Geometry Terms

You must learn these off by heart, and in some cases, be able to give examples.

TERM	DEFINITION
Axiom	A statement that we accept without any proof
Theorem	A rule that has been proved by following a certain number of logical steps or by using a previous theorem or axiom you already know
Proof	A series of logical steps that we use to prove a theorem
Corollary	A statement that follows readily from a previous theorem
Converse	The reverse of a theorem Eg. **Statement:** *The interior angles of a square each measure 90°* (TRUE) **Converse:** *If the interior angles each measure 90°, then the figure is a square* (FALSE)
Implies	Used in a proof when a statement follows on from previous proved statements **Symbol:** $\Rightarrow$

TERM	DEFINITION
Is equivalent to	Two things are equivalent if they have the same value but different forms eg. $\frac{2}{3}=\frac{4}{6}$ or $2 = €1.50
If and only if	Eg. If and only if means that X will only be true when Y is true and Y will only be true when X is true. An example would be "The light will come on if and only if the switch is in the on position" $\Leftrightarrow$ Can be shortened to iff
Proof by contradiction	A proof where an assumption is made. Then, by using valid arguments, a statement is arrived at which is clearly false, so the original assumption must have been false. We prove that a statement or assumption is true by showing that the statement or assumption being false would imply a contradiction (impossibility).

Prove that $\sqrt{2}$ is irrational

Assume the contrary: √2 is rational

there exists integers p and q with no common factors such that:

$\frac{p}{q}=\sqrt{2}$ *(Square both sides)*

$\Rightarrow \frac{p^2}{q^2}=2$

$\Rightarrow p^2=2q^2$

$\Rightarrow p^2$ *is even* *(......it's a multiple of 2)*

$\Rightarrow p$ *is even* *(......even^2 = even)*

$\therefore p=2k$ for some k

$p^2=4k^2$ *(Square both sides)*

$p^2=2q^2$ *and* $p^2=4k^2$

$\Rightarrow 4k^2=2q^2$ *(Divide both sides by 2)*

$\Rightarrow 2k^2=q^2$

Then similarly $q=2m$ for some m

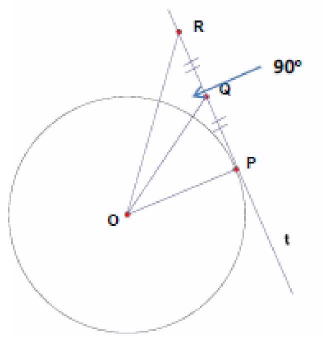
$\Rightarrow \frac{p}{q}=\frac{2k}{2m}\Rightarrow\frac{p}{q}$ has a factor of 2 in common.

This contradicts the original assumption.

√2 is irrational **Q.E.D.**

Theorem 20: Proof by Contradiction

(i) Each tangent is perpendicular to the radius that goes to the point of contact.



Suppose the point of contact is P and the tangent t is not on the perpendicular to OP

Let the perpendicular to the tangent from O meet it at Q.

Pick R on PQ, on the other side of Q from P, with |QR| = |PQ|

Then triangle OQR is congruent to triangle OQP

|OR| = |OP|, so R is a second point where t meets the circle.

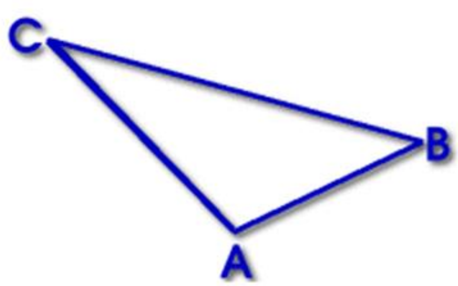
This contradicts the given fact that t is a tangent.

Thus t must be a perpendicular to OP, as required.

Example:

Triangle ABC has no more than one right angle.

Can you complete a proof by contradiction for this statement?



1. Assume ∠A and ∠B are right angles
2. We know ∠A + ∠B + ∠C = 180^0
3. By substitution $90^0 + 90^0$ + ∠C = 180^0
4. ∴ ∠C = 0^0 which is a contradiction
5. ∴ ∠A and ∠B cannot both be right angles
6. ⇒ A triangle can have at most one right angle