Question 1 (2017)

(a)	$P = \frac{A}{1+i} + \frac{A}{(1+i)^2} + \cdots\cdots + \frac{A}{(1+i)^t}$ $P = \frac{\left(\frac{A}{1+i}\right)\left(1-\left(\frac{1}{1+i}\right)^t\right)}{1-\frac{1}{1+i}}$ $= \frac{A\left(1-\frac{1}{(1+i)^t}\right)}{1+i-1}$ $= \frac{A((1+i)^t-1)}{i(1+i)^t}$ $A = \frac{P(i)(1+i)^t}{(1+i)^t-1}$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • $P = \frac{A}{1+i}$ • $A = P(1+i)$ • S_n formula with some substitution *High Partial Credit:* • full substitution for P (or A) into S_n formula.
(b) (i)	$2{\cdot}5\% \times 5000 = 125$	**Scale 10B (0, 4, 10)** *Partial Credit* • Any one unknown
(b) (ii)	$(1+i)^{\frac{1}{12}} = (1{\cdot}2175)^{\frac{1}{12}} = 1{\cdot}016535$ $Rate = 1{\cdot}65\%$	**Scale 10B (0, 4, 10)** *Partial Credit* • Formula with some substitution

(b) (iii)

Payment number	Fixed monthly payment, €A	€A		New balance of debt (€)
		Interest	Previous balance reduced by (€)	
0				5000
1	125	82·50	42·50	4957·50
2	125	81·80	43·20	4914·30
3	125	81·09	43·91	4870·39

(b) (iii)

Scale 10C (0, 5, 8, 10)

Low Partial Credit:
- One correct additional entry

High Partial Credit:
- 6 correct additional entries

Note: Where interest rate in b(ii) is not 1·65%, then check the validity of all values given.

(iv)	$A = p\left[\frac{i(1+i)^t}{(1+i)^t-1}\right]$ $A[(1+i)^t - 1] = pi(1+i)^t$ $A(1+i)^t - A = pi(1+i)^t$ $A = (1+i)^t[A - pi]$ $\frac{A}{A-pi} = (1+i)^t$ $\frac{125}{125 - 5000\left(\frac{1·65}{100}\right)} = \left(1 + \frac{1·65}{100}\right)^t$ $\frac{125}{42·5} = (1·0165)^t$ $\log\left(\frac{125}{42·5}\right) = t\log(1·0165)$ $t = \frac{\log\left(\frac{125}{42·5}\right)}{\log(1·0165)}$ $t = 65·920$ $t = 66$ months **OR** $A = p\left[\frac{i(1+i)^t}{(1+i)^t-1}\right]$ $125 = \frac{5000(0·0165)(1·0165)^t}{(1·0165)^t - 1}$ $125 = \frac{82·5(1·0165)^t}{(1·0165)^t - 1}$ $\frac{125}{82·5} = \frac{1·0165^t}{1·0165^t - 1}$ $\frac{50}{33} = \frac{1·0165^t}{1·0165^t - 1}$ $50(1 \cdot 0165^t - 1) = 33(1·0165^t)$ $50(1·0165^t) - 50 = 33(1·0165^t)$ $50(1·0165^t) - 33(1·0165^t) = 50$ $1·0165^t(50 - 33) = 50$ $1·0165^t(17) = 50$ $1·0165^t = \frac{50}{17}$ $t\log 1·0165 = \log\frac{50}{17}$ $t = \frac{\log\left(\frac{50}{17}\right)}{\log 1·0165} = 65·92$ $t = 66$ months	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • Formula with some substitution • Some relevant manipulation of formula. *High Partial Credit:* • Equation in t (t no longer an index)

(v)	$$A=\frac{pi(1+i)^t}{(1+i)^t-1}$$ $$=\frac{5000\left(1{\cdot}085^{\frac{1}{52}}-1\right)(1{\cdot}085)^3}{(1{\cdot}085)^3-1}$$ $$=€36{\cdot}16$$ **OR** Weekly interest rate $(1+i)^{52}=1{\cdot}085$ $$1+i=1{\cdot}085^{\frac{1}{52}}$$ $$1+i=1{\cdot}00157$$ $$i=0{\cdot}00157$$ $$A=\frac{pi(1+i)^t}{(1+i)^t-1}$$ $$A=\frac{5000(0{\cdot}00157)(1{\cdot}00157)^{156}}{(1{\cdot}00157)^{156}-1}$$ $$=€36{\cdot}16$$	**Scale 10C (0, 5, 8, 10)** *Low Partial Credit:* • r (weekly) found *High Partial Credit:* • Fully substituted equation
(vi)	$$125\times 66-(36{\cdot}16)(156)$$ =€2609·04	**Scale 5B (0, 3, 5)** *Partial Credit:* • Total repayment by either method found

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$P(M,3,3)=\frac{1}{26}\times\frac{1}{10}\times\frac{1}{10}=\frac{1}{2600}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • any correct relevant probability *High Partial credit* • correct probabilities but not expressed as single fraction or equivalent **Note:** Accept correct answer without supporting work
(b)	see tables below	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • 1 correct entry to table *High Partial Credit* • all entries correct but fails to finish or finishes incorrectly • no conclusion

Event	Payout	Prob (P(x))	x.P(x)
Win	1000	$\frac{1}{2600}$	$\frac{1000}{2600}$
letter 1 No.	50	$\frac{9}{2600}$	$\frac{450}{2600}$
letter 2nd No	50	$\frac{9}{2600}$	$\frac{450}{2600}$
letter only	50	$\frac{81}{2600}$	$\frac{4050}{2600}$
Fail to win	0		0

$$\sum x.P(x)=\frac{5950}{2600}=2{\cdot}29$$

Club loses 29 cent per play

Or

Event	Pay out	Prob (P(x)	x.P(x)
Win	-998	$^{1}/_{2600}$	$^{-998}/_{2600}$
letter + 1st No.	-48	$^{9}/_{2600}$	$^{-432}/_{2600}$
Letter + 2nd No	-48	$^{9}/_{2600}$	$^{-432}/_{2600}$
letter only	-48	$^{81}/_{2600}$	$^{-3888}/_{2600}$
Fail to Win	$+2$	$^{2500}/_{2600}$	$^{5000}/_{2600}$

$\sum x.P(x)=-\frac{750}{2600}=-29$ cent

(c)	Profit = Revenue – Pay-out $600 = 845(x - 2{\cdot}29)$ $x = \dfrac{600 + 845(2{\cdot}29)}{845}$ $x = 3$ **or** $\dfrac{600}{845} = 0{\cdot}71$ $0{\cdot}71 + 2{\cdot}29 = 3$	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • links profit, revenue and payout *High partial Credit* • formula fully substituted

Q9	Model Solution – 50 Marks	Marking Notes
(a) (i)	$\mu = 39400, \ \ \sigma = 12920$ $z = \dfrac{x-\mu}{\sigma} = \dfrac{60000-39400}{12920}$ $z = 1{\cdot}59$ $P(z > 1{\cdot}59) = 1 - P(z < 1{\cdot}59)$ $= 1 - 0{\cdot}9441 = 0{\cdot}0559$ $= 5{\cdot}59\%$ = 5·6%	Scale 10D (0, 3, 5, 8, 10) *Low Partial Credit* • μ and σ identified *Mid Partial Credit* • z = 1·59 *High Partial Credit* • identifies 0·9441
(a) (ii)	$P(z \le z_1) = 0{\cdot}9$ $z_1 = 1{\cdot}28$ $\Rightarrow z_2 = -1{\cdot}28$ $\Rightarrow \dfrac{x-39400}{12920} = -1{\cdot}28$ $x = 22862{\cdot}40$ = €22 862	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • identifies 1·28 but fails to progress *High Partial Credit* • formula for x fully substituted
(a) (iii)	$\mu = 39400, \ \ \sigma = 12920,$ $\bar{x} = 38280, \ \ n = 1000$ $H_0 \Rightarrow \mu = 39400$ $H_1 \Rightarrow \mu \neq 39400$ $z = \dfrac{38280-39400}{\frac{12920}{\sqrt{1000}}} = -2{\cdot}74$ $-2{\cdot}74 < \ -1{\cdot}96$ Result is significant. There is evidence to reject the null hypothesis The mean income has changed.	Scale 15D (0, 4, 7, 11,15) *Low Partial Credit* • z formulated with some substitution • states null and/or alternative hypothesis only • reference to 1·96 Mid Partial Credit • z fully substituted *High Partial Credit* • $z = -2{\cdot}74$ and stops • fails to state the null and alternative hypothesis correctly • fails to contextualise the answer

or

Confidence Interval:

$$\bar{x} \pm 1{\cdot}96\frac{\sigma}{\sqrt{n}}$$

$$39400 \pm 1{\cdot}96\frac{12920}{\sqrt{1000}}$$

$$[38599{\cdot}2,\ 40200{\cdot}8]$$

38280 outside range

Result is significant. There is evidence to reject the null hypothesis

The mean income has changed.

or

Confidence Interval:

$$\bar{x} \pm 1{\cdot}96\frac{\sigma}{\sqrt{n}}$$

$$38280 \pm 1{\cdot}96\frac{12920}{\sqrt{1000}}$$

$$38280 \pm 1{\cdot}96(408{\cdot}57)$$

$$[37479{\cdot}2,\ 39080{\cdot}8]$$

39400 outside range

Result is significant. There is evidence to reject the null hypothesis

The mean income has changed.

(b)	$26974 - 1{\cdot}96\left(\frac{5120}{\sqrt{400}}\right) \leq \mu \leq 26974 + 1{\cdot}96\left(\frac{5120}{\sqrt{400}}\right)$ $26472{\cdot}24 \leq \mu \leq 27475{\cdot}76$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • interval formulated with some correct substitution *High Partial Credit* • interval formulated with fully correct substitution
(c)	The distribution of sample means will be normally distributed	Scale 5B (0, 2, 5) *Partial Credit* • mentions 30 (or more) but not contextualised
(d)	$\frac{1}{\sqrt{n}} = 0{\cdot}045$ $\frac{1}{0{\cdot}045} = \sqrt{n}$ $n = \left(\frac{1}{0{\cdot}045}\right)^2 = 493{\cdot}827$	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • $\frac{1}{\sqrt{n}}$ *High Partial Credit* • n formulated with fully correct substitution **Note:** Accept 493 farmers or 494 farmers

Question 4 (2015)

(a) Donagh is arranging a loan and is examining two different repayment options.

(i) Bank A will charge him a monthly interest rate of 0·35%. Find, correct to three significant figures, the annual percentage rate (APR) that is equivalent to a monthly interest rate of 0·35%.

$$F = P(1+i)^t = 1(1+0\cdot0035)^{12} = 1\cdot042818$$
$$\Rightarrow i = 4\cdot28\%$$

(ii) Bank B will charge him a rate that is equivalent to an APR of 4·5%. Find, correct to three significant figures, the monthly interest rate that is equivalent to an APR of 4·5%.

$$F = P(1+i)^t$$
$$1\cdot045 = 1(1+i)^{12}$$
$$1+i = \sqrt[12]{1\cdot045} = 1\cdot0036748$$
$$\Rightarrow i = 0\cdot367\%$$

(b) Donagh borrowed €80 000 at a monthly interest rate of 0·35%, fixed for the term of the loan, from Bank A. The loan is to be repaid in equal monthly repayments over ten years. The first repayment is due one month after the loan is issued. Calculate, correct to the nearest euro, the amount of each monthly repayment.

$$A = P\left[\frac{i(1+i)^t}{(1+i)^t - 1}\right]$$

$$= 80000\left[\frac{0\cdot0035(1\cdot0035)^{120}}{(1\cdot0035)^{120} - 1}\right]$$

$$= 80000\left[\frac{0\cdot00532296}{0\cdot520846}\right]$$

$$= 817\cdot59 = €818$$

or

$$80000 = \frac{A}{1\cdot0035} + \frac{A}{1\cdot0035^2} + \ldots + \frac{A}{1\cdot0035^{120}}$$

$$= A\left[\frac{1}{1\cdot0035} + \frac{1}{1\cdot0035^2} + \ldots + \frac{1}{1\cdot0035^{120}}\right]$$

$$= A\left[\frac{\frac{1}{1\cdot0035}\left(1 - \left(\frac{1}{1\cdot0035}\right)^{120}\right)}{1 - \frac{1}{1\cdot0035}}\right]$$

$$= A\left[\frac{0\cdot342471198}{0.0035}\right]$$

$$= A\left[97\cdot8489137\right]$$

$$A = 817\cdot58 = €818$$

We have

$$P = \frac{F}{1+i} = \frac{20000}{1.03} = 19417.48$$

So the present value is €19417.48 to the nearest cent.

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(b) Write down, in terms of t, the present value of a future payment of €$20,000$ in t years' time.

We have

$$P = \frac{F}{(1+i)^t} = \frac{20000}{(1.03)^t}.$$

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(c) Pádraig wants to have a fund that could, from the date of his retirement, give him a payment of €$20,000$ at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value, on the date of retirement, of the final fund required.

Using the solution to part (b), we see that the amount required on the date of requirement is given by

$$A = 20000 + \frac{20000}{1.03} + \cdots \frac{20000}{(1.03)^{24}}$$

Using the notation of the formula on page 22 of the Formula and Tables book, we have a geometric series with $a = 20000$, $r = \frac{1}{1.03}$ and $n = 25$. Therefore

$$A = \frac{20000\left(1 - \left(\frac{1}{1.03}\right)^{25}\right)}{1 - \frac{1}{1.03}}$$

Using a calculator we obtain

$$A = €358711$$

to the nearest euro.

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(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part **(c)**. His retirement is $40 \times 12 = 480$ months away.

(i) Find, correct to four significant places, the rate of interest per month that would, if paid and compounded annually, be equivalent to an effective annual rate of 3%.

We must solve $(1+i)^{12} = 1.03$. So $(1+i) = \sqrt[12]{1.03}$. Therefore

$$i = \sqrt[12]{1.03} - 1 = 0.002466$$

correct to 4 significant places. So the answer is

$$0.2466\%.$$

(ii) Write dowm, in terms of n and P, the value on the retirement date of a payment of €P made n months before the retirement date.

Using the formula on page 30 of the Formula and Tables booklet we obtain

$$P(1.002466)^n.$$

(iii) If Pádraig makes 480 equal payments of €P from now until his retirement, what value of P will give him the fund he requires?

We must solve

$$P(1.002466)^{480} + P(1.002466)^{479} + \cdots + P(1.002466) = 358711$$

or

$$P(1.002466 + (1.002466)^2 + \cdots + (1.002466)^{480}) = 358711$$

Using the formula for the sum of a geometric series, we obtain

$$P\left(\frac{1.002466(1-(1.002466)^{480})}{1-1.002466}\right) = 358711$$

or

$$P(919.38) = 358711.$$

Therefore $P = \frac{358711}{919.38} =$ €390.17 to the nearest cent.

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(e) If Pádraig waits for ten years before starting his pension fund, how much will he then have to pay each month in order to generate the same pension fund?

Now the number of months until his retirement date is $30 \times 12 = 360$. So as above we must solve

$$P(1.002466)^{360} + P(1.002466)^{359} + \cdots + P(1.002466) = 358711$$

or

$$P(1.002466 + (1.002466)^2 + \cdots + (1.002466)^{360}) = 358711$$

Using the formula for the sum of a geometric series, we obtain

$$P\left(\frac{1.002466(1-(1.002466)^{360})}{1-1.002466}\right) = 358711$$

or

$$P(580.11) = 358711.$$

Therefore, in this case, $P = \frac{358711}{580.11} =$ €618.35 to the nearest cent.

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$(1+i)^{12} = 1\cdot 04 \Rightarrow 1+i = \sqrt[12]{1\cdot 04} = 1\cdot 003273 \Rightarrow i = 0\cdot 003274$

Hence, $i = 0{\cdot}327\%$

OR

$(1.00327)^{12} = 1{\cdot}039953481$

$= 1{\cdot}0400$

$r = 4\%$

$$15000 = P\left(1\cdot 00327^{36} + 1\cdot 00327^{35} + \cdots + 1\cdot 00327^{2} + 1\cdot 00327\right)$$

$$\Rightarrow P\left[\frac{1\cdot 00327\left(1\cdot 00327^{36} - 1\right)}{1\cdot 00327 - 1}\right] = 15000$$

$$\Rightarrow P[38\cdot 26326387] = 15000$$

$$\Rightarrow P = 392\cdot 02 \ = \ €392$$

Question 7 (2013)

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$= 15000\left[\frac{0\cdot00866(1+0\cdot00866)^{36}}{1\cdot00866^{36} - 1}\right]$$

$$= 486\cdot77$$

Monthly payment €487

OR

$$15000 = P\left(\frac{1}{1\cdot00866} + \frac{1}{1\cdot00866^2} + \cdots + \frac{1}{1\cdot00866^{36}}\right)$$

$$\Rightarrow P\left[\frac{\frac{1}{1\cdot00866}\left(1 - \frac{1}{1\cdot00866^{36}}\right)}{1 - \frac{1}{1\cdot00866}}\right] = 15000$$

$$\Rightarrow P[30\cdot8151777] = 15000$$

$$\Rightarrow P = 486\cdot77$$

Monthly payment €487

Question 8 (2012)