



- (d) The predicted population in Avalon at the beginning of 2011 is 3 709 795 people.  
Write down and solve an equation in  $k$  to show that  $k = -0.05$ , correct to 2 decimal places.

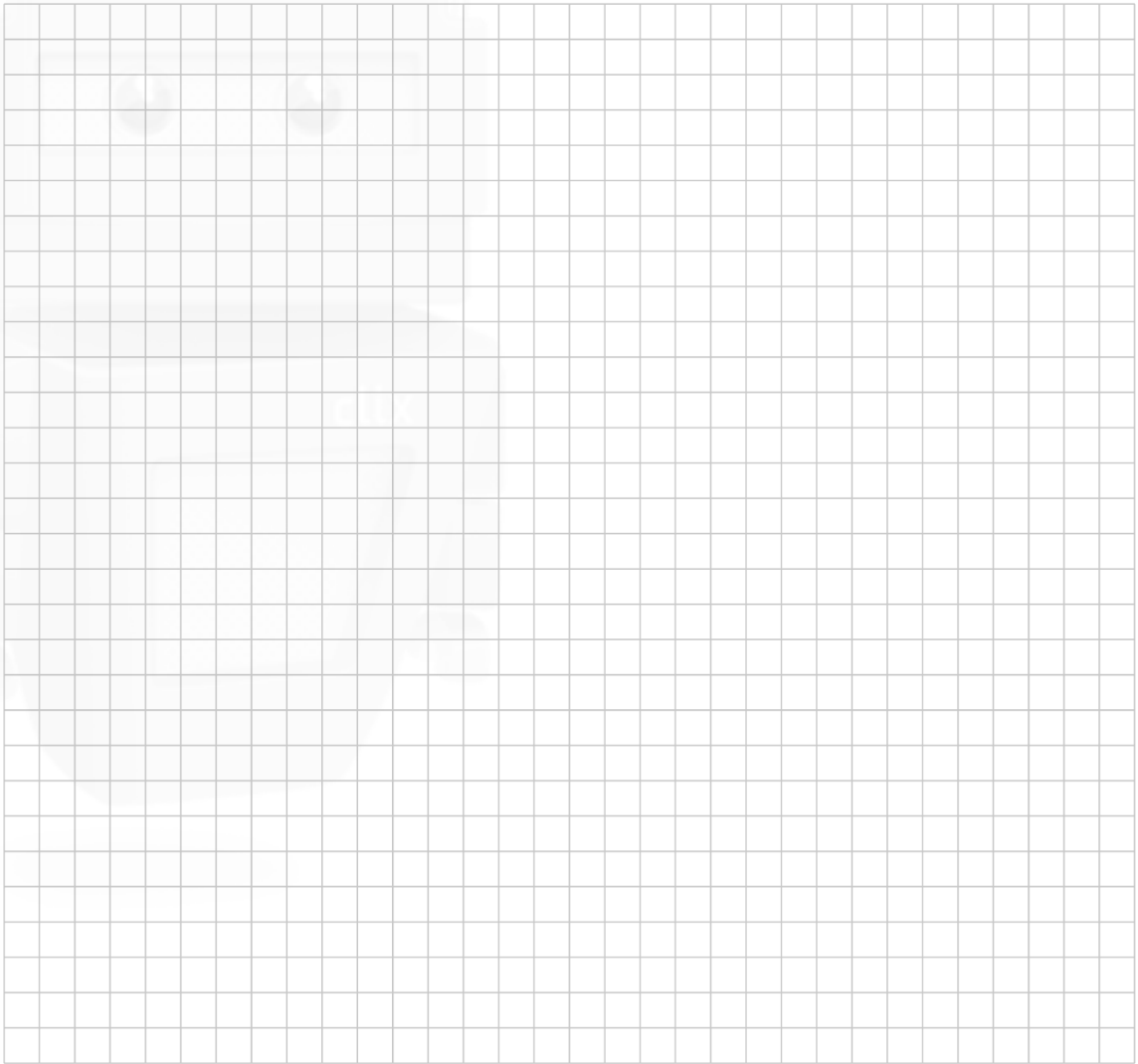
- (e) Find the year during which the populations in both cities will be equal.

- (f) Find the predicted average population in Avalon from the beginning of 2010 to the beginning of 2025.

*This question is continued on the next page*

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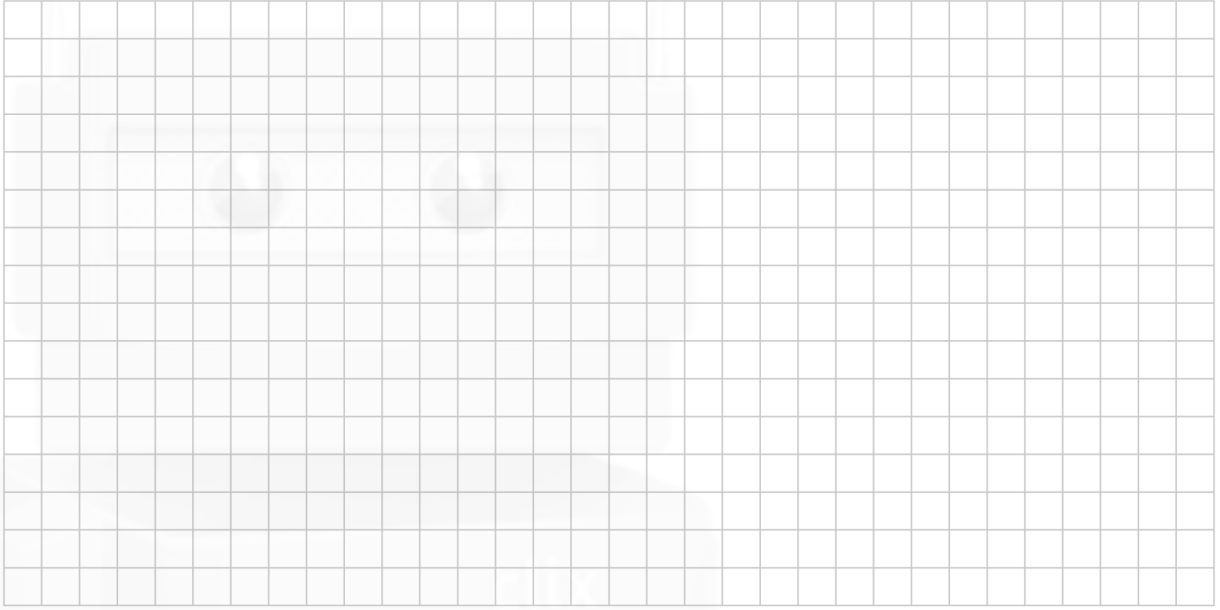
- (g) Use the function  $q(t) = 3.9e^{-0.05t} \times 10^6$  to find the predicted rate of change of the population in Avalon at the beginning of 2018.



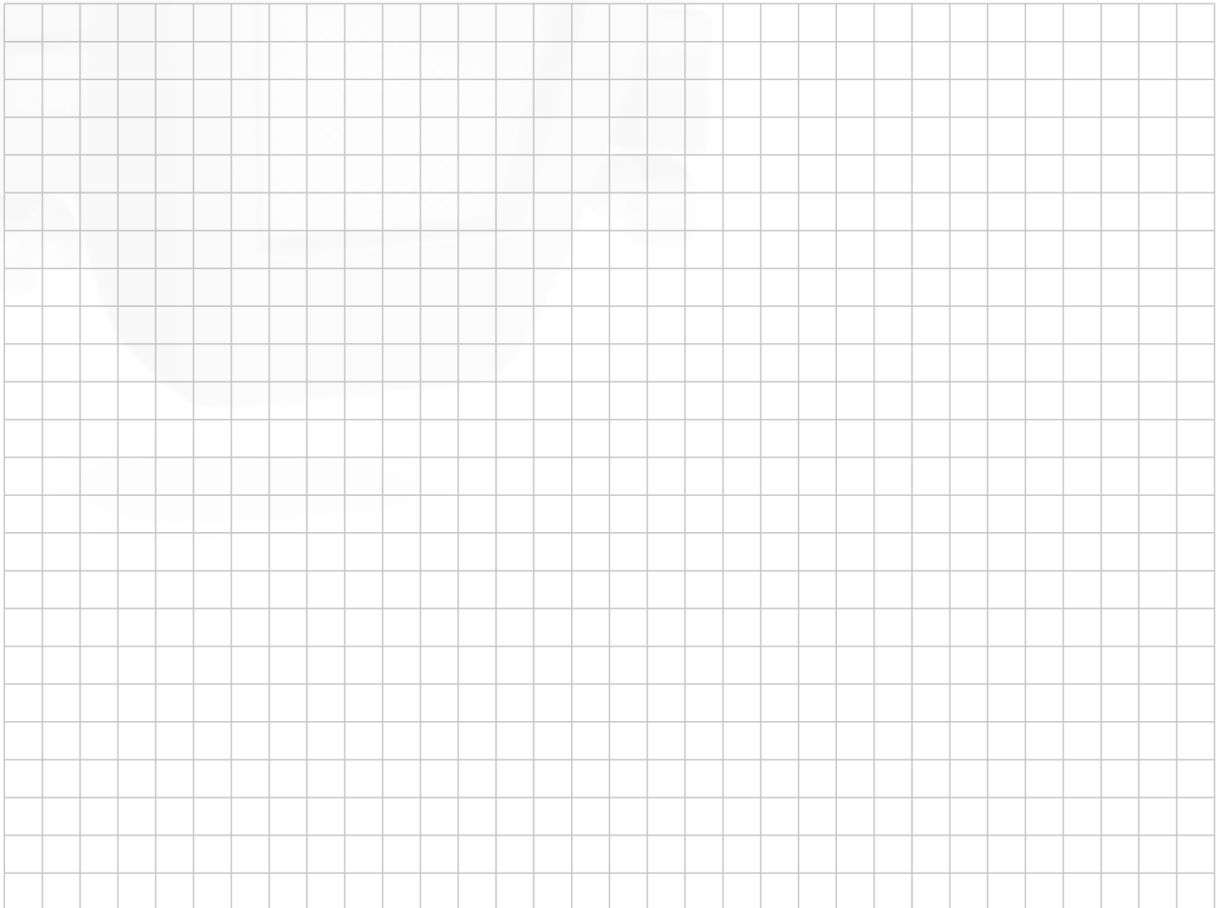
## Question 2

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- (a) Show that  $\frac{\cos 7A + \cos A}{\sin 7A - \sin A} = \cot 3A$ .



- (b) Given that  $\cos 2\theta = \frac{1}{9}$ , find  $\cos \theta$  in the form  $\pm \frac{\sqrt{a}}{b}$ , where  $a, b \in \mathbb{N}$ .



### Question 3

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(b) Given  $\log_a 2 = p$  and  $\log_a 3 = q$ , where  $a > 0$ , write each of the following in terms of  $p$  and  $q$ :

(i)  $\log_a \frac{8}{3}$



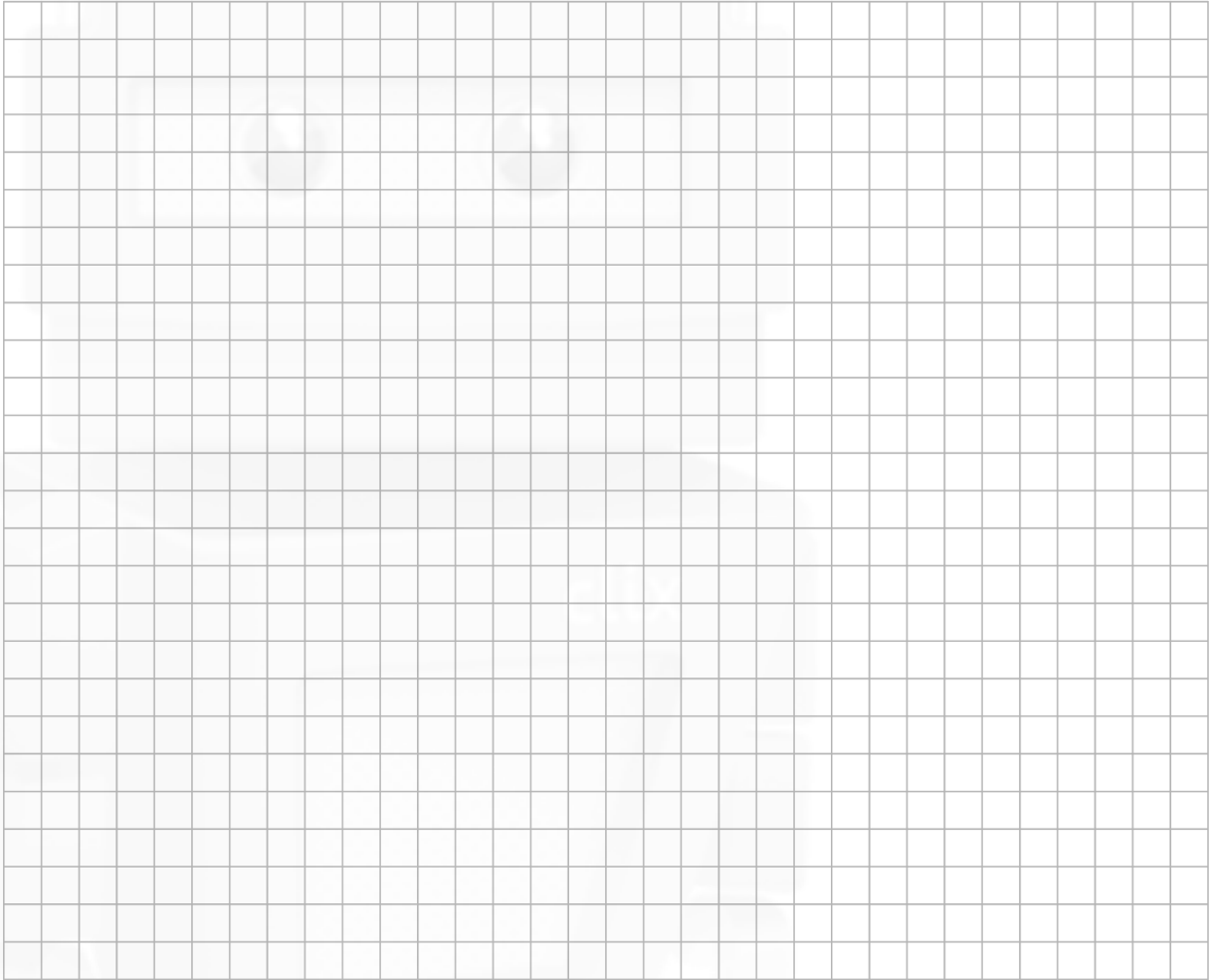
(ii)  $\log_a \frac{9a^2}{16}$ .



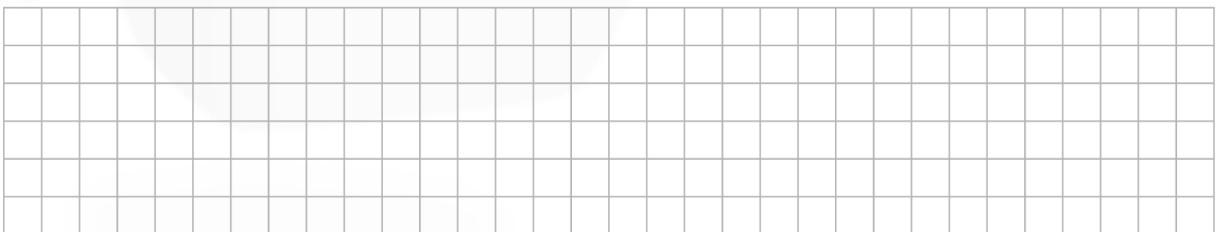
## Question 2

(25 marks)

- (a) (i) Prove by induction that, for any  $n$ , the sum of the first  $n$  natural numbers is  $\frac{n(n+1)}{2}$ .



- (ii) Find the sum of all the natural numbers from 51 to 100, inclusive.



- (b) Given that  $p = \log_c x$ , express  $\log_c \sqrt{x} + \log_c (cx)$  in terms of  $p$ .



## Question 5

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Scientists can estimate the age of certain ancient items by measuring the proportion of carbon-14, relative to the total carbon content in the item. The formula used is  $Q = e^{-\frac{0.693t}{5730}}$ , where  $Q$  is the proportion of carbon-14 remaining and  $t$  is the age, in years, of the item.

- (a) An item is 2000 years old. Use the formula to find the proportion of carbon-14 in the item.

- (b) The proportion of carbon-14 in an item found at Lough Boora, County Offaly, was 0.3402. Estimate, correct to two significant figures, the age of the item.

