

# InductionH

## Question 1

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- (a) Prove by induction that  $8^n - 1$  is divisible by 7 for all  $n \in \mathbb{N}$ .



## Question 2

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- (a) Prove, by induction, that the sum of the first  $n$  natural numbers,

$$1 + 2 + 3 + \dots + n, \text{ is } \frac{n(n+1)}{2}.$$

- (b) Hence, or otherwise, prove that the sum of the first  $n$  even natural numbers,

$$2 + 4 + 6 + \dots + 2n, \text{ is } n^2 + n.$$

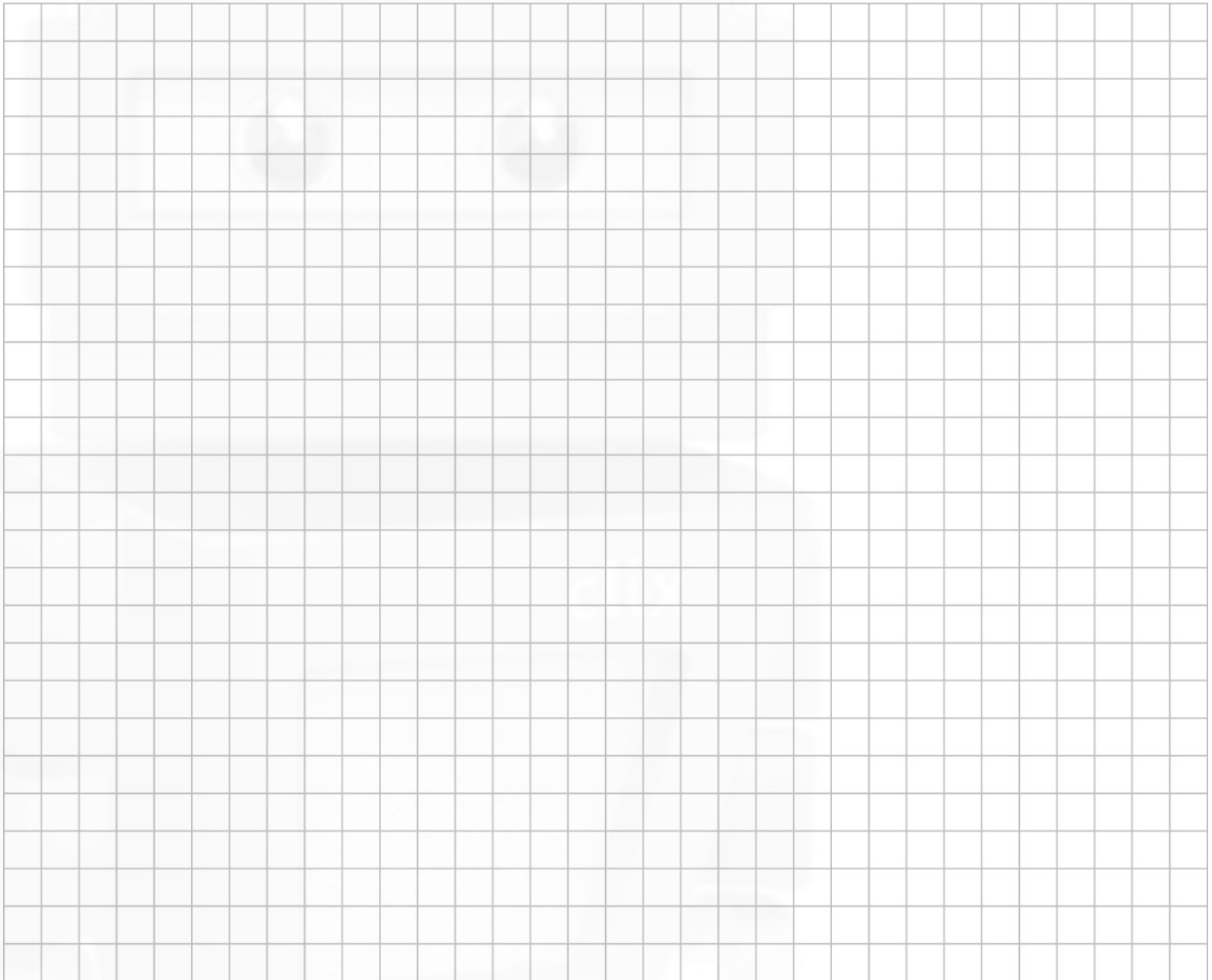
- (c) Using the results from (a) and (b) above, find an expression for the sum of the first  $n$  odd natural numbers in its simplest form.



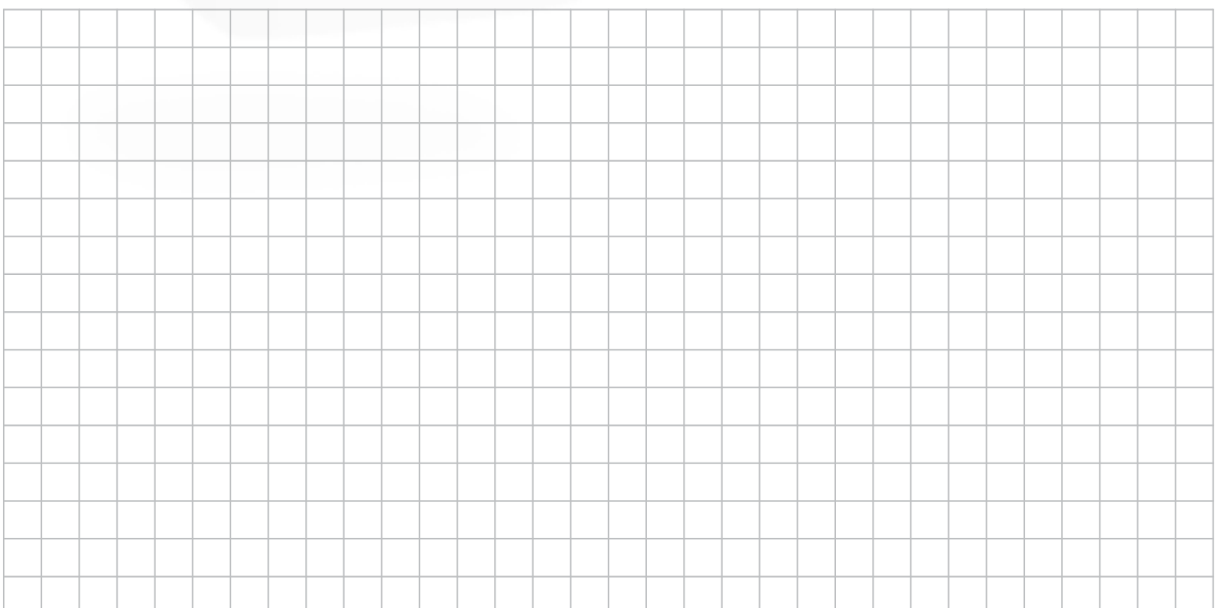
## Question 2

(25 marks)

- (a) Prove by induction that  $\sum_{r=1}^n r = \frac{n(n+1)}{2}$ , for any  $n \in \mathbb{N}$ .



- (b) State the range of values of  $x$  for which the series  $\sum_{r=2}^{\infty} (4x-1)^r$  is convergent, and write the infinite sum in terms of  $x$ .

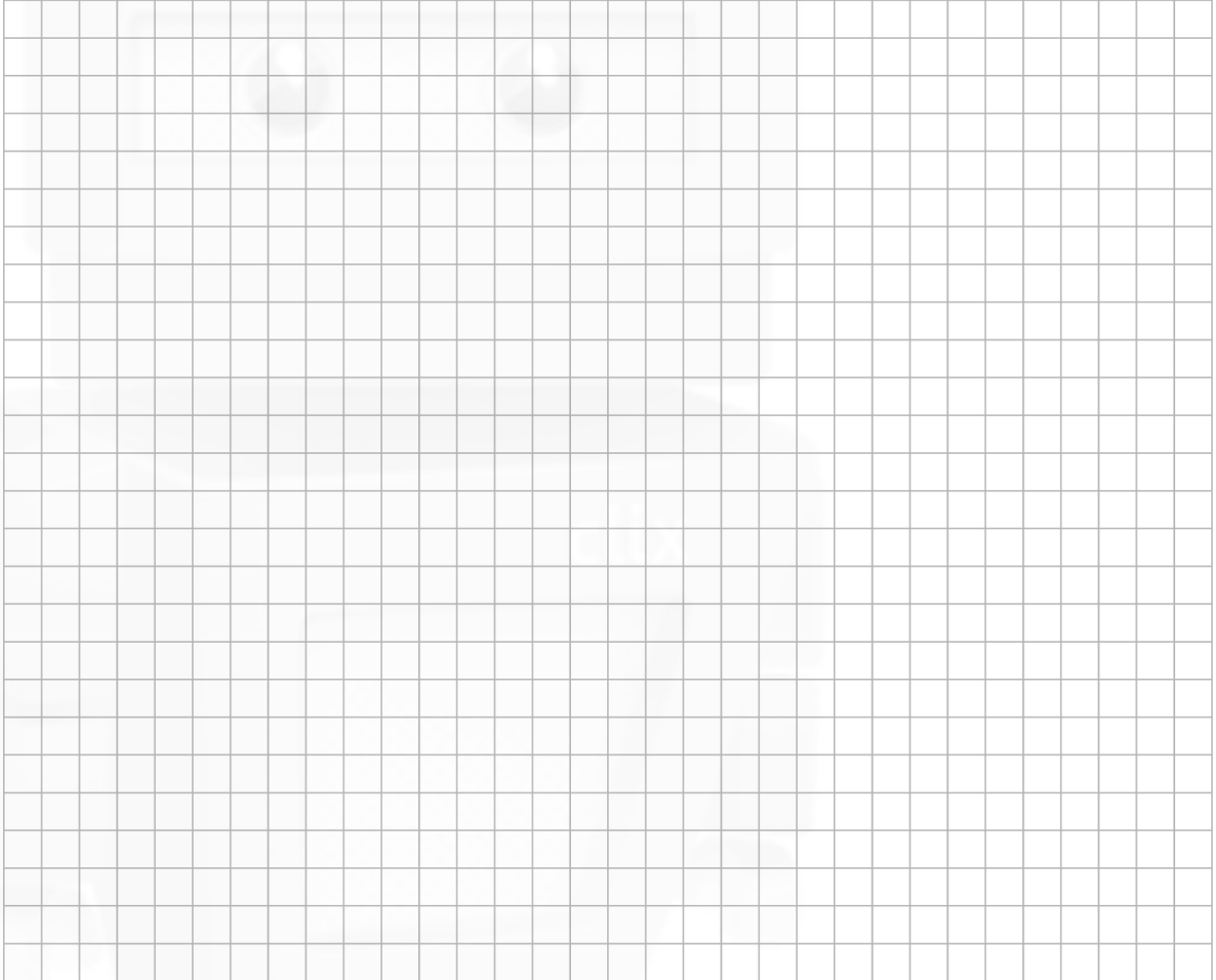


## Question 4

(25 marks)

- (a) Prove, by induction, the formula for the sum of the first  $n$  terms of a geometric series. That is, prove that, for  $r \neq 1$ :

$$a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1-r^n)}{1-r}.$$



- (b) By writing the recurring part as an infinite geometric series, express the following number as a fraction of integers:

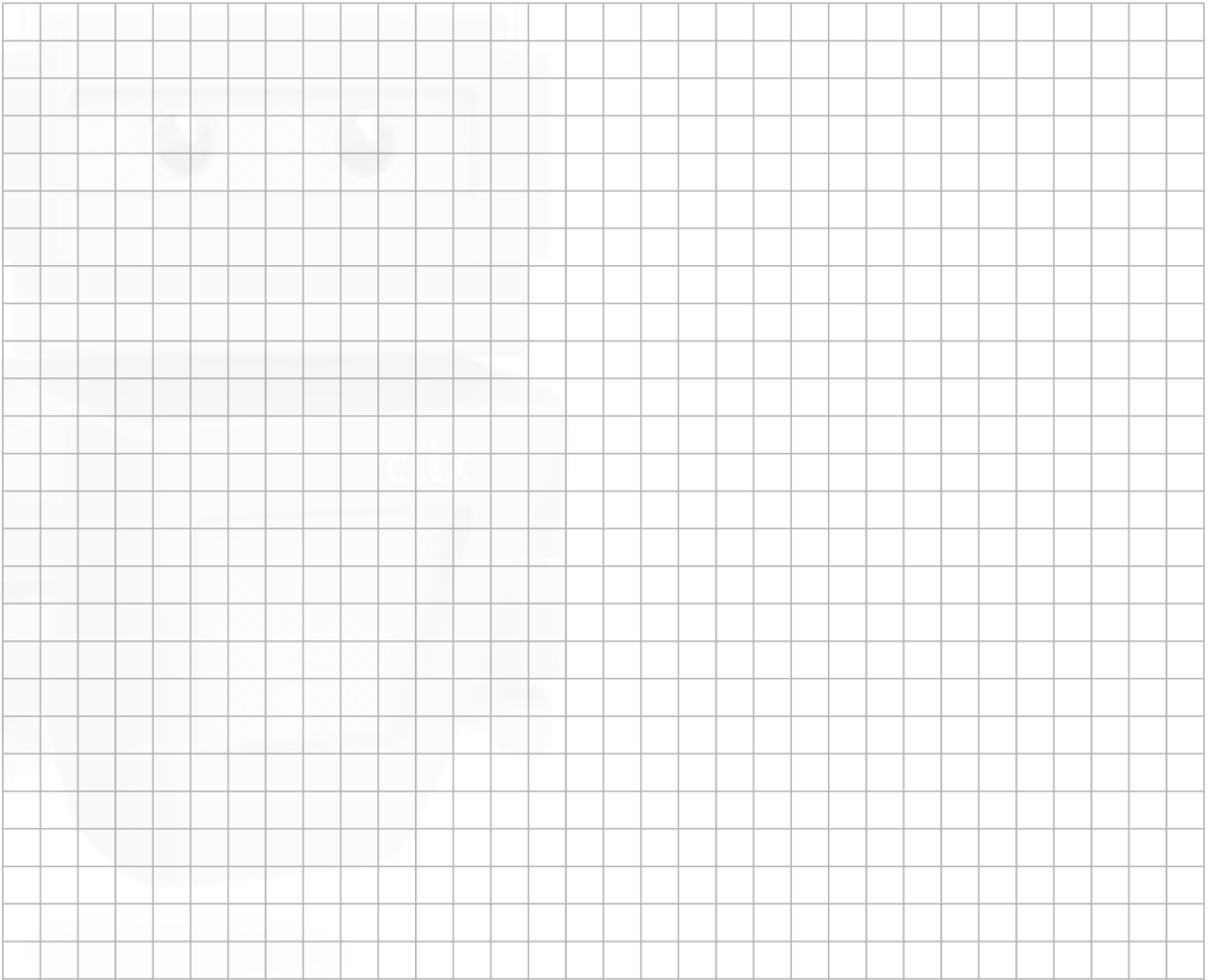
$$5.\dot{2}1 = 5.2121212121\dots$$



Question 6

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- (a) (i) Prove by induction that, for any  $n$ , the sum of the first  $n$  natural numbers is  $\frac{n(n+1)}{2}$ .



- (ii) Find the sum of all the natural numbers from 51 to 100, inclusive.

