Question 2 **(25 marks)**

$z = -\sqrt{3} + i$, where $i^2 = -1$.

(a) Use De Moivre's Theorem to write z^4 in the form $a + b\sqrt{c}\, i$, where a, b, and $c \in \mathbb{Z}$.

(b) The complex number w is such that $|w| = 3$ and w makes an angle of $30°$ with the positive sense of the real axis. If $t = zw$, write t in its simplest form.

Question 1 **(25 marks)**

(a) $(-4+3i)$ is one root of the equation $az^2+bz+c=0$, where $a, b, c \in \mathbb{R}$, and $i^2=-1$.
Write the other root.

(b) Use De Moivre's Theorem to express $(1+i)^8$ in its simplest form.

(c) $(1+i)$ is a root of the equation $z^2+(-2+i)z+3-i=0$.
Find its other root in the form $m+ni$, where $m, n \in \mathbb{R}$, and $i^2=-1$.

Question 4 **(25 marks)**

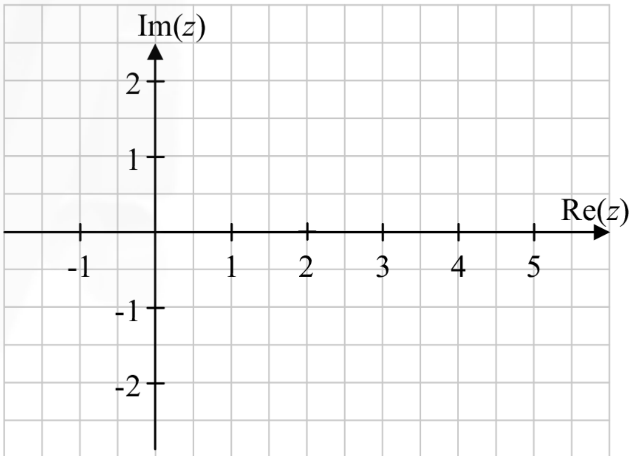
(a) The complex numbers z_1, z_2 and z_3 are such that $\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3}$, $z_2 = 2 + 3i$ and $z_3 = 3 - 2i$, where $i^2 = -1$. Write z_1 in the form $a + bi$, where $a, b \in \mathbb{Z}$.

Question 4

Let $z_1 = 1 - 2i$, where $i^2 = -1$.

(a) The complex number z_1 is a root of the equation $2z^3 - 7z^2 + 16z - 15 = 0$.
Find the other two roots of the equation.

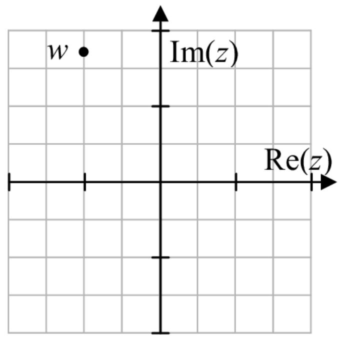
(b) **(i)** Let $w = z_1 \overline{z_1}$, where $\overline{z_1}$ is the conjugate of z_1. Plot z_1, $\overline{z_1}$ and w on the Argand diagram and label each point.



(ii) Find the measure of the acute angle, $\overline{z_1}wz_1$, formed by joining $\overline{z_1}$ to w to z_1 on the diagram above. Give your answer correct to the nearest degree.

Question 1 **(25 marks)**

(a) $w = -1 + \sqrt{3}i$ is a complex number, where $i^2 = -1$.



(i) Write w in polar form.

(ii) Use De Moivre's theorem to solve the equation $z^2 = -1 + \sqrt{3}i$. Give your answer(s) in rectangular form.

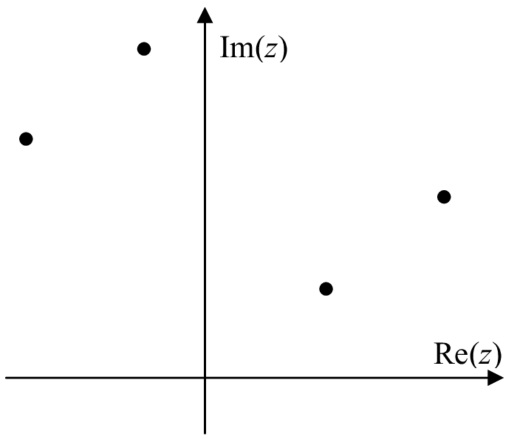
(b) Four complex numbers z_1, z_2, z_3 and z_4 are shown on the Argand diagram. They satisfy the following conditions:

$z_2 = iz_1$

$z_3 = kz_1$, where $k \in \mathbb{R}$

$z_4 = z_2 + z_3$.

The same scale is used on both axes.



(i) Identify which number is which, by labelling the points on the diagram.

(ii) Write down the approximate value of k.

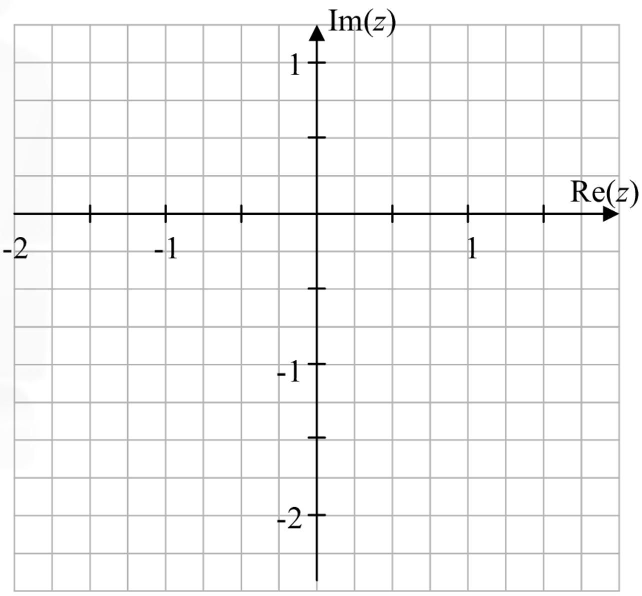
Answer: _______________

Question 6

$z = \dfrac{4}{1+\sqrt{3}i}$ is a complex number, where $i^2 = -1$.

(a) Verify that z can be written as $1-\sqrt{3}i$.

(b) Plot z on an Argand diagram and write z in polar form.



(c) Use De Moivre's theorem to show that $z^{10} = -2^9\left(1-\sqrt{3}i\right)$.

Question 3 **(25 marks)**

The complex number z has modulus $5\frac{1}{16}$ and argument $\dfrac{4\pi}{9}$.

(a) Find, in polar form, the four complex fourth roots of z.
(That is, find the four values of w for which $w^4 = z$.)

(b) z is marked on the Argand diagram below.
On the same diagram, show the four answers to part **(a)**.