Strand 1: Statistics and Probability

Topic	Description of topic Students learn about	Learning outcomes Students should be able to
1.1 Counting	Listing outcomes of experiments in a systematic way, such as in a table, using sample spaces, tree diagrams.	– list all possible outcomes of an experiment – apply the fundamental principle of counting
1.2 Concepts of probability	The probability of an event occurring: students progress from informal to formal descriptions of probability. Predicting and determining probabilities. Difference between experimental and theoretical probability.	– decide whether an everyday event is likely or unlikely to occur – recognise that probability is a measure on a scale of 0-1 of how likely an event is to occur **– use set theory to discuss experiments, outcomes, sample spaces** – use the language of probability to discuss events, including those with equally likely outcomes – estimate probabilities from experimental data – recognise that, if an experiment is repeated, there will be different outcomes and that increasing the number of times an experiment is repeated generally leads to better estimates of probability – associate the probability of an event with its long-run, relative frequency
1.3 Outcomes of simple random processes	Finding the probability of equally likely outcomes.	– construct sample spaces for two independent events – apply the principle that, in the case of equally likely outcomes, the probability is given by the number of outcomes of interest divided by the total number of outcomes (examples using coins, dice, spinners, containers with different coloured objects, playing cards, sports results, etc.) **– use binary / counting methods to solve problems involving successive random events where only two possible outcomes apply to each event**
1.4 Statistical reasoning with an aim to becoming a statistically aware consumer	Situations where statistics are misused and learn to evaluate the reliability and quality of data and data sources. Different types of data.	– engage in discussions about the purpose of statistics and recognise misconceptions and misuses of statistics – work with different types of data: categorical: nominal or ordinal numerical: discrete or continuous in order to clarify the problem at hand **– evaluate reliability of data and data sources**
1.5 Finding, collecting and organising data	The use of statistics to gather information from a selection of the population with the intention of making generalisations about the whole population. Formulating a statistics question based on data that vary, allowing for distinction between different types of data.	– clarify the problem at hand – formulate questions that can be answered with data – explore different ways of collecting data – generate data, or source data from other sources including the internet **– select a sample from a population (Simple Random Sample)** **– recognise the importance of representativeness so as to avoid biased samples** – design a plan and collect data on the basis of above knowledge – summarise data in diagrammatic form including data presented in spreadsheets

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1.6 Representing data graphically and numerically	Methods of representing data. Students develop a sense that data can convey information and that organising data in different ways can help clarify what the data have to tell us. They see a data set as a whole and so are able to use proportions and measures of centre to describe the data. **Mean of a grouped frequency distribution.**	Graphical – select appropriate methods to represent and describe the sample (univariate data only) – evaluate the effectiveness of different displays in representing the findings of a statistical investigation conducted by others – use pie charts, bar charts, line plots, histograms (equal intervals), stem and leaf plots **and back-to-back stem and leaf plots** to display data – use appropriate graphical displays to compare data sets Numerical – use a variety of summary statistics to describe the data: central tendency – mean, median, mode variability – range, **quartiles and inter-quartile range** **– recognise the existence of outliers**
1.7 Analysing, interpreting and drawing conclusions from data	Drawing conclusions from data; limitations of conclusions.	– interpret graphical summaries of data – relate the interpretation to the original question **– recognise how sampling variability influences the use of sample information to make statements about the population** **– draw conclusions from graphical and numerical summaries of data, recognising assumptions and limitations**

Students learn about	Students should be able to
1.8 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Strand 2: Geometry and Trigonometry

<table>
<tr><th>Topic</th><th>Description of topic
Students learn about</th><th>Learning outcomes
Students should be able to</th></tr>
<tr><td>2.1 Synthetic geometry</td><td>Concepts (see Geometry Course section 9.1 for OL and 10.1 for HL)
Axioms (see Geometry Course section 9.3 for OL and 10.3 for HL):
1. [Two points axiom] There is exactly one line through any two given points.
2. [Ruler axiom] The properties of the distance between points
3. [Protractor Axiom] The properties of the degree measure of an angle
4. Congruent triangles (SAS, ASA and SSS)
5. [Axiom of Parallels] Given any line l and a point P, there is exactly one line through P that is parallel to l.

Theorems: [Formal proofs are not examinable at OL. Formal proofs of theorems 4, 6, 9, 14 and 19 are examinable at HL.]
1. Vertically opposite angles are equal in measure.
2. In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.
3. If a transversal makes equal alternate angles on two lines then the lines are parallel, (and converse).
4. The angles in any triangle add to 180°.
5. Two lines are parallel if and only if, for any transversal, the corresponding angles are equal.
6. Each exterior angle of a triangle is equal to the sum of the interior opposite angles.
9. In a parallelogram, opposite sides are equal and opposite angles are equal (and converses).
10. The diagonals of a parallelogram bisect each other.
11. If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.
12. Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio s:t, then it also cuts [AC] in the same ratio (and converse).
13. If two triangles are similar, then their sides are proportional, in order (and converse).
14. [Theorem of Pythagoras] In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.
15. If the square of one side of a triangle is the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.
19. The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.</td><td>– recall the axioms and use them in the solution of problems
– use the terms: theorem, proof, axiom, corollary, converse and implies

– apply the results of all theorems, converses and corollaries to solve problems
– prove the specified theorems</td></tr>
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Topic	Description of topic Students learn about	Learning outcomes Students should be able to
	Corollaries: **1. A diagonal divides a parallelogram into 2 congruent triangles.** **2. All angles at points of a circle, standing on the same arc, are equal, (and converse).** 3. Each angle in a semi-circle is a right angle. 4. If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter. **5. If ABCD is a cyclic quadrilateral, then opposite angles sum to 180˚, (and converse).**	
	Constructions: 1. Bisector of a given angle, using only compass and straight edge. 2. Perpendicular bisector of a segment, using only compass and straight edge. **3. Line perpendicular to a given line *l*, passing through a given point not on *l*.** 4. Line perpendicular to a given line *l*, passing through a given point on *l*. 5. Line parallel to a given line, through a given point. 6. Division of a line segment into 2 or 3 equal segments, without measuring it. **7. Division of a line segment into any number of equal segments, without measuring it.** 8. Line segment of a given length on a given ray. 9. Angle of a given number of degrees with a given ray as one arm. 10. Triangle, given lengths of three sides 11. Triangle, given SAS data 12. Triangle, given ASA data 13. Right-angled triangle, given the length of the hypotenuse and one other side. 14. Right-angled triangle, given one side and one of the acute angles (several cases). 15. Rectangle, given side lengths.	– complete the constructions specified

Topic	Description of topic Students learn about	Learning outcomes Students should be able to
2.2 Co-ordinate geometry	Co-ordinating the plane. Properties of lines and line segments including midpoint, slope, distance and the equation of a line in the form. $y - y_1 = m(x - x_1)$. $y = mx + c$. $\boldsymbol{ax + by + c = 0}$ where a, b, c, are integers and m is the slope of the line. Intersection of lines. **Parallel and perpendicular lines and the relationships between the slopes.**	– explore the properties of points, lines and line segments including the equation of a line – find the point of intersection of two lines **– find the slopes of parallel and perpendicular lines**
2.3 Trigonometry	Right-angled triangles. Trigonometric ratios. **Working with trigonometric ratios in surd form for angles of 30˚, 45˚ and 60˚** **Right-angled triangles.** **Decimal and DMS values of angles.**	– apply the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances – use trigonometric ratios to solve problems involving angles (integer values) between 0˚ and 90˚ **– solve problems involving surds** **– solve problems involving right-angled triangles** **– manipulate measure of angles in both decimal and DMS forms**
2.4 Transformation geometry	Translations, central symmetry, axial symmetry and rotations.	– locate axes of symmetry in simple shapes – recognise images of points and objects under translation, central symmetry, axial symmetry and rotations

Students learn about	Students should be able to
2.5 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Topic	Description of topic Students learn about	Learning outcomes Students should be able to
3.1 Number systems **N**: the set of natural numbers, **N** = {1,2,3,4….} **Z**: the set of integers, including 0 **Q**: the set of rational numbers **R**: the set of real numbers **R/Q**: the set of irrational numbers	The binary operations of addition, subtraction, multiplication and division and the relationships between these operations, beginning with whole numbers and integers. They explore some of the laws that govern these operations and use mathematical models to reinforce the algorithms they commonly use. Later, they revisit these operations in the context of rational numbers **and irrational numbers (R/Q)** and refine, revise and consolidate their ideas. Students learn strategies for computation that can be applied to any numbers; implicit in such computational methods are generalisations about numerical relationships involving the operations being used. Students articulate the generalisation that underlies their strategy, firstly in the vernacular and then in symbolic language. Problems set in context, using diagrams to solve the problems so they can appreciate how the mathematical concepts are related to real life. Algorithms used to solve problems involving fractional amounts.	– investigate models such as decomposition, skip counting, arranging items in arrays and accumulating groups of equal size to make sense of the operations of addition, subtraction, multiplication and division in **N** where the answer is in **N,** including the inverse operations – investigate the properties of arithmetic: commutative, associative and distributive laws and the relationships between them – appreciate the order of operations, including the use of brackets – investigate models such as the number line to illustrate the operations of addition, subtraction, multiplication and division in **Z** – use the number line to order numbers in **N, Z, Q (and R for HL)** – generalise and articulate observations of arithmetic operations – investigate models to help think about the operations of addition, subtraction, multiplication and division of rational numbers – consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value – analyse solution strategies to problems – engage with the idea of mathematical proof – calculate percentages – use the equivalence of fractions, decimals and percentages to compare proportions – consolidate their understanding and their learning of factors, multiples and prime numbers in **N** – consolidate their understanding of the relationship between ratio and proportion – check a result by considering whether it is of the right order of magnitude – check a result by working the problem backwards – justify approximations and estimates of calculations – present numerical answers to the degree of accuracy specified

Topic	Description of topic Students learn about	Learning outcomes Students should be able to
3.2 Indices	Binary operations of addition, subtraction, multiplication and division in the context of numbers in index form.	– use and apply the rules for indices (where $a \in \mathbf{Z}$, $a \neq 0$; $p, q \in \mathbf{N}$): • $a^p a^q = a^{p+q}$ • $\frac{a^p}{a^q} = a^{p-q}$ $p > q$ • $(a^p)^q = a^{pq}$ – **use and apply rules for indices (where $a, b \in \mathbf{R}$, $a, b \neq 0$; $p, q \in \mathbf{Q}$; $a^p, a^q, \in \mathbf{R}$; complex numbers not included):** • $a^p a^q = a^{p+q}$ • $\frac{a^p}{a^q} = a^{p-q}$ • $a^0 = 1$ • $(a^p)^q = a^{pq}$ • $a^{1/q} = \sqrt[q]{a}$, $q \in Z$, $q \neq 0$, $a > 0$ • $a^{p/q} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$ $p, q \in Z$, $q \neq 0$, $a > 0$ • $a^{-p} = \frac{1}{a^p}$ • $(ab)^p = a^p b^p$ • $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$ – **operate on the set of irrational numbers R \ Q** – use the notation $a^{1/2}$, $a \in \mathbf{N}$ – express rational numbers ≥1 in the approximate form $a \times 10^n$, where a is in decimal form correct to a specified number of places and where $n = 0$ or $n \in \mathbf{N}$ – **express non-zero positive rational numbers in the approximate form $a \times 10^n$, where $n \in \mathbf{Z}$ and $1 \leq a < 10$** – compute reciprocals
3.3 Applied arithmetic	Solving problems involving, e.g., mobile phone tariffs, currency transactions, shopping, VAT and meter readings. Making value for money calculations and judgments. Using ratio and proportionality.	– solve problems that involve finding profit or loss, % profit or loss (on the cost price), discount, % discount, selling price, compound interest for not more than 3 years, income tax (standard rate only), net pay (including other deductions of specified amounts) **– solve problems that involve cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price) compound interest, income tax and net pay (including other deductions)**

Topic	Description of topic Students learn about	Learning outcomes Students should be able to
3.4 Applied measure	Measure and time. 2D shapes and 3D solids, including nets of solids (two-dimensional representations of three-dimensional objects). Using nets to analyse figures and to distinguish between surface area and volume. Problems involving perimeter, surface area and volume. Modelling real-world situations and solve a variety of problems (including multi-step problems) involving surface areas, and volumes of cylinders **and prisms.** The circle and develop an understanding of the relationship between its circumference, diameter and π.	– calculate, interpret and apply units of measure and time – solve problems that involve calculating average speed, distance and time – investigate the nets of rectangular solids – find the volume of rectangular solids and cylinders – find the surface area of rectangular solids – identify the necessary information to solve a problem – select and use suitable strategies to find length of the perimeter and the area of the following plane figures: disc, triangle, rectangle, square, and figures made from combinations of these – draw and interpret scaled diagrams **– investigate nets of prisms (polygonal bases) cylinders and cones** **– solve problems involving surface area of triangular base prisms (right angle, isosceles, equilateral), cylinders and cones** **– solve problems involving curved surface area of cylinders, cones and spheres** **– perform calculations to solve problems involving the volume of rectangular solids, cylinders, cones, triangular base prisms (right angle, isosceles, equilateral), spheres and combinations of these**

Topic	Description of topic Students learn about	Learning outcomes Students should be able to
3.5 Sets	Set language as an international symbolic mathematical tool; the concept of a set as being a well-defined collection of objects or elements. They are introduced to the concept of the universal set, null set, subset, cardinal number; the union, intersection, set difference operators, and Venn diagrams. They investigate the properties of arithmetic as related to sets and solve problems involving sets.	– use suitable set notation and terminology – list elements of a finite set – describe the rule that defines a set – consolidate the idea that equality of sets is a relationship in which two equal sets have the same elements – perform the operations of intersection, union (for two sets), set difference and complement – investigate the commutative property for intersection, union and difference **– explore the operations of intersection, union (for three sets), set difference and complement** **– investigate the associative property in relation to intersection, union and difference** **– investigate the distributive property of union over intersection and intersection over union.**

Students learn about	Students should be able to
3.6 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

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4.1 Generating arithmetic expressions from repeating patterns	Patterns and the rules that govern them; students construct an understanding of a relationship as that which involves a set of inputs, a set of outputs and a correspondence from each input to each output.	– use tables to represent a repeating-pattern situation – generalise and explain patterns and relationships in words and numbers – write arithmetic expressions for particular terms in a sequence
4.2 Representing situations with tables, diagrams and graphs	Relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks. Students look at various patterns and make predictions about what comes next.	– use tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations (exponential relations limited to doubling and tripling) – develop and use their own generalising strategies and ideas and consider those of others – present and interpret solutions, explaining and justifying methods, inferences and reasoning
4.3 Finding formulae	Ways to express a general relationship arising from a pattern or context.	– find the underlying formula written in words from which the data are derived (linear relations) **– find the underlying formula algebraically from which the data are derived (linear, quadratic relations)**
4.4 Examining algebraic relationships	Features of a relationship and how these features appear in the different representations. Constant rate of change: linear relationships. Non-constant rate of change: quadratic relationships. Proportional relationships.	– show that relations have features that can be represented in a variety of ways – distinguish those features that are especially useful to identify and point out how those features appear in different representations: in tables, graphs, physical models, and formulas expressed in words, and algebraically – use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others – recognise that a distinguishing feature of quadratic relations is the way change varies – discuss rate of change and the y-intercept; consider how these relate to the context from which the relationship is derived, and identify how they can appear in a table, in a graph and in a formula – decide if two linear relations have a common value – investigate relations of the form $y=mx$ and $y=mx+c$ – recognise problems involving direct proportion and identify the necessary information to solve them

Topic	**Description of topic** Students learn about	**Learning outcomes** Students should be able to
4.5 Relations without formulae	Using graphs to represent phenomena quantitatively.	– explore graphs of motion – make sense of quantitative graphs and draw conclusions from them – make connections between the shape of a graph and the story of a phenomenon – describe both quantity and change of quantity on a graph
4.6 Expressions	Using letters to represent quantities that are variable. Arithmetic operations on expressions; applications to real life contexts. Transformational activities: collecting like terms, simplifying expressions, substituting, expanding and factoring.	– evaluate expressions of the form • $ax + by$ • $a(x + y)$ • $x^2 + bx + c$ • $\frac{ax + by}{cx + dy}$ • axy where $a, b, c, d, x, y \in \mathbf{Z}$ • **$ax^2 + bx + c$** • **$x^3 + bx^2 + cx + d$** **where $a, b, c, d, x, y \in \mathbf{Q}$** – add and subtract simple algebraic expressions of forms such as: • $(ax + by + c) \pm (dx + ey + f)$ • $(ax^2 + bx + c) \pm (dx^2 + ex + f)$ • $\frac{ax + b}{c} \pm \frac{dx + e}{f}$ where $a, b, c, d, e, f \in \mathbf{Z}$ • **$\frac{ax + b}{c} \pm \ldots \pm \frac{dx + e}{f}$** • **$(ax + by + c) \pm \ldots \pm (dx + ey + f)$** • **$(ax^2 + bx + c) \pm \ldots \pm (dx^2 + ex + f)$** **where $a, b, c, d, e, f \in Z$** • **$\frac{a}{bx + c} \pm \frac{p}{qx + r}$ where $a, b, c, p, q, r \in Z$.** – use the associative and distributive property to simplify such expressions as: • $a(bx + cy + d) + e(fx + gy + h)$ • **$a(bx + cy + d) + \ldots + e(fx + gy + h)$** • $a(bx^2 + cx + d)$ • $ax(bx^2 + c)$ where $a, b, c, d, e, f, g, h \in Z$ • **$(x+y)(x+y)$; $(x-y)(x-y)$** – multiply expressions of the form: • $(ax + b)(cx + d)$ • **$(ax + b)(cx^2 + dx + e)$ where $a, b, c, d, e \in \mathbf{Z}$** – divide expressions of the form: • $ax^2 + bx + c \div dx + e$, where $a, b, c, d, e \in \mathbf{Z}$ • **$ax^3 + bx^2 + cx + d \div ex + f$, where $a, b, c, d, e \in \mathbf{Z}$** – factorise expressions such as ax, axy where $a \in \mathbf{Z}$ $abxy + ay$, where $a, b \in \mathbf{Z}$ $sx - ty + tx - sy$, where s, t, x, y are variable $ax^2 + bx$, where $a, b, c \in \mathbf{Z}$ $x^2 + bx + c$, where $b, c \in \mathbf{Z}$ $x^2 - a^2$ **$ax^2 + bx + c$, $a \in \mathrm{N}$ $b, c \in \mathbf{Z}$** **difference of two squares $a^2x^2 - b^2y^2$ where $a, b \in \mathrm{N}$** **– rearrange formulae**

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4.7 Equations and inequalities	Selecting and using suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations and inequalities. They identify the necessary information, represent problems mathematically, making correct use of symbols, words, diagrams, tables and graphs.	– consolidate their understanding of the concept of equality – solve first degree equations in one or two variables, with coefficients elements of **Z** and solutions also elements of **Z** – **solve first degree equations in one or two variables with coefficients elements of Q and solutions also in Q** – solve quadratic equations of the form $x^2 + bx + c = 0$ where $b, c \in \mathbf{Z}$ and $x^2 + bx + c$ is factorisable $\boldsymbol{ax^2 + bx + c = 0}$ **where** $\boldsymbol{a, b, c \in \mathbf{Q}}$ $\boldsymbol{x \in \mathbf{R}}$ – **form quadratic equations given whole number roots** – solve simple problems leading to quadratic equations – solve equations of the form $\frac{ax+b}{c} \pm \frac{dx+e}{f} = \frac{g}{h}$, where $a, b, c, d, e, f, g, h \in \mathbf{Z}$ – solve linear inequalities in one variable of the form $g(x) \leq k$ where $g(x) = ax+b$, $a \in \mathbf{N}$ and $b, k \in \mathbf{Z}$; $\boldsymbol{k \leq g(x) \leq h}$ **where** $\boldsymbol{g(x) = ax+b}$**, and** $\boldsymbol{k, a, b, h, \in Z}$ **and** $\boldsymbol{x \in R}$

Students should learn about	Students should be able to
4.8 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

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5.1 Functions	The meaning and notation associated with functions.	– engage with the concept of a function, domain, co-domain and range – make use of function notation $f(x) =$, $f : x \rightarrow$, and $y =$
5.2 Graphing functions	Interpreting and representing linear, quadratic and exponential functions in graphical form.	– interpret simple graphs – plot points and lines – draw graphs of the following functions and interpret equations of the form $f(x) = g(x)$ as a comparison of functions • $f(x) = ax + b$, where $a, b \in \mathbf{Z}$ • $f(x) = ax^2 + bx + c$, where $a \in \mathbf{N}$; $b, c \in \mathbf{Z}$; $x \in \mathbf{R}$ **• $f(x) = ax^2 + bx + c$, where $a, b, c \in \mathbf{Z}$, $x \in \mathbf{R}$** **• $f(x) = a2^x$ and $f(x) = a3^x$, where $a \in \mathbf{N}$, $x \in \mathbf{R}$** – use graphical methods to find approximate solutions where $f(x) = g(x)$ and interpret the results – **find maximum and minimum values of quadratic functions from a graph** – **interpret inequalities of the form $f(x) \leq g(x)$ as a comparison of functions of the above form; use graphical methods to find approximate solution sets of these inequalities and interpret the results** – graph solution sets on the number line for linear inequalities in one variable

Students learn about	*Students should be able to*
5.3 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.