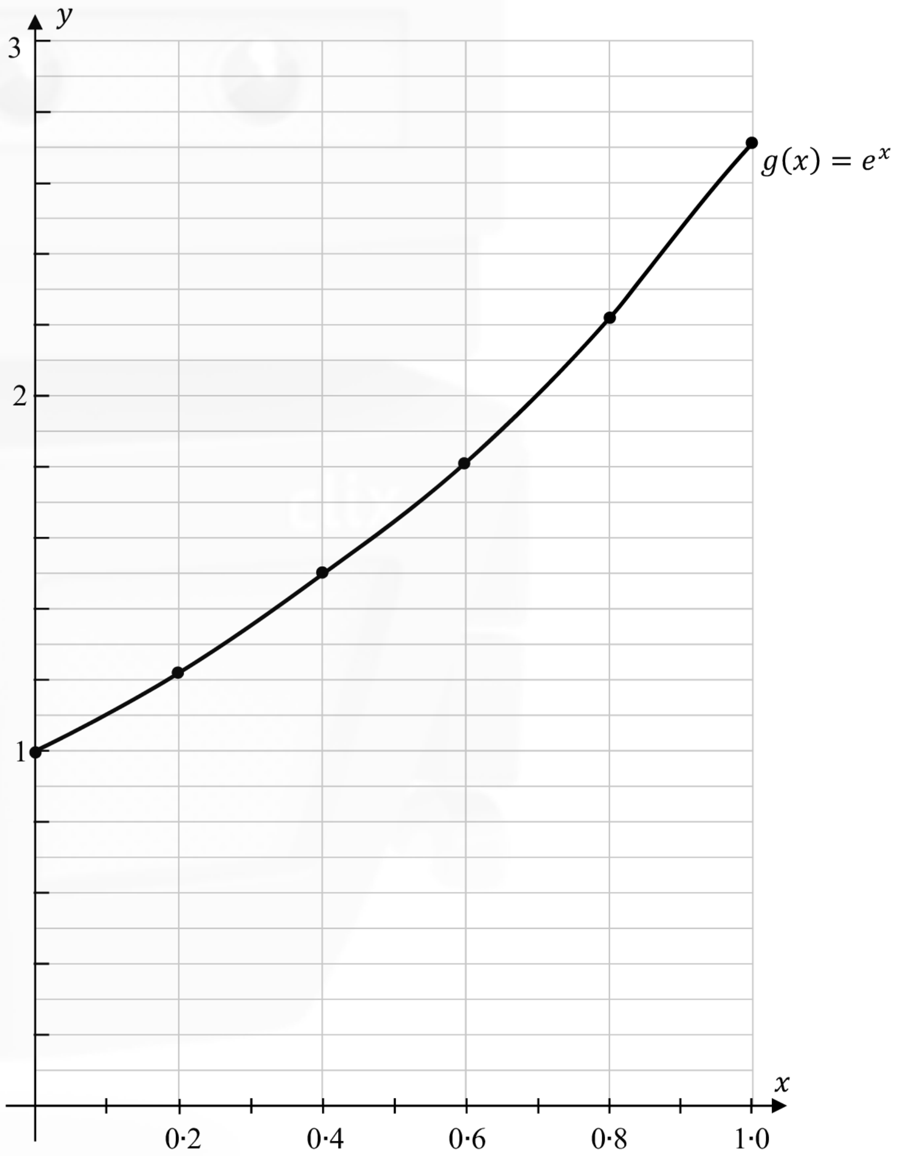
Question 1

Question 6 **(25 marks)**

The graph of the function $g(x) = e^x, x \in \mathbb{R},\ 0 \leq x \leq 1$, is shown on the diagram below.

(a) On the same diagram, draw the graph of $h(x) = e^{-x}, x \in \mathbb{R}$, in the domain $0 \leq x \leq 1$.



(b) Find the area enclosed by $g(x) = e^x$, $h(x) = e^{-x}$, and the line $x = 0{\cdot}75$.
Give your answer correct to 4 decimal places.

Question 2

The height of the water in a port was measured over a period of time. The average height was found to be 1·6 m. The height measured in metres, $h(t)$, was modelled using the function

$$h(t) = 1{\cdot}6 + 1{\cdot}5\cos\left(\frac{\pi}{6}t\right)$$

where t represents the number of hours since the last recorded high tide and $\left(\frac{\pi}{6}t\right)$ is expressed in radians.

(a) Find the period and range of $h(t)$.

Period:

Range:

(b) Find the maximum height of the water in the port.

(c) Find the rate at which the height of the water is changing when $t = 2$, correct to two decimal places. Explain your answer in the context of the question.

Rate:

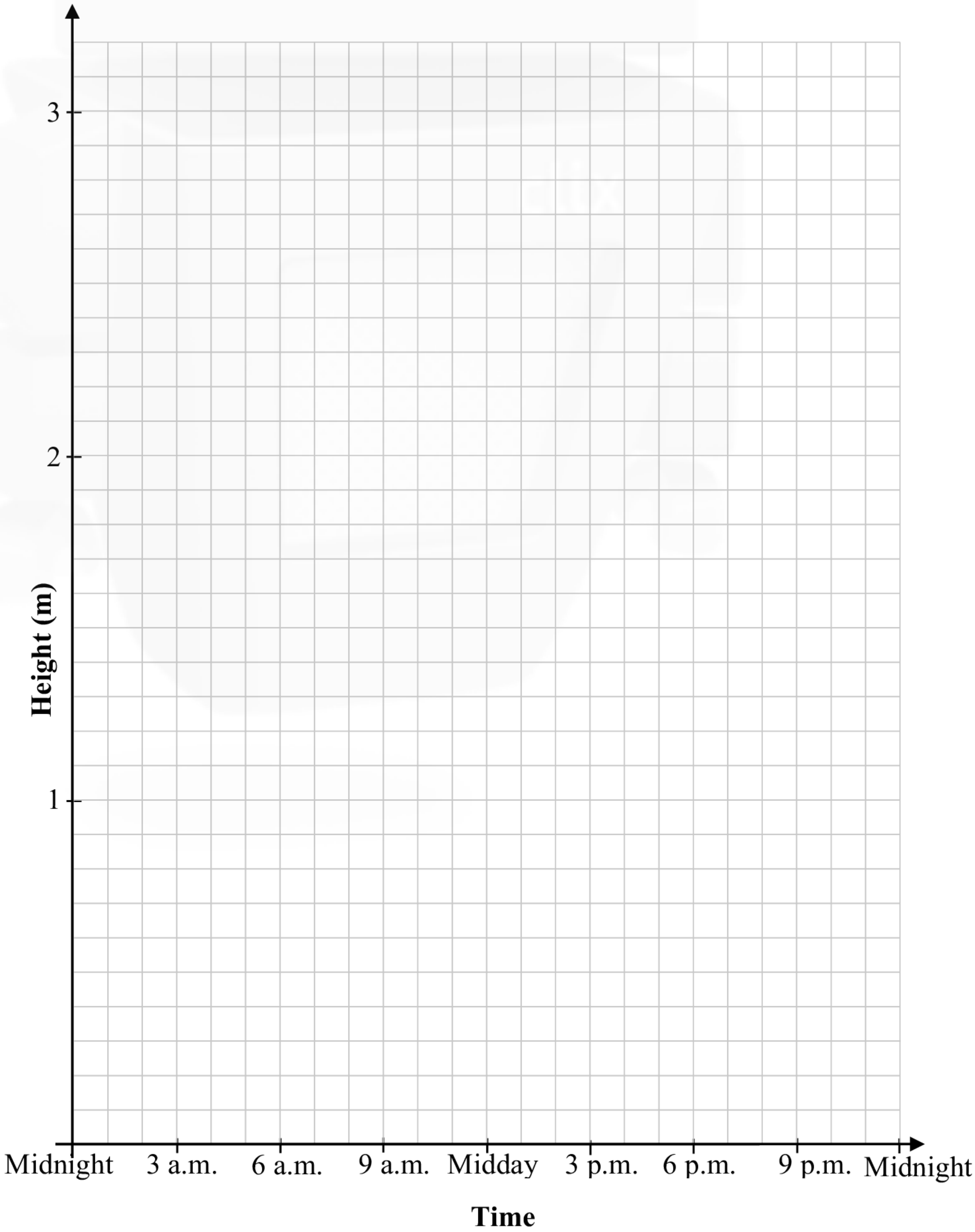
Explanation:

Previous	page	running

(d) (i) On a particular day the high tide occurred at midnight (i.e. $t = 0$). Use the function to complete the table and show the height, $h(t)$, of the water between midnight and the following midnight.

$h(t) = 1{\cdot}6 + 1{\cdot}5\cos\left(\frac{\pi}{6}t\right)$									
Time	Midnight	3 a.m.	6 a.m.	9 a.m.	12 noon	3 p.m.	6 p.m.	9 p.m.	Midnight
***t* (hours)**	0	3							
$h(t)$ (m)									

(ii) Sketch the graph of $h(t)$ between midnight and the following midnight.



(e) Find, from your sketch, the difference in water height between low tide and high tide.

(f) A fully loaded barge enters the port, unloads its cargo and departs some time later.
The fully loaded barge requires a minimum water level of 2 m.
When the barge is unloaded it only requires 1·5 m.
Use your graph to estimate the **maximum** amount of time that the barge can spend in port, without resting on the sea-bed.

Question 3 **(25 marks)**

Let $f(x) = -x^2 + 12x - 27,\ x \in \mathbb{R}$.

(a) **(i)** Complete Table 1 below.

Table 1							
x	3	4	5	6	7	8	9
$f(x)$	0	5			8		

(ii) Use Table 1 and the trapezoidal rule to find the approximate area of the region bounded by the graph of f and the x-axis.

(b) **(i)** Find $\int_3^9 f(x)dx$.

(ii) Use your answers above to find the percentage error in your approximation of the area, correct to one decimal place.

Page	Running

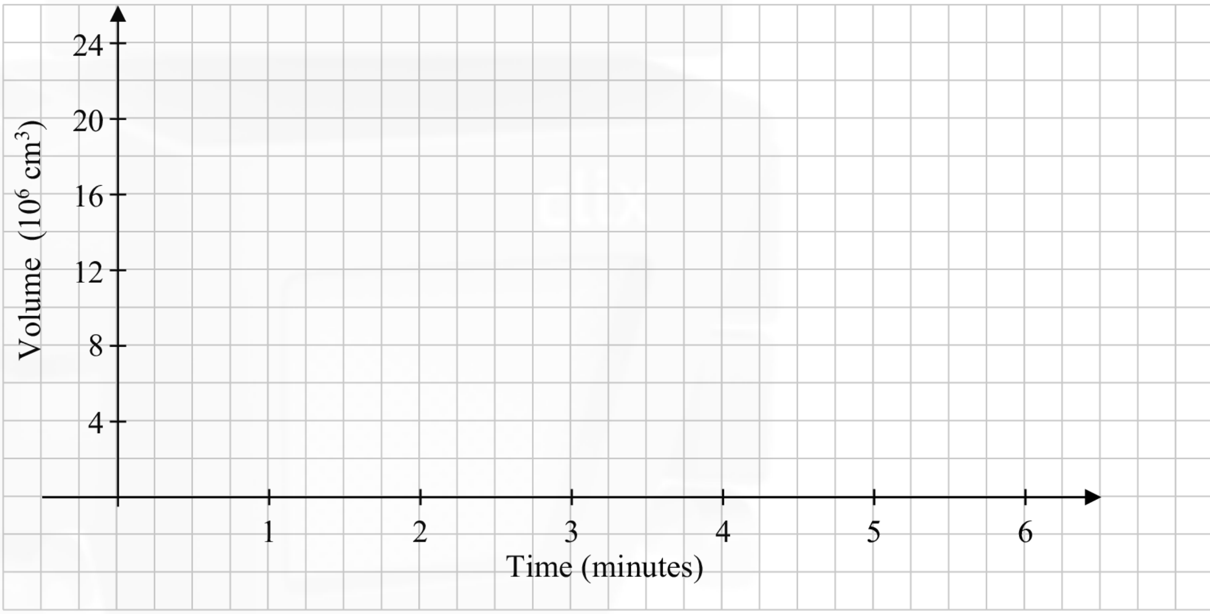
Question 8 **(50 marks)**

An oil-spill occurs off-shore in an area of calm water with no currents. The oil is spilling at a rate of 4×10^6 cm ^{3} per minute. The oil floats on top of the water.

(a) **(i)** Complete the table below to show the total volume of oil on the water after each of the first 6 minutes of the oil-spill.

Time (minutes)	1	2	3	4	5	6
Volume (10^6 cm^3)		8				

(ii) Draw a graph to show the total volume of oil on the water over the first 6 minutes.



(iii) Write an equation for $V(t)$, the volume of oil on the water, in cm^3, after t minutes.

(b) The spilled oil forms a circular oil slick **1 millimetre** thick.

(i) Write an equation for the volume of oil in the slick, in cm^3, when the radius is r cm.

(ii) Find the rate, in cm per minute, at which the radius of the oil slick is increasing when the radius is 50 m.

(c) Show that the area of water covered by the oil slick is increasing at a constant rate of 4×10^7 cm^2 per minute.

(d) The nearest land is 1 km from the point at which the oil-spill began. Find how long it will take for the oil slick to reach land. Give your answer correct to the nearest hour.

Page	Running

Question 9 **(50 marks)**

The approximate length of the day in Galway, measured in hours from sunrise to sunset, may be calculated using the function

$$f(t)=12\cdot 25+4\cdot 75\sin\left(\frac{2\pi}{365}t\right),$$

where t is the number of days after March 21st and $\left(\frac{2\pi}{365}t\right)$ is expressed in radians.

(a) Find the length of the day in Galway on June 5th (76 days after March 21st). Give your answer in hours and minutes, correct to the nearest minute.

(b) Find a date on which the length of the day in Galway is approximately 15 hours.

(c) Find $f'(t)$, the derivative of $f(t)$.

(d) Hence, or otherwise, find the length of the longest day in Galway.

(e) Use integration to find the average length of the day in Galway over the six months from March 21^{st} to September 21^{st} (184 days). Give your answer in hours and minutes, correct to the nearest minute.

Page	Running

Question 6

(a) Find $\int 5\cos 3x\,dx$.

(b) The slope of the tangent to a curve $y = f(x)$ at each point (x, y) is $2x - 2$.
The curve cuts the x-axis at $(-2, 0)$.

(i) Find the equation of $f(x)$.

(ii) Find the average value of f over the interval $0 \le x \le 3$, $x \in \mathbb{R}$.

page	running

Question 6 **(25 marks)**

(a) **(i)** Write down three distinct anti-derivatives of the function

$$g : x \mapsto x^3 - 3x^2 + 3, \qquad x \in \mathbb{R}.$$

1. ______________________________

2. ______________________________

3. ______________________________

(ii) Explain what is meant by the indefinite integral of a function *f*.

(iii) Write down the indefinite integral of *g*, the function in part **(i)**.

Answer: ______________________________

(b) **(i)** Let $h(x) = x \ln x$, for $x \in \mathbb{R}$, $x > 0$.
Find $h'(x)$.

(ii) Hence, find $\int \ln x \, dx$.

Question 9 **(50 marks)**

(a) Let $f(x) = -0{\cdot}5x^2 + 5x - 0{\cdot}98$, where $x \in \mathbb{R}$.

(i) Find the value of $f(0{\cdot}2)$.

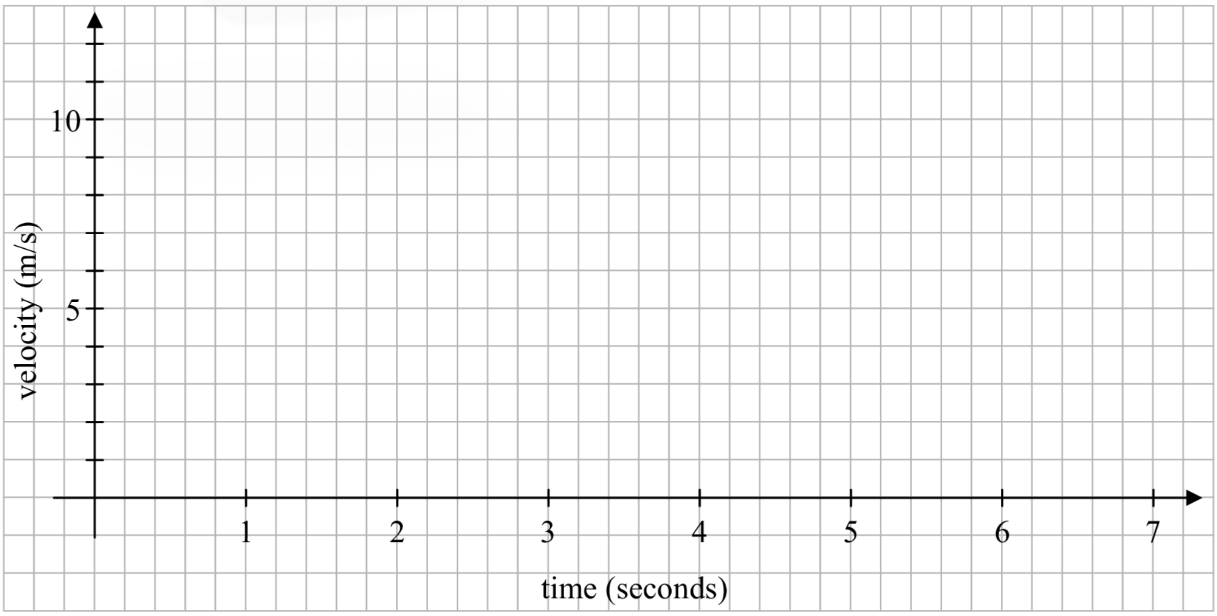
(ii) Show that f has a local maximum point at $(5,\ 11{\cdot}52)$.

(b) A sprinter's velocity over the course of a particular 100 metre race is approximated by the following model, where v is the velocity in metres per second, and t is the time in seconds from the starting signal:

$$v(t) = \begin{cases} 0, & \text{for } 0 \le t < 0{\cdot}2 \\ -0{\cdot}5t^2 + 5t - 0{\cdot}98, & \text{for } 0{\cdot}2 \le t < 5 \\ 11{\cdot}52, & \text{for } t \ge 5 \end{cases}$$

Note that the function in part **(a)** is relevant to $v(t)$ above.

(i) Sketch the graph of v as a function of t for the first 7 seconds of the race.



(ii) Find the distance travelled by the sprinter in the first 5 seconds of the race.

(iii) Find the sprinter’s finishing time for the race. Give your answer correct to two decimal places.

(c) A spherical snowball is melting at a rate proportional to its surface area. That is, the rate at which its volume is decreasing at any instant is proportional to its surface area at that instant.

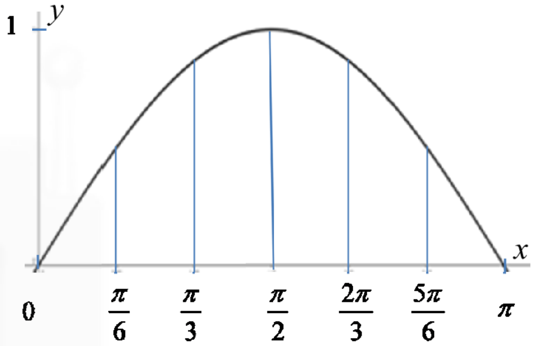
(i) Prove that the radius of the snowball is decreasing at a constant rate.

(ii) If the snowball loses half of its volume in an hour, how long more will it take for it to melt completely?

Give your answer correct to the nearest minute.

Question 9

The diagram shows the graph of the function $y = \sin x$ in the domain $0 \leq x \leq \pi,\ x \in \mathbb{R}$.



(a) Complete the table below, correct to three decimal places.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y							

(b) Use the trapezoidal rule to find the approximate area of the region enclosed between the curve and the x-axis in the given domain.

(c) Use integration to find the actual area of the region shown above.

(d) Find the percentage error in your answer to **(b)** above.

page	running