L.17

NAME

SCHOOL

TEACHER

Pre-Leaving Certificate Examination, 2019

Mathematics

Paper 1

Higher Level

Time: 2 hours, 30 minutes

300 marks

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

School stamp

Running total	

Grade

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) $2 - i$ is a root of $z^3 - kz^2 + 21z - 20 = 0$, where $k \in \mathbb{R}$ and $i^2 = -1$.

(i) Find the value of k.

(ii) Find the other two roots of the equation.

(b) Find $\sqrt{-1 - \sqrt{3}i}$. Give your answers in the form $a + bi$, where $a, b \in \mathbb{R}$.

Question 2 **(25 marks)**

(a) Solve the equation $\sqrt{2x-1}+\sqrt{x-1}=5$ and verify your answers.

(b) Solve the simultaneous equations:

$$\log(x+y) = 2\log x$$
$$\log y = \log 2+\log(x-1), \text{ where } x>1, y>0.$$

Question 3 **(25 marks)**

(a) **(i)** Solve the equation $|x+2| = |x-13|$.

(ii) Hence solve $|3^y+2| = |3^y-13|$.
Give your answer correct to three significant figures.

(b) Find the range of values of k for which the equation $k(x^2+1)-1 = x(x+6)$ has real roots.

Question 4 **(25 marks)**

(a) A circular disc is divided into sectors whose acute angles form an arithmetic series.
The first two sectors have acute angles of $3°$ and $5°$, respectively.
Find the number of sectors in the disc.

(b) The sequence $u_1, u_2, u_3, \ldots$, is defined by $u_1 = 4$ and $u_{n+1} = (-1)^n u_n + 3$.

(i) Find the values of u_2, u_3, u_4, u_5 and u_6.

(ii) Find S_{2019}, the sum of the first 2019 terms.

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Question 5

(25 marks)

(a) Differentiate the function $-2x^2 + 3x - 7$ from first principles, with respect to x.

(b) **(i)** Given that $y = e^x \cos kx$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(ii) Hence find the values of k for which $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.

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Question 6 **(25 marks)**

Tara is planning for her retirement and wishes to calculate the value of the pension fund she requires when she retires. She wants to have a pension fund that could, from one month after the date of her retirement, give her a monthly payment of €2000 for 25 years, assuming the fund will earn an annual equivalent rate (AER) of 3·5% over the period of the payments.

(a) **(i)** Show that the rate of interest, compounded monthly, which is equivalent to an AER of 3·5% is 0·287%, correct to three decimal places.

(ii) Find, correct to the nearest euro, the value of the pension fund that Tara requires when she retires.

(b) Tara decides that she wants the pension fund to give her a fixed monthly payment indefinitely. Assuming the fund will continue to earn an AER of 3·5%, find the amount of each payment, correct to the nearest cent.

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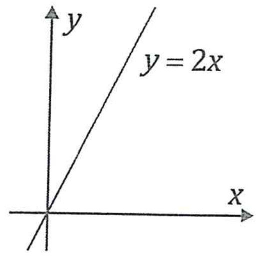
Section B	**Contexts and Applications**	**150 marks**

Answer **all three** questions from this section.

Question 7 **(50 marks)**

(a) A rectangle has four vertices, two of which are on the positive x-axis at x_1 and x_2, and the other two, (x_1, y) and (x_2, y), are on the lines $y = 2x$ and $y = -3x + 6$, respectively.

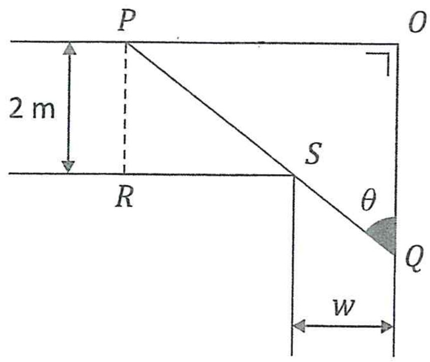


(i) Find x_1 and x_2 in terms of y.

(ii) Find the area of the rectangle in terms of y.

(iii) Hence find the maximum possible area of the rectangle and justify your answer.

(b) A corridor of width 2 m turns 90° into another corridor, of width w, as shown. A ladder, PQ, of length 5 m, is to be carried horizontally around the corner. Let $|\angle PQO| = \theta$.



(i) Find $|RS|$ in terms of $\tan\theta$.

(ii) Find $|PO|$ in terms of θ.

(iii) Hence find the width of the second corridor, w, in terms of θ.

(iv) Find, in metres, the minimum width of the second corridor that will allow the ladder to be carried horizontally around the corner. Give your answer correct to two decimal places.

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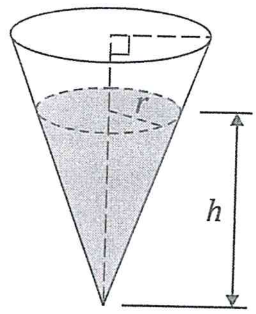
Question 8

(50 marks)

(a) A disposable water paper cup is in the shape of an inverted right circular cone, as shown. The height of the paper cup is 80 mm and the diameter is 50 mm.



The cup is filled with water to a depth of h mm. However, the cup is leaking from its vertex at a constant rate of 8 mm^3 per second.

(i) Given that r is the radius of the free surface of the water in the paper cup when the depth is h, express h in terms of r.

(ii) Find the volume of water in the paper cup in terms of r.

(iii) Find the rate at which the **radius** of the free surface of the water is decreasing when the depth is 48 mm. Give your answer in terms of π.

(iv) Hence find the rate at which the **area** of the free surface of the water is decreasing at the same depth.

(b) The number of people infected by a viral infection in a town can be modelled by the function:

$$N = \frac{20000e^{2t}}{199 + e^{2t}}$$

where t is the time in days from when the infection first appears.

(i) How many people are infected initially?

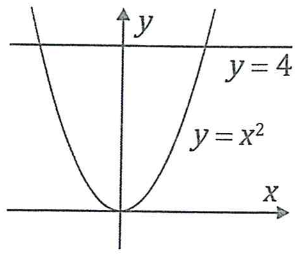
(ii) Find an expression for t in terms of N and hence deduce the range of values for N.

(iii) By finding $\frac{dt}{dN}$, show that $\frac{dN}{dt}$, the rate at which the infection is increasing, can be written as $\frac{N(20000 - N)}{10000}$.

(iv) Hence find the number of people infected when the infection is spreading most rapidly.

Question 9 **(50 marks)**

(a) The area bounded by the curve $y = x^2$ and the line $y = 4$ is divided into two regions of equal area by the line $y = a$, where $0 < a < 4$.



(i) Use the trapezoidal rule to estimate the area enclosed between the curve and the line $y = 4$.

(ii) Find the actual area enclosed between the curve and the line and hence find the percentage error in your answer to part **(a)(i)**.

(iii) Hence show that $a^3 = 16$.

(b) A plant is kept at a constant temperature in a greenhouse. The rate of growth of the plant depends only on the length of the day and can be modelled using the function:

$$\frac{dh}{dt} = k\left[12 + 3\cos\left(\frac{2\pi}{365}t\right)\right]$$

where $h(t)$ is the height of the plant, measured in cm, t is the number of days after 21 June (i.e. $t = 0$), k is a constant and $\left(\frac{2\pi}{365}t\right)$ is expressed in radians.

(i) The height of the plant on 21 June is 84 cm and the height on 4 September (75 days later) is 91·5 cm.
Use integration to show that $k = 0{\cdot}007$, correct to three decimal places.

(ii) Find the expected height of the plant after one year (365 days).

This question continues on the next page

(iii) Find the average height of the plant over the period of one year.
Give your answer in cm, correct to two decimal places.