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Question 1 (2017)

(a)	$A(0,6) \to G\left(\frac{2}{3},\ \frac{4}{3}\right)$ $\to P\left(\frac{2}{3}+\frac{1}{2}\left(\frac{2}{3}\right), \frac{4}{3}+\frac{1}{2}\left(\frac{-14}{3}\right)\right)$ $=\left(\frac{3}{3},\ -\frac{3}{3}\right)$ $P=(1,-1)$ **or** $P=(x,y)$ $\left(\frac{2x+1(0)}{3},\frac{2y+6}{3}\right)=\left(\frac{2}{3},\frac{4}{3}\right)$ $x=1,\quad y=-1$ **or** $P=(x,y)$ $\left(\frac{3(\frac{2}{3})-1(0)}{3-1},\frac{3\left(\frac{4}{3}\right)-1(6)}{3-1}\right)$ $=\left(\frac{2}{2},\frac{-2}{2}\right)=(1,-1)$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • $P\left(\frac{4}{3},\ -\frac{10}{3}\right)$ or equivalent, i.e ratio 1:1 • $\frac{2}{3}$ or $\frac{1}{3}$ identified as part of change in x ordinate • $-\frac{14}{3}$ or $-\frac{7}{3}$ identified as part of change in y ordinate • Ratio formula with some substitution *High Partial Credit:* • one relevant co-ordinate of P found
(b)	$C(4,2) \to P(1,\ -1) \to B(1-3,-1-3)$ $=(-2,\ -4)$ $B(x,y) \to \left(\frac{4+x}{2},\frac{2+y}{2}\right)=(1,-1)$ $x=-2,\quad y=-4$ $B=(-2,-4)$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • P as mid-point of BC *High Partial Credit:* • one relevant co-ordinate of B found ***Note:*** Accept (–2, –4) without work Accept correct graphical solution

(c)		
	$AC \perp BC$ $AC = \frac{2-6}{4-0} = -1$ $BC = \frac{2+4}{4+2} = 1$ $-1 \times 1 = -1$ lines are perpendicular **or** $\text{Slope AB} = 5.$ $\text{Altitude from C}: y - 2 = -\frac{1}{5}(x-4)$ $\rightarrow x + 5y = 14 \ldots.(\text{i}).$ $\text{Slope AC} = -1.$ $\text{Altitude from B}:$ $y + 4 = 1(x+2)$ $\rightarrow x - y = 2 \ldots\ldots(\text{ii})$ $\rightarrow \text{ Solving (i)and (ii)}$ $x = 4$ $y = 2$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • Identifies significance of right-angled triangle • one equation of perpendicular from vertex to opposite side found *High Partial Credit:* • slope of AC and slope of BC found but no conclusion • two equations of perpendiculars from vertex to opposite side found

<table>
<tr><th>Q1</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$\text{Slope } AC = -\frac{2}{3}$
$\text{perp. slope} = \frac{3}{2}$
$y - 3 = \frac{3}{2}(x - 5)$

$3x - 2y = 9$</td><td>Scale 10C (0, 3, 7, 10)
Low Partial Credit
• slope formula with some relevant substitution
• 3 = 5m+c
• $y - y_1 = m(x - x_1)$ with $x_1$or y_1 or both substituted

High Partial Credit
• perpendicular slope
• equation of line through B parallel to AC</td></tr>
<tr><td>(b)</td><td>Point of intersection of the altitudes
$\text{Slope } AB = \frac{3+2}{5-6} = -\frac{5}{1}$
$\text{perp. slope} = \frac{1}{5}$
$y - 4 = \frac{1}{5}(x + 3)$
$x - 5y + 23 = 0$

Orthocentre:
$3x - 2y = 9 \cap x - 5y = -23$

$\Rightarrow y = 6 \quad x = 7$
$(7, 6)$

or

If BC chosen:
$\text{Slope } BC = \frac{3-4}{5+3} = -\frac{1}{8}$

$\text{perp. slope} = 8$
Equation of altitude: $y + 2 = 8(x - 6)$
Equation: $8x - y = 50$
Orthocentre:
$3x - 2y = 9 \cap 8x - y = 50$

$\Rightarrow y = 6 \quad x = 7$
$(7, 6)$</td><td>Scale 15D (0, 4, 7,11,1 5)
Low Partial Credit
• demonstration of understanding of orthocentre (e.g. mentions altitude)
• slope formula with some relevant substitution
• altitude from part (a)

Mid Partial Credit
• equation of an altitude other than (a)
• some relevant substitution towards finding a second altitude and altitude from (a)
• correct construction

High Partial Credit
• two correct altitudes
• correct construction with orthocentre (7, 6)</td></tr>
</table>

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$y-6=\frac{1}{7}(x+1)$ $x-7y+43=0$	Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • equation of line formula with some relevant substitution *High Partial Credit:* • equation of line not in required form
(b)	$D=\frac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ $D=\frac{\lvert 3(-g)+4(-f)-21\rvert}{\sqrt{3^2+4^2}}$ $25=\lvert -3g-4f-21\rvert$ $-3g-4f-21=\pm 25$ $\Longrightarrow 3g+4f=-46$ ... (i) and $3g+4f=4$ ... (ii) But $(-g,-f)\in x-7y+43=0$ $\Rightarrow -g+7f+43=0$ ... (iii) $\Rightarrow g=7f+43$ Solving : $g=7f+43$ and $3g+4f=-46$ $f=-7$ and $g=-6$ Centre (6, 7) $(x-6)^2+(y-7)^2=25$ **or** Solving: $g=7f+43$ and $3g+4f=4$ $f=-5$ and $g=8$ Centre (- 8, 5) $(x+8)^2+(y-5)^2=25$	Scale 15D (0, 4, 7,11,1 5) *Low Partial Credit* • some correct substitution into relevant formula (line, circle, perpendicular distance). *Mid Partial Credit* • one relevant equation in *g* and *f* • (either(i) or (ii) or (iii)) *High Partial Credit* • two relevant equations (either (i) and (iii) or (ii) and (iii))

(a) The co-ordinates of two points are $A(4,\ -1)$ and $B(7,\ t)$.

The line $l_1 : 3x-4y-12=0$ is perpendicular to AB. Find the value of t.

$$\text{Slope } AB=\frac{t+1}{7-4}=\frac{t+1}{3} \qquad \text{Slope } l_1=\frac{3}{4}$$

$$AB\perp l_1 \Rightarrow \frac{t+1}{3}\times\frac{3}{4}=-1\Rightarrow t+1=-4\Rightarrow t=-5$$

or

$$AB: 4x+3y+c=0$$
$$(4,-1)\in 4x+3y+c=0\Rightarrow 16-3+c=0\Rightarrow c=-13$$
$$\therefore 4(7)+3(t)-13=0\Rightarrow t=-5$$

(b) Find, in terms of k, the distance between the point $P(10,\ k)$ and l_1.

$$d=\left|\frac{3(10)-4k-12}{\sqrt{3^2+4^2}}\right|=\left|\frac{18-4k}{5}\right|$$

(c) $P(10,\ k)$ is on a bisector of the angles between the lines l_1 and $l_2 : 5x+12y-20=0$.

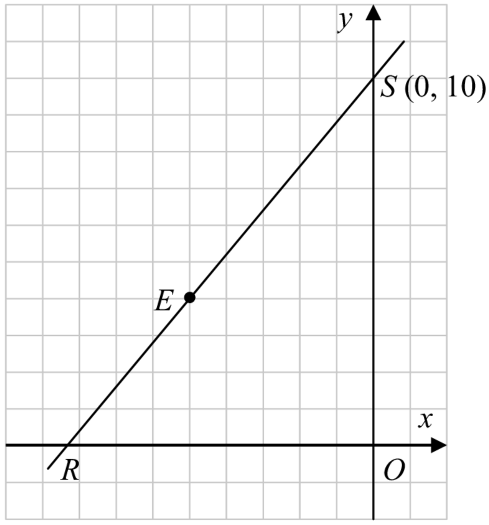
(i) Find the possible values of k.

$$\left|\frac{18-4k}{5}\right|=\left|\frac{50+12k-20}{\sqrt{5^2+12^2}}\right|$$
$$\Rightarrow\left|\frac{18-4k}{5}\right|=\left|\frac{30+12k}{13}\right|$$
$$\Rightarrow 13(18-4k)=\pm5(30+12k)$$
$$\Rightarrow -112k=-84 \quad \text{or} \quad 8k=-384$$
$$\Rightarrow k=\tfrac{3}{4} \quad \text{or} \quad k=-48$$

(ii) If $k>0$, find the distance from P to l_1.

$$k=\frac{3}{4}\Rightarrow d=\left|\frac{18-4\left(\frac{3}{4}\right)}{5}\right|=3$$

The line RS cuts the x-axis at the point R and the y-axis at the point $S(0, 10)$, as shown. The area of the triangle ROS, where O is the origin, is $\frac{125}{3}$.



(a) Find the co-ordinates of R.

Area $ROS = \frac{1}{2}|RO|.|OS| = \frac{125}{3}$
$\Rightarrow \frac{1}{2}|RO|(10) = \frac{125}{3}$
$\Rightarrow |RO| = \frac{25}{3}$
$R\left(-\frac{25}{3}, 0\right)$

(b) Show that the point $E(-5, 4)$ is on the line RS.

Slope RS $= \dfrac{10-0}{0+\frac{25}{3}} = \dfrac{6}{5}$ Slope ES $= \dfrac{10-4}{0+5} = \dfrac{6}{5}$ Slope ER $= \dfrac{4-0}{-5+\frac{25}{3}} = \dfrac{6}{5}$

Any two slopes correct $\Rightarrow (-5, 4) \in RS$

Or

RS: $y - 10 = \frac{6}{5}(x-0) \Rightarrow 6x - 5y + 50 = 0$

$6(-5) - 5(4) + 50 = -30 - 20 + 50 = 0 \Rightarrow (-5, 4) \in RS$

(c) A second line $y = mx + c$, where m and c are positive constants, passes through the point E and again makes a triangle of area $\frac{125}{3}$ with the axes. Find the value of m and the value of c.

$y = mx + c$ cuts x-axis at $P\left(-\frac{c}{m}, 0\right)$ and cuts y-axis at Q(0, c)

Area $\Delta POQ = \frac{1}{2}\left|0 - \left(-\frac{c}{m}\right)c\right| = \frac{1}{2}\left|\frac{c^2}{m}\right| = \frac{125}{3} \Rightarrow m = \frac{3c^2}{250}$

$(-5, 4) \in y = mx + c \Rightarrow 4 = -5m + c \Rightarrow 4 = -5\left(\frac{3c^2}{250}\right) + c \Rightarrow 3c^2 - 50c + 200 = 0$

$\Rightarrow (3c - 20)(c - 10) = 0 \Rightarrow c = \frac{20}{3}$ or $c = 10$

$c = \frac{20}{3}$

Hence, $m = \dfrac{3c^2}{250} = \dfrac{3\left(\frac{20}{3}\right)^2}{250} = \dfrac{400}{750} = \dfrac{8}{15}$

We can subtract the second equation from the first:

$$\begin{array}{rcl} 2x+8y-3z &=& -1 \\ 2x-3y+2z &=& 2 \\ \hline 11y-5z &=& -3 \end{array}$$

Similarly, we subtract the third equation from the first:

$$\begin{array}{rcl} 2x+8y-3z &=& -1 \\ 2x+y+z &=& 5 \\ \hline 7y-4z &=& -6 \end{array}$$

Now we solve the simultaneous equations

$$\begin{array}{rcl} 11y-5z &=& -3 \\ 7y-4z &=& -6 \end{array}$$

Multiply the first by 7, the second by 11 and subtract:

$$\begin{array}{rcl} 77y-35z &=& -21 \\ 77y-44z &=& -66 \\ \hline 9z &=& 45 \end{array}$$

Therefore $z = \frac{45}{9} = 5$. Now substitute $z = 5$ into $7y - 4z = -6$ to get $7y - 4(5) = -6$ or $7y = -6 + 20 = 14$ Therefore $y = 2$.
Finally substitute $y = 2$ and $z = 5$ into $2x + 8y - 3z = -1$ to get $2x + 8(2) - 3(5) = -1$ or $2x = -1 - 8(2) + 3(5) = -2$ So $x = -1$.
So the solution is

$$x = -1, y = 2, z = 5.$$

Now we can check this by substituting into the original equations and verifying that they are all true:

$$\begin{array}{rcl} 2(-1)+8(2)-3(5) &=& -1 \\ 2(-1)-3(2)+2(5) &=& 2 \\ 2(-1)+(2)+(5) &=& 5. \end{array}$$

(b) The graphs of the functions $f : x \mapsto |x-3|$ and $g : x \mapsto 2$ are shown in the diagram.

(i) Find the co-ordinates of the points A, B, C and D.

D is on the y-axis, so its x-co-ordinate is 0. Now $f(0) = |0-3| = |-3| = 3$. So $D = (0,3)$.
$C = (,0)$ (on the x-axis), so we solve $|x-3| = 0$ to find the x-co-ordinate. Now $|x-3| = 0 \Leftrightarrow x-3 = 0 \Leftrightarrow x = 3$. So $C = (3,0)$.
A and B both have y-co-ordinate 2, so we solve $|x-3| = 2$. Now $|x-3| = 2 \Leftrightarrow \pm(x-3) = 2$. So either

$$(x-3) = 2 \text{ or } -(x-3) = 2.$$

In the first case $x = 5$ and in the second case $-x+3 = 2$ or $x = 1$. So $A = (1,2)$ and $B = (5,2)$.

$$A = (1,2) \quad B = (5,2)$$
$$C = (3,0) \quad D = (0,3)$$

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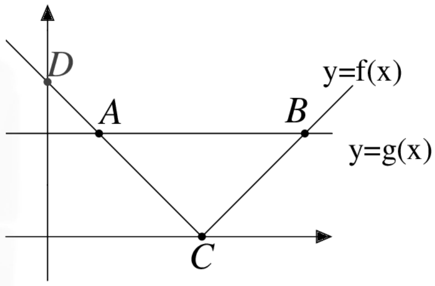

(ii) Hence, or otherwise, solve the inequality $|x-3| < 2$.

The solution set of the inequality corresponds to the values of x for which the graph of f is below the graph of g. From the diagram and calculations above, we see that the solution set is

$$1 < x < 5.$$

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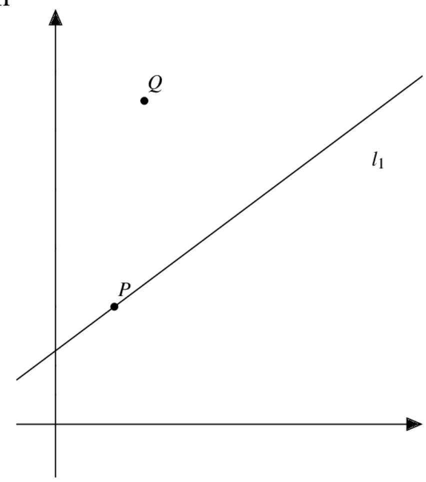
(a) Show that, for all $k \in \mathbb{R}$, the point $P(4k-2, 3k+1)$ lies on the line $l_1 : 3x - 4y + 10 = 0$.



If $(x,y) = (4k-2, 3k+1)$ then

$$\begin{aligned} 3x - 4y + 10 &= 3(4k-2) - 4(3k+1) + 10 \\ &= 12k - 6 - 12k - 4 + 10 \\ &= 0 \end{aligned}$$

So the equation of l_1 is satisfied. Therefore $(4k-2, 3k+1)$ lies on l_1.

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(b) The line l_2 passes through P and is perpendicular to l_1. Find the equation of l_2 in terms of k.

We have

$$\begin{aligned} 3x - 4y + 10 &= 0 \\ &\Leftrightarrow \\ -4y &= -3x - 10 \\ &\Leftrightarrow \\ y &= \frac{3}{4}x + \frac{5}{2} \end{aligned}$$

Therefore the slope of l_1 is $\frac{3}{4}$. Therefore the slope of l_2 is $\frac{1}{\frac{3}{4}} = -\frac{4}{3}$. So l_2 has slope $-\frac{4}{3}$ and passes through $(4k-2, 3k+1)$. So it has equation

$$y - (3k+1) = -\frac{4}{3}(x - (4k-2))$$

or

$$3y - 3(3k+1) = -4(x - (4k-2)).$$

Rearranging this gives

$$4x + 3y - 25k + 5 = 0.$$

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(c) Find the value of k for which l_2 passes through the point $Q(3, 11)$.

The equation of l_2 is

$$4x+3y-25k+5=0.$$

Now $(3,11)$ lies on l_2 if and only if $4(3)+3(11)-25k+5=0 \Leftrightarrow 25k=50 \Leftrightarrow k=2$. So the $k=2$ is the required value.

(d) Hence, or otherwise, find the co-ordinates of the foot of the perpendicular from Q to l_1.

When $k=2$ the equation of l_2 is

$$4x+3y-45=0.$$

So to find the required point, we solve

$$\begin{aligned} 3x-4y+10 &= 0 \\ 4x+3y-45 &= 0 \end{aligned}$$

simultaneously.
This is equivalent to

$$\begin{aligned} 12x-16y+40 &= 0 \\ 12x+9y-135 &= 0 \end{aligned}$$

Subtracting yields

$$-25y+175=0.$$

Therefore $25y=175$ and $y=\frac{175}{25}=7$.
Now $3x-4(7)+10=0 \Leftrightarrow 3x=4(7)-10=18 \Leftrightarrow x=6$. So the foot of the perpendicular from Q to l_1 has co-ordinates $(6,7)$.

Description	Line(s)
A line with a slope of 2.	l
A line which intersects the y-axis at $(0, -2\frac{1}{2})$.	l
A line which makes equal intercepts on the axes.	h
A line which makes an angle of $150°$ with the positive sense of the x-axis.	m
Two lines which are perpendicular to each other.	l and k

(b) Find the acute angle between the lines m and n.

Slope of m: $m_1 = -\frac{1}{\sqrt{3}}$

Slope of n: $m_2 = -\sqrt{3}$

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2} = \pm\frac{-\frac{1}{\sqrt{3}} + \sqrt{3}}{1 - \frac{1}{\sqrt{3}}\left(-\sqrt{3}\right)} = \pm\frac{\frac{-1+3}{\sqrt{3}}}{1+1} = \pm\frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \theta = 30°$$

1. Check whether both pairs of opposite sides have the same slope (slope formula).
2. Check whether both pairs of opposite sides are equal in length (distance formula).
3. Check whether the midpoints of the diagonals coincide (diagonals bisecting each other).
4. Check whether the translation from A to B is the same as the translation from D to C [or equivalent.]
5. Check whether a pair of opposite sides have the same slope and are equal in length (slope and distance formulae).
6. Use slopes and the formula for the angle between two lines to check whether both pairs of opposite angles are equal.
7. Use slopes and the formula for the angle between two lines to check whether $|\angle A| + |\angle B| = 180°$, and $|\angle C| + |\angle B| = 180°$. [or equivalent]

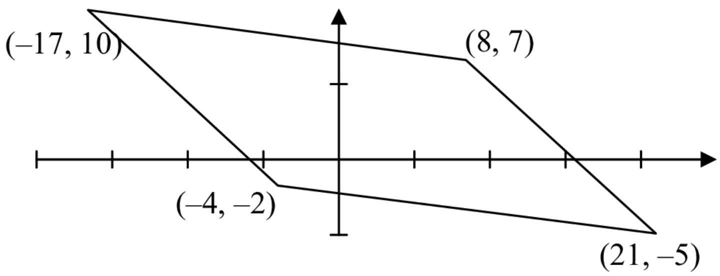
(b) Using **one** of the methods you described, determine whether the quadrilateral with vertices (–4, –2), (21, –5), (8, 7) and (–17, 10) is a parallelogram.

Midpoints of diagonals:

$$\left(\frac{-4+8}{2}, \frac{-2+7}{2}\right) = \left(2, \frac{5}{2}\right)$$

$$\left(\frac{-17+21}{2}, \frac{10-5}{2}\right) = \left(2, \frac{5}{2}\right)$$

Equal $\Rightarrow$ parallelogram.



For other methods: slopes are $\frac{-12}{13}$ and $\frac{-3}{25}$; side-lengths are $\sqrt{313}$ and $\sqrt{634}$, translations are $(x, y) \rightarrow (x+25, y-3)$ and $(x, y) \rightarrow (x+13, y-12)$, or reverse.

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) Given the co-ordinates of the vertices of a quadrilateral *ABCD*, describe **three** different ways to determine, using co-ordinate geometry techniques, whether the quadrilateral is a parallelogram.

1. Check whether both pairs of opposite sides have the same slope (slope formula).
2. Check whether both pairs of opposite sides are equal in length (distance formula).
3. Check whether the midpoints of the diagonals coincide (diagonals bisecting each other).
4. Check whether the translation from *A* to *B* is the same as the translation from *D* to *C* [or equivalent.]
5. Check whether a pair of opposite sides have the same slope and are equal in length (slope and distance formulae).
6. Use slopes and the formula for the angle between two lines to check whether both pairs of opposite angles are equal.
7. Use slopes and the formula for the angle between two lines to check whether $|\angle A| + |\angle B| = 180°$, and $|\angle C| + |\angle B| = 180°$. [or equivalent]

(b) Using **one** of the methods you described, determine whether the quadrilateral with vertices (–4, –2), (21, –5), (8, 7) and (–17, 10) is a parallelogram.

Midpoints of diagonals:

$$\left(\frac{-4+8}{2}, \frac{-2+7}{2}\right) = \left(2, \frac{5}{2}\right)$$

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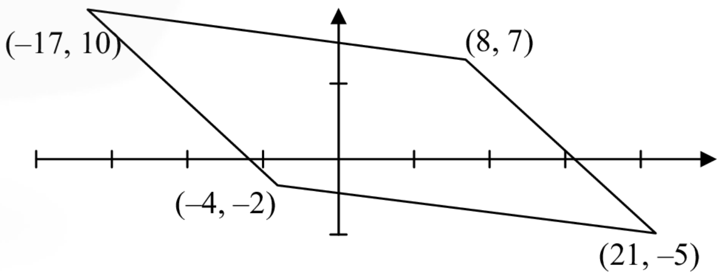
Equal ⇒ parallelogram.



For other methods: slopes are $\frac{-12}{13}$ and $\frac{-3}{25}$; side-lengths are $\sqrt{313}$ and $\sqrt{634}$, translations are $(x, y) \to (x+25, y-3)$ and $(x, y) \to (x+13, y-12)$, or reverse.

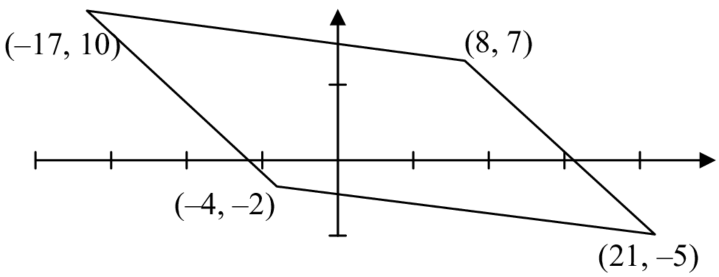
1. Check whether both pairs of opposite sides have the same slope (slope formula).
2. Check whether both pairs of opposite sides are equal in length (distance formula).
3. Check whether the midpoints of the diagonals coincide (diagonals bisecting each other).
4. Check whether the translation from A to B is the same as the translation from D to C [or equivalent.]
5. Check whether a pair of opposite sides have the same slope and are equal in length (slope and distance formulae).
6. Use slopes and the formula for the angle between two lines to check whether both pairs of opposite angles are equal.
7. Use slopes and the formula for the angle between two lines to check whether $|\angle A| + |\angle B| = 180°$, and $|\angle C| + |\angle B| = 180°$. [or equivalent]

(b) Using **one** of the methods you described, determine whether the quadrilateral with vertices (–4, –2), (21, –5), (8, 7) and (–17, 10) is a parallelogram.

Midpoints of diagonals:

$$\left(\frac{-4+8}{2}, \frac{-2+7}{2}\right) = \left(2, \frac{5}{2}\right)$$
$$\left(\frac{-17+21}{2}, \frac{10-5}{2}\right) = \left(2, \frac{5}{2}\right)$$

Equal ⇒ parallelogram.



For other methods: slopes are $\frac{-12}{13}$ and $\frac{-3}{25}$; side-lengths are $\sqrt{313}$ and $\sqrt{634}$, translations are $(x, y) \to (x + 25, y - 3)$ and $(x, y) \to (x + 13, y - 12)$, or reverse.

Question 11 (2012)

Question 3 **(25 marks)**

(a) Find the equation of AB.

Here $(x_1, y_1) = (2, 2)$ and $(x_2, y_2) = (6, -6)$. First find the slope of AB

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-2}{6-2} = \frac{-8}{4} = -2$$

Now use the equation of a line formula

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= -2(x-2) \\ y - 2 &= -2x + 4 \\ 2x + y - 6 &= 0 \end{aligned}$$

(b) The line AB intersects the y-axis at D. Find the coordinates of D.

Use the equation of AB and set $x = 0$ to get

$$2(0) + y - 6 = 0$$

$$y - 6 = 0$$

which means $y = 6$ and so $D = (0, 6)$.

(c) Find the perpendicular distance from C to AB.

Using the perpendicular distance formula:

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

From the equation of AB we have $a = 2, b = 1, c = -6$ and from the coordinates of C we have $x_1 = -2, y_1 = -3$. Then the perpendicular distance between C and AB is

$$\frac{|2(-2) + 1(-3) - 6|}{\sqrt{2^2 + 1^2}} = \frac{|-4-3-6|}{\sqrt{4+1}} = \frac{|-13|}{\sqrt{5}} = \frac{13}{\sqrt{5}}$$

(d) Hence, find the area of the triangle ADC.

If AD is the base of the triangle then the perpendicular height is the answer from part **(c)**. The distance from A to D is

$$|AD| = \sqrt{(0-2)^2+(6-2)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

So the area of ADC is

$$\begin{aligned} \frac{1}{2}\text{base} \times \text{perpendicular height} &= \frac{1}{2}(2\sqrt{5})\left(\frac{13}{\sqrt{5}}\right) \\ &= 13 \text{ units squared} \end{aligned}$$

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Question 12 (2012)

Question 4 **(25 marks)**

(a) Write down the equation of the circle with centre $(-3,2)$ and radius 4.

Let the centre of the circle $(h,k) = (-3,2)$ and $r = 4$. So the equation of the circle is

$$\begin{aligned} (x-h)^2+(y-k)^2 &= r^2 \\ (x+3)^2+(y-2)^2 &= 4^2 \\ x^2+6x+9+y^2-4y+4 &= 16 \\ x^2+y^2+6x-4y-3 &= 0 \end{aligned}$$

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(b) A circle has equation $x^2+y^2-2x+4y-15=0$. Find the values of m for which the line $mx+2y-7=0$ is a tangent line.

Re-write this equation as $x^2+y^2+2(-1)x+2(2)y-15=0$ which matches the equation of a circle with $g=-1, f=2, c=-15$. So this circle has centre $(-g,-f)=(1,-2)$ and radius $r=\sqrt{g^2+f^2-c}=\sqrt{1+4+15}=\sqrt{20}$.

For a line to be tangent to this circle, the perpendicular distance from that line to the centre $(1,-2)$ must be equal to the radius. The distance from the line $mx+2y-7=0$ to $(1,-2)$ is

$$\frac{|m(1)+2(-2)-7|}{\sqrt{m^2+2^2}}$$

This must be equal to the radius in order to be tangent which means

$$\begin{aligned}
\frac{|m(1)+2(-2)-7|}{\sqrt{m^2+2^2}} &= \sqrt{20}\\
|m-4-7| &= \sqrt{20}\sqrt{m^2+4}\\
|m-11| &= \sqrt{20m^2+80}\\
(m-11)^2 &= 20m^2+80\\
m^2-22m+121 &= 20m^2+80\\
0 &= 19m^2+22m-41
\end{aligned}$$

We can solve this quadratic to get solutions $m=1$ and $m=-\dfrac{41}{19}$

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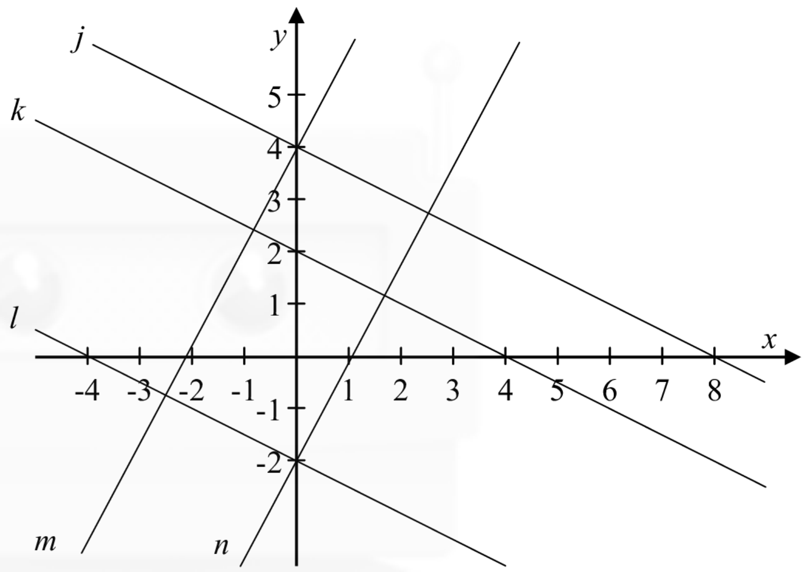

Equation	Line
$x+2y=-4$	l
$2x-y=-4$	m
$x+2y=8$	j
$2x-y=2$	n

(a) Complete the table, by matching four of the lines to their equations.

$x + 2y = -4 \Rightarrow \quad y = -\frac{1}{2}x - 2 \quad \rightarrow l$

$2x - y = -4 \Rightarrow \quad y = 2x + 4 \quad \rightarrow m$

$x + 2y = 8 \Rightarrow \quad y = -\frac{1}{2}x + 4 \quad \rightarrow j$

$2x - y = 2 \Rightarrow \quad y = 2x - 2 \quad \rightarrow n$

(b) Hence, insert scales on the x-axis and y-axis.

Shown above

(c) Hence, find the equation of the remaining line, given that its x-intercept and y-intercept are both integers.

Equation of k: $y = -\frac{1}{2}x + 2$

or

$x + 2y = 4$