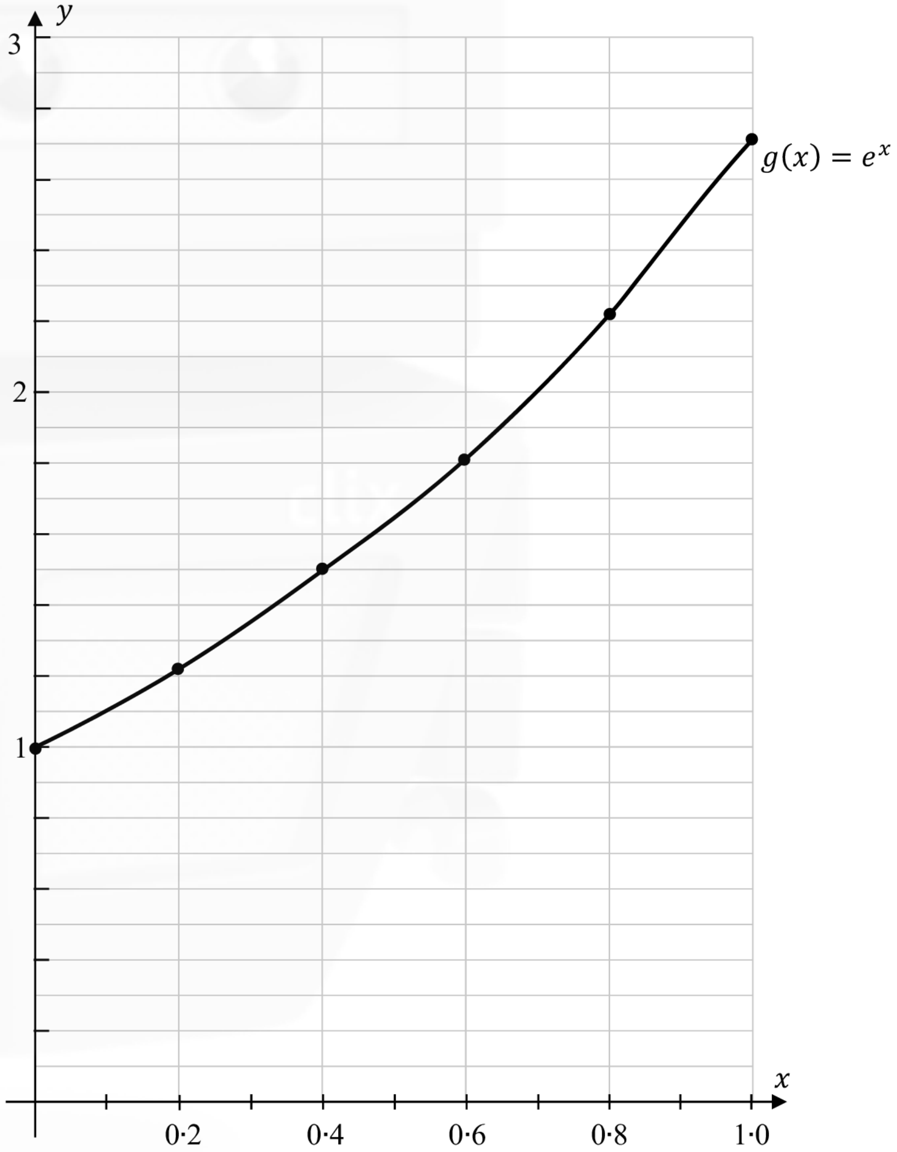
Question 1

Question 6 **(25 marks)**

The graph of the function $g(x) = e^x$, $x \in \mathbb{R}$, $0 \leq x \leq 1$, is shown on the diagram below.

(a) On the same diagram, draw the graph of $h(x) = e^{-x}$, $x \in \mathbb{R}$, in the domain $0 \leq x \leq 1$.



(b) Find the area enclosed by $g(x) = e^{x},\ h(x) = e^{-x}$, and the line $x = 0{\cdot}75$.
Give your answer correct to 4 decimal places.

Question 2

The height of the water in a port was measured over a period of time. The average height was found to be 1·6 m. The height measured in metres, $h(t)$, was modelled using the function

$$h(t) = 1{\cdot}6 + 1{\cdot}5\cos\left(\frac{\pi}{6}t\right)$$

where t represents the number of hours since the last recorded high tide and $\left(\frac{\pi}{6}t\right)$ is expressed in radians.

(a) Find the period and range of $h(t)$.

Period:

Range:

(b) Find the maximum height of the water in the port.

(c) Find the rate at which the height of the water is changing when $t = 2$, correct to two decimal places. Explain your answer in the context of the question.

Rate:

Explanation:

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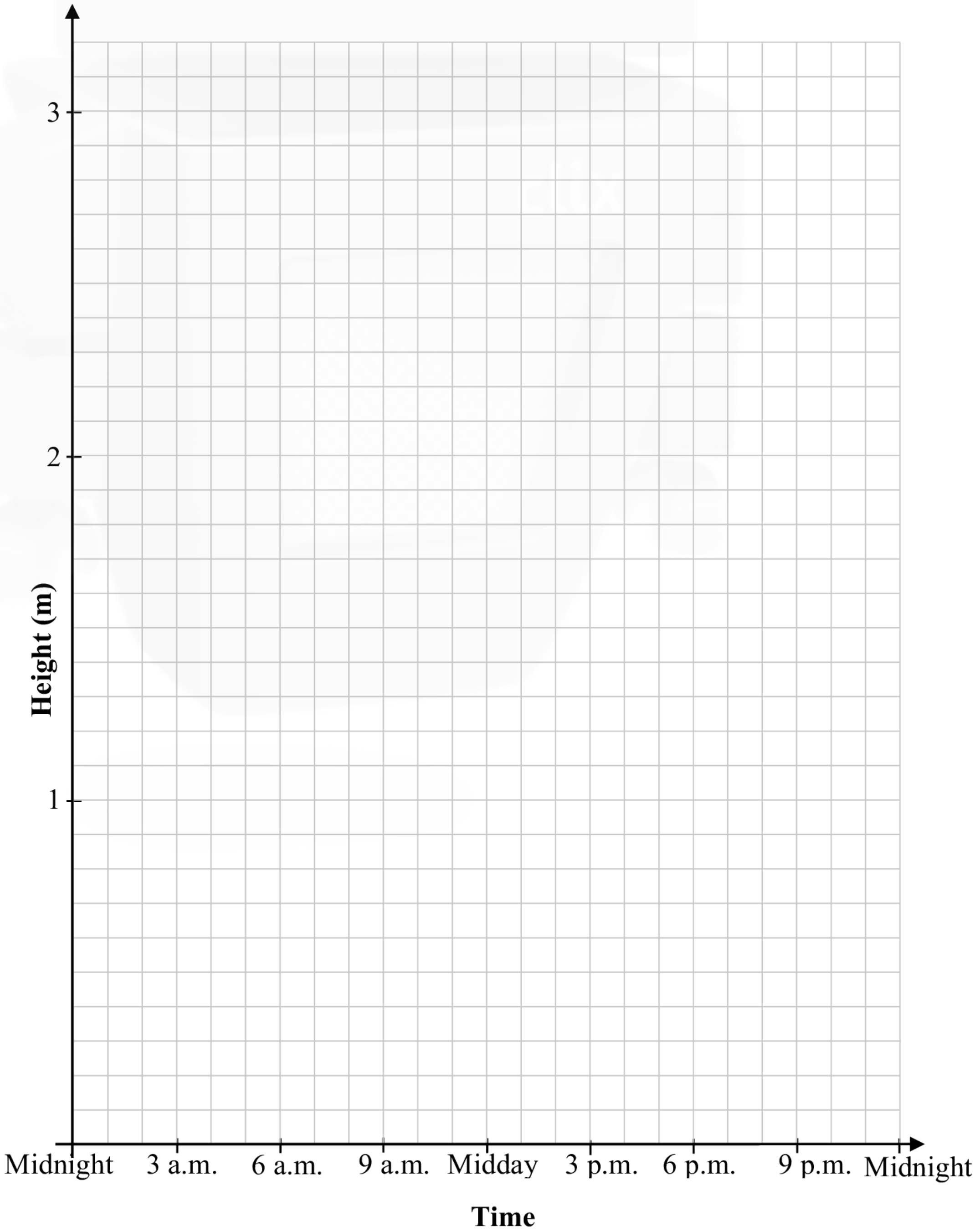
(d) (i) On a particular day the high tide occurred at midnight (i.e. $t = 0$). Use the function to complete the table and show the height, $h(t)$, of the water between midnight and the following midnight.

$h(t) = 1{\cdot}6 + 1{\cdot}5\cos\left(\frac{\pi}{6}t\right)$									
Time	Midnight	3 a.m.	6 a.m.	9 a.m.	12 noon	3 p.m.	6 p.m.	9 p.m.	Midnight
***t* (hours)**	0	3							
$h(t)$ **(m)**									

(ii) Sketch the graph of $h(t)$ between midnight and the following midnight.



(e) Find, from your sketch, the difference in water height between low tide and high tide.

(f) A fully loaded barge enters the port, unloads its cargo and departs some time later.
The fully loaded barge requires a minimum water level of 2 m.
When the barge is unloaded it only requires 1·5 m.
Use your graph to estimate the **maximum** amount of time that the barge can spend in port, without resting on the sea-bed.

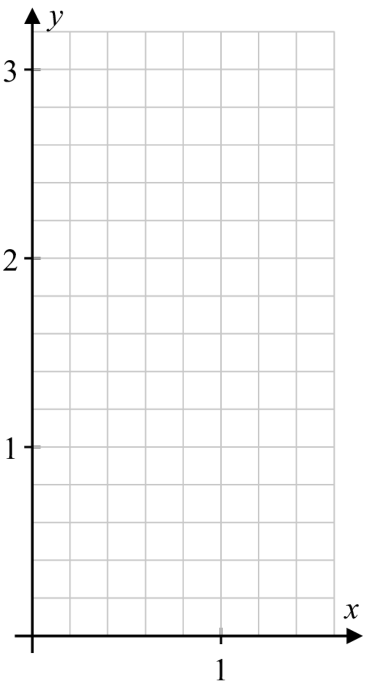
Question 3

(a) **(i)** $f(x) = \dfrac{2}{e^x}$ and $g(x) = e^x - 1$, where $x \in \mathbb{R}$.

Complete the table below. Write your values correct to two decimal places where necessary.

x	**0**	**0·5**	**1**	**ln(4)**
$f(x) = \dfrac{2}{e^x}$				
$g(x) = e^x - 1$				

(ii) In the grid on the right, use the table to draw the graphs of $f(x)$ and $g(x)$ in the domain $0 \le x \le \ln(4)$. Label each graph clearly.



(iii) Use your graphs to estimate the value of x for which $f(x) = g(x)$.

(b) Solve $f(x) = g(x)$ using algebra.

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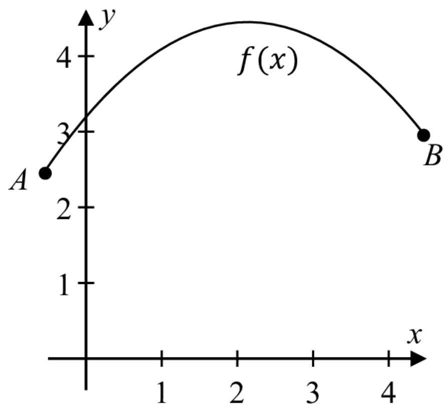
Question 4

(a) The diagram shows Sarah's first throw at the basket in a basketball game. The ball left her hands at A and entered the basket at B. Using the co-ordinate plane with $A(-0{\cdot}5,\ 2{\cdot}565)$ and $B(4{\cdot}5,\ 3{\cdot}05)$, the equation of the path of the centre of the ball is

$$f(x) = -0{\cdot}274x^2 + 1{\cdot}193x + 3{\cdot}23,$$

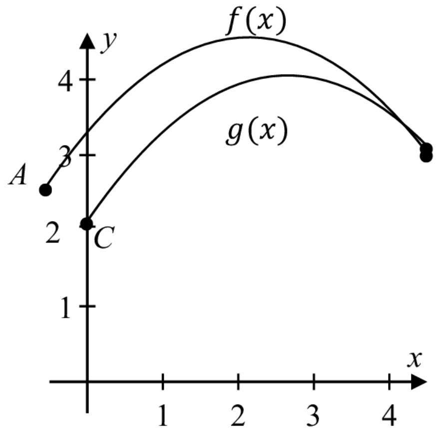
where both x and $f(x)$ are measured in metres.



(i) Find the maximum height reached by the centre of the ball, correct to three decimal places.

(ii) Find the acute angle to the horizontal at which the ball entered the basket. Give your answer correct to the nearest degree.

(iii) Sarah took a second throw. This throw followed the path of the parabola $g(x)$ as shown.
The ball left Sarah's hands at the point $C(0, 2)$.
The graph $y = g(x)$ is the image of the graph $y = f(x)$ under the translation which maps A onto C.
Using your result from part **a(i)**, show that the centre of this ball reached its maximum height at the point $(2{\cdot}677, 3{\cdot}964)$, correct to three decimal places.



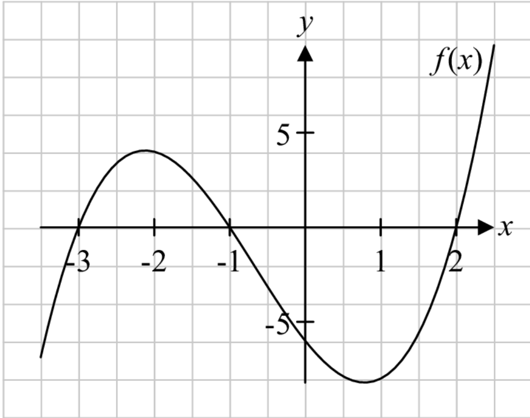
(iv) Hence, or otherwise, find the equation of the parabola $g(x)$.

Question 1 **(25 marks)**

(a) The graph of a cubic function $f(x)$ cuts the x-axis at $x=-3$, $x=-1$ and $x=2$, and the y-axis at $(0, -6)$, as shown.

Verify that $f(x)$ can be written as

$$f(x) = x^3 + 2x^2 - 5x - 6.$$



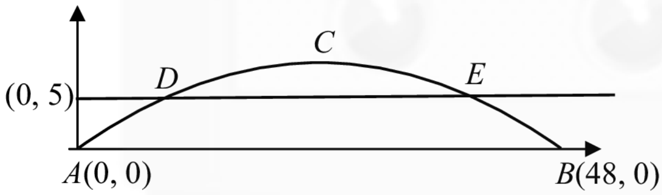
(b) **(i)** The graph of the function $g(x) = -2x - 6$ intersects the graph of the function $f(x)$ above. Let $f(x) = g(x)$ and solve the resulting equation to find the co-ordinates of the points where the graphs of $f(x)$ and $g(x)$ intersect.

(ii) Draw the graph of the function $g(x) = -2x - 6$ on the diagram above.

Question 8 **(50 marks)**

In 2011, a new footbridge was opened at Mizen Head, the most south-westerly point of Ireland.

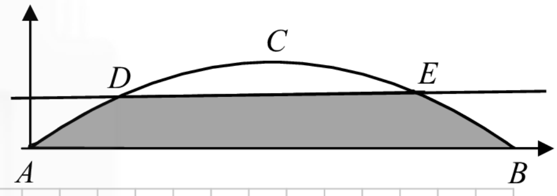
The arch of the bridge is in the shape of a parabola, as shown. The length of the span of the arch, $[AB]$, is 48 metres.



(a) Using the co-ordinate plane, with $A(0,0)$ and $B(48,0)$, the equation of the parabola is $y = -0\cdot013x^2 + 0\cdot624x$. Find the co-ordinates of C, the highest point of the arch.

(b) The perpendicular distance between the walking deck, $[DE]$, and $[AB]$ is 5 metres. Find the co-ordinates of D and of E. Give your answers correct to the nearest whole number.

(c) Using integration, find the area of the shaded region, *ABED,* shown in the diagram below. Give your answer correct to the nearest whole number.



(d) Write the equation of the parabola in part **(a)** in the form $y - k = p(x - h)^2$, where k, p, and h are constants.

(e) Using what you learned in part **(d)** above, or otherwise, write down the equation of a parabola for which the coefficient of x^2 is -2 and the co-ordinates of the maximum point are $(3, -4)$.

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Ciarán is preparing food for his baby and must use cooled boiled water. The equation $y = Ae^{kt}$ describes how the boiled water cools. In this equation:

- t is the time, in minutes, from when the water boiled,
- y is the *difference* between the water temperature and room temperature at time t, measured in degrees Celsius,
- A and k are constants.

The temperature of the water when it boils is $100°C$ and the room temperature is a constant $23°C$.

(a) Write down the value of the temperature difference, y, when the water boils, and find the value of A.

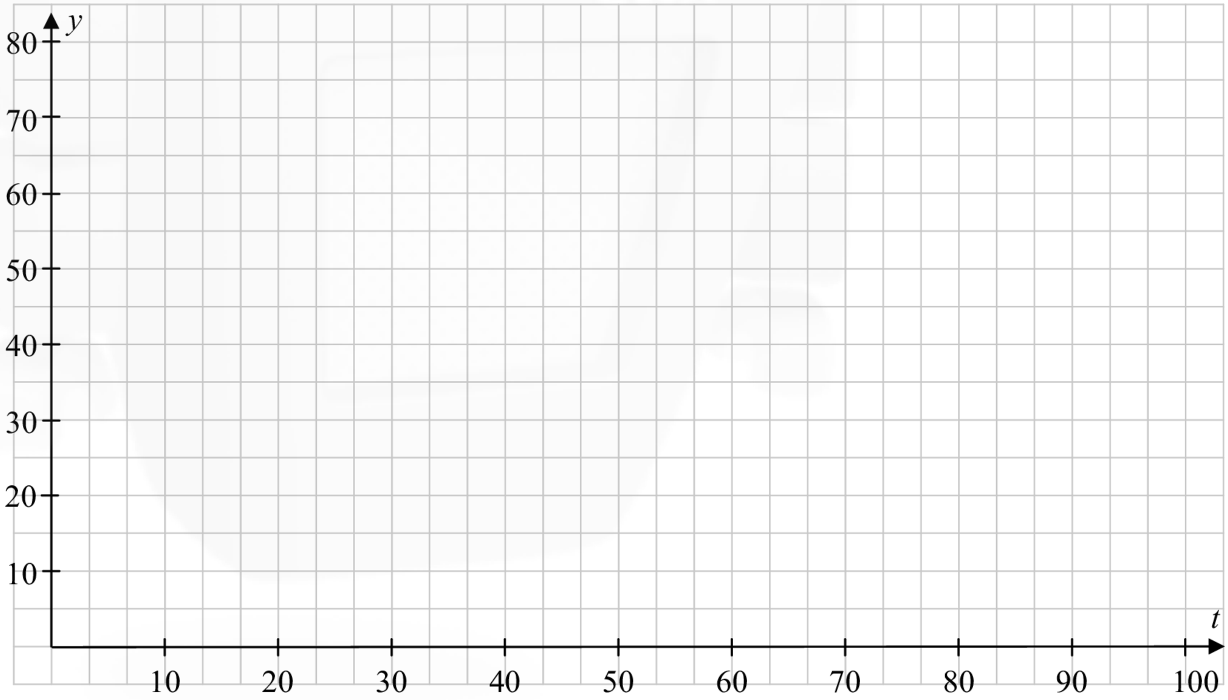
$y =$

$A =$

(b) After five minutes, the temperature of the water is $88°C$.
Find the value of k, correct to three significant figures.

(c) Ciarán prepares the food for his baby when the water has cooled to $50°C$. How long does it take, correct to the nearest minute, for the water to cool to this temperature?

(d) Using your values for A and k, sketch the curve $f(t) = Ae^{kt}$ for $0 \le t \le 100,\ t \in \mathbb{R}$.



(e) **(i)** On the same diagram, sketch a curve $g(t) = Ae^{mt}$, showing the water cooling at a *faster* rate, where A is the value from part **(a)**, and m is a constant. Label each graph clearly.

(ii) Suggest one possible value for m for the sketch you have drawn and give a reason for your choice.

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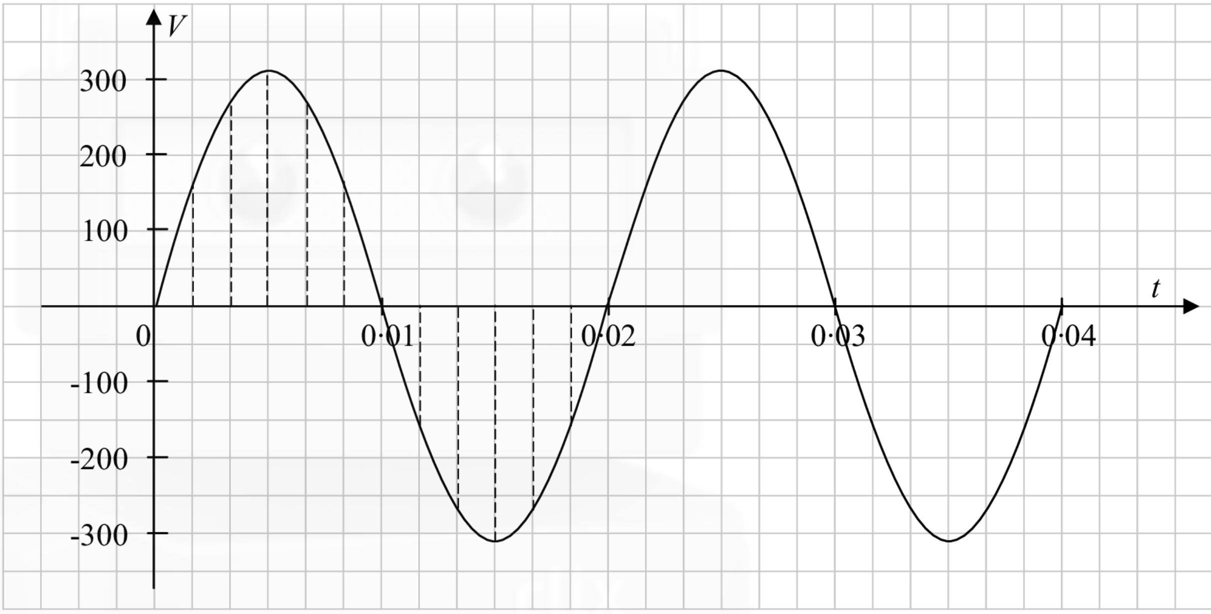
(f) **(i)** Find the rates of change of the function $f(t)$ after 1 minute and after 10 minutes. Give your answers correct to two decimal places.

(ii) Show that the rate of change of $f(t)$ will always increase over time.

The graph below shows the voltage, V, in an electric circuit as a function of time, t. The voltage is given by the formula $V = 311\sin(100\pi t)$, where V is in volts and t is in seconds.



(a) **(i)** Write down the range of the function.

(ii) How many complete periods are there in one second?

(b) **(i)** The table below gives the voltage, correct to the nearest whole number, at equally spaced intervals from t_1 to t_{12} over one complete period (as shown by the dashed lines on the diagram). Use the entries given in the table and the properties of the function to complete the table.

t	t_1	t_2	t_3	t_4	t_5	$t_6 = 0{\cdot}01$	t_7	t_8	t_9	t_{10}	t_{11}	$t_{12} = 0{\cdot}02$
V	156	269	311									

(ii) Using a calculator, or otherwise, calculate the standard deviation, σ, of the twelve values of V in the table, correct to the nearest whole number.

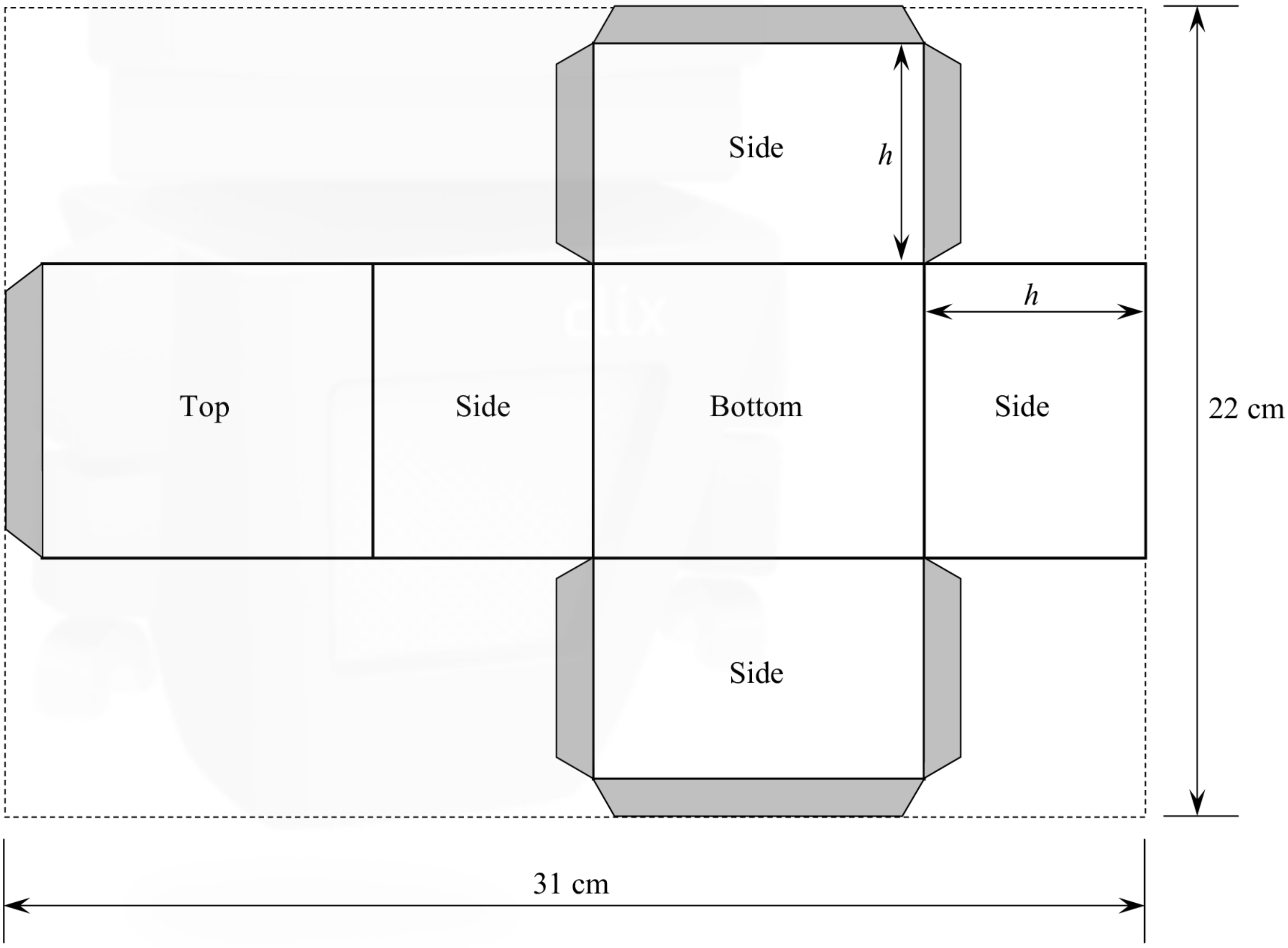
(c) **(i)** The standard deviation, σ, of closely spaced values of any function of the form $V = a\sin(bt)$, over 1 full period, is given by $k\sigma = V_{\max}$, where k is a constant that does not depend on a or b, and $V_{\max}$ is the maximum value of the function. Use the function $V = 311\sin(100\pi t)$ to find an approximate value for k correct to three decimal places.

(ii) Using your answer in part **(c)** **(i)**, or otherwise, find the value of b required so that the function $V = a\sin(bt)$ has 60 complete periods in one second and the approximate value of a so that it has a standard deviation of 110 volts.

Question 7 **(50 marks)**

A company has to design a rectangular box for a new range of jellybeans. The box is to be assembled from a single piece of cardboard, cut from a rectangular sheet measuring 31 cm by 22 cm. The box is to have a capacity (volume) of 500 cm^3.

The net for the box is shown below. The company is going to use the full length and width of the rectangular piece of cardboard. The shaded areas are flaps of width 1 cm which are needed for assembly. The height of the box is *h* cm, as shown on the diagram.



(a) Write the dimensions of the box, in centimetres, in terms of *h*.

height = *h* cm

length = ______________ cm

width = ______________ cm

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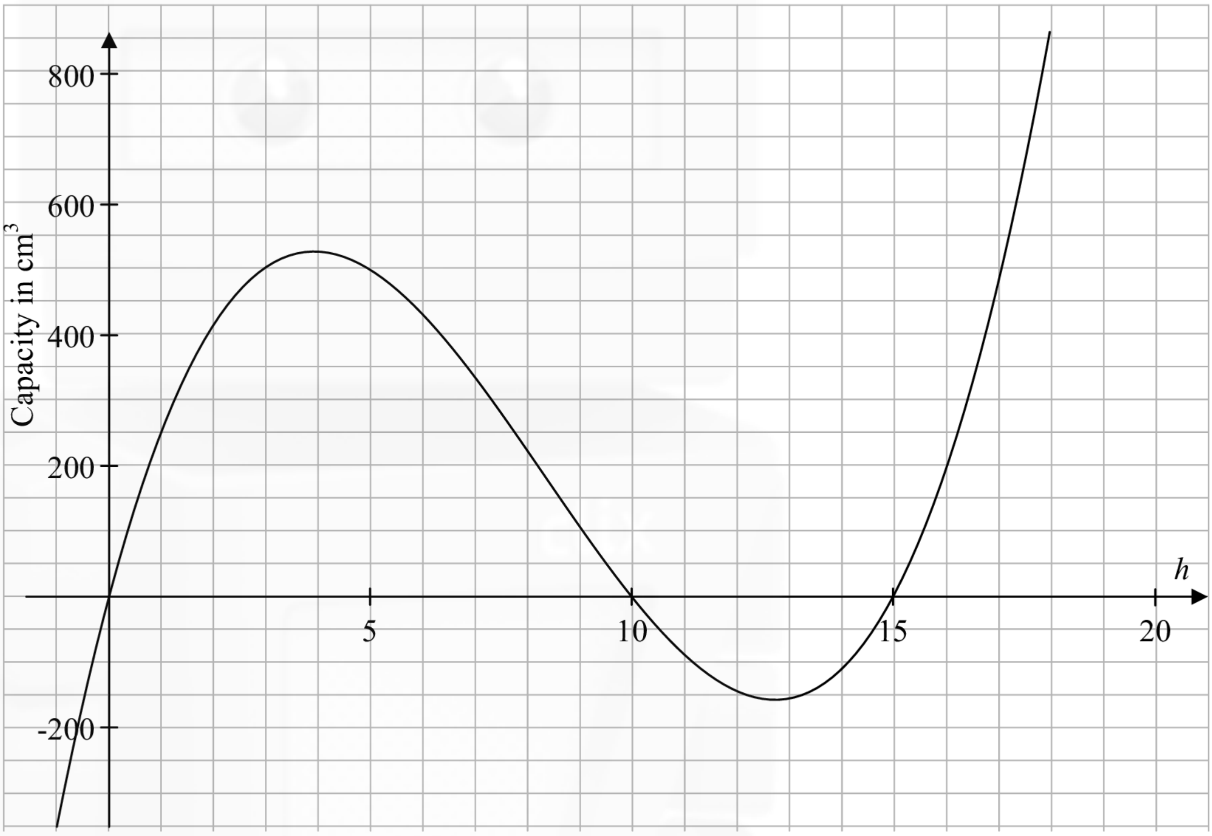
(b) Write an expression for the capacity of the box in cubic centimetres, in terms of h.

(c) Show that the value of h that gives a box with a square bottom will give the correct capacity.

(d) Find, correct to one decimal place, the other value of h that gives a box of the correct capacity.

(e) The client is planning a special "10% extra free" promotion and needs to increase the capacity of the box by 10%. The company is checking whether they can make this new box from a piece of cardboard the same size as the original one (31 cm × 22 cm). They draw the graph below to represent the box's capacity as a function of h. Use the graph to explain why it is *not* possible to make the larger box from such a piece of cardboard.



Explanation:

Question 9 **(50 marks)**

(a) Let $f(x) = -0{\cdot}5x^2 + 5x - 0{\cdot}98$, where $x \in \mathbb{R}$.

(i) Find the value of $f(0{\cdot}2)$.

(ii) Show that f has a local maximum point at $(5, 11{\cdot}52)$.

(b) A sprinter's velocity over the course of a particular 100 metre race is approximated by the following model, where v is the velocity in metres per second, and t is the time in seconds from the starting signal:

$$v(t) = \begin{cases} 0, & \text{for } 0 \le t < 0{\cdot}2 \\ -0{\cdot}5t^2 + 5t - 0{\cdot}98, & \text{for } 0{\cdot}2 \le t < 5 \\ 11{\cdot}52, & \text{for } t \ge 5 \end{cases}$$

Note that the function in part **(a)** is relevant to $v(t)$ above.

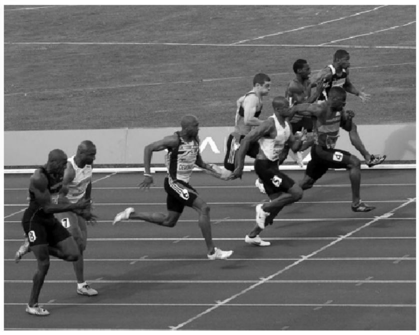
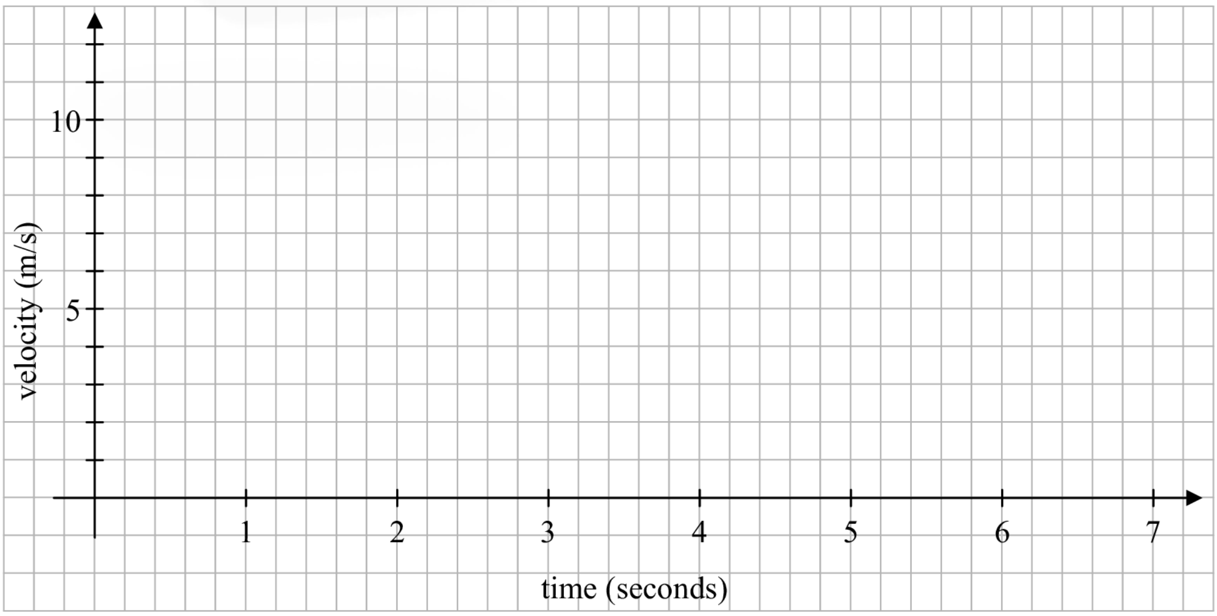


Photo: *William Warby*. Wikimedia Commons. CC BY 2.0

(i) Sketch the graph of v as a function of t for the first 7 seconds of the race.



(ii) Find the distance travelled by the sprinter in the first 5 seconds of the race.

(iii) Find the sprinter's finishing time for the race. Give your answer correct to two decimal places.

(c) A spherical snowball is melting at a rate proportional to its surface area. That is, the rate at which its volume is decreasing at any instant is proportional to its surface area at that instant.

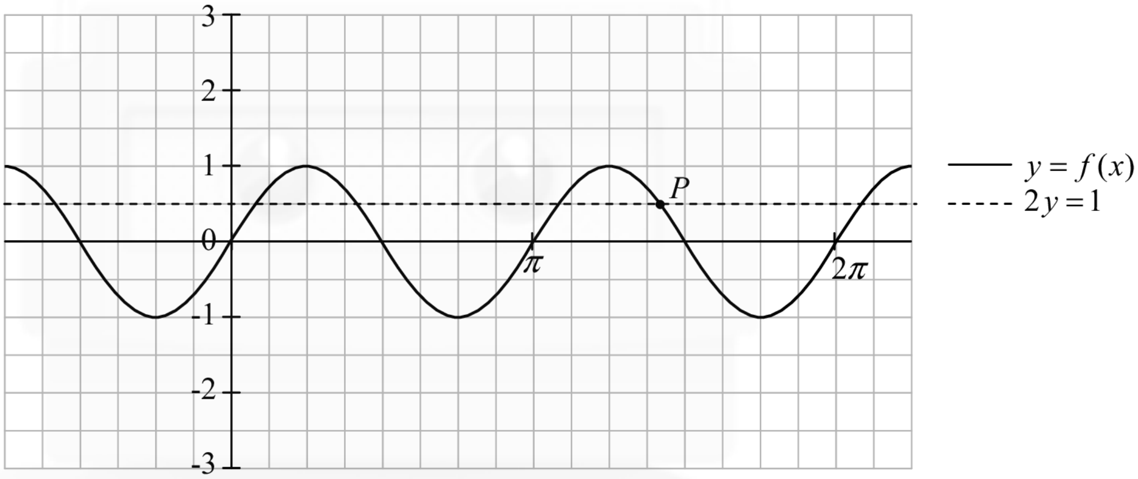
(i) Prove that the radius of the snowball is decreasing at a constant rate.

(ii) If the snowball loses half of its volume in an hour, how long more will it take for it to melt completely?

Give your answer correct to the nearest minute.

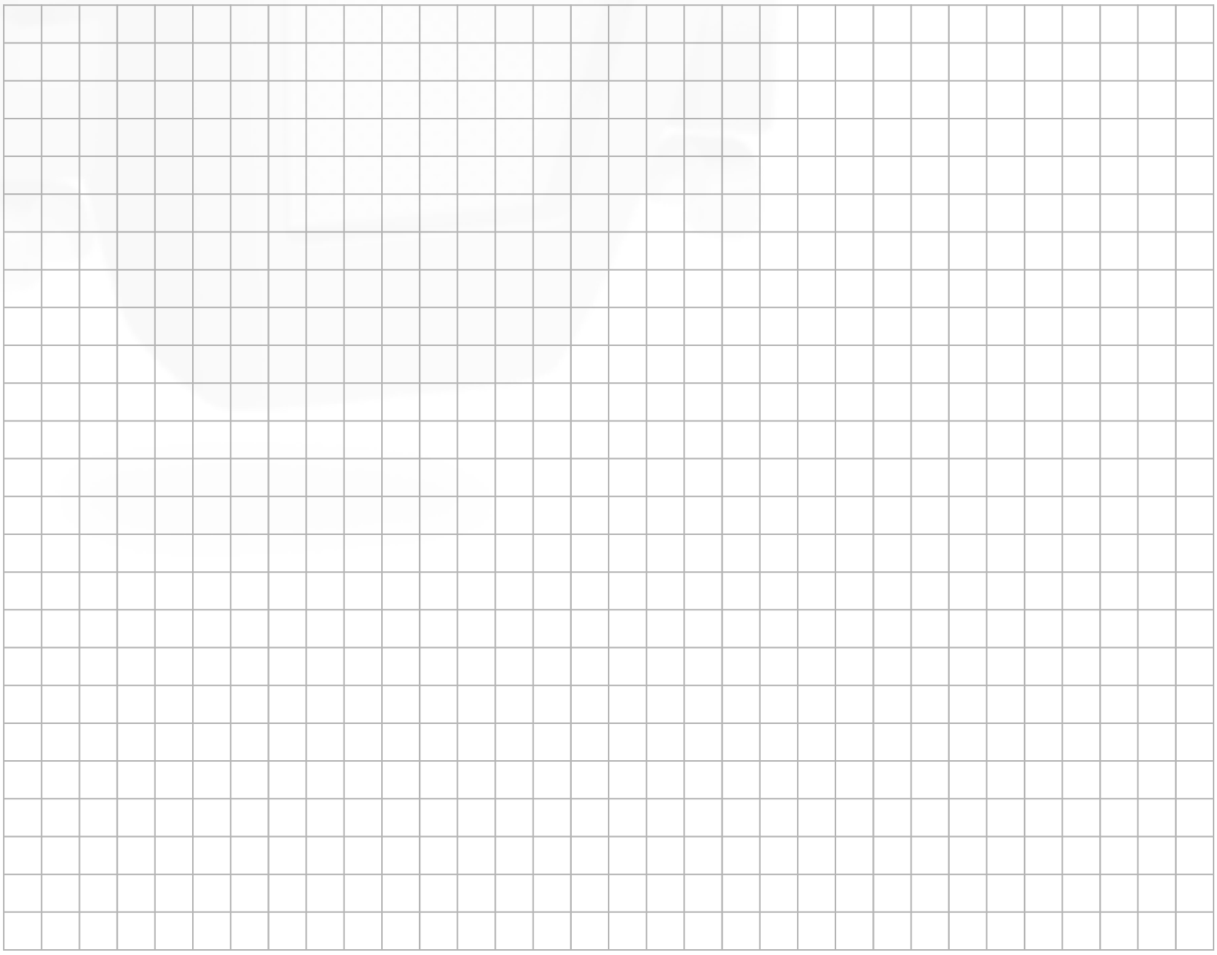
Question 5 **(25 marks)**

The diagram below shows the graph of the function $f : x \mapsto \sin 2x$. The line $2y = 1$ is also shown.



(a) On the same diagram above, sketch the graphs of $g : x \mapsto \sin x$ and $h : x \mapsto 3\sin 2x$. Indicate clearly which is g and which is h.

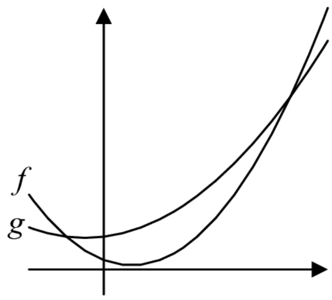
(b) Find the co-ordinates of the point P in the diagram.



Question 5 **(25 marks)**

The functions *f* and *g* are defined for $x \in \mathbb{R}$ as

$$f : x \mapsto 2x^2 - 3x + 2 \quad \text{and}$$
$$g : x \mapsto x^2 + x + 7 .$$

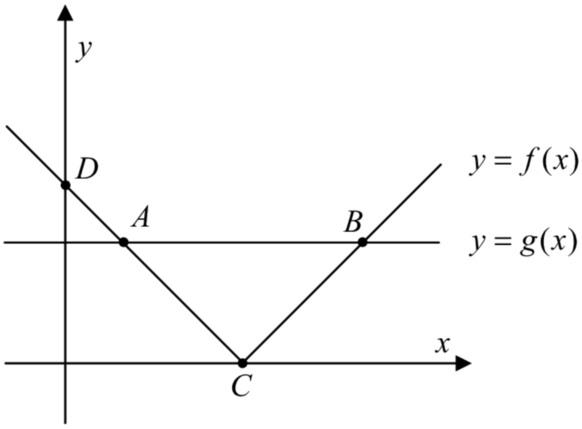


(a) Find the co-ordinates of the two points where the curves $y = f(x)$ and $y = g(x)$ intersect.

(b) Find the area of the region enclosed between the two curves.

(b) The graphs of the functions $f : x \mapsto |x-3|$ and $g : x \mapsto 2$ are shown in the diagram.

(i) Find the co-ordinates of the points A, B, C and D.



$A = (\quad , \quad)$ $B = (\quad , \quad)$

$C = (\quad , \quad)$ $D = (\quad , \quad)$

(ii) Hence, or otherwise, solve the inequality $|x-3| < 2$.

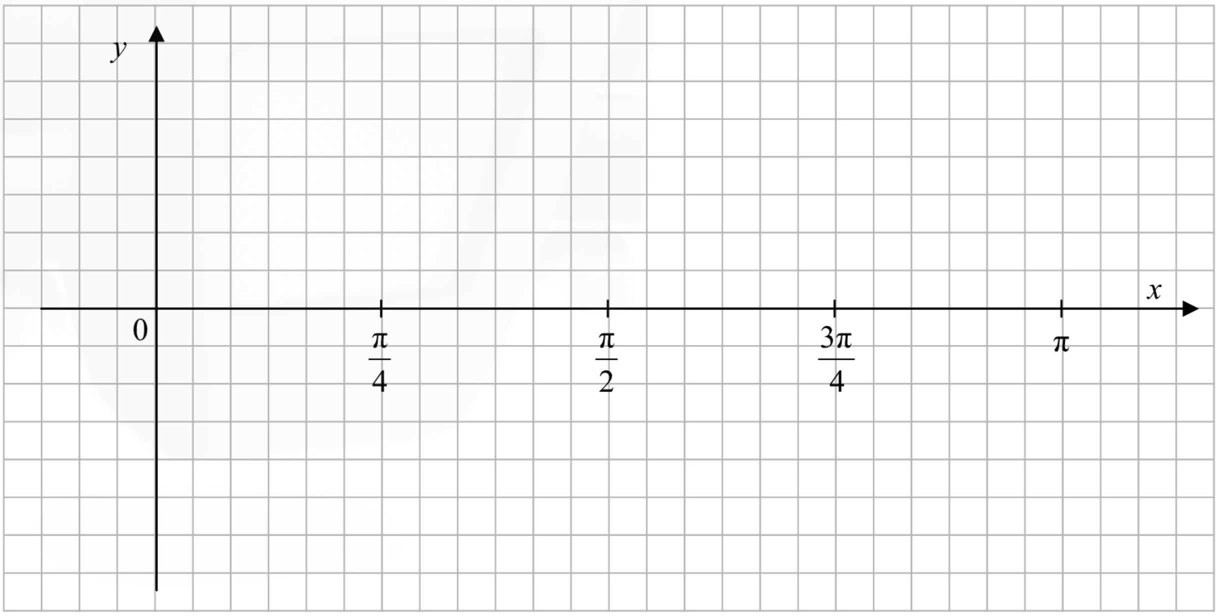
Question 5 **(25 marks)**

The function $f : x \mapsto 3\sin(2x)$ is defined for $x \in \mathbb{R}$.

(a) Complete the table below

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$2x$					
$\sin(2x)$					
$3\sin(2x)$					

(b) Draw the graph of $y = f(x)$ in the domain $0 \le x \le \pi,\ x \in \mathbb{R}$.



(c) Write down the range and the period of *f*.

Range = ________________ Period = ________________

Question 5 **(25 marks)**

The function $f : x \mapsto 3\sin(2x)$ is defined for $x \in \mathbb{R}$.

(a) Complete the table below

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
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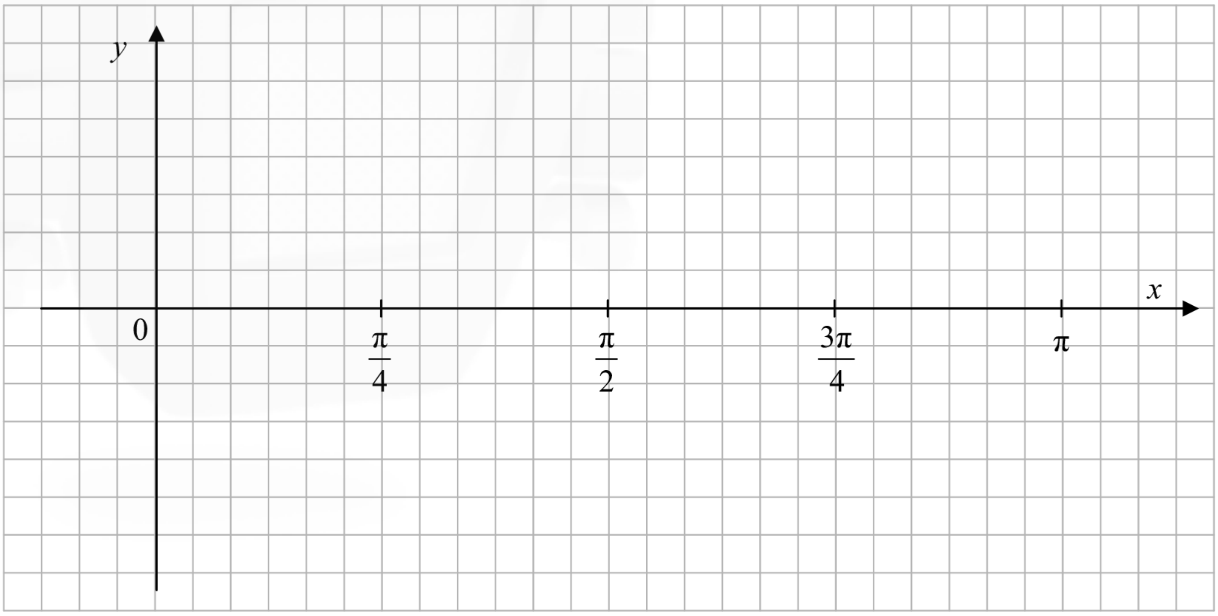
$\sin(2x)$					
$3\sin(2x)$					

(b) Draw the graph of $y = f(x)$ in the domain $0 \le x \le \pi,\ x \in \mathbb{R}$.



(c) Write down the range and the period of *f*.

Range = ________________ Period = ________________