## Question 5

$A B C D$ is a rectangle.
$F \in[A B], \quad G \in[B C], \quad[F D] \cap[A G]=\{E\}$, and $F D \perp A G$.
$|A E|=12 \mathrm{~cm},|E G|=27 \mathrm{~cm}$, and $|F E|=5 \mathrm{~cm}$.
(a) Prove that $\triangle A F E$ and $\triangle D A E$ are similar (equiangular).

(b) Find $|A D|$.

(c) $\triangle A F E$ and $\triangle A G B$ are similar. Show that $|A B|=36 \mathrm{~cm}$.

(d) Find the area of the quadrilateral GCDE.

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The diagram shows a semi-circle standing on a diameter $[A C]$, and $[B D] \perp[A C]$.
(a) (i) Prove that the triangles $A B D$ and $D B C$ are similar.

(ii) If $|A B|=x,|B C|=1$, and $|B D|=y$, write $y$ in terms of $x$.

(b) Use your result from part (a)(ii) to construct a line segment equal in length (in centimetres) to the square root of the length of the line segment [TU] which is drawn below.


## Question 6

(a) Construct the centroid of the triangle $A B C$ below. Show all construction lines. (Where measurement is used, show all relevant measurements and calculations clearly.)

(b) Prove that, if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

## Diagram:

Given:

To Prove:

Construction:


(a) Prove that, if two triangles $\triangle A B C$ and $\triangle A^{\prime} B^{\prime} C^{\prime}$ are similar, then their sides are proportional, in order:

$$
\frac{|A B|}{\left|A^{\prime} B^{\prime}\right|}=\frac{|B C|}{\left|B^{\prime} C^{\prime}\right|}=\frac{|C A|}{\left|C^{\prime} A^{\prime}\right|} .
$$

Diagram:

## Given:

## To Prove:

$\qquad$
Construction:

Proof:
(b) Given the line segment $[B C]$, construct, without using a protractor or set square, a point $A$ such that $|\angle A B C|=60^{\circ}$. Show your construction lines.

$[A B]$ and $[C D]$ are chords of a circle that intersect externally at $E$, as shown.

(a) Name two similar triangles in the diagram above and give reasons for your answer.

(b) Prove that $|E A| .|E B|=|E C| .|E D|$.

(c) Given that $|E B|=6 \cdot 25,|E D|=5 \cdot 94$ and $|C B|=10$, find $|A D|$.


## Question 6A

Explain, with the aid of an example, what is meant by proof by contradiction.
Note: you do not need to provide the full proof involved in your example. Give sufficient outline to illustrate how contradiction is used.

## Explanation:

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## Example:

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(a) Complete each of the following statements.
(i) The circumcentre of a triangle is the point of intersection of $\qquad$
$\qquad$
(ii) The incentre of a triangle is the point of intersection of $\qquad$
$\qquad$
(iii) The centroid of a triangle is the point of intersection of $\qquad$
(b) In an equilateral triangle, the circumcentre, the incentre and the centroid are all in the same place. Explain why this is the case.

(c) Construct the orthocentre of the triangle $A B C$ below. Show all construction lines clearly.

(a) A quadrilateral (four sided figure) has two sides which are parallel and equal in length. Prove that the quadrilateral is a parallelogram.

(b) In the parallelogram $A B C D$, $D E$ is perpendicular to $A C$. $B F$ is perpendicular to $A C$. Prove that $E B F D$ is a parallelogram.


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## Question 6

Answer either 6A or 6B.

## Question 6A

Explain, with the aid of an example, what is meant by proof by contradiction.
Note: you do not need to provide the full proof involved in your example. Give sufficient outline to illustrate how contradiction is used.

## Explanation:

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## Example:

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## OR

## Question 6B

$A B C$ is a triangle.
$D$ is the point on $B C$ such that $A D \perp B C$. $E$ is the point on $A C$ such that $B E \perp A C$. $A D$ and $B E$ intersect at $O$.

Prove that $|\angle D O C|=|\angle D E C|$.


Prove that if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

## Diagram:

Given:

To prove:
Construction:

## Proof:

## Question 6B

In the diagram, $P_{1} Q_{1}, P_{2} Q_{2}$, and $P_{3} Q_{3}$ are parallel and so also are $Q_{1} P_{2}$ and $Q_{2} P_{3}$.

Prove that $\left|P_{1} Q_{1}\right| \times\left|P_{3} Q_{3}\right|=\left|P_{2} Q_{2}\right|^{2}$.


(a) Prove that if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

## Diagram:

Given: $\qquad$

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| Proof: |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Proof:

