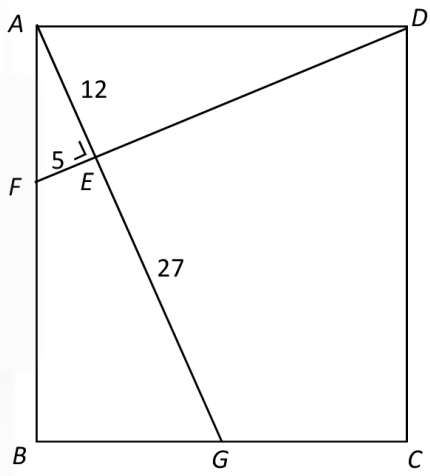
Question 5 **(25 marks)**

$ABCD$ is a rectangle.

$F \in [AB]$, $G \in [BC]$, $[FD] \cap [AG] = \{E\}$, and $FD \perp AG$.

$|AE| = 12$ cm, $|EG| = 27$ cm, and $|FE| = 5$ cm.



(a) Prove that ΔAFE and ΔDAE are similar (equiangular).

(b) Find $|AD|$.

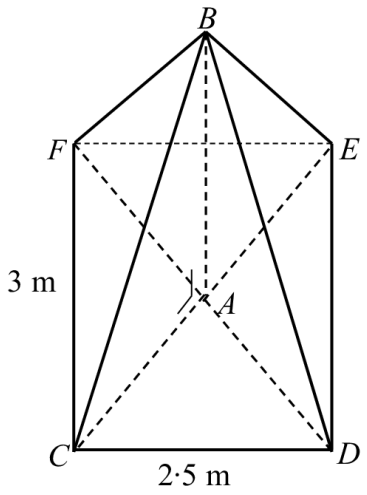
(c) ΔAFE and ΔAGB are similar. Show that $|AB| = 36$ cm.

(d) Find the area of the quadrilateral $GCDE$.

Question 2

A glass Roof Lantern in the shape of a pyramid has a rectangular base $CDEF$ and its apex is at B as shown. The vertical height of the pyramid is $|AB|$, where A is the point of intersection of the diagonals of the base as shown in the diagram.
Also $|CD| = 2{\cdot}5$ m and $|CF| = 3$ m.

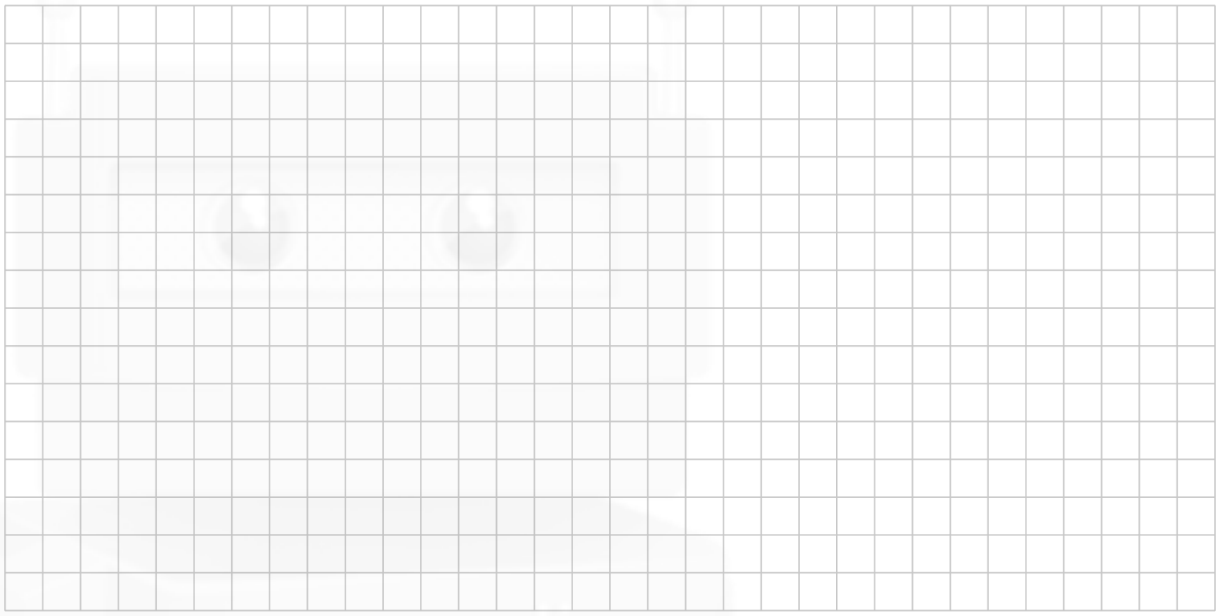
(a) **(i)** Show that $|AC| = 1{\cdot}95$ m, correct to two decimal places.



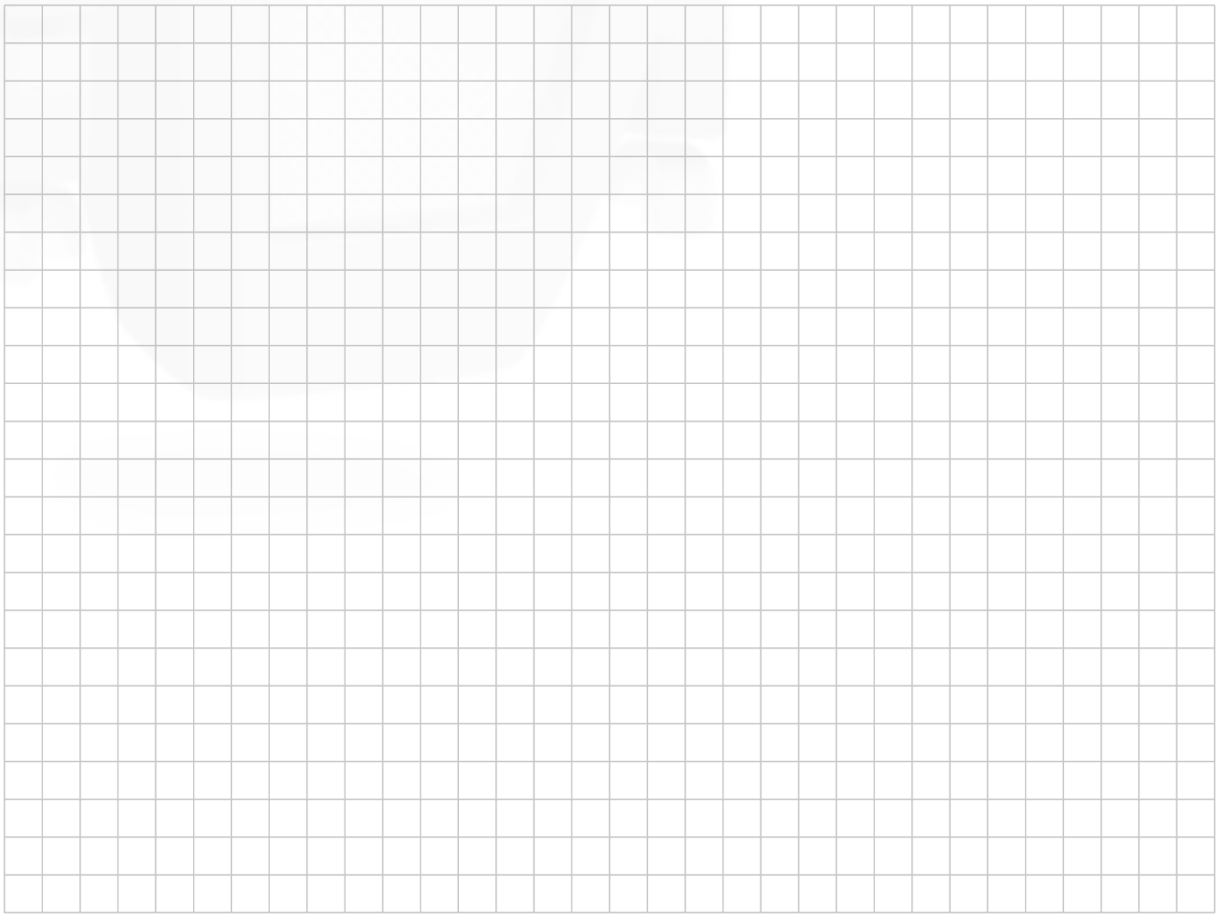
(ii) The angle of elevation of B from C is $50°$ (i.e. $|\angle BCA| = 50°$).
Show that $|AB| = 2{\cdot}3$ m, correct to one decimal place.

(iii) Find $|BC|$, correct to the nearest metre.

(iv) Find $|\angle BCD|$, correct to the nearest degree.



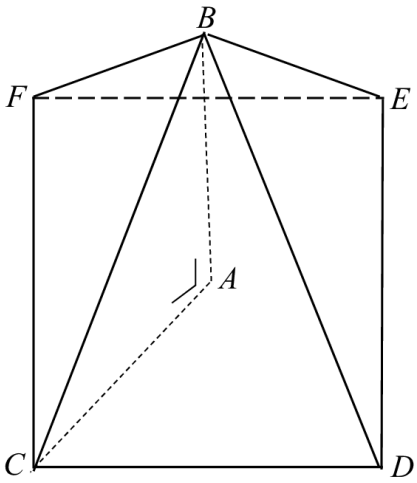
(v) Find the area of glass required to glaze all four triangular sides of the pyramid.
Give your answer correct to the nearest m^2.



(b) Another Roof Lantern, in the shape of a pyramid, has a square base $CDEF$. The vertical height $|AB| = 3$ m, where A is the point of intersection of the diagonals of the base as shown.

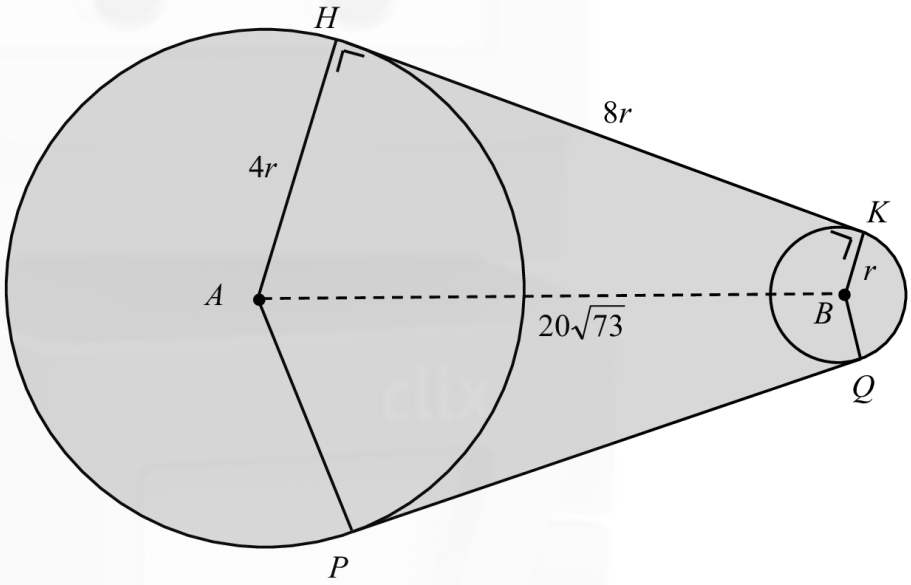
The angle of elevation of B from C is 60°
(i.e. $|\angle BCA| = 60°$).

Find the length of the side of the square base of the lantern.
Give your answer in the form $\sqrt{a}$ m, where $a \in \mathbb{N}$.



Question 7 **(40 marks)**

A flat machine part consists of two circular ends attached to a plate, as shown (diagram not to scale).
The sides of the plate, HK and PQ, are tangential to each circle.
The larger circle has centre A and radius $4r$ cm.
The smaller circle has centre B and radius r cm.
The length of $[HK]$ is $8r$ cm and $|AB| = 20\sqrt{73}$ cm.



(a) Find r, the radius of the smaller circle. (Hint: Draw $BT \parallel KH$, $T \in AH$.)

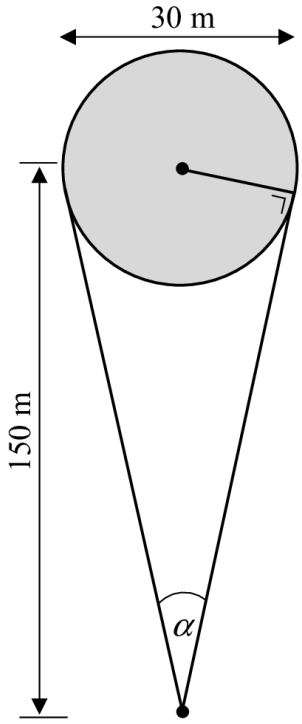
(b) Find the area of the quadrilateral $ABKH$.

(c) **(i)** Find $|\angle HAP|$, in degrees, correct to one decimal place.

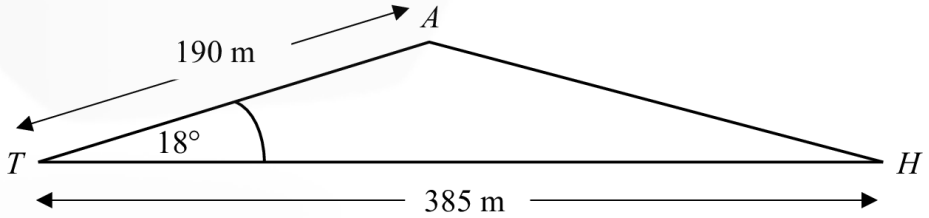
(ii) Find the area of the machine part, correct to the nearest cm^2.

Question 9 **(45 marks)**

(a) Joan is playing golf. She is 150 m from the centre of a circular green of diameter 30 m. The diagram shows the range of directions in which Joan can hit the ball so that it could land on the green. Find α, the measure of the angle of this range of directions. Give your answer, in degrees, correct to one decimal place.



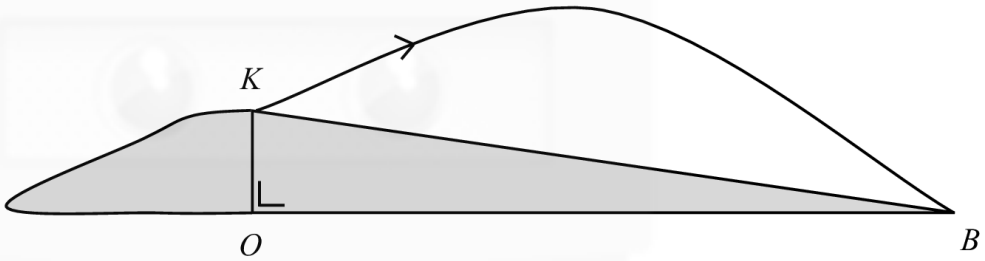
(b) At the next hole, Joan, at T, attempts to hit the ball in the direction of the hole H. Her shot is off target and the ball lands at A, a distance of 190 metres from T, where $|\angle ATH| = 18°$. $|TH|$ is 385 metres. Find $|AH|$, the distance from the ball to the hole, correct to the nearest metre.



(c) At another hole, where the ground is not level, Joan hits the ball from K, as shown. The ball lands at B. The height of the ball, in metres, above the horizontal line OB is given by

$$h = -6t^2 + 22t + 8$$

where t is the time in seconds after the ball is struck and h is the height of the ball.



(i) Find the height of K above OB.

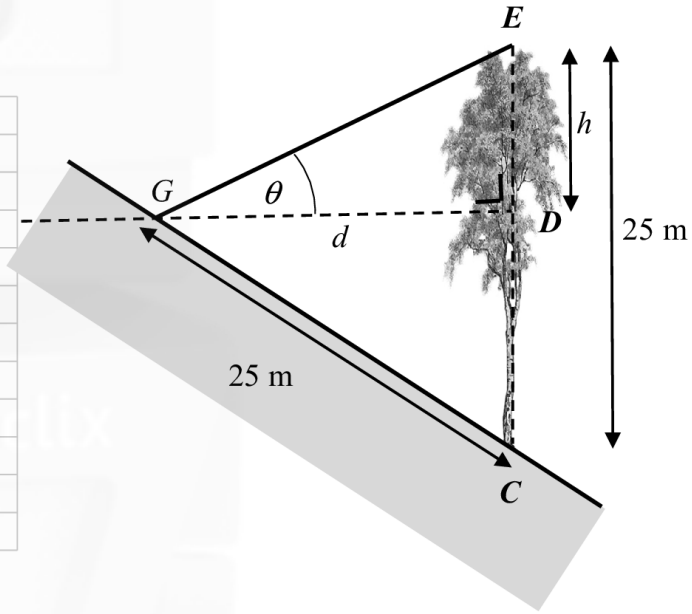
(ii) The horizontal speed of the ball over the straight distance $[OB]$ is a constant 38 m s^{-1}. Find the angle of elevation of K from B, correct to the nearest degree.

(d) At a later hole, Joan's first shot lands at the point G, on ground that is sloping downwards, as shown. A vertical tree, $[CE]$, 25 metres high, stands between G and the hole. The distance, $|GC|$, from the ball to the bottom of the tree is also 25 metres.

The angle of elevation at G to the top of the tree, E, is θ, where $\theta = \tan^{-1}\frac{1}{2}$.

The height of the top of the tree above the horizontal, GD, is h metres and $|GD| = d$ metres.

(i) Write d and $|CD|$ in terms of h.



$d =$

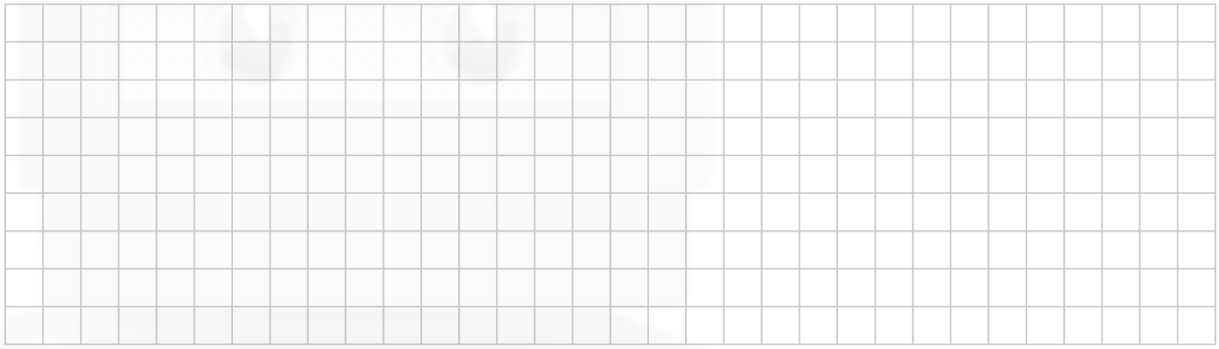
$|CD| =$

(ii) Hence, or otherwise, find h.

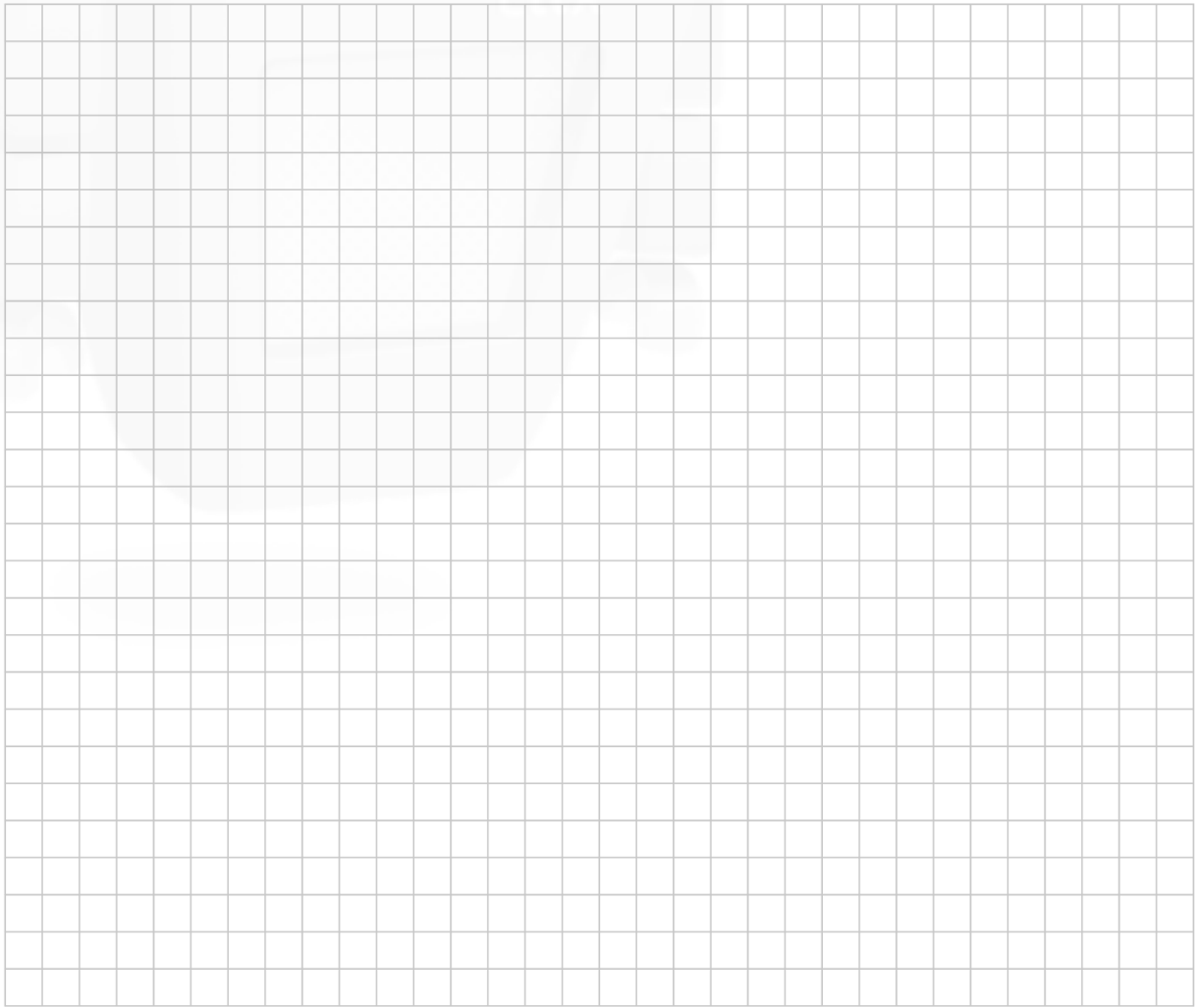
Question 7 **(40 marks)**

(a) Three natural numbers a, b and c, such that $a^2 + b^2 = c^2$, are called a Pythagorean triple.

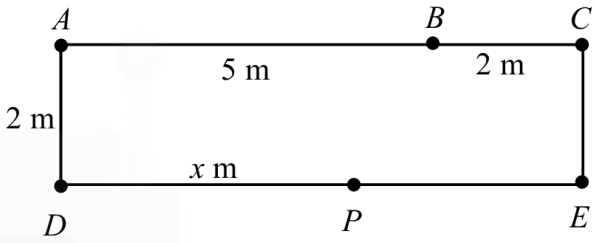
(i) Let $a = 2n+1$, $b = 2n^2 + 2n$ and $c = 2n^2 + 2n + 1$.
Pick one natural number n and verify that the corresponding values of a, b and c form a Pythagorean triple.



(ii) Prove that $a = 2n+1$, $b = 2n^2 + 2n$ and $c = 2n^2 + 2n + 1$, where $n \in \mathbb{N}$, will always form a Pythagorean triple.



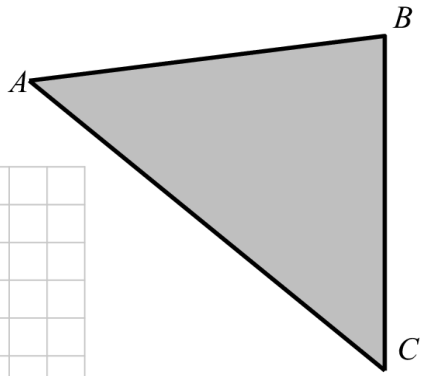
(b) $ADEC$ is a rectangle with $|AC| = 7$ m and $|AD| = 2$ m, as shown.
B is a point on $[AC]$ such that $|AB| = 5$ m.
P is a point on $[DE]$ such that $|DP| = x$ m.



(i) Let $f(x) = |PA|^2 + |PB|^2 + |PC|^2$.
Show that $f(x) = 3x^2 - 24x + 86$, for $0 \le x \le 7$, $x \in \mathbb{R}$.

(ii) The function $f(x)$ has a minimum value at $x = k$.
Find the value of k and the minimum value of $f(x)$.

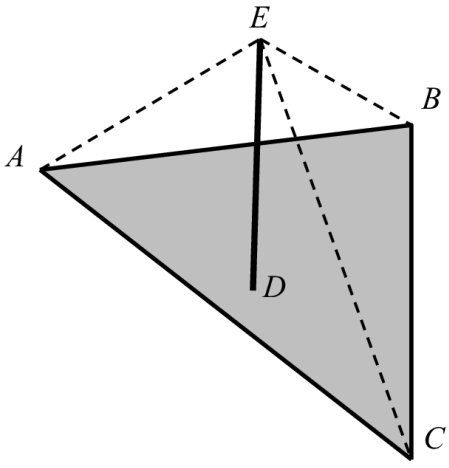
The lengths of the sides of a flat triangular field ACB are, $|AB| = 120$ m, $|BC| = 134$ m and $|AC| = 150$ m.



(a) **(i)** Find $|\angle CBA|$. Give your answer, in degrees, correct to two decimal places.

(ii) Find the area of the triangle ACB correct to the nearest whole number.

(b) A vertical mast, $[DE]$, is fixed at the circumcentre, D, of the triangle. The mast is held in place by three taut cables $[EA]$, $[EB]$ and $[EC]$. Explain why the three cables are equal in length.



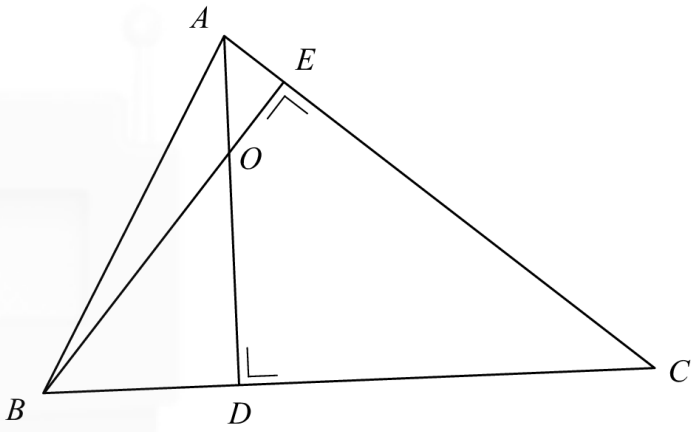
Question 6B

ABC is a triangle.

D is the point on BC such that $AD \perp BC$.
E is the point on AC such that $BE \perp AC$.

AD and BE intersect at O.

Prove that $|\angle DOC| = |\angle DEC|$.



Question 5 **(25 marks)**

(a) In a triangle ABC, the lengths of the sides are a, b and c. Using a formula for the area of a triangle, or otherwise, prove that

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}.$$

(b) In a triangle XYZ, $|XY| = 5$ cm, $|XZ| = 3$ cm and $|\angle XYZ| = 27°$.

(i) Find the two possible values of $|\angle XZY|$. Give your answers correct to the nearest degree.

(ii) Draw a sketch of the triangle XYZ, showing the two possible positions of the point Z.

(c) In the case that $|\angle XZY| < 90°$, write down $|\angle ZXY|$, and hence find the area of the triangle XYZ, correct to the nearest integer.

$|\angle ZXY| =$

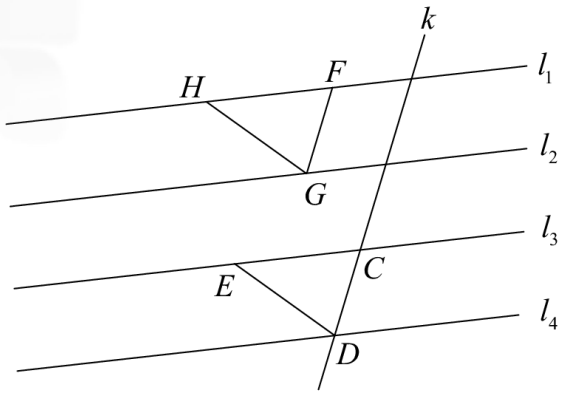
Question 6A

(a) **(i)** Given the points B and C below, construct, without using a protractor or setsquare, a point A such that $|\angle ABC| = 60°$.

(ii) Hence construct, on the same diagram above, and using a compass and straight edge only, an angle of 15°.

(b) In the diagram, $l_1, l_2, l_3,$ and l_4 are parallel lines that make intercepts of equal length on the transversal k. FG is parallel to k, and HG is parallel to ED.

Prove that the triangles ΔCDE and ΔFGH are congruent.

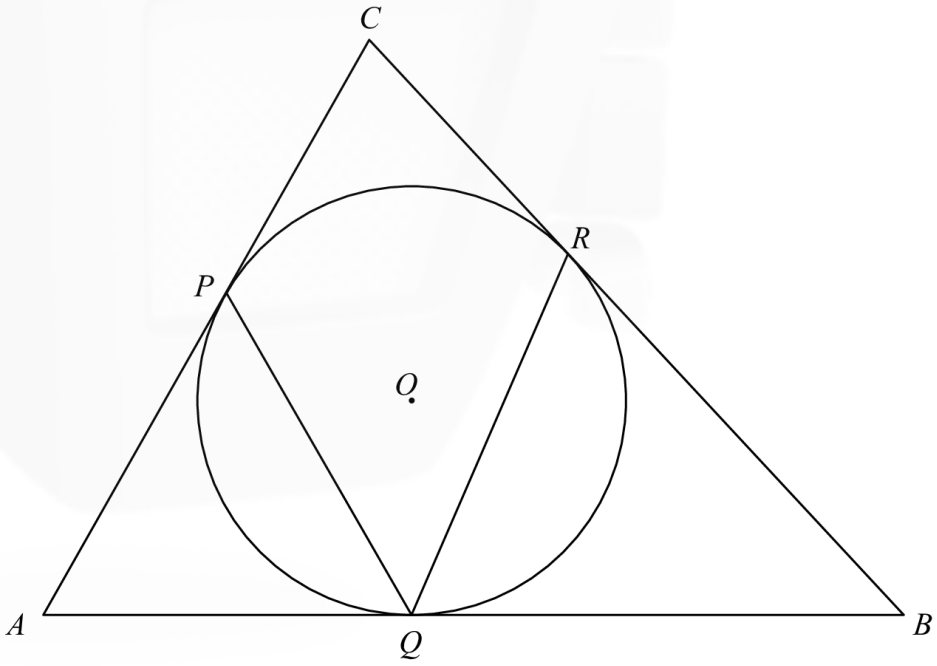


OR

Question 6B

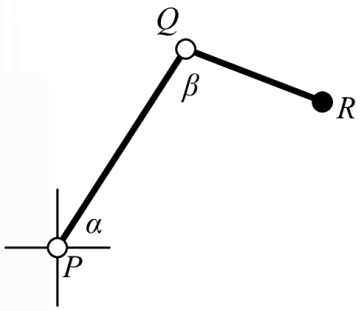
The incircle of the triangle ABC has centre O and touches the sides at P, Q and R, as shown.

Prove that $|\angle PQR| = \frac{1}{2}\left(|\angle CAB| + |\angle CBA|\right)$.



Question 8 **(75 marks)**

The diagram is a representation of a robotic arm that can move in a vertical plane. The point P is fixed, and so are the lengths of the two segments of the arm. The controller can vary the angles α and β from 0° to 180°.



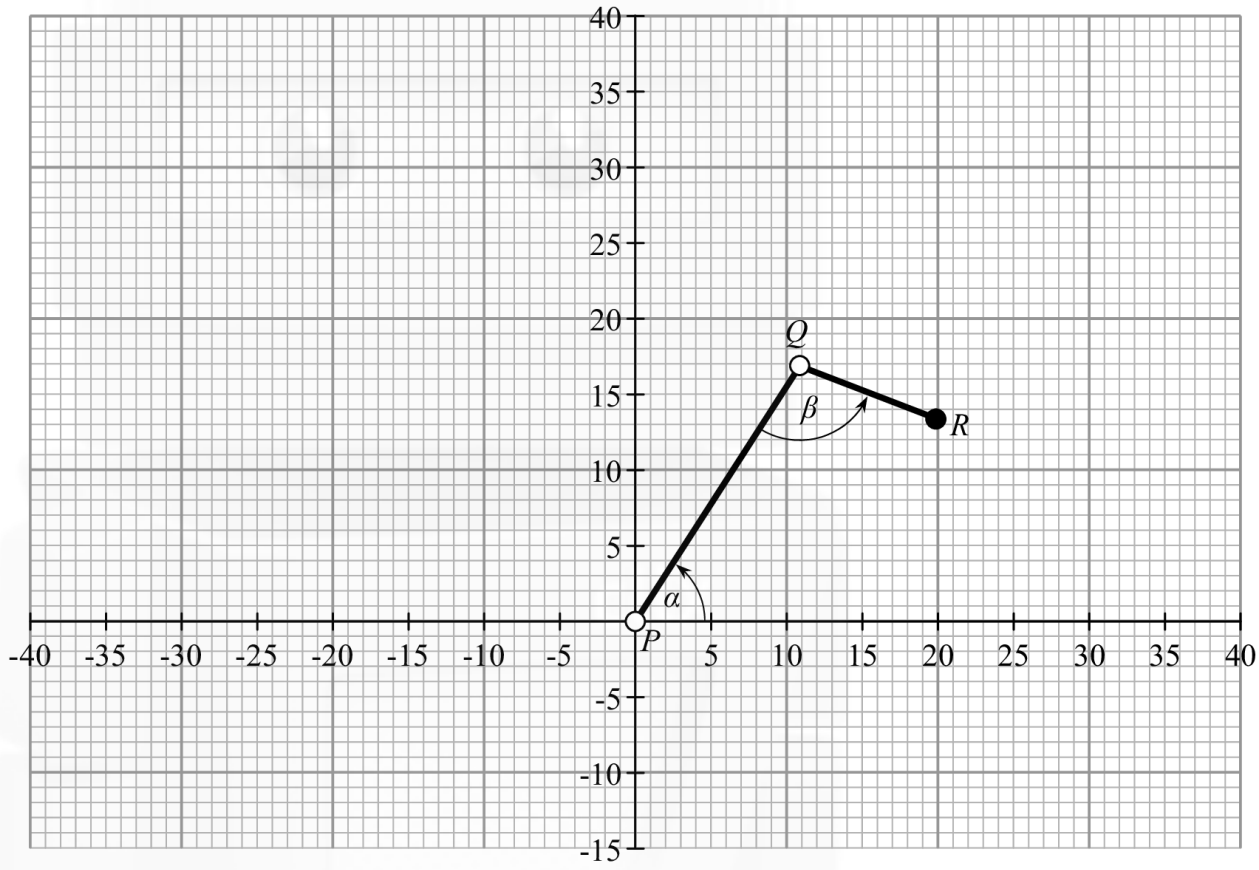
(a) Given that $|PQ| = 20$ cm and $|QR| = 12$ cm, determine the values of the angles α and β so as to locate R, the tip of the arm, at a point that is 24 cm to the right of P, and 7 cm higher than P. Give your answers correct to the nearest degree.

(b) In setting the arm to the position described in part **(a)**, which will cause the greater error in the location of R: an error of $1°$ in the value of α or an error of $1°$ in the value of β?

Justify your answer. You may assume that if a point moves along a circle through a small angle, then its distance from its starting point is equal to the length of the arc travelled.

(c) The answer to part **(b)** above depends on the particular position of the arm. That is, in certain positions, the location of R is more sensitive to small errors in α than to small errors in β, while in other positions, the reverse is true. Describe, with justification, the conditions under which each of these two situations arises.

(d) Illustrate the set of all possible locations of the point R on the coordinate diagram below. Take P as the origin and take each unit in the diagram to represent a centimetre in reality. Note that α and β can vary only from 0° to 180°.



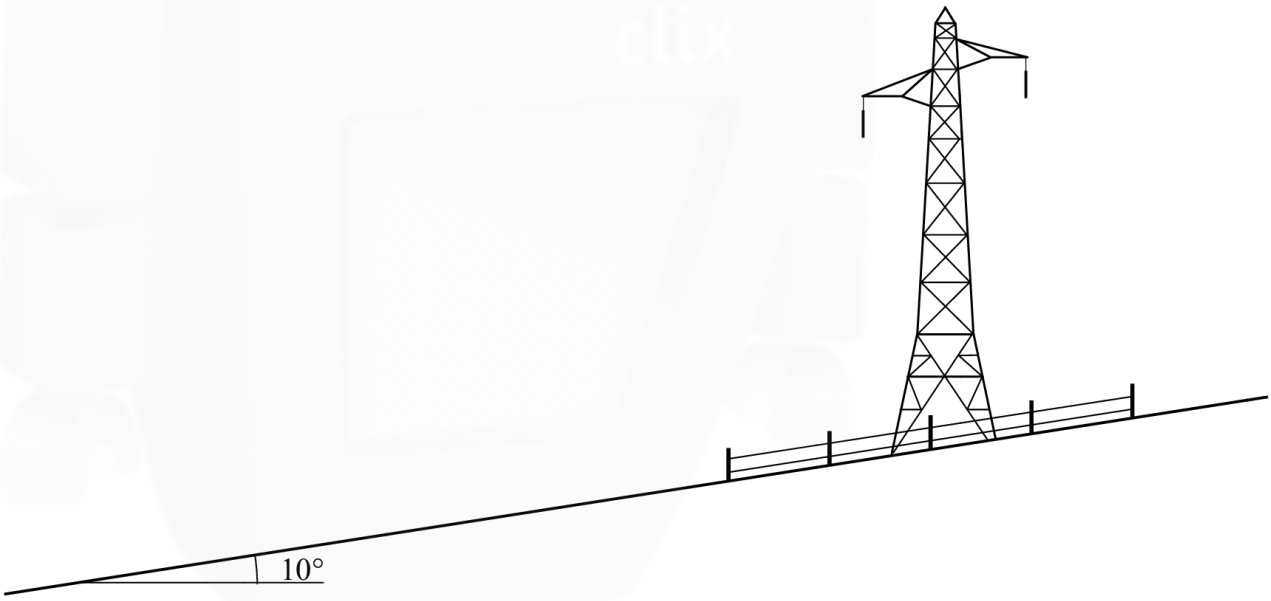
Question 8 **(75 marks)**

(a) Two surveyors want to find the height of an electricity pylon. There is a fence around the pylon that they cannot cross for safety reasons. The ground is inclined at an angle. They have a clinometer (for measuring angles of elevation) and a 100 metre tape measure. They have already used the clinometer to determine that the ground is inclined at 10° to the horizontal.

(i) Explain how they could find the height of the pylon.

Your answer should be illustrated on the diagram below. Show the points where you think they should take measurements, write down clearly what measurements they should take, and outline briefly how these can be used to find the height of the pylon.

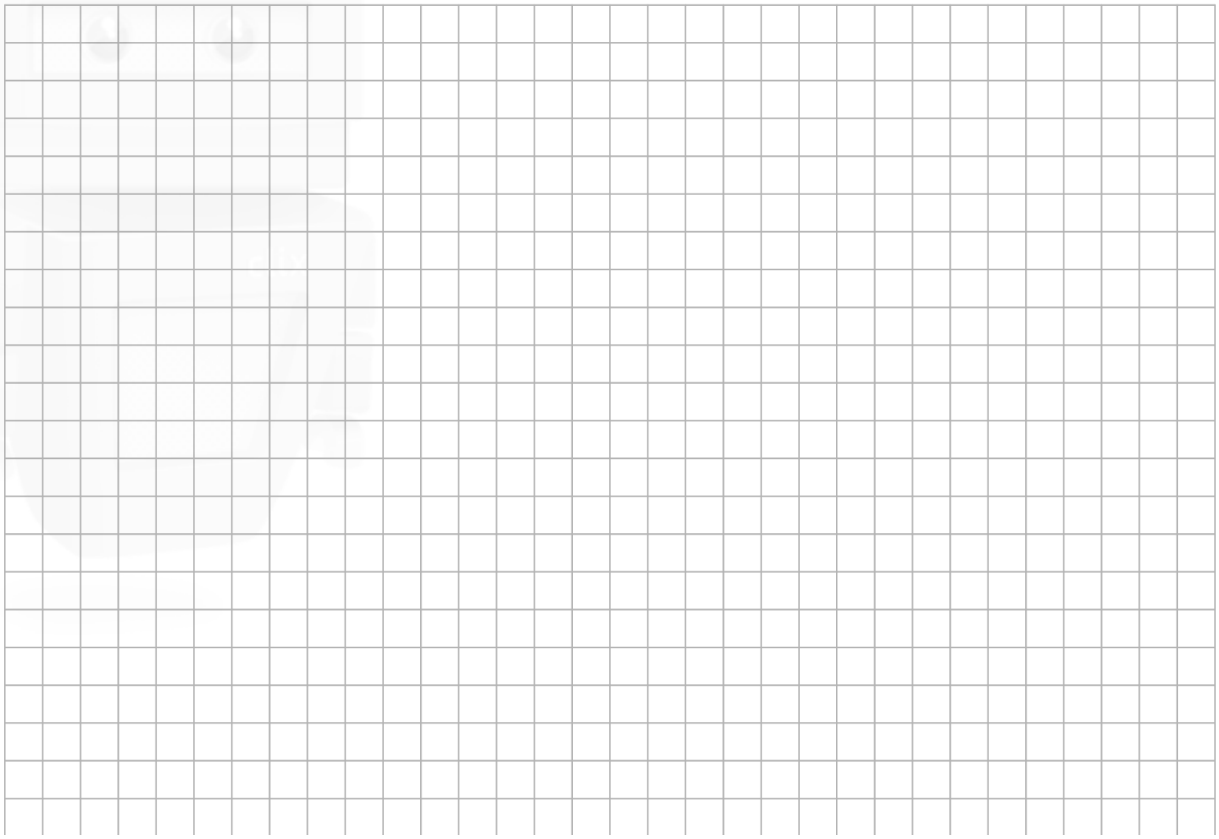
Diagram:



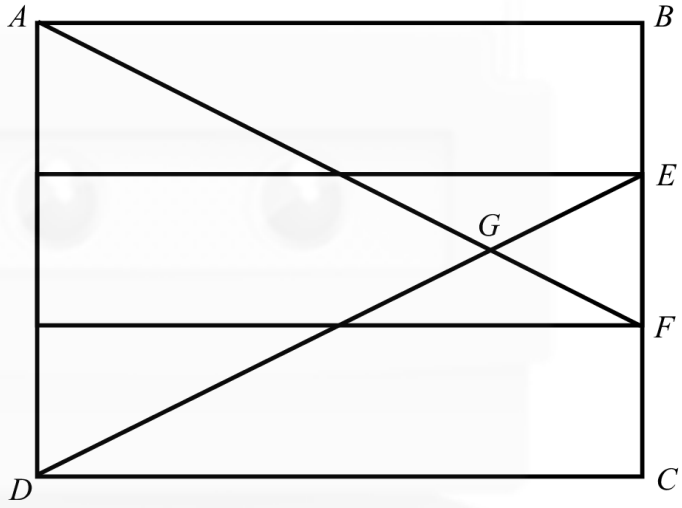
Measurements to be taken:

Procedure used to find the height:

(ii) Write down possible values for the measurements taken, and use them to show how to find the height of the pylon. (That is, find the height of the pylon using your measurements, and showing your work.)



(b) Anne is having a new front gate made and has decided on the design below.



The gate is 2 metres wide and 1·5 metres high. The horizontal bars are 0·5 metres apart.

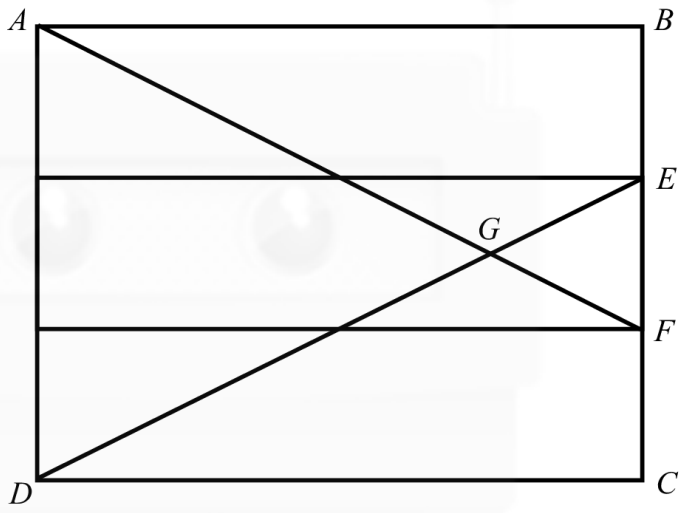
(i) Calculate the common length of the bars $[AF]$ and $[DE]$, in metres, correct to three decimal places.

(ii) In order to secure the bar $[AF]$ to $[DE]$, the manufacturer needs to know:

- the measure of the angle EGF, and
- the common distance $|AG| = |DG|$.

Find these measures. Give the angle correct to the nearest degree and the length correct to three decimal places.

(b) Anne is having a new front gate made and has decided on the design below.



The gate is 2 metres wide and 1·5 metres high. The horizontal bars are 0·5 metres apart.

(i) Calculate the common length of the bars $[AF]$ and $[DE]$, in metres, correct to three decimal places.

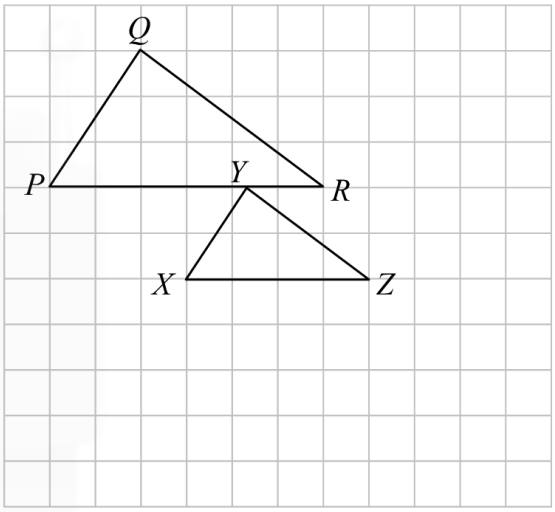
(ii) In order to secure the bar $[AF]$ to $[DE]$, the manufacturer needs to know:

- the measure of the angle EGF, and
- the common distance $|AG| = |DG|$.

Find these measures. Give the angle correct to the nearest degree and the length correct to three decimal places.

Question 14

Two triangles are drawn on a square grid as shown. The points P, Q, R, X, and Z are on vertices of the grid, and the point Y lies on $[PR]$. The triangle PQR is an enlargement of the triangle XYZ.



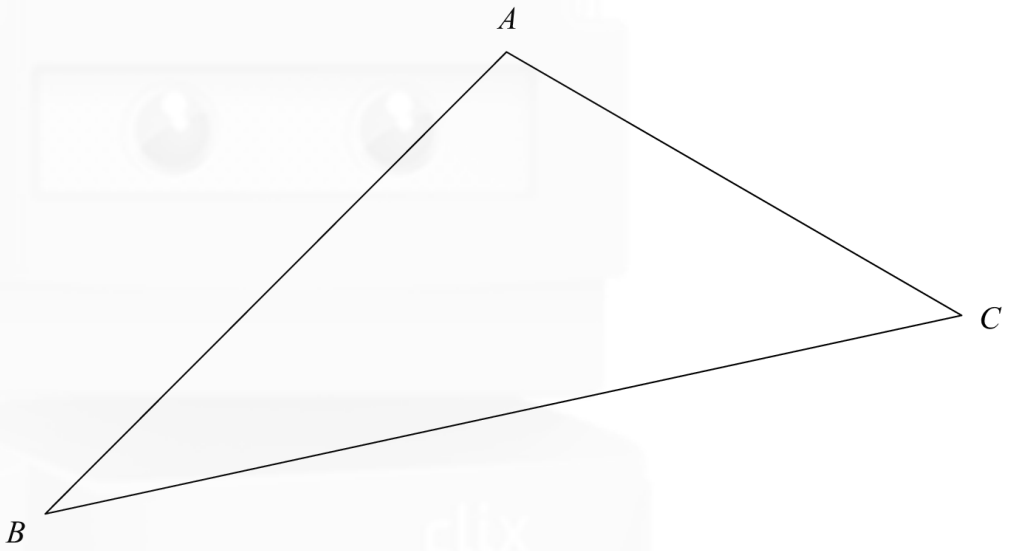
(a) Calculate the scale factor of the enlargement, showing your work.

(b) By construction or otherwise, locate the centre of enlargement on the diagram above.

(c) Calculate $|YR|$ in grid units.

Question 15

(a) Construct the incircle of the triangle ABC below using only a compass and straight edge.
Show all construction lines clearly.

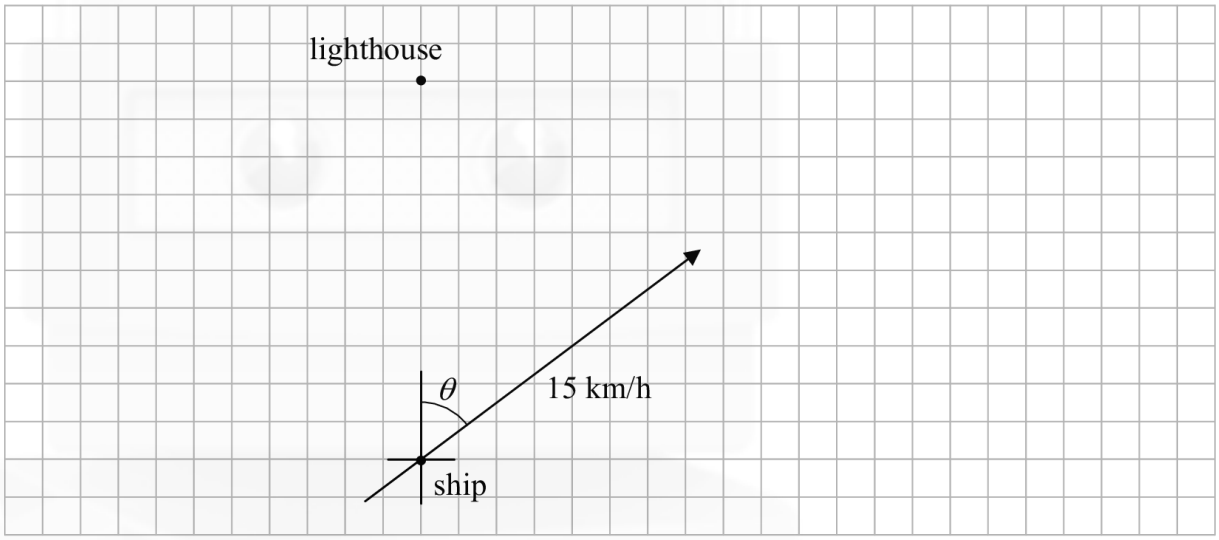


(b) An equilateral triangle has sides of length 2 units.
Find the area of its incircle.

Question 16

A ship is 10 km due South of a lighthouse at noon.

The ship is travelling at 15 km/h on a bearing of θ, as shown below, where $\theta = \tan^{-1}\left(\frac{4}{3}\right)$.



(a) On the diagram above, draw a set of co-ordinate axes that takes the lighthouse as the origin, the line East-West through the lighthouse as the x-axis, and kilometres as units.

(b) Find the equation of the line along which the ship is moving.

(c) Find the shortest distance between the ship and the lighthouse during the journey.

(d) At what time is the ship closest to the lighthouse?

(e) Visibility is limited to 9 km. For how many minutes in total is the ship visible from the lighthouse?