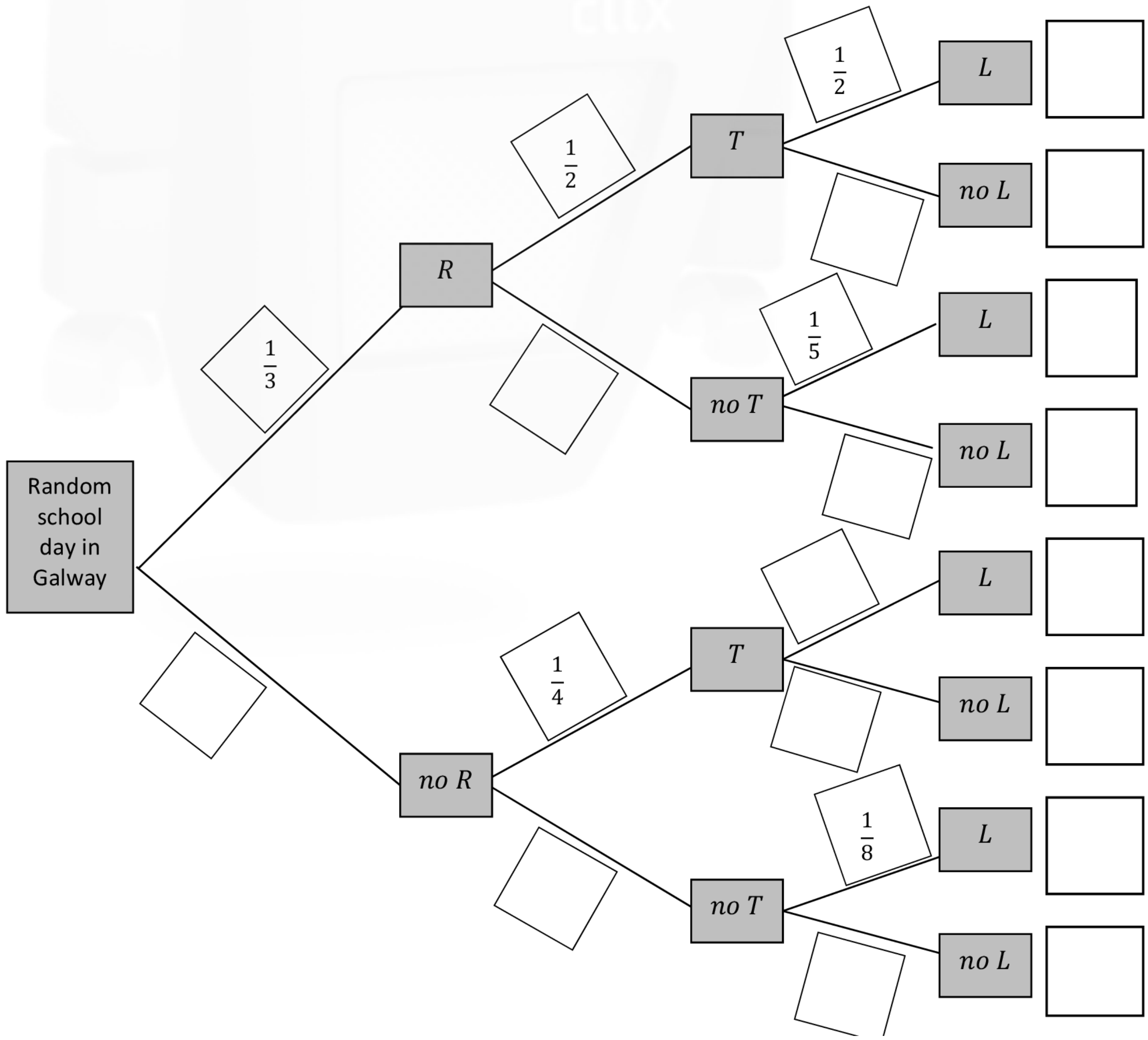
In any other situation the probability of being late for school is $\frac{1}{5}$.

Some of this information is shown in the tree diagram below.

(i) Write the probability associated with each branch of the tree diagram **and** the probability of each outcome into the blank boxes provided.

Give each answer in the form $\frac{a}{b}$, where $a, b \in \mathbb{N}$.

Key	Rain $= R$	Heavy traffic $= T$	Late $= L$
	No rain $= no\ R$	Not heavy traffic $= no\ T$	Not late $= no\ L$



(ii) On a random school day in Galway, find the probability of being late for school.

(iii) On a random school day in Galway, find the probability that it rained in the morning, given that you were late for school.

Question 3

(a) **(i)** In an archery competition, the team consisting of John, David, and Mike will win 1st prize if at least two of them hit the bullseye with their last arrows. From past experience, they know that the probability that John, David, and Mike will hit the bullseye on their last arrow is $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{4}$ respectively.
Complete the table below to show all the ways in which they could win 1st prize.

	Way 1	**Way 2**	**Way 3**	**Way 4**
John	✓			
David	✓			
Mike	×			

✓	= Hit
×	= Miss

(ii) Hence or otherwise find the probability that they will win the competition.

Question 4

A local sports club is planning to run a weekly lotto. To win the Jackpot of €1000, contestants must match one letter chosen from the 26 letters in the alphabet and two numbers chosen, in the correct order, from the numbers 0 to 9. In this lotto, repetition of numbers is allowed (e.g. M, 3, 3 is an outcome).

(a) Calculate the probability that M, 3, 3 would be the winning outcome in a particular week.

(b) If a contestant matches the letter only, or the letter and one number (but not both numbers), they will win €50. Using the table below, or otherwise, find how much the club should expect to make or lose on each play, correct to the nearest cent, if they charge €2 per play.

Event	**Payout** (x) €	**Probability** (P(x))	x.P(x)
Win Jackpot			
Match letter and first number only			
Match letter and second number only			
Match letter and neither number			
Fail to win			

(c) The club estimates that the average number of plays per week will be 845. If the club wants to make an average profit of €600 per week from the lotto, how much should the club charge per play, correct to the nearest cent?

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Question 1 **(25 marks)**

An experiment consists of throwing two fair, standard, six-sided dice and noting the sum of the two numbers thrown. If the sum is 9 or greater it is recorded as a "win" (W). If the sum is 8 or less it is recorded as a "loss" (L).

(a) Complete the table below to show all possible outcomes of the experiment.

		Die 2					
		1	2	3	4	5	6
Die 1	1		L				
	2						
	3						
	4						
	5						W
	6						

(b) **(i)** Find the probability of a win on one throw of the two dice.

(ii) Find the probability that each of 3 successive throws of the two dice results in a loss. Give your answer correct to four decimal places.

(c) The experiment is repeated until a total of 3 wins occur. Find the probability that the third win occurs on the tenth throw of the two dice. Give your answer correct to four decimal places.

Question 8 **(65 marks)**

In basketball, players often have to take free throws. When Michael takes his first free throw in any game, the probability that he is successful is 0·7.
For all subsequent free throws in the game, the probability that he is successful is:

- 0·8 if he has been successful on the previous throw
- 0·6 if he has been unsuccessful on the previous throw.

(a) Find the probability that Michael is successful (S) with all three of his first three free throws in a game.

P(S, S, S) =

(b) Find the probability that Michael is unsuccessful (U) with his first two free throws and successful with the third.

P(U, U, S) =

(c) List all the ways that Michael could be successful with his third free throw in a game and hence find the probability that Michael is successful with his third free throw.

(d) **(i)** Let p_n be the probability that Michael is successful with his n^{th} free throw in the game (and hence $(1 - p_n)$ is the probability that Michael is unsuccessful with his n^{th} free throw). Show that $p_{n+1} = 0{\cdot}6 + 0{\cdot}2p_n$.

(ii) Assume that p is Michael's success rate in the long run; that is, for large values of n, we have $p_{n+1} \approx p_n \approx p$.
Using the result from part **(d) (i)** above, or otherwise, show that $p = 0{\cdot}75$.

(e) For all positive integers n, let $a_n = p - p_n$, where $p = 0{\cdot}75$ as above.

(i) Use the ratio $\frac{a_{n+1}}{a_n}$ to show that a_n is a geometric sequence with common ratio $\frac{1}{5}$.

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(ii) Find the smallest value of n for which $p - p_n < 0{\cdot}00001$.

(f) You arrive at a game in which Michael is playing. You know that he has already taken many free throws, but you do not know what pattern of success he has had.

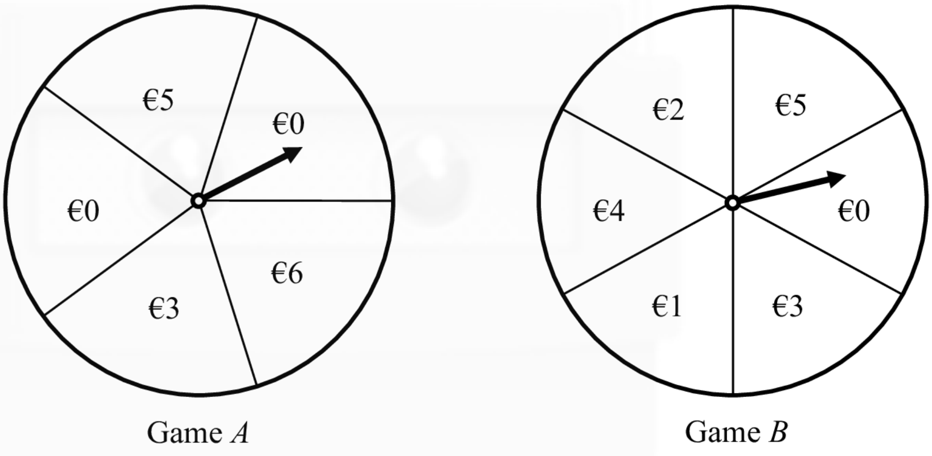
(i) Based on this knowledge, what is your estimate of the probability that Michael will be successful with his next free throw in the game?

Answer: ________________

(ii) Why would it **not** be appropriate to consider Michael's subsequent free throws in the game as a sequence of Bernoulli trials?

Question 7

Two different games of chance, shown below, can be played at a charity fundraiser. In each game, the player spins an arrow on a wheel and wins the amount shown on the sector that the arrow stops in. Each game is fair in that the arrow is just as likely to stop in one sector as in any other sector on that wheel.



(a) John played Game *A* four times and tells us that he has won a total of €8. In how many different ways could he have done this?

(b) To spin either arrow once, the player pays €3. Which game of chance would you expect to be more successful in raising funds for the charity? Give a reason for your answer.

(c) Mary plays Game *B* six times. Find the probability that the arrow stops in the €4 sector exactly twice.

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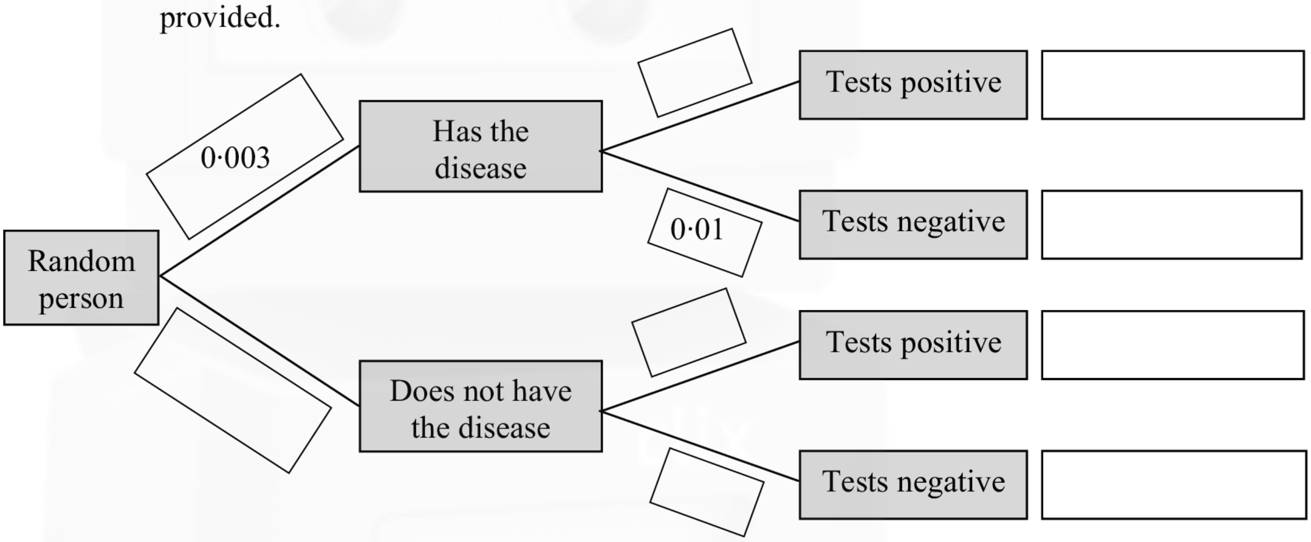
Blood tests are sometimes used to indicate if a person has a particular disease. Sometimes such tests give an incorrect result, either indicating the person has the disease when they do not (called a false positive) or indicating that they do not have the disease when they do (called a false negative).

It is estimated that 0·3% of a large population have a particular disease. A test developed to detect the disease gives a false positive in 4% of tests and a false negative in 1% of tests. A person picked at random is tested for the disease.

(a) **(i)** Write the probability associated with each branch of the tree diagram in the blank boxes provided.



(ii) Hence, or otherwise, calculate the probability that a person selected at random from the population tests positive for the disease.

(iii) A person tests positive for the disease. What is the probability that the person actually has the disease? Give your answer correct to three significant figures.

(iv) The health authority is considering using a test on the general population with a view to treatment of the disease. Based on your results, do you think that the above test would be an effective way to do this? Give a reason for your answer.

(b) A generic drug used to treat a particular condition has a success rate of 51%. A company is developing two new drugs, *A* and *B*, to treat the condition. They carried out clinical trials on two groups of 500 patients suffering from the condition. The results showed that Drug *A* was successful in the case of 296 patients. The company claims that Drug *A* is more successful in treating the condition than the generic drug.

(i) Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to justify the company's claim. State the null hypothesis and state your conclusion clearly.

(ii) The null hypothesis was accepted for Drug *B*. Estimate the greatest number of patients in that trial who could have been successfully treated with Drug *B*.

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Question 1 **(25 marks)**

The random variable X has a discrete distribution. The probability that it takes a value other than 13, 14, 15 or 16 is negligible.

(a) Complete the probability distribution table below and hence calculate $E(X)$, the expected value of X.

x	13	14	15	16
$P(X = x)$	0·383	0·575		0·004

(b) If X is the age, in complete years, on 1 January 2013 of a student selected at random from among all second-year students in Irish schools, explain what $E(X)$ represents.

(c) If ten students are selected at random from this population, find the probability that exactly six of them were 14 years old on 1 January 2013. Give your answer correct to three significant figures.

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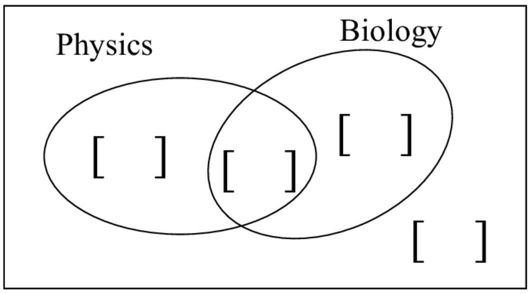
(b) In a class of 30 students, 20 study Physics, 6 study Biology and 4 study both Physics and Biology.

(i) Represent the information on the Venn Diagram.



A student is selected at random from this class.
The events E and F are:

E: The student studies Physics

F: The student studies Biology.

(ii) By calculating probabilities, investigate if the events E and F are independent.

Question 4 **(25 marks)**

A certain basketball player scores 60% of the free-throw shots she attempts. During a particular game, she gets six free throws.

(a) What assumption(s) must be made in order to regard this as a sequence of Bernoulli trials?

(b) Based on such assumption(s), find, correct to three decimal places, the probability that:

(i) she scores on exactly four of the six shots

(ii) she scores for the second time on the fifth shot.

Question 1 **(25 marks)**

The events A and B are such that $P(A) = 0{\cdot}7$, $P(B) = 0{\cdot}5$ and $P(A \cap B) = 0{\cdot}3$.

(a) Find $P(A \cup B)$

(b) Find $P(A \mid B)$

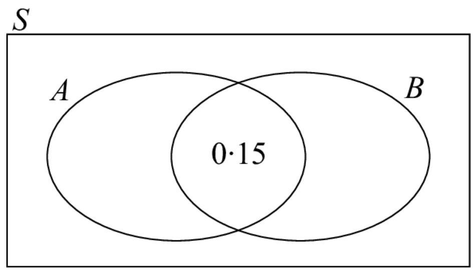
(c) State whether A and B are independent events, and justify your answer.

(b) There are 16 girls and 8 boys in a class. Half of these 24 students study French. The probability that a randomly selected girl studies French is 1·5 times the probability that a randomly selected boy studies French. How many of the boys in the class study French?

Question 14

Two events A and B are such that $P(A) = 0{\cdot}2$, $P(A \cap B) = 0{\cdot}15$ and $P(A' \cap B) = 0{\cdot}6$.

(a) Complete this Venn diagram.

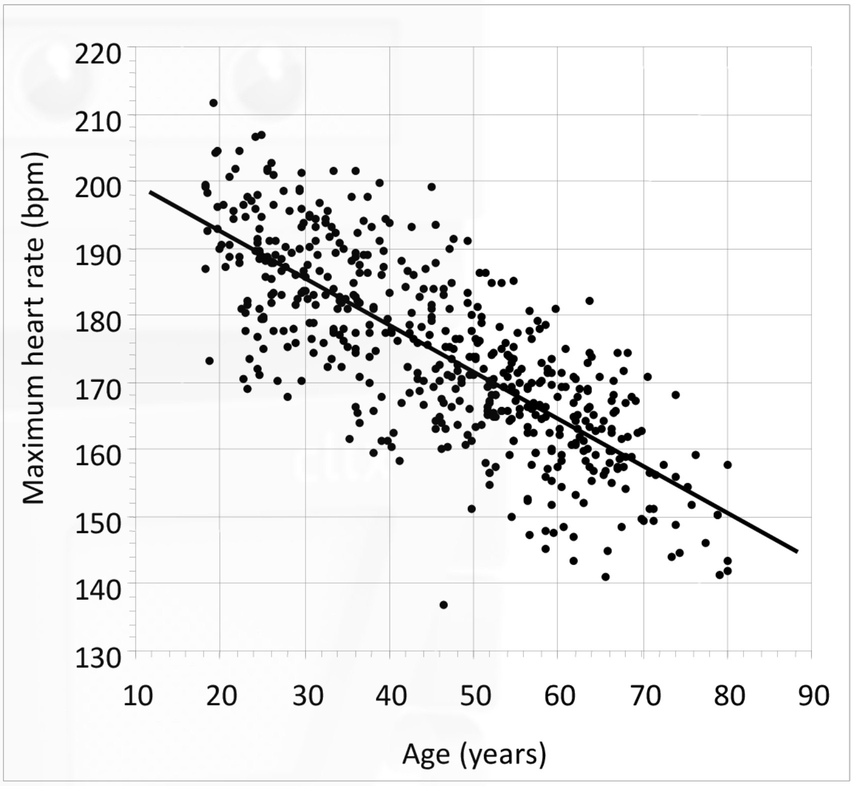


(b) Find the probability that neither A nor B happens.

(c) Find the conditional probability $P(A \mid B)$.

(d) State whether A and B are independent events and justify your answer.

A person's *maximum heart rate* is the highest rate at which their heart beats during certain extreme kinds of exercise. It is measured in beats per minute (bpm). It can be measured under controlled conditions. As part of a study in 2001, researchers measured the maximum heart rate of 514 adults and compared it to each person's age. The results were like those shown in the scatter plot below.



Source: Simulated data based on:Tanaka H, Monaghan KD, and Seals DR. *Age-predicted maximal heart rate revisited*, J. Am. Coll. Cardiol. 2001;37;153-156.

(a) From the diagram, estimate the correlation coefficient.

Answer: [ ]

(b) Circle the *outlier* on the diagram and write down the person's age and maximum heart rate.

Age = [ ] Max. heart rate = [ ]

(c) The line of best fit is shown on the diagram. Use the line of best fit to estimate the maximum heart rate of a 44-year-old person.

Answer: [ ]

(d) By taking suitable readings from the diagram, calculate the slope of the line of best fit.

(e) Find the equation of the line of best fit and write it in the form: $MHR = a - b \times (\text{age})$, where MHR is the maximum heart rate.

(f) The researchers compared their new rule for estimating maximum heart rate to an older rule. The older rule is: $MHR = 220 - \text{age}$. The two rules can give different estimates of a person's maximum heart rate. Describe how the level of agreement between the two rules varies according to the age of the person. Illustrate your answer with two examples.

(g) A particular exercise programme is based on the idea that a person will get most benefit by exercising at 75% of their estimated *MHR*. A 65-year-old man has been following this programme, using the old rule for estimating *MHR*. If he learns about the researchers' new rule for estimating *MHR*, how should he change what he is doing?

A factory manufactures aluminium rods. One of its machines can be set to produce rods of a specified length. The lengths of these rods are normally distributed with mean equal to the specified length and standard deviation equal to 0·2 mm.

The machine has been set to produce rods of length 40 mm.

(a) What is the probability that a randomly selected rod will be less than 39·7 mm in length?

(b) Five rods are selected at random. What is the probability that at least two of them are less than 39·7 mm in length?

(c) The operators want to check whether the setting on the machine is still accurate. They take a random sample of ten rods and measure their lengths. The lengths in millimetres are:

39·5	40·0	39·7	40·2	39·8
39·7	40·2	39·9	40·1	39·6

Conduct a hypothesis test at the 5% level of significance to decide whether the machine's setting has become inaccurate. You should start by clearly stating the null hypothesis and the alternative hypothesis, and finish by clearly stating what you conclude about the machine.