MarkingScheme

RootsOfFunctionH

Question 1 (2017)

(a)	$f(x) = 2x^3 + 5x^2 - 4x - 3$ $f(-3) = 2(-3)^3 + 5(-3)^2 - 4(-3) - 3$ $= -54 + 45 + 12 - 3$ $f(-3) = 0$ $\Rightarrow (x+3)$ is a factor $\begin{array}{r} 2x^2 - x - 1 \\ x+3 \overline{) 2x^3 + 5x^2 - 4x - 3} \\ \underline{2x^3 + 6x^2} \qquad\qquad \\ -x^2 - 4x \quad\;\; \\ \underline{-x^2 - 3x} \quad\;\; \\ -x - 3 \\ \underline{-x - 3} \end{array}$ $f(x) = (x+3)(2x^2 - x - 1)$ $f(x) = (x+3)(2x+1)(x-1)$ $x = -3 \quad x = -\frac{1}{2} \quad x = 1$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* • Shows $f(-3) = 0$ *High Partial Credit:* • quadratic factor of $f(x)$ found **Note:** No remainder in division may be stated as reason for $x = -3$ as root
(b)	$y = 2x^3 + 5x^2 - 4x - 3$ $\frac{dy}{dx} = 6x^2 + 10x - 4 = 0$ $3x^2 + 5x - 2 = 0$ $(x+2)(3x-1) = 0$ $3x - 1 = 0 \quad x + 2 = 0$ $x = \frac{1}{3} \quad x = -2$ $f\left(\frac{1}{3}\right) = \frac{-100}{27} \quad f(-2) = 9$ $Max = (-2, 9) \quad Min = \left(\frac{1}{3}, \frac{-100}{27}\right)$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • $\frac{dy}{dx}$ found (Some correct differentiation) *High Partial Credit* • roots and one y value found **Note:** One of Max/Min must be identified for full credit
(c)	$a > \frac{100}{27}$ **or** $a < -9$	**Scale 5B (0, 3, 5)** *Partial Credit:* • one value identified • no range identified (from 2 values)

Q7	Model Solution – 40 Marks	Marking Notes
(a) (i)	$v=\frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dr}=4\pi r^2$ $\frac{dv}{dt}=250\ \mathrm{cm^3/s}$ $\frac{dr}{dt}=\frac{dr}{dv}.\frac{dv}{dt}=\frac{1}{4\pi r^2}.250$ $\frac{dr}{dt}=\frac{250}{4\pi 400}=\frac{5}{32\pi}\ \ \mathrm{cm/s}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • work towards $\frac{dv}{dr}$ or $\frac{dv}{dt}$ or $\frac{dr}{dt}$ *High Partial Credit* • correct expression for $\frac{dr}{dt}$
(ii)	$a=4\pi r^2 \Rightarrow \frac{da}{dr}=8\pi r$ $\frac{da}{dt}=\frac{da}{dr}.\frac{dr}{dt}=8\pi r.\frac{5}{32\pi}$ $=\frac{5(20)}{4}$ $=25\ \mathrm{cm^2/s}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • work towards $\frac{da}{dr}$ or $\frac{da}{dt}$ *High Partial Credit* • correct expression for $\frac{da}{dt}$
(b) (i)	$-x^2+10x=0$ $x(-x+10)=0$ $x=0 \quad \text{or} \quad x=10$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • quadratic equation formed • gets $x=0$ only *High Partial Credit* • quadratic factorised **Note:** $f'(x)=0 \Rightarrow 2x-10=0 \Rightarrow x=5$ *merits 0 marks*
(ii)	$\frac{1}{10-0}\int_0^{10}(-x^2+10x)dx$ $=\frac{1}{10}\left[\frac{-x^3}{3}+5x^2\right]_0^{10}$ $=\frac{1}{10}\left[\left(\frac{-1000}{3}+500\right)-0\right]$ $=\frac{-100}{3}+50=\frac{50}{3}\mathrm{m}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • integration set up *High Partial Credit* • correct integration with some substitution

Q8	Model Solution – 55 Marks	Marking Notes
(a) **(i)**	$f(x) = -0{\cdot}274x^2 + 1{\cdot}193x + 3{\cdot}23$ $f'(x) = -0{\cdot}548x + 1{\cdot}193 = 0$ $x = 2{\cdot}177 \text{ m}$ $f(2{\cdot}177) = -0{\cdot}274(2{\cdot}177)^2 + 1{\cdot}193(2{\cdot}177) + 3{\cdot}23$ $= -1{\cdot}2986 + 2{\cdot}5972 + 3{\cdot}23$ $= 4{\cdot}529 \text{ m}$ *or* $-0{\cdot}274(x^2 - \frac{1193}{274}x - \frac{1615}{137})$ $-0{\cdot}274(x - \frac{1193}{548})^2 + 4.5285$ Max Height $= 4{\cdot}529$ m	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • any correct differentiation • effort made at completing square • trial and error with more than one value of x tested *High Partial Credit* • x value correct **Note:** *if correct answer by trial and error, must show points on each side of max point to be lower to earn full credit*
(ii)	$\tan\theta = -0{\cdot}548(4{\cdot}5)+1{\cdot}193$ $\tan\theta = -1{\cdot}273$ $\theta = 51{\cdot}8° = 52°$	Scale 5B (0, 2, 5) *Partial Credit* • tan **Note:** *right angled triangles may appear in diagram given in equation*
(iii)	Map $A \to C$ $(-0{\cdot}5, 2{\cdot}565) \to (0, 2)$ $2{\cdot}177 — (-0{\cdot}5) = 2{\cdot}677$ $4{\cdot}529 - 0{\cdot}565 = 3{\cdot}964$ $(2{\cdot}177, 4{\cdot}529) \to (2{\cdot}677, 3{\cdot}964)$	Scale 5B (0, 2, 5) *Partial Credit* • $(-0{\cdot}5, 2{\cdot}565) \to (0, 2)$

(iv)	$$g(x) = ax^2 + bx + c$$ $$C(0,2) \in g(x) => c = 2$$ $B(4{\cdot}5, 3{\cdot}05) \in g(x)$ $3{\cdot}05 = a(4{\cdot}5)^2 + b(4{\cdot}5) + 2$ $\Longrightarrow 20{\cdot}25a + 4{\cdot}5b = 1{\cdot}05$ ... **(i)** $g'(x) = 2ax + b = 0$ $\Longrightarrow 2a(2{\cdot}677) + b = 0$ $5{\cdot}354a + b = 0$ ... **(ii)** From **(i)** and **(ii)** $a = -0{\cdot}273$ $b = 1{\cdot}462$ $$g(x) = -0{\cdot}273x^2 + 1{\cdot}462x + 2$$ *[Note: a third equation that could be used is* $3{\cdot}964 = a(2{\cdot}677)^2 + b(2{\cdot}677) + 2$ ... **(iii)**]	Scale 10D (0, 2, 5, 8, 10) *Low Partial Credit* • c value found • relevant equation in a, b and/or c *Mid Partial Credit* • formulated correctly any two equations *High Partial Credit* • formulated correctly any three equations **Note**: $ax^2 + bx + c$ *not in an equation merits 0 marks*
	Or	**Or**
	Equation of parabola with vertex (h,k): $$g(x) = a(x-h)^2 + k$$ $C(0,2)$ on curve: $(h,k) = (2{\cdot}677, 3.964)$ $$2 = a(-2{\cdot}677)^2 + 3{\cdot}964$$ $$-1{\cdot}964 = a(7{\cdot}166329)$$ $$a = -0{\cdot}27405 = -0{\cdot}274$$ Parabola: $$g(x) = -0{\cdot}274[(x - 2{\cdot}677)^2] + 3{\cdot}964$$ **or** $$g(x) = f(x - 0{\cdot}5) - 0{\cdot}565$$ $$g(x) = -0{\cdot}274(x-0{\cdot}5)^2 + 1{\cdot}193(x - 0{\cdot}5) + 3{\cdot}23 - 0{\cdot}565$$ $$g(x) = -0{\cdot}274x^2 + 1{\cdot}467x + 2$$	Scale 10D (0, 2, 5, 8, 10) *Low Partial Credit* • equation of curve • use of C *Mid Partial Credit* • using peak value *High Partial Credit* • value of a found

Question 4 (2015)

Question 2 **(25 marks)**

$f(x) = x^3 - 3x^2 - 9x + 11$

$f(1) = 1^3 - 3(1)^2 - 9 + 11 = 0$

$\Rightarrow x = 1$ is a solution.

$(x - 1)$ is a factor

$$
\begin{array}{r|l}
 & x^2 - 2x \quad -11 \\
\hline
x - 1 & x^3 - 3x^2 - 9x + 11 \\
 & \underline{x^3 - x^2} \\
 & \quad -2x^2 - \ 9x + 11 \\
 & \quad \underline{-2x^2 + 2x} \\
 & \qquad\quad -11x + 11 \\
 & \qquad\quad \underline{-11x + 11}
\end{array}
$$

or

$(x-1)\left(x^2 + Ax - 11\right) = x^3 - 3x^2 - 9x + 11$

$\Rightarrow x^3 + Ax^2 - x - x^2 - Ax + 1 = x^3 - 3x^2 - 9x + 11$

$\Rightarrow A - 1 = -3$

$\Rightarrow A = -2$

or

	x^2	$-2x$	-11
x	x^3	$-2x^2$	$-11x$
-1	$-x^2$	$2x$	11

Hence, other factor is $x^2 - 2x - 11$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)} = \frac{2 \pm \sqrt{48}}{2} = \frac{2 \pm 4\sqrt{3}}{2} = 1 \pm 2\sqrt{3}$$

Solutions: $\{1,\ 1 + 2\sqrt{3},\ 1 - 2\sqrt{3}\}$

Question 5 (2015)

$x = \sqrt{x+6}$

$\Rightarrow x^2 = x + 6$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x+2)(x-3) = 0$

$\Rightarrow x = -2, \quad x = 3$

$x = -2: \quad -2 \neq \sqrt{-2+6} = \sqrt{4} = 2$ ✗

$x = 3: \quad 3 = \sqrt{3+6} = \sqrt{9} = 3$ ✓

Question 6 (2012)