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Question 1 (2017)

(a)	Proof: $\lvert\angle AEF\rvert = \lvert\angle AED\rvert$ ... *right angles* $\lvert\angle FAE\rvert + \lvert\angle EAD\rvert = 90^o$ $\lvert\angle EAD\rvert + \lvert\angle ADE\rvert = 90^o$ *remaining angles in* ΔAED $\therefore \lvert\angle FAE\rvert = \lvert\angle ADE\rvert$ ***or*** $\therefore \lvert\angle AFE\rvert = \lvert\angle DAE\rvert$ $\therefore \Delta AFE$ *and* ΔDAE *equiangular* $\therefore$ *similar*	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • Identifies one angle of same size in each triangle *High Partial Credit:* • Identifies second angle of same size in each triangle • Implies triangles are similar without justifying $\lvert\angle FAE\rvert = \lvert\angle ADE\rvert$
(b)	$\frac{\lvert AD\rvert}{13} = \frac{12}{5}$ $\lvert AD\rvert = 31{\cdot}2$ cm	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • $\lvert AF\rvert = 13$ • One set of corresponding sides identified, e.g . $\frac{\lvert AD\rvert}{13}$ **or** $\frac{12}{5}$ *High Partial Credit:* • $\frac{\lvert AD\rvert}{13} = \frac{12}{5}$ or equivalent
(c)	$\frac{39}{13} = \frac{\lvert AB\rvert}{12}$ $\lvert AB\rvert = 3 \times 12 = 36$ cm	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • $\lvert AG\rvert = 39$ • One set of corresponding sides identified *High Partial Credit:* • $\frac{39}{13} = \frac{\lvert AB\rvert}{12}$ or equivalent

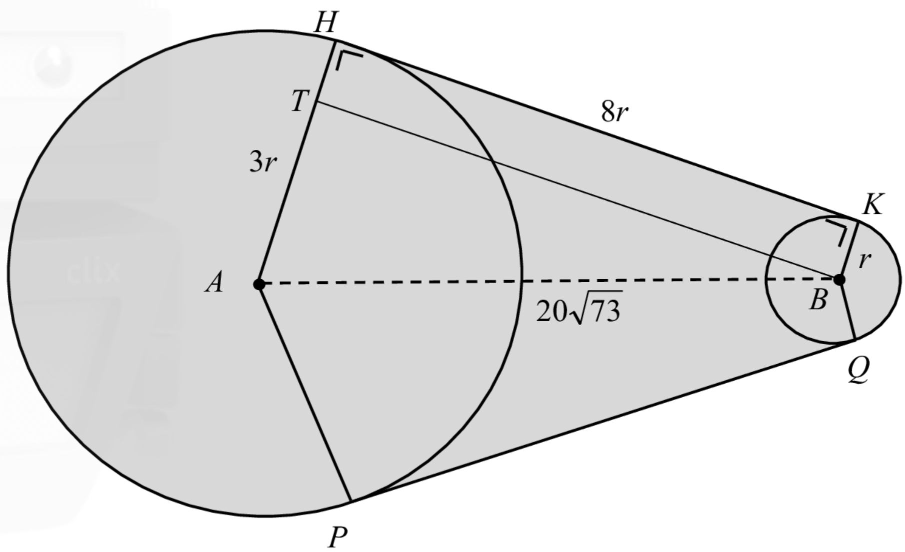
(d)	Area = Area$ABCD$ − AreaΔAFD $-\Delta$AreaABG + AreaΔAFE $= (31{\cdot}2)(36) - \frac{1}{2}(31{\cdot}2)(13)$ $-\frac{1}{2}(36)(15) + \frac{1}{2}(5)(12)$ $= 680{\cdot}4$ cm^2 **or** (method 2) Area = Area$ABCD$ − AreaΔABG − AreaΔAED $= (31{\cdot}2)(36) - \frac{1}{2}(36)(15)$ $-\frac{1}{2}(12)\sqrt{31{\cdot}2^2 - 12^2}$ $= 1123{\cdot}2 - 270 - 172{\cdot}8$ $= 680{\cdot}4$ cm^2 **or** (method 3) Area = AreaΔDCG + AreaΔGED $= \frac{1}{2}(36)(16{\cdot}2) + \frac{1}{2}(27)\sqrt{31{\cdot}2^2 - 12^2}$ $= 291{\cdot}6 + 388{\cdot}8$ $= 680{\cdot}4$ cm^2	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • One relevant area formulated • Relevant equation for area *GCDE* *High Partial Credit:* • Relevant individual areas found but fails to finish • Area calculated but with one relevant area omitted (except method 3)

Q7	Model Solution – 55 Marks	Marking Notes
(a) (i)	$\|EC\|^2 = 3^2 + 2{\cdot}5^2 = 15{\cdot}25$ $\|EC\| = \sqrt{15{\cdot}25}$ $\|EC\| = 3{\cdot}905$ $\Rightarrow \|AC\| = 1{\cdot}9525$ $= 1{\cdot}95$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • Pythagoras with relevant substitution *High Partial Credit* • $\|EC\|$ correct • $\|AC\| = \frac{1}{2}\sqrt{15{\cdot}25}$
(a) (ii)	$\tan 50° = \frac{\|AB\|}{1{\cdot}95}$ $\|AB\| = 1{\cdot}95(1{\cdot}19175) = 2{\cdot}23239$ $\|AB\| = 2{\cdot}3$	Scale 10B (0, 5, 10) *Partial Credit* • tan formulated correctly
(a) (iii)	$\|BC\|^2 = 1{\cdot}95^2 + 2{\cdot}3^2$ $\|BC\| = 3 \cdot 015377$ $\|BC\| = 3$ Also: $\sin 40° = \frac{1{\cdot}95}{\|BC\|}$ **or** $\cos 40° = \frac{2{\cdot}3}{\|BC\|}$ **or** $\cos 50° = \frac{1{\cdot}95}{\|BC\|}$ **or** $\sin 50° = \frac{2{\cdot}3}{\|BC\|}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • Pythagoras with relevant substitution *High Partial Credit* • Pythagoras fully substituted • $\|BC\| = \frac{1{\cdot}95}{\sin 40°}$ (i.e. $\|BC\|$isolated)
(a) (iv)	$3^2 = 3^2 + 2{\cdot}5^2 - 2(3)(2{\cdot}5)\cos \propto$ $15\cos \propto = 6{\cdot}25$ $\propto = 65°$ **or** $\cos \propto = \frac{1{\cdot}25}{3}$ $\propto = 65°$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • cosine rule with some relevant substitution • cosine ratio with some relevant substitutions • identifies three sides of triangle *BCD* *High Partial Credit* • cosine rule with full relevant substitutions • cosine ratio with full relevant substitutions

(a) **(v)**	$A = 2\times$ isosceles triangle $+2\times$ equilateral triangle $= 2\times\left[\frac{1}{2}(2\cdot5)(3)\sin 65°\right] +$ $2\times\left[\frac{1}{2}(3)(3)\sin 60°\right]$ $= 14\cdot59$ A=15	Scale 10D (0,3,5,8,10) *Low Partial Credit* • recognises area of 4 triangles *Mid Partial Credit* • Area of 1 triangle correct *High Partial Credit* • area of isosceles triangle and equilateral triangle **Note**: Area = 4 isosceles or 4 equilateral triangles merit *HPC* at most
(b)	$\tan 60° = \frac{3}{\lvert CA\rvert}$ $\Rightarrow \lvert CA\rvert = \sqrt{3}$ $\lvert CE\rvert = 2\sqrt{3}$ $x^2 + x^2 = (2\sqrt{3})^2$ $x = \sqrt{6}$	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • effort at Pythagoras but without $\lvert CA\rvert$ (or $\lvert CE\rvert$) • $\lvert CA\rvert$ found *High Partial Credit* • $\lvert CE\rvert = 2\sqrt{3}$

Question 3 (2015)

(a) Find r, the radius of the smaller circle. (Hint: Draw $BT \parallel KH$, $T \in AH$.)



$$|AT|^2 + |BT|^2 = |AB|^2 \Rightarrow (3r)^2 + (8r)^2 = (20\sqrt{73})^2$$

$$\Rightarrow 9r^2 + 64r^2 = 29200$$

$$\Rightarrow r^2 = 400 \Rightarrow r = 20 \text{ cm}$$

(b) Find the area of the quadrilateral *ABKH*.

$$|ABKH| = |BKHT| + |\Delta ABT|$$
$$= 20\times 160 + \tfrac{1}{2}(60)(160)$$
$$= 8000 \text{ cm}^2$$

(c) **(i)** Find $|\angle HAP|$, in degrees, correct to one decimal place.

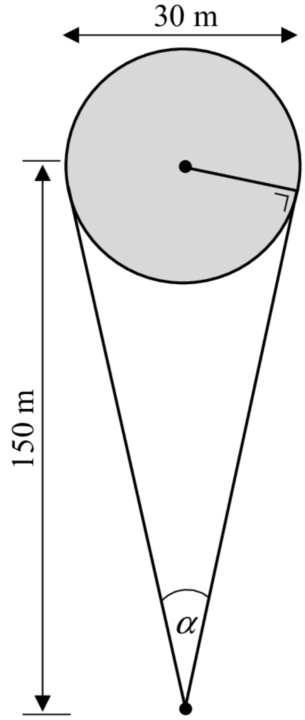
$$\tan|\angle HAB| = \frac{160}{60} \Rightarrow |\angle HAB| = 69\cdot 44°$$
$$\Rightarrow |\angle HAP| = 138\cdot 9°$$

(ii) Find the area of the machine part, correct to the nearest cm^2.

Area large sector *HAP* + 2 area *HABK* + area sector *KBQ*

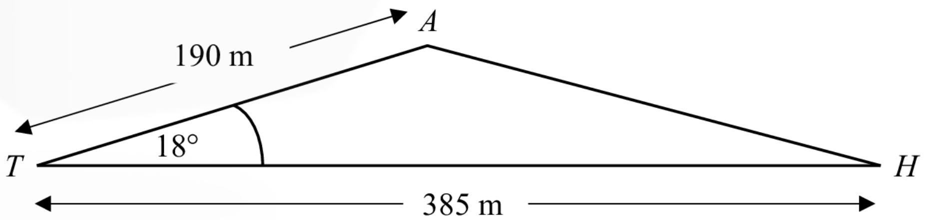
$$= \pi(80)^2\left(\frac{221\cdot 1}{360}\right) + 2\times 8000 + \pi(20)^2\left(\frac{138\cdot 9}{360}\right)$$
$$= 12348\cdot 55 + 16000 + 484\cdot 85$$
$$= 28833\cdot 4$$
$$= 28833$$

(a) Joan is playing golf. She is 150 m from the centre of a circular green of diameter 30 m. The diagram shows the range of directions in which Joan can hit the ball so that it could land on the green. Find α, the measure of the angle of this range of directions. Give your answer, in degrees, correct to one decimal place.



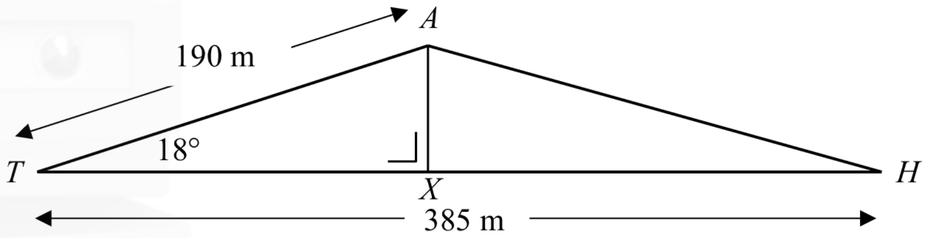
$$\sin \tfrac{1}{2}\alpha = \frac{15}{150} = 0 \cdot 1$$
$$\Rightarrow \tfrac{1}{2}\alpha = 5 \cdot 739°$$
$$\Rightarrow \alpha = 11 \cdot 478°$$
$$\alpha = 11 \cdot 5°$$

(b) At the next hole, Joan, at T, attempts to hit the ball in the direction of the hole H. Her shot is off target and the ball lands at A, a distance of 190 metres from T, where $|\angle ATH| = 18°$. $|TH|$ is 385 metres. Find $|AH|$, the distance from the ball to the hole, correct to the nearest metre.



$$|AH|^2 = 190^2 + 385^2 - 2(190)(385)\cos 18°$$
$$= 36100 + 148225 - 139139 \cdot 5683$$
$$= 45185 \cdot 4317$$
$$|AH| = 212 \cdot 57 = 213$$

or



Draw AX perpendicular to TH

triangle ATX: $\sin 18° = \frac{|AX|}{190} \Rightarrow |AX| = 58\cdot71$

$$\cos 18° = \frac{|TX|}{190} \Rightarrow |TX| = 180\cdot7$$

$$\Rightarrow |XH| = 204\cdot3$$

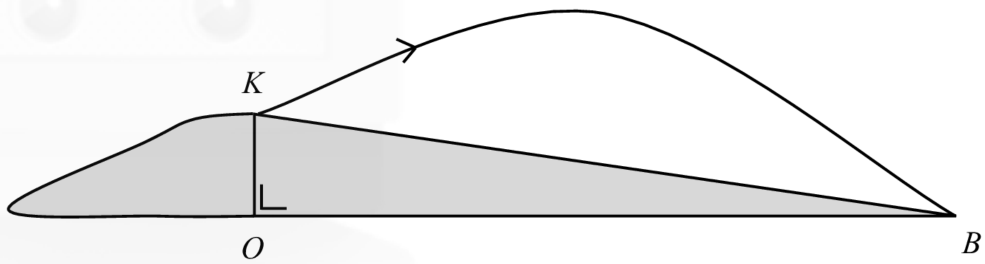
$$\Rightarrow |AH|^2 = (58\cdot71)^2 + (204\cdot3)^2$$

$$\Rightarrow |AH| = 212\cdot566 = 213$$

(c) At another hole, where the ground is not level, Joan hits the ball from *K*, as shown. The ball lands at *B*. The height of the ball, in metres, above the horizontal line *OB* is given by

$$h = -6t^2 + 22t + 8$$

where *t* is the time in seconds after the ball is struck and *h* is the height of the ball.



(i) Find the height of *K* above *OB*.

$h = -6t^2 + 22t + 8$

$t = 0 \Rightarrow h = 8 \text{ m}$

(ii) The horizontal speed of the ball over the straight distance [*OB*] is a constant 38 m s^{-1}. Find the angle of elevation of *K* from *B*, correct to the nearest degree.

$h = 0 \Rightarrow -6t^2 + 22t + 8 = 0$

$\Rightarrow (t-4)(-6t-2) = 0$

$\Rightarrow t = 4, \quad t = -\frac{1}{3}$

$t = 4 \Rightarrow |OB| = 38 \times 4 = 152 \text{ m}$

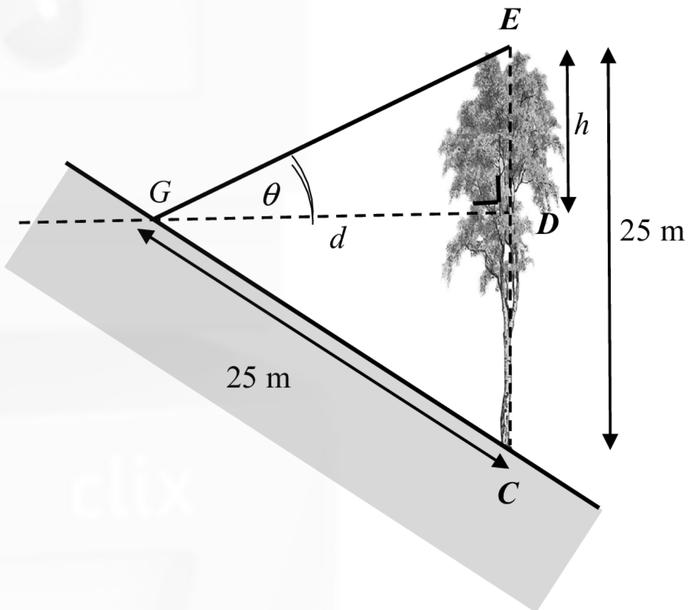
$\tan|\angle OBK| = \dfrac{8}{152} = \dfrac{1}{19} \quad \Rightarrow \quad |\angle OBK| = 3\cdot 01° = 3°$

(d) At a later hole, Joan's first shot lands at the point G, on ground that is sloping downwards, as shown. A vertical tree, $[CE]$, 25 metres high, stands between G and the hole. The distance, $|GC|$, from the ball to the bottom of the tree is also 25 metres.

The angle of elevation at G to the top of the tree, E, is θ, where $\theta = \tan^{-1}\frac{1}{2}$.

The height of the top of the tree above the horizontal, GD, is h metres and $|GD| = d$ metres.

(i) Write d and $|CD|$ in terms of h.



$$\tan\theta = \frac{h}{d} = \frac{1}{2}$$
$$\Rightarrow d = 2h$$

$$|CD| = 25 - h$$

(ii) Hence, or otherwise, find h.

$$d^2 + |CD|^2 = 25^2$$
$$(2h)^2 + (25-h)^2 = 25^2$$
$$4h^2 + 625 - 50h + h^2 = 625$$
$$5h^2 - 50h = 0$$
$$h = 0, \quad h = 10$$
$$h = 10 \text{ m}$$

or

$$\theta = \tan^{-1}\tfrac{1}{2} = 26\cdot565°$$
$$\Rightarrow \quad |GED| = 63\cdot435°$$
$$\Rightarrow \quad |CGE| = 63\cdot435°$$
$$\Rightarrow \quad |CGD| = 63\cdot435° - 26\cdot565° = 36\cdot87°$$
$$\sin 36\cdot87 = \frac{25-h}{25} = 0\cdot6$$
$$\Rightarrow 25 - h = 15$$
$$\Rightarrow h = 10 \text{ m}$$

or

$$|\angle GCE = 53\cdot14°| \Rightarrow \sin 53\cdot14° = \frac{2h}{25}$$
$$\Rightarrow 0\cdot8 = \frac{2h}{25} \Rightarrow h = 10 \text{ m}$$

(i) Let $a = 2n+1$, $b = 2n^2 + 2n$ and $c = 2n^2 + 2n + 1$.
Pick one natural number n and verify that the corresponding values of a, b and c form a Pythagorean triple.

Let $n = 1$:

$a = 2n+1 \Rightarrow a = 2(1)+1 = 3$

$b = 2n^2 + 2n \Rightarrow b = 2(1)^2 + 2(1) = 4$

$c = 2n^2 + 2n + 1 \Rightarrow c = 2(1)^2 + 2(1) + 1 = 5$

$3^2 + 4^2 = 5^2 \Rightarrow a^2 + b^2 = c^2$

(ii) Prove that $a = 2n+1$, $b = 2n^2 + 2n$ and $c = 2n^2 + 2n + 1$, where $n \in \mathbb{N}$, will always form a Pythagorean triple.

$a^2 = (2n+1)^2 = 4n^2 + 4n + 1$

$b^2 = (2n^2 + 2n)^2 = 4n^4 + 8n^3 + 4n^2$

$a^2 + b^2 = 4n^4 + 8n^3 + 8n^2 + 4n + 1$

$$\begin{aligned} c^2 &= (2n^2 + 2n + 1)^2 \\ &= 4n^4 + 8n^3 + 8n^2 + 4n + 1 \\ &= a^2 + b^2 \end{aligned}$$

(i) Let $f(x) = |PA|^2 + |PB|^2 + |PC|^2$.
Show that $f(x) = 3x^2 - 24x + 86$, for $0 \le x \le 7$, $x \in \mathbb{R}$.

$$\begin{aligned} |PM| &= |PE| - |ME| \\ &= (7-x) - 2 \\ &= (5-x) \end{aligned}$$

$$\begin{aligned} f(x) &= |PA|^2 + |PB|^2 + |PC|^2 \\ &= \left[|PD|^2 + |DA|^2\right] + \left[|PM|^2 + |MB|^2\right] + \left[|PE|^2 + |EC|^2\right] \\ &= x^2 + 2^2 + \left((5-x)^2 + 2^2\right) + \left((7-x)^2 + 2^2\right) \\ &= x^2 + 4 + 25 - 10x + x^2 + 4 + 49 - 14x + x^2 + 4 \\ &= 3x^2 - 24x + 86 \end{aligned}$$

(ii) The function $f(x)$ has a minimum value at $x = k$.
Find the value of k and the minimum value of $f(x)$.

$$f(x) = 3x^2 - 24x + 86$$
$$f'(x) = 6x - 24$$
$$f''(x) = 6 > 0 \quad \Rightarrow \text{ minimum}$$
$$f'(x) = 0 \Rightarrow 6x - 24 = 0 \Rightarrow x = 4 = k$$
$$f(4) = 3(4)^2 - 24(4) + 86 = 38$$

OR

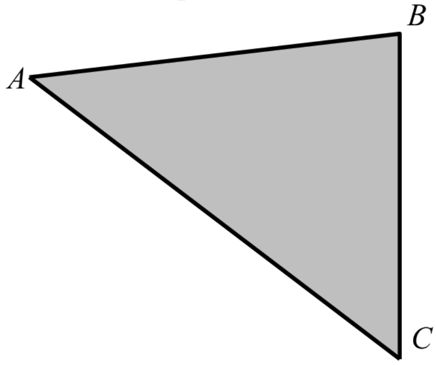
$$\begin{aligned} f(x) &= 3x^2 - 24x + 86 \\ &= 3\left(x^2 - 8x + \frac{86}{3}\right) \\ &= 3\left[\left(x^2 - 8x + 16\right) + \frac{38}{3}\right] \\ &= 3\left[\left(x-4\right)^2 + \frac{38}{3}\right] \end{aligned}$$

At $x = 4 \Rightarrow$ minimum value for $f(x)$

$$\begin{aligned} f(4) &= 3x^2 - 24x + 86 \\ &= 3(4)^2 - 24(4) + 86 \\ &= 48 - 96 + 86 \\ &= 38 \end{aligned}$$

Question 6 (2014)

(a) **(i)** Find $|\angle CBA|$. Give your answer, in degrees, correct to two decimal places.



$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$= \frac{120^2 + 134^2 - 150^2}{2(120)(134)}$$
$$= \frac{9856}{32160}$$
$$= 0 \cdot 306468$$
$$\Rightarrow B = 72 \cdot 15^\circ$$

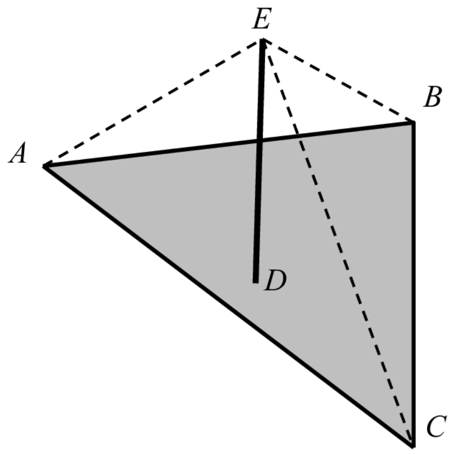
(ii) Find the area of the triangle *ACB* correct to the nearest whole number.

$$\text{Area } \Delta ABC = \tfrac{1}{2} ac \sin B = \tfrac{1}{2}(120)(134)\sin 72 \cdot 15$$
$$= 7652 \cdot 97$$
$$\approx 7653 \text{ m}^2$$

Or

$$\text{Area } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{a+b+c}{2}$$
$$= \sqrt{202(202-134)(202-150)(202-120)}$$
$$= \sqrt{58570304} = 7653 \cdot 12$$
$$\approx 7653 \text{ m}^2$$

(b) A vertical mast, $[DE]$, is fixed at the circumcentre, D, of the triangle. The mast is held in place by three taut cables $[EA]$, $[EB]$ and $[EC]$. Explain why the three cables are equal in length.



Circumcentre at $D \Rightarrow |AD| = |BD| = |CD|$

Each of the triangles *EAD*, *EBD*, *ECD* is right-angled at *D* and has the two sides, the base and the perpendicular, equal.
Hence, by theorem of Pythagoras, the third side of each, the hypotenuse (the cables), must be equal.

Question 7 (2014)

Consider the quadrilateral $DOEC$. We have

$$|\angle CDO| + |\angle OEC| = 90^\circ + 90^\circ = 180^\circ.$$

Therefore $DOEC$ is a cyclic quadrilateral (by the converse of Corollary 5).
Therefore $\angle DOC$ and $\angle DEC$ are angles standing on the same arc of a circle (the circumcircle of $DOEC$).
Therefore, by Theorem 19

$$|\angle DOC| = |\angle DEC|$$

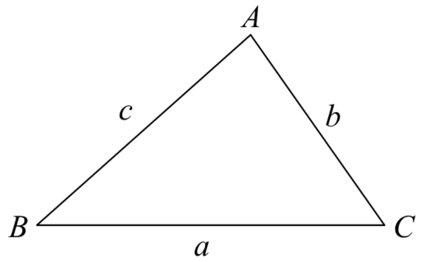
as required.

Question 8 (2013)

$$\frac{1}{2}ac\sin\angle B = \frac{1}{2}ab\sin\angle C$$

Divide by $\frac{1}{2}abc$

$$\frac{\sin\angle B}{b} = \frac{\sin\angle C}{c} \Rightarrow \frac{b}{\sin\angle B} = \frac{c}{\sin\angle C}$$



OR

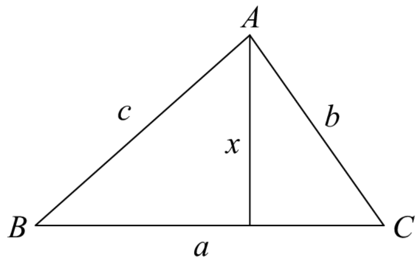
Case 1

$$\sin\angle B = \frac{x}{c} \qquad \sin\angle C = \frac{x}{b}$$

$$x = c\sin\angle B \qquad x = b\sin\angle C$$

$$b\sin\angle C = c\sin\angle B$$

$$\frac{b}{\sin\angle B} = \frac{c}{\sin\angle C}$$



Case 2

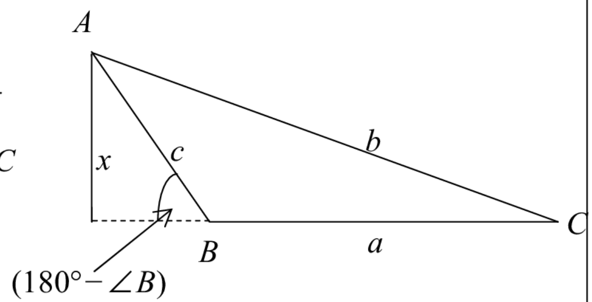
$$\sin(180^\circ - \angle B) = \frac{x}{c} \qquad \sin\angle C = \frac{x}{b}$$

$$x = c\sin(180^\circ - \angle B) \qquad x = b\sin\angle C$$

$$x = c\sin\angle B$$

$$b\sin\angle C = c\sin\angle B$$

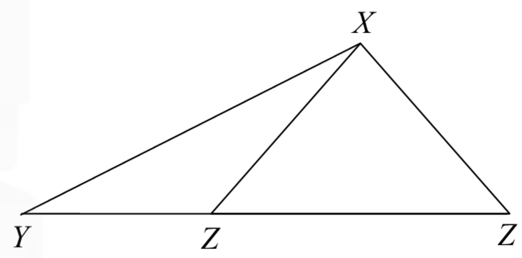
$$\frac{b}{\sin\angle B} = \frac{c}{\sin\angle C}$$



$$\frac{3}{\sin 27^\circ} = \frac{5}{\sin \angle Z} \quad \Rightarrow \quad \sin \angle Z = \frac{5\sin 27^\circ}{3} = 0\cdot 756$$

$$\Rightarrow \quad |\angle Z| = 49^\circ \quad \text{or} \quad |\angle Z| = 131^\circ$$

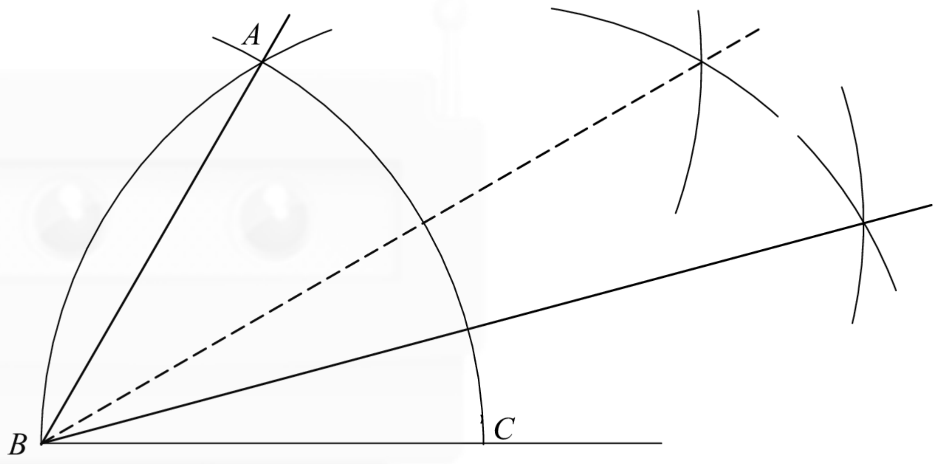
(ii) Draw a sketch of the triangle *XYZ*, showing the two possible positions of the point *Z*.



(c) In the case that $|\angle XZY| < 90^\circ$, write down $|\angle ZXY|$, and hence find the area of the triangle *XYZ*, correct to the nearest integer.

$$|\angle ZXY| = 180^\circ - (27^\circ + 49^\circ) = 104^\circ$$

$$\Delta = \tfrac{1}{2}ab\sin C = \tfrac{1}{2}(5)(3)\sin 104^\circ = 7\cdot 27 = 7\text{ cm}^2$$

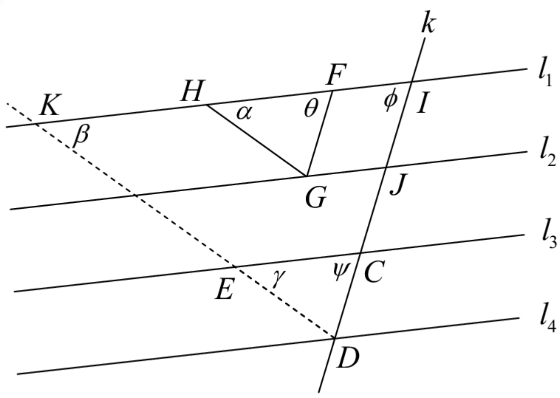


(ii) Hence construct, on the same diagram above, and using a compass and straight edge only, an angle of 15°.

Bisect 60° to get 30°; bisect again to get 15° (as shown above) OR Construct a right angle and use it to construct 45° and combine with 60° to get 15°.

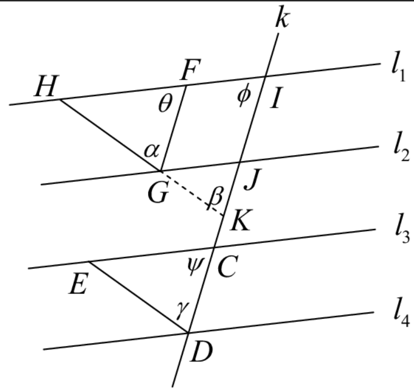
(b) In the diagram, $l_1, l_2, l_3,$ and l_4 are parallel lines that make intercepts of equal length on the transversal k. FG is parallel to k, and HG is parallel to ED.

Prove that the triangles ΔCDE and ΔFGH are congruent.



$\lvert CD\rvert = \lvert IJ\rvert$	(given)
$= \lvert FG\rvert$	(opposite sides of parallelogram
$\theta = \phi = \psi$	(corresponding angles)
$\alpha = \beta = \gamma$	(corresponding angles)
$\Rightarrow \lvert\angle HGF\rvert = \lvert\angle EDC\rvert$	
$\therefore \Delta CDE \equiv \Delta FGH$	(ASA)

OR



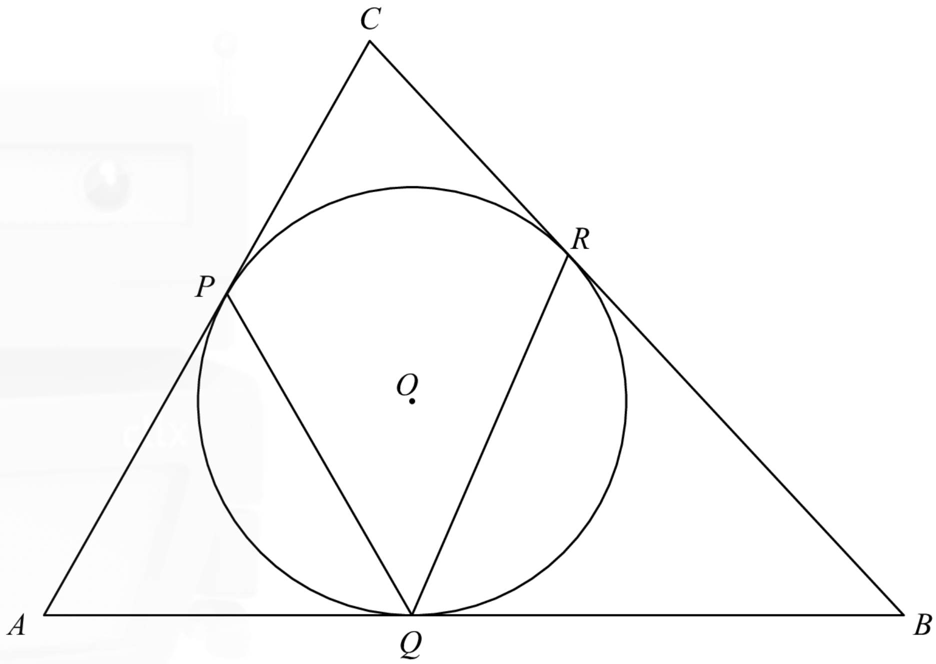
$|CD| = |IJ|$ (given)

$= |FG|$ (opposite sides of parallelogram

$\theta = \phi = \psi$ (corresponding angles)

$\alpha = \beta = \gamma$ (corresponding angles)

$\therefore \Delta CDE \equiv \Delta FGH$ (ASA)



$|\angle OQA| = |\angle OPA| = 90°$ (radius $\perp$ tangent)

$\therefore$ O, Q, A, P are concyclic.

$|\angle OQP| = |\angle OAP|$ (standing on same arc OP)

$= \frac{1}{2}|\angle PAQ|$ (since [AO is the bisector of $\angle PAQ$)

Similarly, $|\angle OQR| = \frac{1}{2}|\angle QBR|$

Adding these two gives the required result.

OR

$|\angle OPC| = |\angle ORC| = 90°$ (radius ⊥ tangent)

$\therefore |\angle PBR| = 180° - |\angle POR|$ (angles in any quadrilateral add up to 360°)

But $|\angle PBR| = 180° - (|\angle CAB| + |\angle CBA|)$ (angles in a triangle)

So $|\angle POR| = |\angle CAB| + |\angle CBA|$

But $|\angle PQR| = \frac{1}{2}|\angle POR|$

So $|\angle PQR| = \frac{1}{2}(|\angle CAB| + |\angle CBA|)$

OR

Let $OA \cap PQ = \{D\}$

$|OP| = |OQ| \Rightarrow |AP| = |AQ|$ (Pythagoras)

$|\angle PAD| = |\angle QAD|$ (bisector)

$\therefore \Delta PDA \equiv \Delta QDA$ (S.A.S.)

$\therefore |\angle PDA| = |\angle QDA| = 90°$

$|\angle DAQ| = 90° - |\angle DQA|$

$= |\angle OQD|$

$\therefore |\angle PAQ| = 2|\angle OQD|$

Similarly, $|\angle RBQ| = 2|OQR|$

Adding these two gives the required result.

OR

Let $OA \cap PQ = \{D\}$

$|OP| = |OQ| \Rightarrow |AP| = |AQ|$ (Pythagoras)

$|\angle APQ| = |\angle AQP|$ (isosceles triangle theorem)

Similarly, $|\angle RQB| = |\angle RBQ|$

$|\angle AQP| + |\angle PQR| + |\angle RQB| = 180^0$

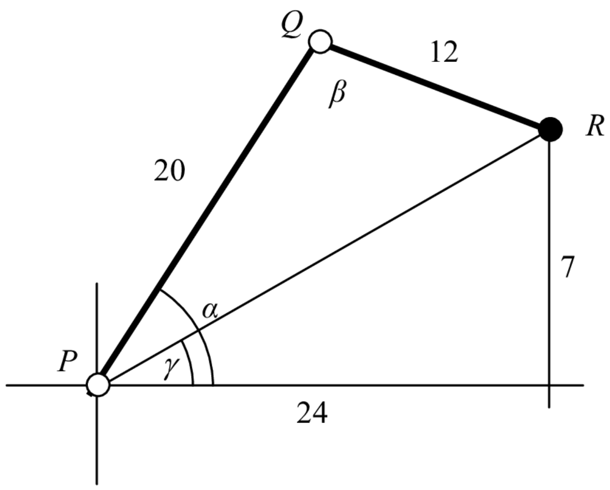
$|\angle PQR| = 180^0 - |\angle AQP| - |\angle RQB|$

$|\angle CAB| = 180^0 - 2|\angle AQP|$

$|\angle CBA| = 180^0 - 2|\angle RQB|$

$\Rightarrow |\angle CAB| + |\angle CBA| = 360^0 - 2\left[|\angle AQP| + |\angle RQB|\right]$

$\Rightarrow \frac{1}{2}\left[|\angle CAB| + |\angle CBA|\right] = 180^0 - |\angle AQP| - |\angle RQB| = |\angle PQR|$



$$|PR|^2 = 7^2 + 24^2$$
$$|PR| = 25$$

$$25^2 = 20^2 + 12^2 - 2(20)(12)\cos\beta$$
$$\cos\beta = -0{\cdot}16875$$
$$\beta \approx 100°$$

$$12^2 = 25^2 + 20^2 - 2(25)(20)\cos(\alpha-\gamma)$$
$$\cos(\alpha-\gamma) = 0{\cdot}881$$
$$\alpha-\gamma \approx 28{\cdot}237°$$

$$\tan\gamma = \frac{7}{24}$$
$$\gamma \approx 16{\cdot}260°$$

$$\therefore \alpha \approx 44°$$

Ans: α

Reason: 1° error in α causes R to move along an arc of radius 25.
1° error in β causes R to move along an arc of radius 12.

So, since $l = r\theta$, and θ is the same in each case, the point moves further in the first case.

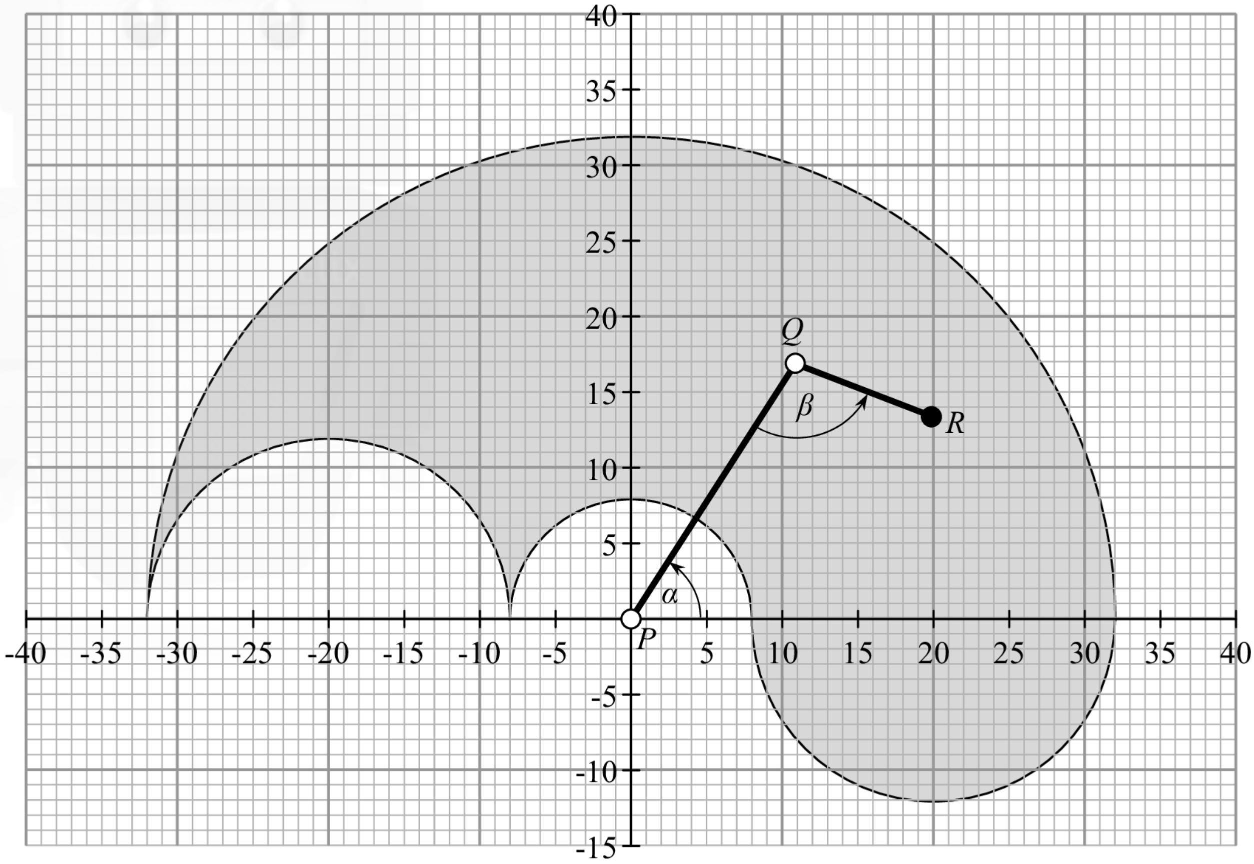
(c) The answer to part **(b)** above depends on the particular position of the arm. That is, in certain positions, the location of R is more sensitive to small errors in α than to small errors in β, while in other positions, the reverse is true. Describe, with justification, the conditions under which each of these two situations arises.

More sensitive to errors in α when $|PR| > 12$
More sensitive to errors in β when $|PR| < 12$

The condition $|PR| > 12$
is true whenever
$$\beta > \cos^{-1}\left(\frac{5}{6}\right) \approx 33{\cdot}6°$$

(Borderline case is when ΔPQR is isosceles with $|QR| = |RP|$.)

(d) Illustrate the set of all possible locations of the point R on the coordinate diagram below. Take P as the origin and take each unit in the diagram to represent a centimetre in reality. Note that α and β can vary only from 0° to 180°.



Question 11 (2012)

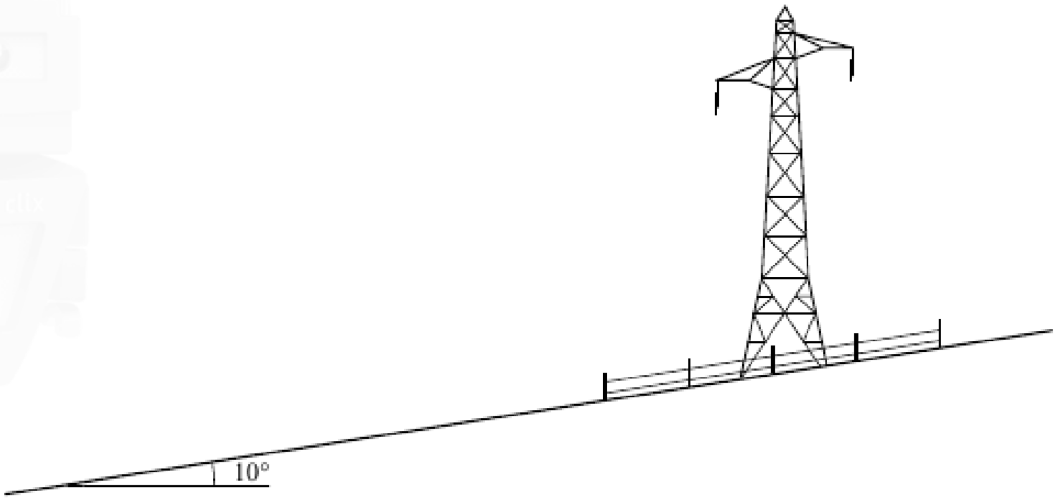
Question 8 **(75 marks)**

(a) Two surveyors want to find the height of an electricity pylon. There is a fence around the pylon that they cannot cross for safety reasons. The ground is inclined at an angle. They have a clinometer (for measuring angles of elevation) and a 100 metre tape measure. They have already used the clinometer to determine that the ground is inclined at 10° to the horizontal.

(i) Explain how they could find the height of the pylon.

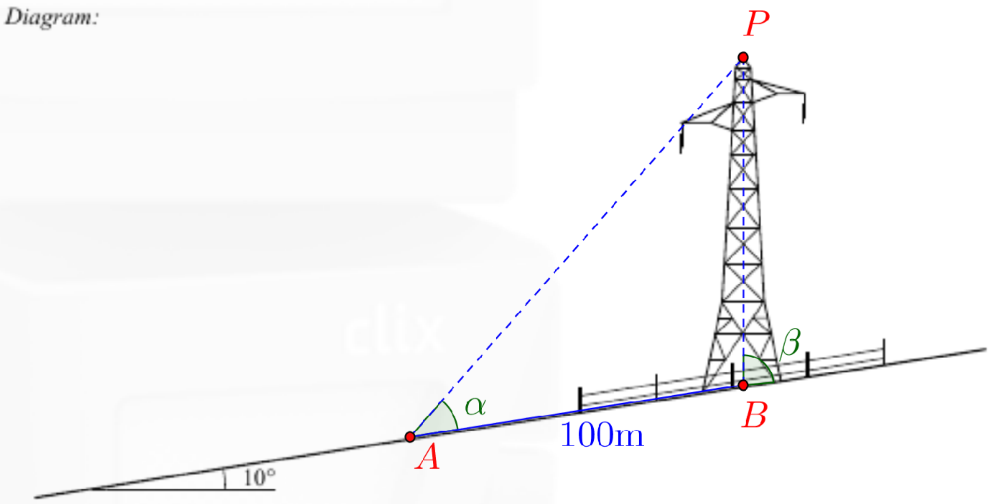
Your answer should be illustrated on the diagram below. Show the points where you think they should take measurements, write down clearly what measurements they should take, and outline briefly how these can be used to find the height of the pylon.

Diagram:



Measurements to be taken:

Mark point B directly under the pylon.
Mark another point A 100 metres downhill from point B.
Use the clinometer to measure the angle of elevation at A.
We know the pylon makes a right-angle with the horizontal.
Subtract 10° from each of these angles to find the measure of the angles α and β marked in green below.



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Procedure used to find the height:

Firstly, the angle $\angle PBA$ is equal to $180^\circ - \angle\beta$. Once this is calculated we know 2 angles inside the triangle $\triangle APB$ and can find the third because these must add up to 180°. Now use the sine rule:

$$\begin{aligned}
\frac{|BP|}{\sin\alpha} &= \frac{|AB|}{\sin\angle APB} \\
\frac{|BP|}{\sin\alpha} &= \frac{100}{\sin\angle APB} \\
|BP| &= \frac{100\sin\alpha}{\sin\angle APB}
\end{aligned}$$

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(ii) Write down the possible values for the measurements taken, and use them to show how to find the height of the pylon. (That is, find the height of the pylon using your measurements, and showing your work.)

From the diagram, a protractor can be used to estimate $\alpha = 42^\circ$ and we know that $\beta = (90 - 10) = 80^\circ$.

Then $\angle PBA = 180^\circ - 80^\circ = 100^\circ$. Now we get

$$\begin{aligned}
\angle APB + \angle PBA + \alpha &= 180^\circ \\
\angle APB + 100^\circ + 42^\circ &= 180^\circ \\
\angle APB &= 180^\circ - 100^\circ - 42^\circ \\
\angle APB &= 38^\circ
\end{aligned}$$

Finally, using the sine rule:

$$\begin{aligned}
\frac{|BP|}{\sin 42^\circ} &= \frac{100}{\sin 38^\circ} \\
\frac{|BP|}{0.6691} &= \frac{100}{0.6157} \\
|BP| &= \frac{(100)(0.6691)}{0.6157} \\
|BP| &= 108.7 \text{ metres}
\end{aligned}$$

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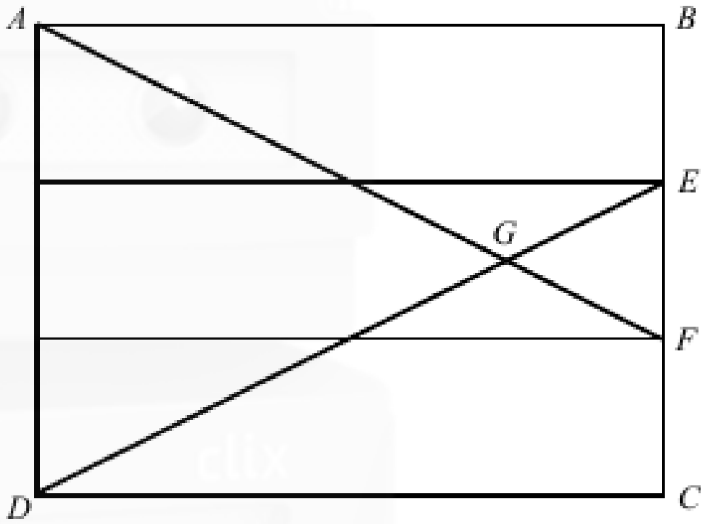
(b) Anne is having a new front gate made and has decided on the design below.



The gate is 2 metres wide and 1.5 metres high. The horizontal bars are 0.5 metres apart.

(i) Calculate the common length of the bars $[AF]$ and $[DE]$, in metres, correct to three decimal places.

Use Pythagoras theorem on the triangle $\triangle AFB$:

$$|AF|^2 = 2^2 + 1^2$$
$$|AF|^2 = 5$$

which means $|AF| = \sqrt{5} = 2.236$ metres correct to three decimal places.

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(ii) In order to secure the bar $[AF]$ and $[DE]$ the manufacturer needs to know:

- The measure of the angle EGF, and
- the common distance $|AG| = |DG|$

Find these measures. Give the angle correct to the nearest degree and the length correct to three decimal places.

- First find the angle $\angle AFB$. Note that

$$\tan\angle AFB = \frac{opposite}{adjacent} = \frac{2}{1}$$

which means $\angle AFB = \tan^{-1}(2) = 63.43^\circ$ to 2 decimal places.

Now focus on the triangle $\triangle GEF$.
The angle $\angle GFE = 63.43^\circ$.
The angle $\angle GEF$ must also be 63.43° since the triangle $\triangle DCE$ is congruent to $\triangle AFB$.
Since the three angles in $\triangle GEF$ must add up to 180° we have

$$\begin{aligned} \angle GFE + \angle GEF + \angle EGF &= 180^\circ \\ 63.43^\circ + 63.43^\circ + \angle EGF &= 180^\circ \\ \angle EGF &= 180^\circ - 63.43^\circ - 63.43^\circ \\ \angle EGF &= 53.14^\circ \\ \angle EGF &= 53^\circ \text{ to the nearest degree} \end{aligned}$$

- Using the sin rule on $\triangle GEF$ we have

$$\frac{\sin 63}{|GE|} = \frac{\sin 53}{0.5}$$

which means

$$\begin{aligned} |GE| &= \frac{0.5\sin 63}{\sin 53} \\ &= 0.558 \end{aligned}$$

correct to three decimal places. Finally we have

$$\begin{aligned} |DG| + 0.558 &= 2.236 \\ |DG| &= 1.678 \text{ metres} \end{aligned}$$

Question 13 (2012)

(i) Calculate the common length of the bars $[AF]$ and $[DE]$, in metres, correct to three decimal places.

Use Pythagoras theorem on the triangle $\triangle AFB$:

$$\begin{aligned} |AF|^2 &= 2^2 + 1^2 \\ |AF|^2 &= 5 \end{aligned}$$

which means $|AF| = \sqrt{5} = 2.236$ metres correct to three decimal places.

- First find the angle $\angle AFB$. Note that

$$\tan \angle AFB = \frac{opposite}{adjacent} = \frac{2}{1}$$

which means $\angle AFB = \tan^{-1}(2) = 63.43^\circ$ to 2 decimal places.

Now focus on the triangle $\triangle GEF$.
The angle $\angle GFE = 63.43^\circ$.
The angle $\angle GEF$ must also be 63.43° since the triangle $\triangle DCE$ is congruent to $\triangle AFB$.
Since the three angles in $\triangle GEF$ must add up to 180° we have

$$\begin{aligned}
\angle GFE + \angle GEF + \angle EGF &= 180^\circ \\
63.43^\circ + 63.43^\circ + \angle EGF &= 180^\circ \\
\angle EGF &= 180^\circ - 63.43^\circ - 63.43^\circ \\
\angle EGF &= 53.14^\circ \\
\angle EGF &= 53^\circ \text{ to the nearest degree}
\end{aligned}$$

- Using the sin rule on $\triangle GEF$ we have

$$\frac{\sin 63}{|GE|} = \frac{\sin 53}{0.5}$$

which means

$$\begin{aligned}
|GE| &= \frac{0.5 \sin 63}{\sin 53} \\
&= 0.558
\end{aligned}$$

correct to three decimal places. Finally we have

$$\begin{aligned}
|DG| + 0.558 &= 2.236 \\
|DG| &= 1.678 \text{ metres}
\end{aligned}$$

Question 14 (2011)

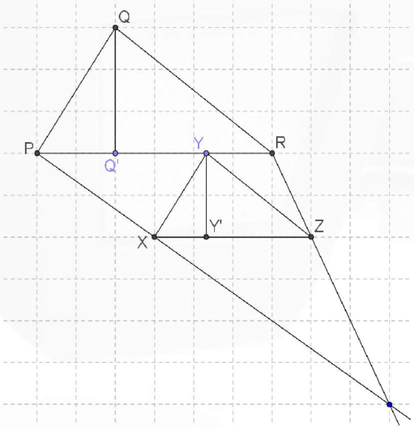
(a) Calculate the scale factor of the enlargement, showing your work.

$$\frac{|PR|}{|XZ|}=\frac{6}{4}=\frac{3}{2}$$

(b) By construction or otherwise, locate the centre of enlargement on the diagram above.

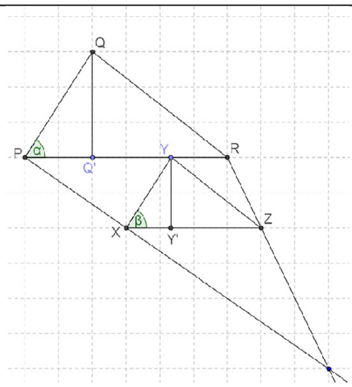
Shown as O above.

(c) Calculate $|YR|$ in grid units.



$$|Y'Z|=\frac{2}{3}|Q'R|=\frac{2}{3}(4)=\frac{8}{3}$$

$$|YR|=\frac{8}{3}-1=\frac{5}{3}$$



$$\tan\alpha=\frac{3}{2}$$

$$\alpha=\beta$$

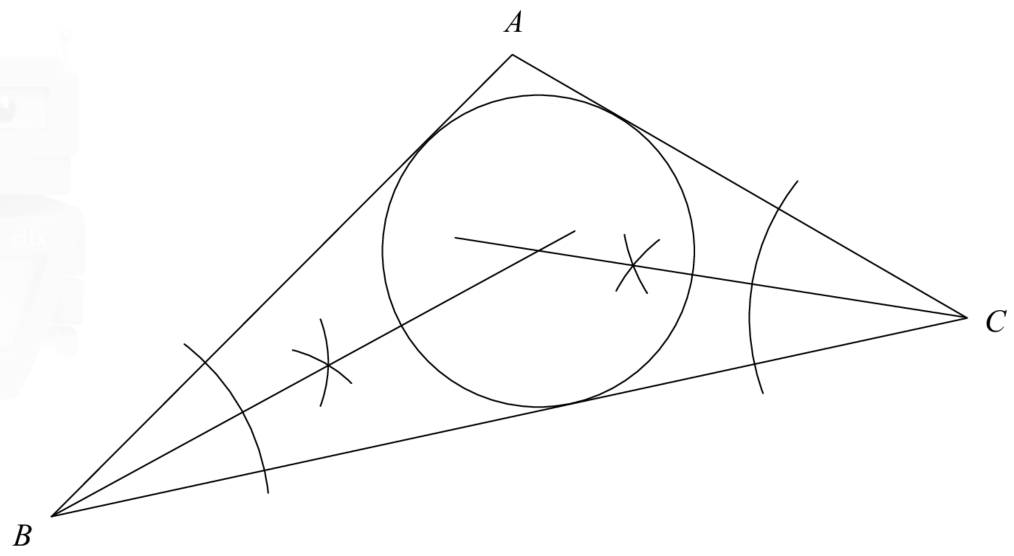
$$\frac{2}{|XY'|}=\frac{3}{2}$$

$$|XY'|=\frac{4}{3}$$

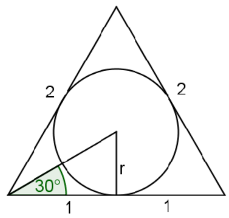
$$|YR|=3-\frac{4}{3}=\frac{5}{3}$$

Question 15 (2010)

(a) Construct the incircle of the triangle ABC below using only a compass and straight edge.
Show all construction lines clearly.



(b) An equilateral triangle has sides of length 2 units.
Find the area of its incircle.



$$\tan 30^\circ = \frac{r}{1}$$

$$r = \frac{1}{\sqrt{3}}$$

$$A = \pi(r)^2$$

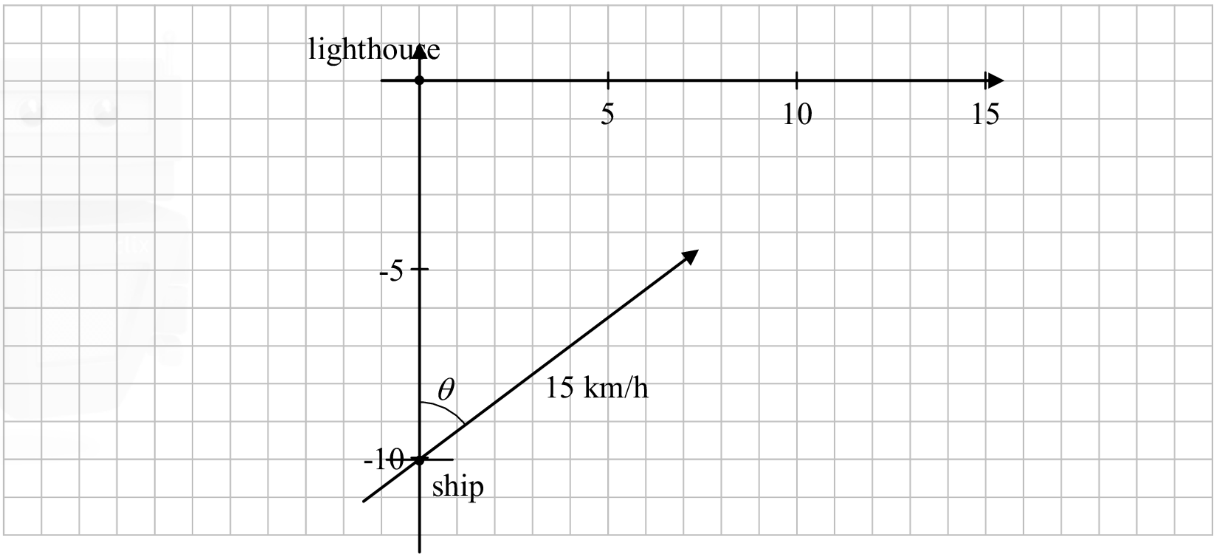
$$A = \pi\left(\frac{1}{\sqrt{3}}\right)^2$$

$$A = \frac{\pi}{3} \text{ square units.}$$

Question 16 (2010)

A ship is 10 km due South of a lighthouse at noon.

The ship is travelling at 15 km/h on a bearing of θ, as shown below, where $\theta = \tan^{-1}\left(\frac{4}{3}\right)$.



(a) On the diagram above, draw a set of co-ordinate axes that takes the lighthouse as the origin, the line East-West through the lighthouse as the x-axis, and kilometres as units.

(b) Find the equation of the line along which the ship is moving.

$$\tan\theta = \frac{4}{3}$$
$$\therefore m = \frac{3}{4}$$
$$y + 10 = \frac{3}{4}(x - 0)$$
$$4y + 40 = 3x$$
$$3x - 4y - 40 = 0$$

Or

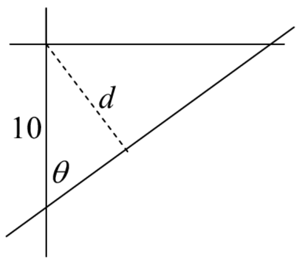
$$y = mx + c$$
$$y = \frac{3}{4}x - 10$$

(c) Find the shortest distance between the ship and the lighthouse during the journey.

$$\sin\theta = \frac{d}{10}$$
$$\frac{4}{5} = \frac{d}{10}$$
$$5d = 40$$
$$x = 8 \text{ km}$$

Or

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$
$$d = \frac{|3(0) - 4(0) - 40|}{\sqrt{3^2 + (-4)^2}}$$
$$d = 8 \text{ km.}$$



(d) At what time is the ship closest to the lighthouse?

$$\tan\theta = \frac{8}{x}$$

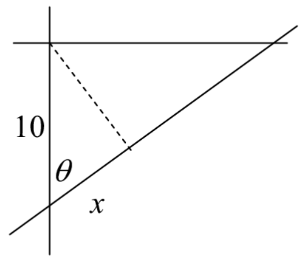
$$\frac{4}{3} = \frac{8}{x}$$

$$4x = 24$$

$$x = 6 \text{ km.}$$

$$\text{Time} = \frac{6}{15} = 0{\cdot}4 \text{ hours} = 24 \text{ minutes.}$$

$\therefore$ closest to the lighthouse at 12:24 pm



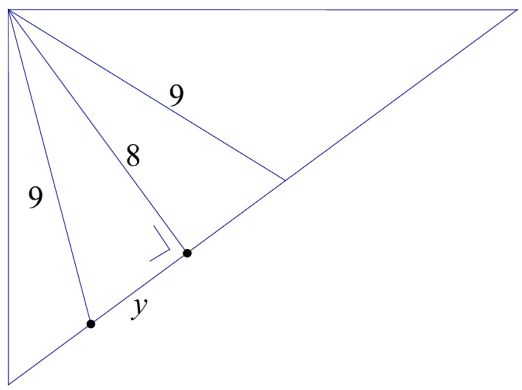
(e) Visibility is limited to 9 km. For how many minutes in total is the ship visible from the lighthouse?

$$8^2 + y^2 = 9^2$$

$$y^2 = 81 - 64$$

$$y^2 = 17$$

$$y = \sqrt{17}$$



Distance travelled by the ship while visible from the lighthouse is $2y = 2\sqrt{17}$ km.

$$\text{Time} = \frac{2\sqrt{17}}{15} \text{ hours.}$$

$$= 8\sqrt{17} \text{ minutes or } 32{\cdot}98 \text{ minutes} \approx 33 \text{ minutes.}$$