InductionH

Question 1

(a) Prove by induction that $8^n - 1$ is divisible by 7 for all $n \in \mathbb{N}$.

Question 2

(a) Prove, by induction, that the sum of the first n natural numbers, $1+2+3+\cdots+n$, is $\frac{n(n+1)}{2}$.

(b) Hence, or otherwise, prove that the sum of the first n even natural numbers, $2+4+6+\cdots+2n$, is n^2+n.

(c) Using the results from **(a)** and **(b)** above, find an expression for the sum of the first n odd natural numbers in its simplest form.

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Question 2 **(25 marks)**

(a) **(i)** Prove by induction that, for any n, the sum of the first n natural numbers is $\dfrac{n(n+1)}{2}$.

(ii) Find the sum of all the natural numbers from 51 to 100, inclusive.

(b) Given that $p = \log_c x$, express $\log_c \sqrt{x} + \log_c (cx)$ in terms of p.

Question 2 **(25 marks)**

(a) Prove by induction that $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$, for any $n \in \mathbb{N}$.

(b) State the range of values of x for which the series $\sum_{r=2}^{\infty} (4x-1)^r$ is convergent, and write the infinite sum in terms of x.

Question 4 **(25 marks)**

(a) Prove, by induction, the formula for the sum of the first n terms of a geometric series. That is, prove that, for $r \neq 1$:

$$a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a(1-r^n)}{1-r}.$$

(b) By writing the recurring part as an infinite geometric series, express the following number as a fraction of integers:

$$5{\cdot}\dot{2}\dot{1} = 5{\cdot}2121212121\ldots$$

Question 6

(a) **(i)** Prove by induction that, for any n, the sum of the first n natural numbers is $\frac{n(n+1)}{2}$.

(ii) Find the sum of all the natural numbers from 51 to 100, inclusive.