## Question 8

(a) When a loan of $€ P$ is repaid in equal repayments of amount $€ A$, at the end of each of $t$ equal periods of time, where $i$ is the periodic compound interest rate (expressed as a decimal), the formula below can be used to find the amount of each repayment.

$$
A=P \frac{i(1+i)^{t}}{\left((1+i)^{t}-1\right)}
$$

Show how this formula is derived. You may use the formula for the sum of a finite geometric series.

(b) Alex has a credit card debt of $€ 5000$. One method of clearing this debt is to make a fixed repayment at the end of each month. The amount of this repayment is $2.5 \%$ of the original debt.
(i) What is the fixed monthly repayment, € $A$, required to pay the debt of $€ 5000$ ?

(ii) The annual percentage rate (APR) charged on debt by the credit card company is $21.75 \%$, fixed for the term of the debt. Find as a percentage, correct to 3 significant figures, the monthly interest rate that is equivalent to an APR of $21.75 \%$.

(iii) Assume Alex pays the fixed monthly repayment, $€ A$, each month and does not have any further transactions on that card. Complete the table below to show how the balance of the debt of $€ 5000$ is reducing each month for the first three months, assuming an APR of $21.75 \%$, charged and compounded monthly.

| Payment number | Fixed monthly payment, $€ A$ | $€ A$ |  | New balance of debt ( $€$ ) |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Interest | Previous balance reduced by ( $€$ ) |  |
| 0 |  |  |  | 5000 |
| 1 |  |  | $42 \cdot 50$ | $4957 \cdot 50$ |
| 2 |  |  |  |  |
| 3 |  |  |  |  |

(iv) Using the formula you derived on the previous page, or otherwise, find how long it would take to pay off a credit card debt of $€ 5000$, using the repayment method outlined at the beginning of part (b) above.
Give your answer in months, correct to the nearest month.

$\square$
(v) Alex decides to borrow $€ 5000$ from the local Credit Union to pay off this credit card debt of $€ 5000$. The APR charge for the Credit Union loan is $8.5 \%$ fixed for the term of the loan. The loan is to be repaid in equal weekly repayments, at the end of each week, for 156 weeks. Find the amount of each weekly repayment.

(vi) How much will Alex save by paying off the credit card debt using the loan from the Credit Union instead of paying the fixed repayment from part (b)(i) each month to the credit card company?

## Question 2

A local sports club is planning to run a weekly lotto. To win the Jackpot of $€ 1000$, contestants must match one letter chosen from the 26 letters in the alphabet and two numbers chosen, in the correct order, from the numbers 0 to 9 . In this lotto, repetition of numbers is allowed (e.g. M, 3, 3 is an outcome).
(a) Calculate the probability that $\mathrm{M}, 3,3$ would be the winning outcome in a particular week.

(b) If a contestant matches the letter only, or the letter and one number (but not both numbers), they will win $€ 50$. Using the table below, or otherwise, find how much the club should expect to make or lose on each play, correct to the nearest cent, if they charge $€ 2$ per play.

| Event | Payout (x) $\quad €$ | Probability $(\mathrm{P}(x))$ | $x . \mathrm{P}(x)$ |
| :--- | :--- | :--- | :--- |
| Win Jackpot |  |  |  |
| Match letter and <br> first number only |  |  |  |
| Match letter and <br> second number only |  |  |  |
| Match letter and <br> neither number |  |  |  |
| Fail to win |  |  |  |


(c) The club estimates that the average number of plays per week will be 845 . If the club wants to make an average profit of $€ 600$ per week from the lotto, how much should the club charge per play, correct to the nearest cent?


Data on earnings were published for a particular country. The data showed that the annual income of people in full-time employment was normally distributed with a mean of $€ 39400$ and a standard deviation of $€ 12920$.
(a) (i) The government intends to impose a new tax on incomes over €60 000 .

Find the percentage of full-time workers who will be liable for this tax, correct to one decimal place.

(ii) The government will also provide a subsidy to the lowest $10 \%$ of income earners. Find the level of income at which the government will stop paying the subsidy, correct to the nearest euro.

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(iii) Some time later a research institute surveyed a sample of 1000 full-time workers, randomly selected, and found that the mean annual income of the sample was $€ 38280$. Test the hypothesis, at the $5 \%$ level of significance, that the mean annual income of full-time workers has changed since the national data were published.
State the null hypothesis and the alternative hypothesis.
Give your conclusion in the context of the question.

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(b) The research institute surveyed 400 full-time farmers, randomly selected from all the full-time farmers in the country, and found that the mean income for the sample was €26 974 and the standard deviation was $€ 5120$.
Assuming that annual farm income is normally distributed in this country, create a $95 \%$ confidence interval for the mean income of full-time farmers.

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(c) It is known that data on farm size are not normally distributed.

The research institute could take many large random samples of farm size and create a sampling distribution of the means of all these samples.
Give one reason why they might do this.

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(d) The research institute also carried out a survey into the use of agricultural land. $n$ farmers were surveyed.
If the margin of error of the survey was $4 \cdot 5 \%$, find the value of $n$.


## Question 6

(a) Donagh is arranging a loan and is examining two different repayment options.
(i) Bank A will charge him a monthly interest rate of $0.35 \%$. Find, correct to three significant figures, the annual percentage rate (APR) that is equivalent to a monthly interest rate of $0.35 \%$.

(ii) Bank B will charge him a rate that is equivalent to an APR of $4 \cdot 5 \%$. Find, correct to three significant figures, the monthly interest rate that is equivalent to an APR of $4.5 \%$.

(b) Donagh borrowed $€ 80000$ at a monthly interest rate of $0 \cdot 35 \%$, fixed for the term of the loan, from Bank A. The loan is to be repaid in equal monthly repayments over ten years. The first repayment is due one month after the loan is issued. Calculate, correct to the nearest euro, the amount of each monthly repayment.


## Question 8

Pádraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65 . First, he calculates how much he wants to have in his pension fund when he retires. Then, he calculates how much he needs to invest in order to achieve this. He assumes that, in the long run, money can be invested at an inflation-adjusted annual rate of $3 \%$. Your answers throughout this question should therefore be based on a 3\% annual growth rate.
(a) Write down the present value of a future payment of $€ 20,000$ in one year's time.

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(b) Write down, in terms of $t$, the present value of a future payment of $€ 20,000$ in $t$ years' time.

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(c) Pádraig wants to have a fund that could, from the date of his retirement, give him a payment of $€ 20,000$ at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value, on the date of retirement, of the fund required.

(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part (c). His retirement is $40 \times 12=480$ months away.
(i) Find, correct to four significant figures, the rate of interest per month that would, if paid and compounded monthly, be equivalent to an effective annual rate of $3 \%$.

(ii) Write down, in terms of $n$ and $P$, the value on the retirement date of a payment of $€ P$ made $n$ months before the retirement date.

(iii) If Pádraig makes 480 equal monthly payments of $€ P$ from now until his retirement, what value of $P$ will give the fund he requires?

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(e) If Pádraig waits for ten years before starting his pension investments, how much will he then have to pay each month in order to generate the same pension fund?

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(a) Niamh has saved to buy a car. She saved an equal amount at the beginning of each month in an account that earned an annual equivalent rate (AER) of 4\%.
(i) Show that the rate of interest, compounded monthly, which is equivalent to an AER of $4 \%$ is $0 \cdot 327 \%$, correct to 3 decimal places.
$\qquad$
(ii) Niamh has $€ 15000$ in the account at the end of 36 months. How much has she saved each month, correct to the nearest euro?

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(b) Conall borrowed to buy a car. He borrowed $€ 15000$ at a monthly interest rate of $0 \cdot 866 \%$. He made 36 equal monthly payments to repay the entire loan. How much, to the nearest euro, was each of his monthly payments?

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## Question 8

Pádraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65 . First, he calculates how much he wants to have in his pension fund when he retires. Then, he calculates how much he needs to invest in order to achieve this. He assumes that, in the long run, money can be invested at an inflation-adjusted annual rate of $3 \%$. Your answers throughout this question should therefore be based on a $3 \%$ annual growth rate.
(a) Write down the present value of a future payment of $€ 20,000$ in one years' time.

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(b) Write down, in terms of $t$, the present value of a future payment of $€ 20,000$ in $t$ years' time.

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(c) Pádraig wants to have a fund that could, from the date of his retirement, give him a payment of $€ 20,000$ at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value on the date of retirement of the fund required.

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(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part (c). His retirement is $40 \times 12=480$ months away.
(i) Find, correct to four significant figures, the rate of interest per month that would, if paid and compounded monthly, be equivalent to an effective annual rate of $3 \%$.

(ii) Write down, in terms of $n$ and $P$, the value on the retirement date of a payment of $€ P$ made $n$ months before the retirement date.

(iii) If Pádraig makes 480 equal monthly payments of $€ P$ from now until his retirement, what value of $P$ will give the fund he requires?

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(e) If Pádraig waits for ten years before starting his pension investments, how much will he then have to pay each month in order to generate the same pension fund?

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