2010 Q10:

(a) $$y\frac{dy}{dx} = x + xy^2 = x(1+y^2)$$

$$\int \frac{y\,dy}{1+y^2} = \int x\,dx$$

$u = 1+y^2$ so $y\,dy = \frac{du}{2}$

$du = 2y\,dy$

$$\int \frac{du}{2u} = \int x\,dx$$

$$\frac{1}{2}\ln u = \frac{x^2}{2} + c$$

$$\ln u = x^2 + k$$

$$\ln(1+y^2) = x^2 + k$$ $y=0$ when $x=0$

$$\ln_e(1) = 0 + k \Rightarrow k = 0$$ since $\ln(1)=0$

$$\ln_e(1+y^2) = x^2$$

$$e^{x^2} = 1+y^2$$

$$\sqrt{e^{x^2}-1} = y$$

2010 Q10 (b.)

$$a = 0.12 - 0.0006v^2 = \frac{v\,dv}{ds} \text{ or } \frac{dv}{dt}$$

$v = 0$ when $t = 0$, $s = 0$

$$0.12 - 0.0006v^2 = \frac{v\,dv}{ds} \Rightarrow \frac{v\,dv =}{(0.12 - 0.0006v^2)} \quad ds$$

$$\int \frac{v\,dv}{0.12 - 0.0006v^2} = \int ds$$

$u = 0.12 - 0.0006v^2$

$du = -0.0012v\,dv$

$$\frac{-du}{0.0012} = v\,dv \Rightarrow \int \frac{-du}{0.0012(u)} = \int ds$$

$$\frac{-1}{0.0012} \ln u = s + c \qquad s = 0 \text{ when } v = 0$$

$$\frac{-1}{0.0012} \ln(0.12 - 0) = c$$

$$\text{so} \quad s = \frac{-1}{0.0012} \ln(0.12 - 0.0006v^2) + \frac{1}{0.0012} \ln(0.12)$$

$$s = \frac{1}{0.0012} \ln\left(\frac{0.12}{0.12 - 0.0006v^2}\right)$$

$$0.0012s = \ln_e\left(\frac{0.12}{0.12 - 0.0006v^2}\right)$$

$$e^{-0.0012s} = \frac{0.12 - 0.0006v^2}{0.12} = 1 - 0.005v$$

$$1 - e^{-0.0012S} = 0.005v^2$$

$$\sqrt{\frac{1 - e^{-0.0012S}}{0.005}} = v = 5.179 m/s$$

(ii) $\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{120}{2.65} = 45.283 s$

2009 Q10 (a.)

$$\frac{dy}{dx} = \frac{1}{xy} + \frac{y}{x} = \frac{1+y^2}{xy}$$

$$\int \frac{y\,dy}{1+y^2} = \int \frac{dx}{x}, \quad \text{let } u = 1+y^2$$

$$du = 2y\,dy$$

$$\frac{du}{2} = y\,dy$$

$$\int \frac{du}{2u} = \int \frac{dx}{x}$$

$$\frac{1}{2}\ln u = \ln x + c$$

$$\frac{1}{2}\ln(1+y^2) = \ln x + c \qquad y=\sqrt{3},\ x=1$$

$$\frac{1}{2}\ln(4) = \underbrace{\ln(1)}_{0} + c \quad \Rightarrow \quad c = \tfrac{1}{2}\ln 4$$

$$c = \ln 4^{1/2}$$

$$\text{or } c = \ln 2$$

$$\frac{1}{2}\ln(1+y^2) = \ln x + \frac{1}{2}\ln 4$$

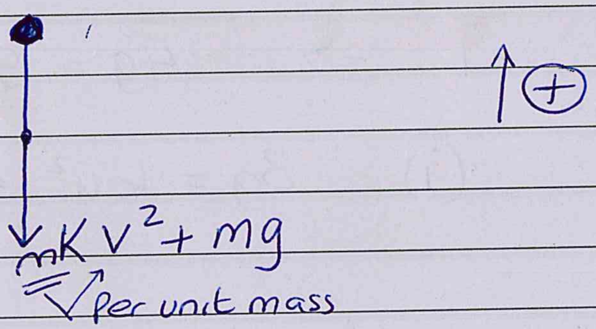
$$\ln(1+y^2) = 2\ln x + \ln 4$$

$$\ln(1+y^2) = \ln x^2 + \ln 4 = \ln 4x^2$$

$$1+y^2 = 4x^2$$

$$y = \sqrt{4x^2-1}$$

2009 Q10 (b.)



$$F = ma$$

$$-mkv^2 - mg = ma$$

$$a = -kv^2 - g = \frac{v\,dv}{ds}$$

$$-\int ds = \int \frac{v\,dv}{+kv^2+g}$$

$$-s + c = \int \frac{v\,dv}{kv^2+g} \qquad \begin{aligned} u &= kv^2+g \\ du &= 2kv\,dv \\ du/2k &= v\,dv \end{aligned}$$

$$-s + c = \int \frac{du}{2ku} = \frac{1}{2k}\ln u = \frac{1}{2k}\ln(kv^2+g)$$

but $V = 0$ when $S = \dfrac{\ln 4}{2k}$

$$-\frac{\ln 4}{2k} + c = \frac{1}{2k}\ln g \Rightarrow c = \frac{1}{2k}(\ln g + \ln 4)$$

$$c = \frac{1}{2k}\ln 4g \Rightarrow S = \frac{-1}{2k}\ln(kv^2+g) + c$$

$$S = \frac{1}{2k}\left[\ln 4g - \ln(kv^2+g)\right] = \frac{1}{2k}\ln\left(\frac{4g}{g+kv^2}\right)$$

When $S = 0$, $V = u$ $\quad$ $S = 0$ when $\dfrac{4g}{g+kv^2} = 1$ $\quad (\ln_e 1 = 0)$

2009 Q10(b) cont'd

So $\dfrac{4g}{g+ku^2} = 1 \Rightarrow 4g = g + ku^2$

$3g = ku^2$

$u = \sqrt{\dfrac{3g}{k}}$

(ii) $a = \dfrac{dv}{dt} = -(g + kv^2)$

REM: Can't use u,v,a,s,t because a is nonlinear

$\displaystyle\int \frac{dv}{g+kv^2} = \int -\,dt$ log tables ...

$=$

$\displaystyle\frac{1}{k}\int \frac{dv}{(\underbrace{g/k}_{a^2} + v^2)} = \frac{1}{k}\left(\frac{1}{\sqrt{g/k}}\,\mathrm{Tan}^{-1}\frac{v}{\sqrt{g/k}}\right) = -t + c$

$V = 0$ when $t = \pi/3$

$\dfrac{1}{k}\cdot\dfrac{1}{\sqrt{g/k}}\,\mathrm{Tan}^{-1} 0 = -\dfrac{\pi}{3} + c \Rightarrow c = \dfrac{\pi}{3}$

So $\dfrac{1}{k}\left(\dfrac{1}{\sqrt{g/k}}\,\mathrm{Tan}^{-1}\dfrac{v}{\sqrt{g/k}}\right) = -t + \pi/3$ but $V = u$ when $t = 0$.

$\dfrac{1}{k}\left(\sqrt{\dfrac{k}{g}}\,\mathrm{Tan}^{-1}\sqrt{\dfrac{\cancel{k}}{g}}\cdot\sqrt{\dfrac{3g}{\cancel{k}}}\right) = 0 + \dfrac{\pi}{3}$

$\dfrac{1}{\sqrt{kg}}\,\mathrm{Tan}^{-1}\sqrt{3} = \dfrac{\pi}{3}$, $\mathrm{Tan}^{-1}\sqrt{3} = \dfrac{\pi}{3}\cdot\sqrt{kg}$

$\mathrm{Tan}^{-1}\sqrt{3} = \dfrac{\pi}{3}$ so $\dfrac{\pi}{3} = \dfrac{\pi}{3}\sqrt{kg} \Rightarrow \sqrt{kg} = 1$

$kg = 1$

$k = 1/g$

2008 Q10 (a)

$$x^2y\frac{dy}{dx} + y\frac{dy}{dx} = 1 \qquad y=0,\ x=0$$

$$y\frac{dy}{dx}\left(x^2+1\right) = 1$$

$$\int y\,dy = \int \frac{dx}{1+x^2}$$

$$\frac{y^2}{2} = \text{Tan}^{-1}x + c \qquad y=0,\ x=0$$

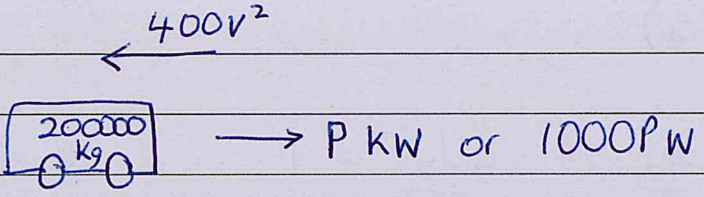
$$0 = \text{Tan}^{-1}0 + c \Rightarrow c = 0$$

$$\frac{y^2}{2} = \text{Tan}^{-1}x \Rightarrow \text{Tan}\frac{y^2}{2} = x$$

$$\text{Tan}\left(\frac{\pi}{2}\Big/2\right) = x = \text{Tan}\frac{\pi}{4} = 1 \qquad \text{Ans}: x = 1$$

2008 (b) Q10

(1)



$$a = \frac{8000 - v^3}{500v}$$

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Force} \times \text{distance}}{\text{time}} = \overset{\text{Engine}}{\text{Force}} \times \text{velocity}$$

$$= F_E v$$

$$\text{so } \frac{1000P}{v} = F_E$$

$$F = ma = \frac{1000P}{v} - 400v^2$$

$$\frac{1000P}{v} - 400v^2 = 200{,}000\left(\frac{8000 - v^3}{500v}\right)$$

$$1000P - 400v^3 = 200{,}000\frac{(8000 - v^3)}{500} = 400(800$$

$$1000P - 400v^3 = 3200000 - 400v^3$$

$$P = \frac{3200\cancel{000}}{1\cancel{000}} = 3200$$

So Power = 3200kW.

2008 (b)(ii)

$s=0,\ t=0,\ v=10\,m/s$ $\qquad$ $s=69.07,\ V=V_1$

$$a = \frac{8000 - v^3}{500v}$$

$$\frac{v\,dv}{ds} = \frac{8000 - v^3}{500v}$$

$$\int \frac{500v^2\,dv}{8000 - v^3} = \int ds \qquad \text{let } u = 8000 - v^3$$

$$du = -3v^2\,dv$$

$$-\frac{du}{3} = v^2\,dv$$

$$00\int -\frac{du}{3} \cdot \frac{1}{u} = \int ds$$

$$-\frac{500}{3} \ln u = s + c$$

$$-\frac{500}{3} \ln (8000 - v^3) = s + C \quad - \quad s = 0,\ v = 10$$

$$-\frac{500}{3} \ln (7000) = c$$

$$s = +\frac{500}{3} \ln(7000) - \frac{500}{3} \ln(8000 - v^3)$$

$$s = \frac{500}{3} \ln \left(\frac{7000}{8000 - v^3} \right), \quad \frac{3s}{500} = \ln_e \frac{7000}{8000 - v^3}$$

$$e^{-\frac{3s}{500}} = \frac{8000 - v^3}{7000}, \quad 7000\,e^{-3s/500} = 8000 - v$$

$$v^3 = 8000 - 7000\,e^{-3s/500} \qquad (v = V_1,\ s = 69.07)$$

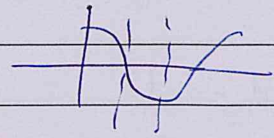
$$V_1 = \sqrt[3]{8000 - 7000\,e^{-3(69.07)/500}} = 15\,!$$

2007 Q10 (a.)

$$\frac{dy}{dx} = y^2 \operatorname{Sin} x$$

$$\int \frac{dy}{y^2} = \int dx \operatorname{Sin} x$$

$$\frac{-1}{y} = -\operatorname{Cos} x + c \qquad y = 1,\ x = \frac{\pi}{2}$$



$$-1 = -\operatorname{Cos}(\pi/2) + c$$

$$-1 = c$$

$$\frac{-1}{y} = -\operatorname{Cos} x - 1$$

$$\frac{1}{y} = \operatorname{Cos} x + 1$$

$$y = \frac{1}{\operatorname{Cos} x + 1}$$

2007 Q10 (b) $a = 1 - \frac{v^2}{3200}$

i)

$a = \frac{3200 - v^2}{3200} = v\frac{dv}{ds}$

$\int \frac{ds}{3200} = \int \frac{v\,dv}{3200 - v^2}$ $u = 3200 - v^2$

$du = -2v\,dv \Rightarrow -du/2 = v\,dv$

$\frac{s}{3200} + c = \int -\frac{du}{2u} = -\frac{1}{2}\ln u = -\frac{1}{2}\ln(3200 - v^2)$

$\frac{s}{3200} + c = -\frac{1}{2}\ln(3200 - v^2)$

$s + K = -1600\ln(3200 - v^2)$

$v = 0,\ s = 0$

$K = -1600\ln(3200)$

$s = +1600\ln 3200 - 1600\ln(3200 - v^2)$

$s = 1600\ln_e\left(\frac{3200}{3200 - v^2}\right)$

$e^{s/1600} = \frac{3200}{3200 - v^2} \Rightarrow e^{-s/1600} = \frac{3200 - v^2}{3200}$

$\sqrt{3200\left(1 - e^{-s/1600}\right)} = v$

$= 44 \cdot 123$ m/s

(ii) max speed ?

$v = \sqrt{3200\left(1 - e^{-s/1600}\right)}$ is a max when?

when accel gets to zero rem $a = \frac{3200 - v^2}{3200}$

so when $3200 = v^2$

$\sqrt{3200} = v = 56 \cdot 57$

2006 $\dfrac{dy}{dx} = \dfrac{xy}{1+x}$

$\displaystyle\int \frac{dy}{y} = \int \frac{x\,dx}{1+x}$ $u = 1+x$ $dx = du$

$u-1 = x$

$\ln y = \displaystyle\int \left(\frac{u-1}{u}\right) du = \int \left(1 - \frac{1}{u}\right) du$

$\ln y = u - \ln u + c$ $y = e$ when $x = 0$

$u = 1+x$

$\ln_e e = 1 + 0 - \ln(1+0) + c$

$1 = 1 - \ln 1 + c$ $(\ln 1 = 0)$

$1 = 1 + c \Rightarrow c = 0$

so $\ln y = 1+x - \ln 1+x$

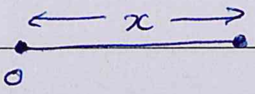
$\ln y + \ln(1+x) = 1+x$

$\ln_e y(1+x) = 1+x$

$e^{1+x} = y(1+x) \Rightarrow y = \dfrac{e^{1+x}}{1+x}$

2006 Q10 (b.)

(i) $a = \frac{1}{x^3}$



$$a = \frac{1}{x^3} = v\frac{dv}{dx}$$

$$\int \frac{dx}{x^3} = \int v\,dv$$

$$-\frac{1}{2x^2} = \frac{v^2}{2} + c \qquad x = 1 \text{ when } v = 0$$

$$-\frac{1}{2} = c$$

$$-\frac{1}{2x^2} = \frac{v^2}{2} - \frac{1}{2} \qquad \times 2 \qquad -\frac{1}{x^2} = v^2 - 1$$

$$v^2 = 1 - \frac{1}{x^2} \qquad \text{when } x = \frac{4}{3}$$

$$v^2 = 1 - \frac{9}{16} = \frac{7}{16} \qquad v = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$$

(ii) $$v = \frac{dx}{dt} = \sqrt{1 - \frac{1}{x^2}} = \sqrt{\frac{x^2-1}{x^2}} = \frac{\sqrt{x^2-1}}{x}$$

$$\text{so} \int \frac{x\,dx}{\sqrt{x^2-1}} = \int dt \qquad u = (x^2-1) \quad du = 2x\,dx \quad du/2 = x\,d$$

$$\int \frac{du}{2\sqrt{u}} = t + c \qquad \sqrt{u} = t + c = \sqrt{x^2-1}$$

$x = 1, t = 0 \Rightarrow c = 0$ so when $x = 2$

$$t = \sqrt{x^2-1} = \sqrt{3}$$

2005 Q10 (a) $\quad x\frac{dy}{dx} - xy - y = 0$

$$x\frac{dy}{dx} = y(1+x)$$

$$\int \frac{dy}{y} = \int \frac{(1+x)}{x}\,dx = \int \left(1 + \frac{1}{x}\right) dx$$

so $\quad \ln y = x + \ln x + c$

$y = 1$ when $x = 1$

$$\ln 1 = 1 + \ln 1 + c$$

$$0 = 1 + c \quad \Rightarrow \quad c = -1$$

So $\quad \ln y = x - 1 + \ln x$

$$\ln y - \ln x = x - 1$$

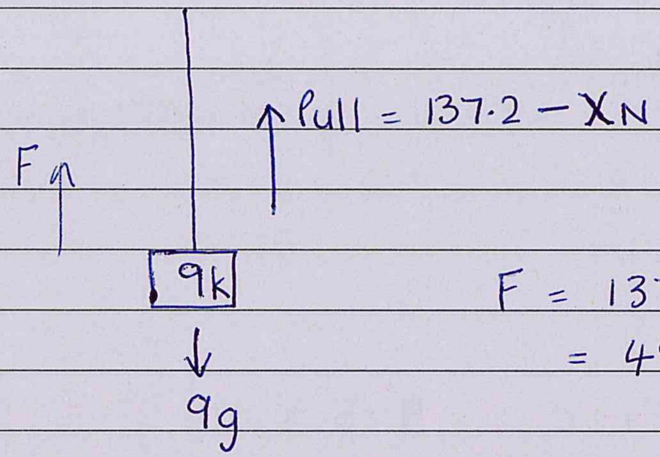
$$\ln_e \frac{y}{x} = x - 1$$

$$\frac{y}{x} = e^{x-1}$$

$$y = xe^{x-1}$$

2005 Q10 (b.)

(i)



$$F = 137.2 - x - 9g$$
$$= 49 - x$$

(ii) $F = ma$ so $a = \dfrac{F}{m} = \dfrac{49-x}{9} = \dfrac{v\,dv}{dx}$

$$\frac{v\,dv}{dx} = \frac{49-x}{9}$$

$$\int v\,dv = \int \left(\frac{49-x}{9}\right) dx \Rightarrow \frac{v^2}{2} + C = \frac{49x - \frac{x^2}{2}}{9}$$

×2 $\quad v^2 + k = \dfrac{98x - x^2}{9}$ $\qquad v = 0$ when $x = 0$

$0 + k = 0 \Rightarrow k = 0$

$$v^2 = \frac{98x - x^2}{9} \Rightarrow v = \sqrt{\frac{98x - x^2}{9}}$$

so when $x = 15$ $\quad v = 11.76\,m/s$

Work done = gain in energy

$= \frac{1}{2}m(11.76)^2 - \frac{1}{2}m(0)^2 = \frac{1}{2}(9)(11.76)^2 = 622.3\,J$

622.5 if you use $v^2 = \sqrt{\dfrac{98(15) - 15^2}{9}}$

(not sure why P.E. not counted here)

2004 : Q10 (a) $\quad x^2 \dfrac{dy}{dx} - y + 4 = 0 \qquad y = 5, x=$

$$x^2 \frac{dy}{dx} = y - 4$$

$$\int \frac{dy}{y-4} = \int \frac{dx}{x^2}$$

$$\ln(y-4) = -x^{-1} + c \qquad y = 5, x = 1$$

$$\ln(5-4) = -1 + c$$

$$0 = -1 + c$$

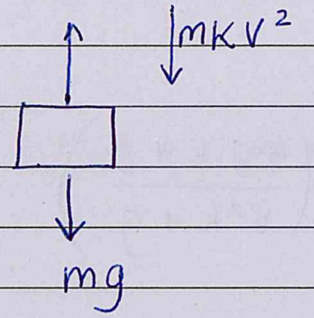
$$c = 1$$

so $\ln_e(y-4) = -\dfrac{1}{x} + 1$

$$y - 4 = e^{1 - 1/x}$$

$$y = 4 + e^{1 - 1/x}$$

2004 Q10 (b.)



$\uparrow \oplus$

$v = 2g$, $s = 0$

$F = -mkv^2 - mg$

$a = F/m$ so $a = -(kv^2 + g) = v\dfrac{dv}{ds}$

(i) $\displaystyle\int \frac{v\,dv}{kv^2+g} = -\int ds$

let $u = kv^2 + g$

$du = 2kv\,dv$

$du/2k = v\,dv$

$\displaystyle\int \frac{du}{2k\,u} = -s + c$

$\dfrac{1}{2k}\ln u = -s + c = \dfrac{1}{2k}\ln(kv^2 + g)$ $\quad v = 2g, s = 0$

$\dfrac{1}{2k}\ln(\cancel{4}4g^2k + g) = c$

$s = c - \dfrac{1}{2k}\ln(kv^2+g) = \dfrac{1}{2k}\left[\ln(4g^2k + g) - \ln(kv^2+g)\right]$

$s = \dfrac{1}{2k}\ln\left(\dfrac{4g^2k+g}{v^2k+g}\right)$, max height is when $v=0$:

$s_{MH} = \dfrac{1}{2k}\ln\left(\dfrac{4g^2k+g}{g}\right) = \dfrac{1}{2k}\ln\left(4gk+1\right)$

(ii) $v = g$ when $s = \frac{1}{4k} \ln(1+4kg)$ ($\frac{1}{2}$ max heigh

From (i) $S = \frac{1}{2k} \ln\left(\frac{4g^2k+g}{v^2k+g}\right)$

subbing in:

$$\frac{1}{4k}\ln(1+4kg) = \frac{1}{2k}\ln\left(\frac{4g^2k+g}{g^2k+g}\right) = \frac{1}{2k}\ln\left(\frac{4gk+1}{gk+1}\right)$$

$$\frac{1}{2k}\ln\sqrt{1+4kg}) = \frac{1}{2k}\ln\frac{(4gk+1)}{gk+1}$$

$$\sqrt{1+4kg} = \frac{1+4kg}{1+gK} \quad \text{sqr both sides}$$

$$1+4kg = \frac{1+8kg+16k^2g^2}{1+2kg+k^2g^2} \quad \text{cross x}$$

$$\cancel{1} + 2kg + k^2g^2 + 4kg + 8k^2g^2 + 4k^3g^3 = \cancel{1} + 8kg + 16k^2g$$

$\div kg$

$$2 + kg + 4 + 8kg + 4k^2g^2 = 8 + 16kg$$

$$4k^2g^2 - 7kg - 2 = 0 \qquad 4x^2 - 7x - 2 = 0$$

$$(4x+1)(x-2) = 0$$

$4x = 1 \Rightarrow x$ is $-$ not possible since $k > 0$

$x = 2$ = ans.

$kg = 2$

$k = \frac{2}{g}$

2003 Q10 (a.) $\frac{dy}{dx} = \frac{xy}{2x^2-3}$

$\int \frac{dy}{y} = \int \frac{x\,dx}{2x^2-3}$ $\quad u = 2x^2-3,\ du = 4x\,dx$

$du/4 = x\,dx$

$\ln y + c = \int \frac{du}{4u} = \frac{1}{4}\ln u$

$\ln y + c = \frac{1}{4}\ln(2x^2-3)$ $\quad y = 1,\ x = \sqrt{2}$

$\ln 1 + c = \frac{1}{4}\ln(4-3) = 0$ since $\ln 1 = 0$

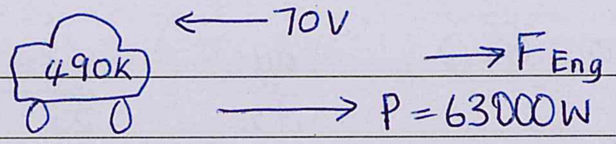
So $c = 0$

$\ln y = \frac{1}{4}\ln(2x^2-3) = \ln(2x^2-3)^{1/4}$

So $y = (2x^2-3)^{1/4}$

2003

Q10 (b.) (i)



$$P = \frac{W}{t} = \frac{F_{Eng}\,d}{t} = F_{Eng}v \quad \text{so} \quad F_{Eng} = \frac{P}{v}$$

$$\text{so overall force} = F_{eng} - 70v = \frac{63000}{v} - 70v$$

equation of motion: $F = ma$

$$\frac{63000}{v} - 70v = 490.a = 490\frac{dv}{dt} \qquad \text{since } a = \frac{dv}{dt}$$

$$\frac{63000 - 70v^2}{v} = 490\frac{dv}{dt} \qquad \div 70$$

$$\frac{900 - v^2}{v} = 7\frac{dv}{dt}$$

(ii) $$\frac{dv}{dt} = \frac{900 - v^2}{7v} \Rightarrow \int \frac{7v\,dv}{900 - v^2} = \int dt$$

let $u = 900 - v^2$, $du = -2v\,dv$, $v\,dv = -du/2$

$$-\int \frac{7\,du}{2\,u} = \int dt$$

$$-\frac{7}{2}\ln u = t + c = \frac{7}{2}\ln(900 - v^2)$$

find c, $v = 10$
find t_2 $v = 20$
ans $= t_2$

$$-\frac{7}{2}\ln(900 - 100) = c = -\frac{7}{2}\ln(800)$$

$$\text{So } t = \frac{7}{2}\left[\ln(800) - \ln(900 - v^2)\right] = \frac{7}{2}\ln\left(\frac{800}{900 - v^2}\right)$$

$$\text{when } v = 20 \quad t = \frac{7}{2}\ln\left(\frac{800}{900 - 400}\right) = 1{\cdot}65$$

2002 : Q10 (a) $\frac{dy}{dx} = e^{x-y} = \frac{e^x}{e^y}$

$\int e^y\,dy = \int e^x\,dx$

$e^y = e^x + c$ $\quad$ $y = \ln 4$ when $x = 0$

$e^{\ln 4} = e^0 + c = 1 + c$

$4 = 1 + c \Rightarrow c = 3$

So $e^y = e^x + c$

or $\ln(e^x + c) = y$

(b.) (i) $\frac{dv}{dt} = 100 - v$ $\quad$ $\int \frac{dv}{100-v} = \int dt$

$u = 100 - v$, $du = -dv$, $-du = dv$

$\int \frac{-du}{100-v} = t + c = \int \frac{-du}{u} = -\ln u$

(easiest is to use relative time)

$-\ln(100 - v) = t + c$ $\quad$ $t = 0,\ v = 25$

$-\ln(100 - 25) = 0 + c \Rightarrow c = -\ln 75$

$-\ln(100 - v) = t - \ln 75$

$t = \ln 75 - \ln 100 - v \Rightarrow \ln \frac{75}{100 - v} = t$ (1)

now find t when $v = 75$

$t = \ln \frac{75}{100 - 75} = \ln \frac{75}{25} = \ln 3$

(ii) $\dfrac{v\,dv}{ds} = 100 - v$

$$\int \frac{v\,dv}{100-v} = \int ds$$

$u = 100 - v$

$du = -dv$, $dv = -du$

$v = 100 - u$

$$-\int \frac{(100-u)\,du}{u} = \int ds = \int \left(1 - \frac{100}{u}\right) du$$

$$s + c = u - 100 \ln u$$

$$s + c = 100 - v - 100 \ln(100 - v)$$

$s = 0$, $v = 0$

$$c = 100 - 100 \ln 100$$

$$s + \cancel{100} - 100 \ln 100 = \cancel{100} - v - 100 \ln(100 - v)$$

$$s = 100 \ln 100 - 100 \ln 100 - v - v$$

$$s = 100 \ln\left(\frac{100}{100-v}\right) - v$$

need to find s when $v = 75$

$$s = 100 \ln\left(\frac{100}{100-75}\right) - 75$$

$$s = 100 \ln\left(\frac{100}{25}\right) - 75 = 100 \ln 4 - 75$$

NOTE: Looks like we Can also do by rearranging ① to get V expression for then let $V = ds/dt$ and integrate but you won't have a condition for finding c because when $t = 0$ we don't know what s is (rem we said $t=0$ was when $v = 25$) avoid having to work out t_1 $(v=25)$ t_2 $(v=75)$ and $t_1 - t_2$

If we had done this way we could have used this method to find S_{75}