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Question 4 **(25 marks)**

$A(0,0)$, $B(6{\cdot}5,0)$ and $C(10,7)$ are three points on a circle.

(a) Find the equation of the circle.

(b) Find $|\angle BCA|$. Give your answer in degrees, correct to 2 decimal places.

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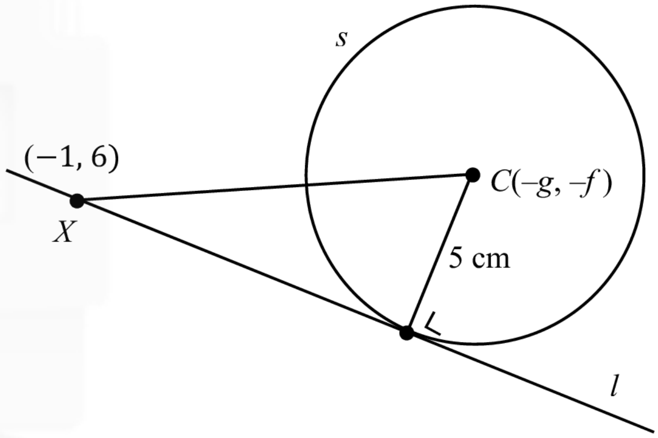
Question 2

A point X has co-ordinates $(-1, 6)$ and the slope of the line XC is $\frac{1}{7}$.

(a) Find the equation of XC. Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.

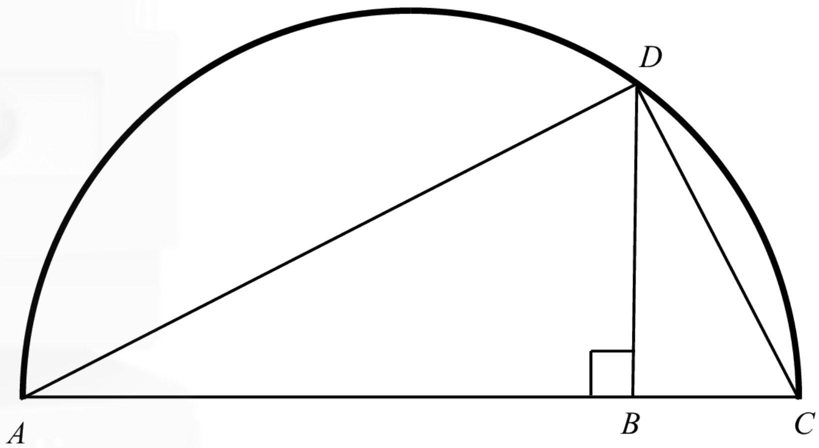


(b) C is the centre of a circle s, of radius 5 cm. The line l: $3x + 4y - 21 = 0$ is a tangent to s and passes through X, as shown. Find the equation of one such circle s.

Question 3

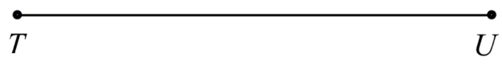
The diagram shows a semi-circle standing on a diameter $[AC]$, and $[BD] \perp [AC]$.

(a) **(i)** Prove that the triangles *ABD* and *DBC* are similar.



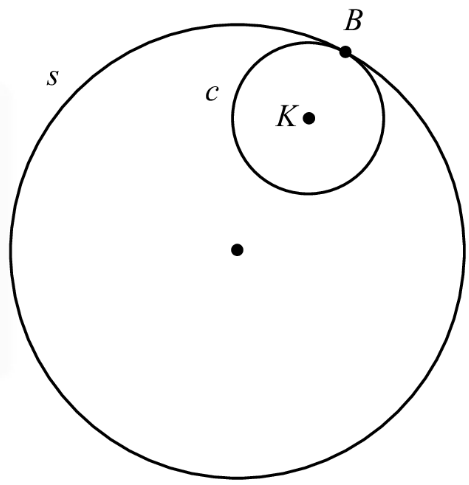
(ii) If $|AB| = x$, $|BC| = 1$, and $|BD| = y$, write y in terms of x.

(b) Use your result from part **(a)(ii)** to **construct** a line segment equal in length (in centimetres) to the square root of the length of the line segment $[TU]$ which is drawn below.



Question 4 **(25 marks)**

Two circles s and c touch internally at B, as shown.



(a) The equation of the circle s is

$(x-1)^2+(y+6)^2=360.$

Write down the co-ordinates of the centre of s.

Centre:_________________

Write down the radius of s in the form $a\sqrt{10}$, where $a\in\mathbb{N}$.

Radius:_________________________

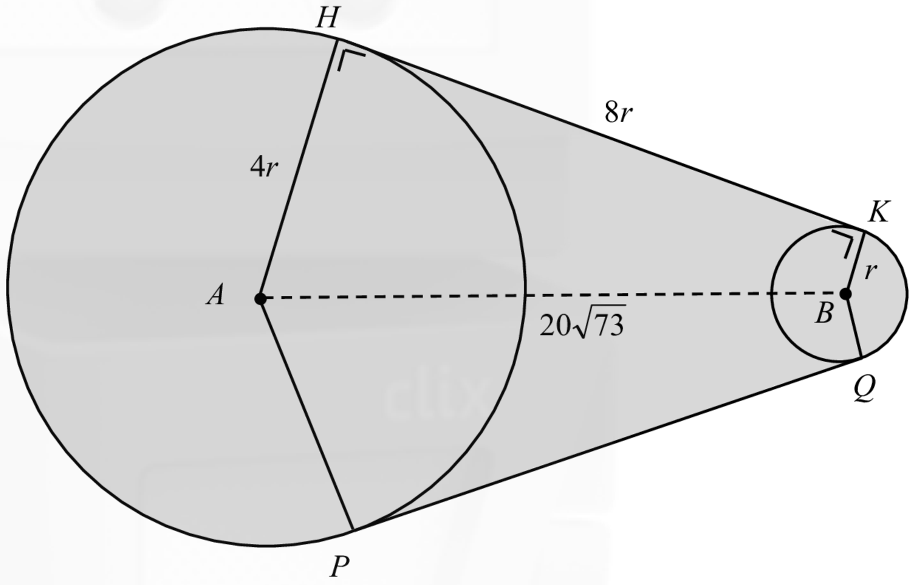
(b) **(i)** The point K is the centre of circle c.
The radius of c is one-third the radius of s.
The co-ordinates of B are (7, 12).
Find the co-ordinates of K.

(ii) Find the equation of c.

(c) Find the equation of the common tangent at B.
Give your answer in the form $ax+by+c=0$, where $a, b, c\in\mathbb{Z}$.

Question 7 **(40 marks)**

A flat machine part consists of two circular ends attached to a plate, as shown (diagram not to scale).
The sides of the plate, HK and PQ, are tangential to each circle.
The larger circle has centre A and radius $4r$ cm.
The smaller circle has centre B and radius r cm.
The length of $[HK]$ is $8r$ cm and $|AB| = 20\sqrt{73}$ cm.



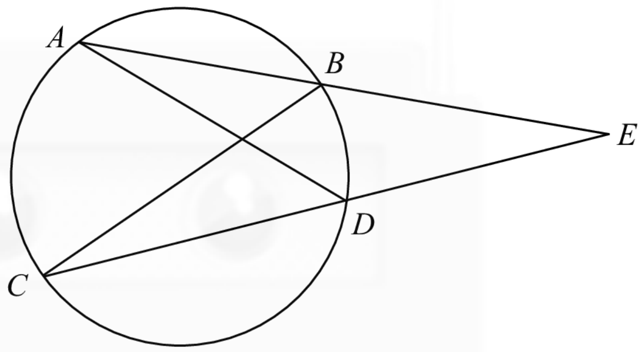
(a) Find r, the radius of the smaller circle. (Hint: Draw $BT \parallel KH$, $T \in AH$.)

(b) Find the area of the quadrilateral *ABKH*.

(c) **(i)** Find $|\angle HAP|$, in degrees, correct to one decimal place.

(ii) Find the area of the machine part, correct to the nearest cm^2.

$[AB]$ and $[CD]$ are chords of a circle that intersect externally at E, as shown.



(a) Name two similar triangles in the diagram above and give reasons for your answer.

(b) Prove that $|EA|.|EB| = |EC|.|ED|$.

(c) Given that $|EB| = 6 \cdot 25$, $|ED| = 5 \cdot 94$ and $|CB| = 10$, find $|AD|$.

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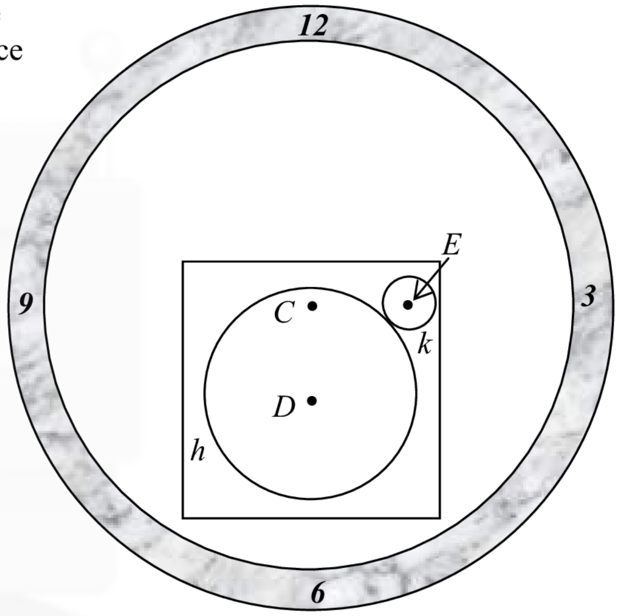
(a) The diagram shows a circular clock face, with the hands not shown. The square part of the clock face is glass so that the mechanism is visible. Two circular cogs, h and k, which touch externally are shown.



The point C is the centre of the clock face. The point D is the centre of the larger cog, h, and the point E is the centre of the smaller cog, k.

(i) In suitable co-ordinates, the equation of the circle h is

$$x^2 + y^2 + 4x + 6y - 19 = 0.$$

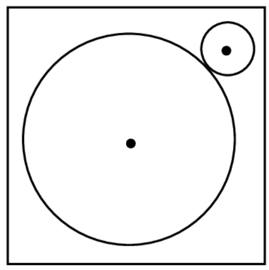
Find the radius of h, and the co-ordinates of its centre, D.

(ii) The point E has co-ordinates $(3, 2)$. Find the radius of the circle k.

(iii) Show that the distance from $C(-2, 2)$ to the line DE is half the length of $[DE]$.

(iv) The translation which maps the midpoint of $[DE]$ to the point C maps the circle k to the circle j. Find the equation of the circle j.

(v) The glass square is of side length l. Find the smallest whole number l such that the two cogs, h and k, are fully visible through the glass.



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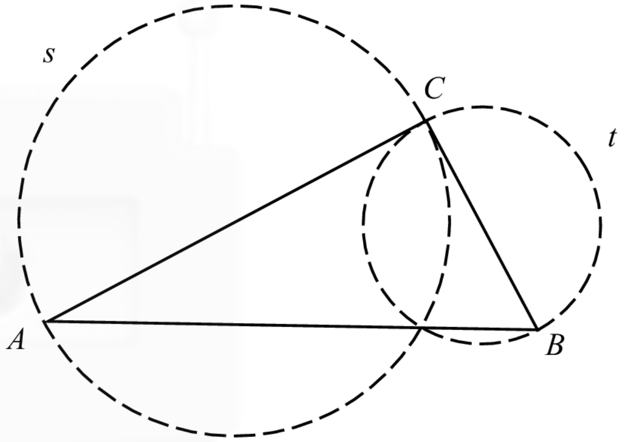
(b) The triangle ABC is right-angled at C.

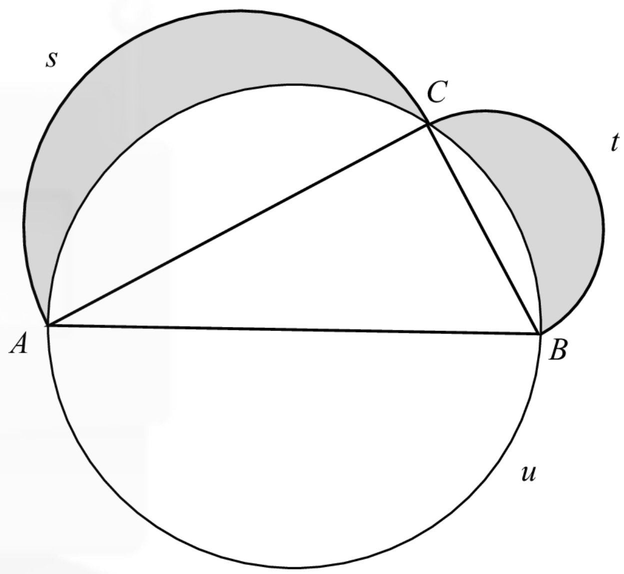
The circle s has diameter $[AC]$ and the circle t has diameter $[CB]$.



(i) Draw the circle u which has diameter $[AB]$.

(ii) Prove that in any right-angles triangle ABC, the area of the circle u equals the sum of the areas of the circles s and t.

(iii) The diagram shows the right-angled triangle ABC and arcs of the circles s, t and u.



Each of the shaded areas in the diagram is called a lune, a crescent-shaped area bounded by arcs of the circles.

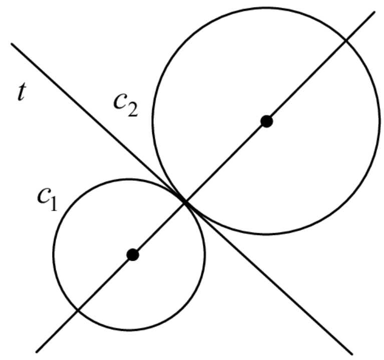
Prove that the sum of the areas of the two shaded lunes is equal to the area of the triangle ABC.

Question 4 **(25 marks)**

The centre of a circle lies on the line $x+2y-6=0$. The x-axis and the y-axis are tangents to the circle. There are two circles that satisfy these conditions. Find their equations.

Question 4 **(25 marks)**

The circles c_1 and c_2 touch externally as shown.



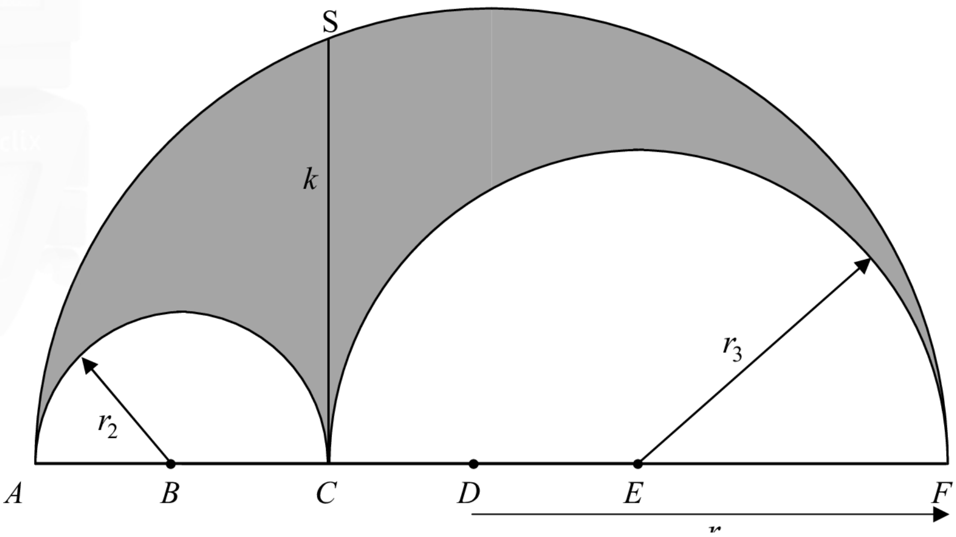
(a) Complete the following table:

Circle	**Centre**	**Radius**	**Equation**
c_1	$(-3, -2)$	2	
c_2			$x^2 + y^2 - 2x - 2y - 7 = 0$

(b) **(i)** Find the co-ordinates of the point of contact of c_1 and c_2.

(ii) Hence, or otherwise, find the equation of the tangent, t, common to c_1 and c_2.

(b) The shaded region in the diagram below is called an **arbelos.** It is a plane semicircular region of radius r_1 from which semicircles of radius r_2 and r_3 are removed, as shown. In the diagram $SC \perp AF$ and $|SC| = k$.



(i) Show that, for fixed r_1, the perimeter of the arbelos is independent of the values of r_2 and r_3.

(ii) If $r_2 = 2$ and $r_3 = 4$, show that the area of the arbelos is the same as the area of the circle of diameter k.

(c) To investigate the area of an arbelos, a student fixed the value of r_1 at 6 cm and completed the following table for different values of r_2 and r_3.

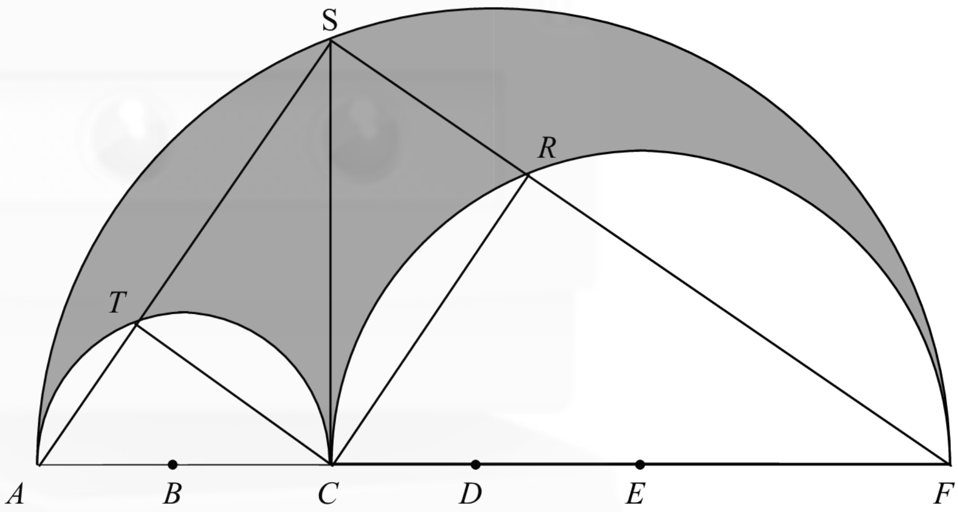
(i) Complete the table.

r_1	r_2	r_3	Area of arbelos
6	1		
6	2		
6	3		
6	4		
6	5		

(ii) In general, for $r_1 = 6$ cm and $r_2 = x$, $0 < x < 6$, $x \in \mathbb{R}$, find an expression in x for the area of the arbelos.

(iii) Hence, or otherwise, find the maximum area of an arbelos that can be formed in a semicircle of radius 6 cm.

(d) *AS* and *FS* cut the two smaller semicircles at *T* and *R* respectively.
Prove that *RSTC* is a rectangle.



Question 2 **(25 marks)**

The equations of two circles are:

$$c_1 : x^2 + y^2 - 6x - 10y + 29 = 0$$
$$c_2 : x^2 + y^2 - 2x - 2y - 43 = 0$$

(a) Write down the centre and radius-length of each circle.

centre of c_1: __________________

radius-length of c_1: _____________

centre of c_2: __________________

radius-length of c_2: _____________

(b) Prove that the circles are touching.

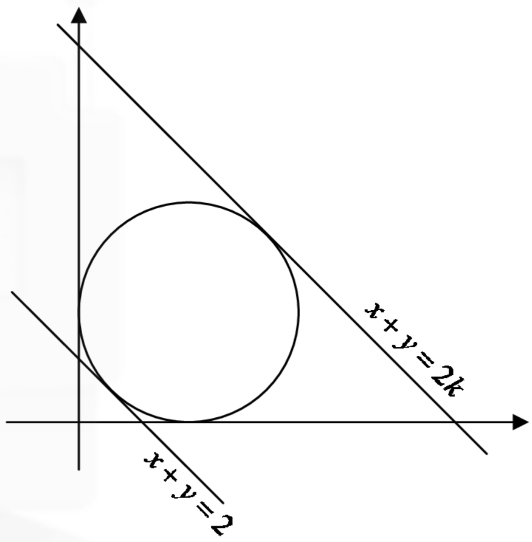
(c) Verify that (4, 7) is the point that they have in common.

(d) Find the equation of the common tangent.

Question 3 **(25 marks)**

The circle shown in the diagram has, as tangents, the x-axis, the y-axis, the line $x + y = 2$ and the line $x + y = 2k$, where $k > 1$.

Find the value of k.



Question 4 **(25 marks)**

(a) Write down the equation of the circle with centre (–3, 2) and radius 4.

(b) A circle has equation $x^2 + y^2 - 2x + 4y - 15 = 0$.
Find the values of m for which the line $mx + 2y - 7 = 0$ is a tangent to this circle.

Question 5 **(25 marks)**

The line $x + 3y = 20$ intersects the circle $x^2 + y^2 - 6x - 8y = 0$ at the points P and Q.

Find the equation of the circle that has $[PQ]$ as diameter.

Question 15

(a) The centre of a circle lies on the line $x - 2y - 1 = 0$. The x-axis and the line $y = 6$ are tangents to the circle. Find the equation of this circle.

(b) A different circle has equation $x^2 + y^2 - 6x - 12y + 41 = 0$. Show that this circle and the circle in part **(a)** touch externally.