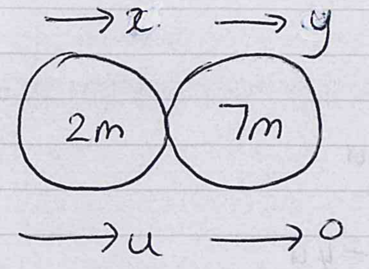
2014 Q5 ⓐ

(i)



$\boxed{2mx + 7my = 2mu}$

$\boxed{2x + 7y = 2u}$

$\boxed{x - y = -e(u)}$

$2x + 7y = 2u$
$-2x + 2y = 2eu$

$9y = 2u(e+1)$

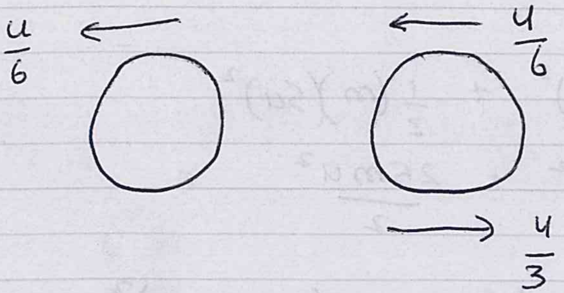
$y = \frac{2u}{9}(e+1)$

$7x - 7y = -7eu$
$2x + 7y = 2u$

$9x = u(2-7e)$

$x = \frac{u}{9}(2-7e)$

$e = \frac{1}{2}$! $y = \frac{2u}{9}\left(\frac{3}{2}\right) = \frac{u}{3}$ $x = \frac{u}{9}\left(2-\frac{7}{2}\right) = -\frac{u}{6}$



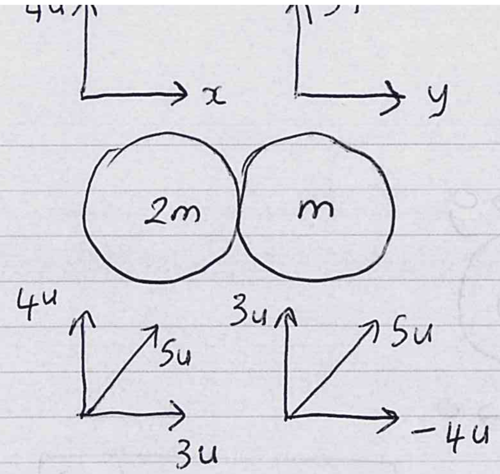
Since they are both going at same speed, they wont collide

(ii) Loss in KE $= \frac{2}{2}mu^2 + \frac{1}{2}7m(0)^2 - \left[\frac{2}{2}m\frac{u^2}{36} + \frac{1}{2}7m\frac{u^2}{36}\right]$

$= mu^2\left[1 - \frac{2}{} - \frac{7}{72}\right] = mu^2\left[1 - \frac{1}{8}\right] = \frac{7}{8}mu^2$

2014

5 (b) :



$x - y = -e(3u - -4u)$

$\boxed{x - y = -7eu}$

$2\cancel{m}x + \cancel{m}y = 6\cancel{m}u - 4\cancel{m}u$

$\boxed{2x + y = 2u}$

$2x + y = 2u$

$-\ 2x + 2y = 14eu$

$3y = 2u + 14eu$

$y = \frac{u}{3}(2 + 14e)$

$\boxed{y = \frac{2u}{3}([illegible] + 7e)}$

$2x + y = 2u$

$x - y = -7eu$

$3x = 2u - 7eu$

$\boxed{x = \frac{u}{3}(2 - 7e)}$

KE Before : $\frac{1}{2}(2m)(5u)^2 + \frac{1}{2}(m)(5u)^2$

$= 25mu^2 + \frac{25mu^2}{2} =$

KE AFTER $= \frac{1}{2}(2m)(4u)^2 + \frac{1}{2}(2m)\left(\frac{u}{3}(2-7e)\right)^2 + \frac{1}{2}(m)(3u)^2$

$+ \frac{1}{2}(m)\left(\frac{2u}{3}(1+7e)\right)^2$

$= u^2m + \frac{mu^2}{9}(4 - 28e + 49e^2) + \frac{9u^2m}{2} + \frac{2mu^2}{9}(\ ^2 + 14e + 49e^2)$

$= 9mu^2 + 8mu^2 - \frac{mu^2}{9}(4 - 28e - 49e^2) - \frac{2mu^2}{9}(1 + 14e + 49e^2)$

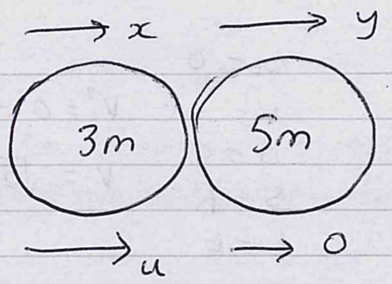
$\frac{25}{2} = 9 + 8 - \frac{(4 - 28e - 49e^2)}{9} - \frac{2}{9}(1 + 14e + 49e^2)$

$25 = 34 - \frac{2}{9}(4 - 28e + 49e^2) - \frac{4}{9}(1 + 14e + 49e^2)$

$294e^2 = 69$

$e = \sqrt{\frac{69}{294}} = 0.484$

2013 Q5(a)



$3mx + 5my = 3mu$

$\boxed{3x + 5y = 3u}$

$\boxed{x - y = -e(u)}$

$-3x + 3y = 3eu$
$3x + 5y = 3u$

$8y = 3u(1+e)$

$\boxed{y = \frac{3u}{8}(1+e)}$

$5x - 5y = -5eu$
$3x + 5y = 3u$

$8x = u(3-5e)$

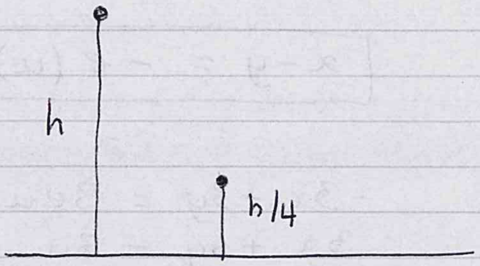
$\boxed{x = \frac{u}{8}(3-5e)}$

Impulse (5m mass) $= 5m\left(\frac{3u}{8}(1+e)\right) = \frac{15mu}{8}(1+e)$

$2\cancel{m}\cancel{u} = \frac{15\cancel{m}\cancel{u}(1+e)}{8}$

$16 = 15 + 15e$
$1 = 15e$
$\frac{1}{15} = e$

(b.)



$u = 0$

$v =$ $\qquad v^2 = 0^2 + 2gh$

$a = g$ $\qquad v = \sqrt{2gh}$

$s = h$

$t = t$

$u = e\sqrt{2gh}$

$v = 0$ $\qquad 0^2 = e^2 . 2gh - 2g\frac{h}{4}$

$a = -g$

$s = h/4$ $\qquad e^2 = \frac{1}{4} \Rightarrow e = \frac{1}{2}$ ~ 0

$u = e\sqrt{2gh}$

$v = 0$ $\qquad 0^2 = e^2 . 2gh - 2g\frac{h}{9}$

$a = -g$

$h = h/9$ $\qquad e^2 = \frac{1}{9}$, $e = \frac{1}{3}$ ~ 2.5cm

$u = e\sqrt{2gh}$

$v = 0$

$a = -g$ $\qquad e^2 = \frac{1}{16}$, $e = \frac{1}{4}$ ~ ?

$h = h/16$

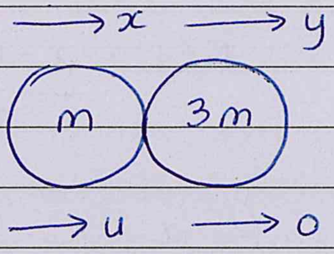
old $\underline{e}$ = kt (new e) $\qquad \frac{\text{old e}}{\text{new e}} = kt$

$\frac{1}{2}/\frac{1}{3} = k(2.5)$ $\qquad k = \frac{3}{2}/2.5$ $\qquad = 0.6$

$\frac{1}{2}/\frac{1}{4} = k(t) = 0.6t$ $\qquad \frac{2}{1} = 0.6t$ $\qquad t = \frac{2}{0.6}$ $= 3.33$

Q5 2010:

(a)



N.E.L: $\frac{x-y}{u} = -e$

① $x - y = -eu$

P.C.M:

$mx + 3my = mu$

② $x + 3y = u$

$$\begin{aligned} 3x - 3y &= -3eu \\ \hline 4x &= u(1-3e) \\ x &= \frac{u}{4}(1-3e) \end{aligned}$$

$$\begin{aligned} x + 3y &= u \\ -x + y &= eu \\ \hline 4y &= u(1+e) \\ y &= \frac{u}{4}(1+e) \end{aligned}$$

(ii)

$e = \frac{1}{4}$ so $x = \frac{u}{4}\left(1 - \frac{3}{4}\right) = \frac{u}{16}$,

$y = \frac{u}{4}\left(1 + \frac{1}{4}\right) = \frac{5u}{16}$,

% loss in K.E $= \left(\frac{\text{KE before} - \text{K.E after}}{\text{K.E. before}}\right) \times 100$

K.E. before $= \frac{1}{2}mu^2$

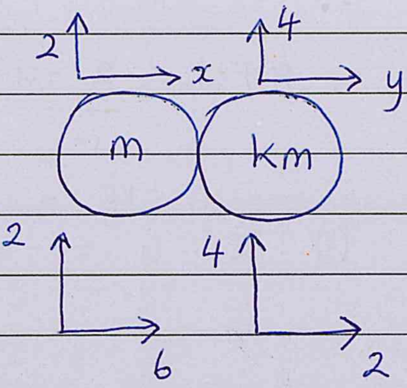
KE after $= \frac{1}{2}mx^2 + \frac{1}{2}my^2$

$= \frac{1}{2}m\left(\frac{u^2}{16^2}\right) + \frac{3}{2}m\left(\frac{25u^2}{16^2}\right) = \frac{1}{2}m\left(\frac{76u^2}{16^2}\right)$

So % loss in KE $= \left(\frac{\cancel{\frac{1}{2}mu^2} - \cancel{\frac{1}{2}m}\left(\frac{76u^2}{16^2}\right)}{\cancel{\frac{1}{2}mu^2}}\right) \times 100$

$=$ ~~$u^2(1-76)$~~ $\left(1 - \frac{76}{16^2}\right) \times 100 = 70.3\%$

2010 Q5 (b)



NEL : $\frac{x-y}{6-2} = -e$

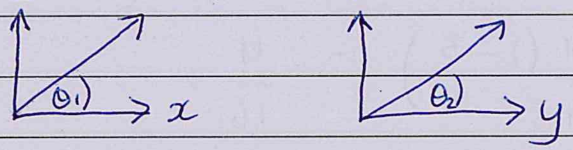
② $x - y = -4e$

P.C.M.:

$\cancel{m}x + k\cancel{m}y = 6\cancel{m} + 2k\cancel{m}$

① $x + ky = 6 + 2k$

If the spheres move off in parallel directions then



are parallel so $\theta_1 = \theta_2$

so $\tan\theta_1 = \tan\theta_2$

So $\frac{2}{x} = \frac{4}{y}$ or $4x = 2y$

$2x = y$ sub this into ①, ②

③ $x + 2kx = 6 + 2k$

④ $x - 2x = -4e \Rightarrow x = 4e$ ④

sub ④ into ③ : $4e + 8ke = 6 + 2k$

$\Rightarrow e(4 + 8k) = 6 + 2k$

$e = \frac{6+2k}{4+8k} = \frac{3+k}{2+4k}$

(ii) e must be ≤ 1 so $\frac{3+k}{2+4k} \leq 1$

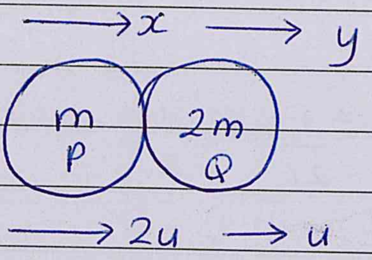
$3 + k \leq 2 + 4k$

$3 - 2 \leq 4k - k$

$1 \leq 3k$

$\frac{1}{3} \leq k$

2009 Q5



N.E.L:

$$\frac{x-y}{2u-u} = -e$$

① $x - y = -eu$

P.C.M:

$mx + 2my = 2mu + 2mu$

② $x + 2y = 4u$

-① $-x + y = +eu$

$3y = (4+e)u$

$y = \frac{u}{3}(4+e)$

$x + 2y = 4u$

$2x - 2y = -2eu$

$3x = u(4-2e)$

$x = \frac{u}{3}(4-2e)$

(ii) If Speed of Q ~~before~~ after - Speed of Q ~~after~~ before > 0 then speed increases

$y - u = \frac{u}{3}(4+e) - u = \frac{u + eu}{3}$ and since $e > 0$ this is > 0

so Q increases its speed.

ii) $x = \frac{10u}{9}$.

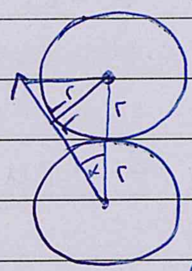
so $\frac{10\cancel{u}}{{}_3\cancel{9}} = \frac{\cancel{u}}{\cancel{3}}(4-2e) \Rightarrow 10 = 12 - 6e$

$-2 = -6e$

$\Rightarrow \frac{1}{3} = e$.

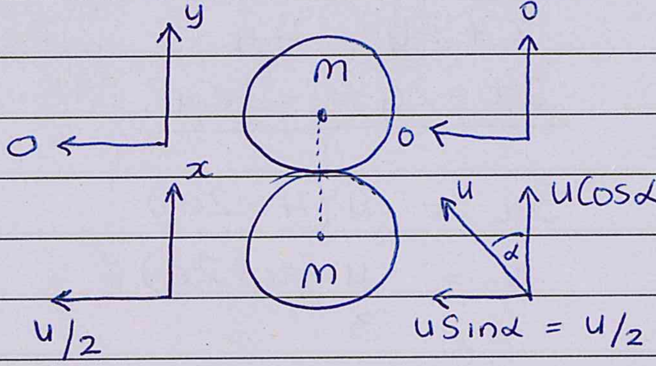
2009 Q5 (b)

(i)



$\sin\alpha = \frac{r}{2r} = \frac{1}{2} \Rightarrow \alpha = 30°$

After: Before:



$u\cos\alpha = \frac{u\sqrt{3}}{2}$

$u\sin\alpha = u/2$

NEL:

$\frac{x-y}{\frac{u\sqrt{3}}{2}} = -e$

① $2x-2y = -eu\sqrt{3}$

P.C.M: $\cancel{m}x + \cancel{m}y = \frac{\cancel{m}u\sqrt{3}}{2}$ ② $2x+2y = u\sqrt{3}$

$2x-2y = -eu\sqrt{3}$

$2x+2y = u\sqrt{3}$

$4x = u\sqrt{3}(1-e)$ just realised $e = \frac{4}{5}$

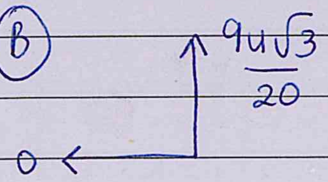
$4x = u\sqrt{3}\left(\frac{1}{5}\right)$

$x = \frac{u\sqrt{3}}{20}$, $y = \frac{u\sqrt{3}-2x}{2} = \frac{u\sqrt{3}}{2} - \frac{u\sqrt{3}}{20}$

$y = \frac{9u\sqrt{3}}{20}$,

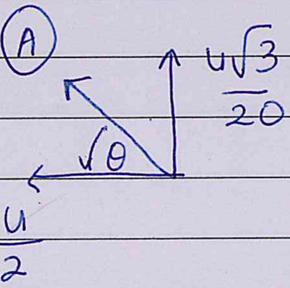
so after:

Ⓑ



and

Ⓐ



$\theta = \tan^{-1}\frac{\frac{u\sqrt{3}}{20}}{\frac{u}{2}}$

$\theta = \tan^{-1}\frac{\sqrt{3}}{10}$

2009 Q5 (b)

(iii) % loss in KE $= \left(\frac{\text{KE before} - \text{KE after}}{\text{KE before}}\right) \times 100$

K.E. before $= \frac{1}{2}m(u)^2$

K.E after $= \frac{1}{2}m\left(\frac{9u\sqrt{3}}{20}\right)^2 + \frac{1}{2}m\left(\left(\frac{u}{2}\right)^2 + \left(\frac{u\sqrt{3}}{20}\right)^2\right)$

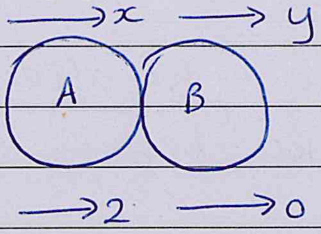
$= \frac{1}{2}mu^2\left[\frac{243}{400} + \frac{1}{4} + \frac{3}{400}\right]$

$= \frac{1}{2}mu^2\left[\frac{243 + 100 + 3}{400}\right] = \frac{1}{2}mu^2\left[\frac{346}{400}\right]$

So % loss in KE $= \frac{\frac{1}{2}mu^2 - \frac{1}{2}mu^2\left[\frac{346}{400}\right]}{\frac{1}{2}mu^2}$

$= 1 - \frac{346}{400} = \frac{54}{400} = 13.5\%$

2008 Q5: (a)



N.E.L. $\frac{x-y}{2} = -e$

① $x - y = -2e$

② $mx + my = 2m + 0$

② $x + y = 2$

$x - y = -2e$

$2x = 2 - 2e$

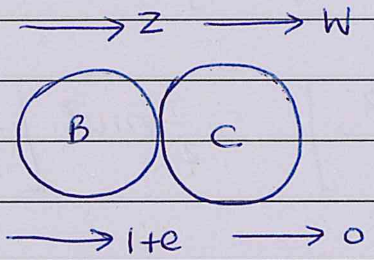
$x = 1 - e$

$x + y = 2$

$-x + y = 2e$

$2y = 2 + 2e$

$y = 1 + e$



N.E.L. $\frac{z-w}{1+e} = -e$

① $z - w = -e - e^2$

$mz + mw = m(1+e)$

② $z + w = 1 + e$

$z - w = -e - e^2$

$2z = 1 + e - e - e^2$

$2z = 1 - e^2$

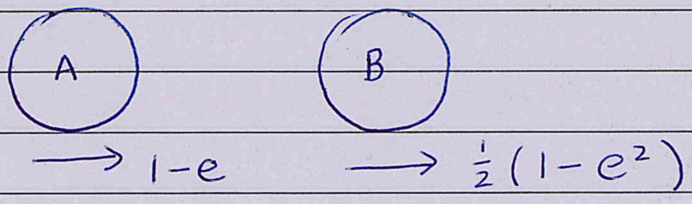
$z = \frac{1}{2}(1 - e^2)$

$z + w = 1 + e$

$-z + w = e + e^2$

$2w = e^2 + 2e + 1$

$w = \frac{1}{2}(e+1)^2$



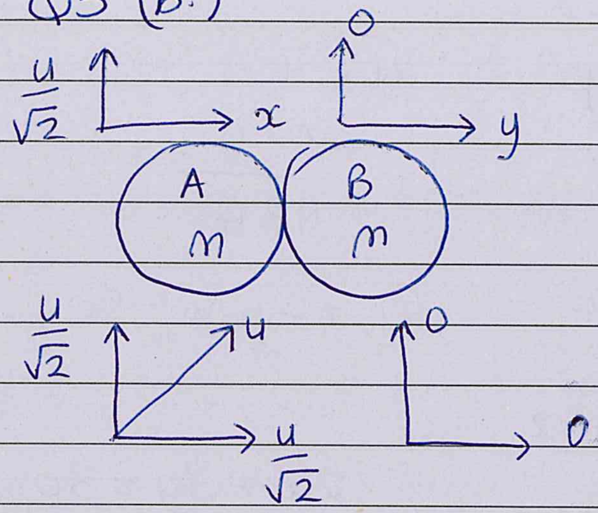
if $1 - e > \frac{1}{2}(1 - e^2)$ then A will collide with B again.

$2 - 2e > 1 - e^2$

$e^2 - 2e + 1 > 0$

$(e-1)^2 > 0$ since this is always true, there WILL be a collision.

2008 Q5 (b.)



NEL:

$$\frac{x-y}{\frac{u}{\sqrt{2}}-0} = -e$$

$$x-y = -\frac{eu}{\sqrt{2}}$$

② $\sqrt{2}x - \sqrt{2}y = -eu$

P.C.M.

$$\cancel{m}x + \cancel{m}y = \frac{\cancel{m}u}{\sqrt{2}}$$

① $\sqrt{2}x + \sqrt{2}y = u$
② $\sqrt{2}x - \sqrt{2}y = -eu$

$$2\sqrt{2}\,x = u(1-e)$$

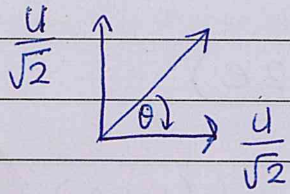
$$x = \frac{u}{2\sqrt{2}}(1-e)$$

$$\sqrt{2}x + \sqrt{2}y = u$$
$$-\sqrt{2}x + \sqrt{2}y = eu$$

$$2\sqrt{2}\,y = u(1+e)$$

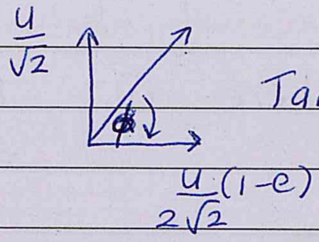
$$y = \frac{u}{2\sqrt{2}}(1+e)$$

A: Before



$\tan\theta = 1$

After



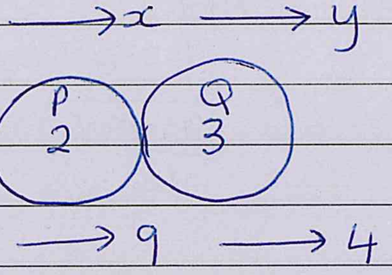
$$\tan\phi = \frac{\frac{u}{\sqrt{2}}}{\frac{u(1-e)}{2\sqrt{2}}} = \frac{2}{1-e}$$

$\alpha = \phi - \theta$

$$\tan(\alpha) = \tan(\phi - \theta) = \frac{\tan\phi - \tan\theta}{1+\tan\theta\tan\phi}$$

$$= \frac{\frac{2}{1-e} - 1}{1+\frac{2}{1-e}} = \frac{2-1+e}{1-e+2} = \frac{1+e}{3-e}$$

2007: Q5 (a)



N.E.L:

$$\frac{x-y}{9-4} = -e$$

② $x - y = -5e$

P.C.M.

$2x + 3y = 18 + 12$

① $2x + 3y = 30$

② ×3 $3x - 3y = -15e$

$5x = 30 - 15e$

$x = 6 - 3e$

$2x + 3y = 30$

$-\ 2x + 2y = 10e$

$5y = 30 + 10e$

$y = 6 + 2e$

(ii) momentum change for sphere P:

$m(9) - m(6-3e)$

$2(9) - 2(6-3e)$

$18 - 12 + 6e$

$= 6 + 6e = 6(1+e)$

for sphere Q: $m(4) - m(6+2e)$

$= 12 - 18 - 6e$

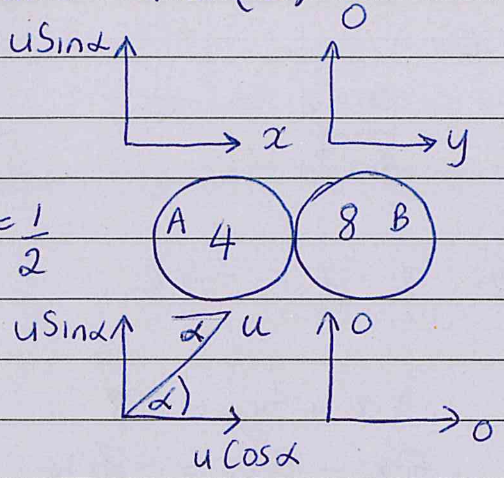
$= -6 - 6e = -6(1+e)$

So Q transferred $6(1+e)$ to P.

2007: Q5 (b)

(i)

$e = \frac{1}{2}$



NEL : $\frac{x - y}{u \cos\alpha} = -\frac{1}{2}$

① $-2x + 2y = u \cos\alpha$

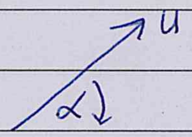
P.C.M:

$$4x + 8y = 4u\cos\alpha$$
$$x + 2y = \cancel{4}\, u\cos\alpha$$
$$2x - 2y = -u\cos\alpha$$
$$3x = 0, \quad x = 0$$

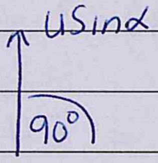
$$2x + 4y = 2u\cos\alpha$$
$$-2x + 2y = u\cos\alpha$$
$$6y = 3u\cos\alpha$$
$$y = \frac{u\cos\alpha}{2}$$

(ii)

4 kg : Before :



After



deflected through $90° - \alpha°$

(iii) loss in KE = KE before − KE after

KE before = $\frac{1}{2}(4)u^2$

K.E. after = $\frac{1}{2}(4)u^2\sin^2\alpha + \frac{1}{2}(8)\frac{u^2\cos^2\alpha}{4}$

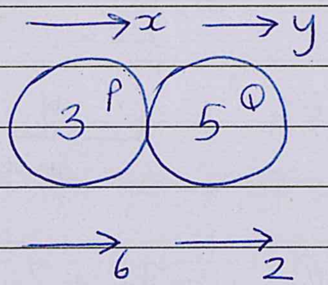
loss in KE = $2u^2 - (2u^2\sin^2\alpha + u^2\cos^2\alpha)$

$= 2u^2 - u^2(2 - 2\cos^2\alpha + \cos^2\alpha)$

$= 2u^2 - 2u^2 + u^2\cos^2\alpha$

$= u^2\cos^2\alpha$

2006 Q5 (a.)



N.E.L:

$$\frac{x-y}{6-2} = -e$$

① $x - y = -4e$

P.C.M:

$3x + 5y = 18 + 10$

② $3x + 5y = 28$

$-3x + 3y = +12e$

$8y = 28 + 12e$

$2y = 7 + 3e$

$y = \frac{1}{2}(7+3e)$

$3x + 5y = 28$

$5x - 5y = -20e$

$8x = 28 - 20e$

$2x = 7 - 5e$

$x = \frac{1}{2}(7-5e)$

(ii) loss of KE = KE before − KE after

KE before = $\frac{1}{2}(3)6^2 + \frac{1}{2}(5)2^2$

$= 54 + 10 = 64$

KE after = $\frac{1}{2}(3)\left(\frac{7-5e}{4}\right)^2 + \frac{1}{2}(5)\left(\frac{7+3e}{4}\right)^2$

$= \frac{3}{8}(49 - 70e + 25e^2) + \frac{5}{8}(49 + 42e + 9e$

$= \frac{147 \cancel{-210e} + 75e^2 + 245 \cancel{+210e} + 45e}{8}$

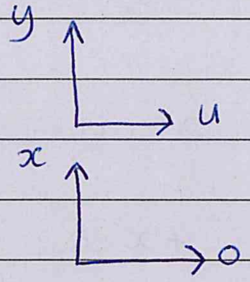
$= \frac{120e^2 + 392}{8}$

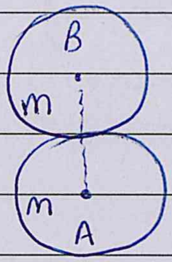
So loss in KE = $\frac{64(8) - 120e^2 - 392}{8}$

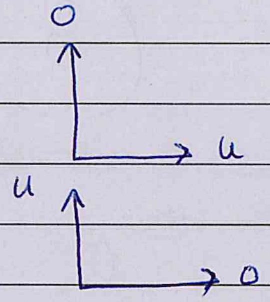
$= \frac{120 - 120e^2}{8} = 15 - 15e^2$

So $k = 15$

2006 Q5 (b.)







NEL:

$\frac{x-y}{u} = -e$ ① $x-y = -eu$

P.C.M:

$mx + my = mu$ ② $x+y = u$

$x - y = -eu$

$2x = u(1-e)$

$x = \frac{u}{2}(1-e)$

$x+y = u$

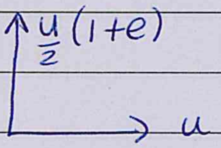
$-x+y = eu$

$2y = u(1+e)$

$y = \frac{u}{2}(1+e)$

Speed of A $= \frac{u}{2}(1-e) \uparrow$

Speed of B?



$= \sqrt{u^2 + \frac{u^2}{4}(1-2e+e^2)}$

$= \frac{u}{2}\sqrt{e^2+2e+5}$

(ii) If these were equal:

$\frac{u}{2}(1-e) = \frac{u}{2}\sqrt{e^2+2e+5}$

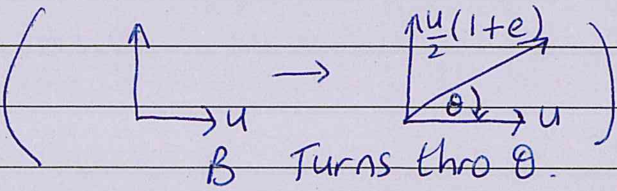
$(1-e)^2 = e^2+2e+5$

$1-2e+e^2 = e^2+2e+5 \Rightarrow 4e+4=0$

$4(e+1)=0 \Rightarrow e=-1$ not possible

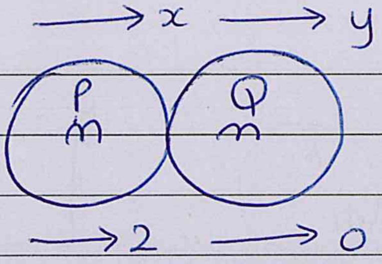
(iii) Angle thro which B turns?

$\theta = \mathrm{Tan}^{-1} \frac{\frac{u}{2}(1+e)}{u} = \frac{1+e}{2}$



B Turns thro θ.

2005 Q5 (a)



P.C.M.

$mx + my = 2m$

$x + y = 2$ ①

NEL

$\frac{x-y}{2} = -\frac{3}{4}$

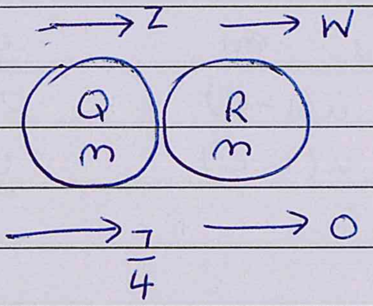
$x - y = -\frac{6}{4}$

$4x - 4y = -6$

$2x - 2y = -3$ ②

$$\begin{array}{l} 2x + 2y = 4 \\ 2x - 2y = -3 \\ \hline 4x = 1 \\ x = 1/4 \end{array}$$

$$\begin{array}{l} 2x + 2y = 4 \\ -2x + 2y = 3 \\ \hline 4y = 7 \\ y = 7/4 \end{array}$$



$\cancel{m}z + \cancel{m}W = \frac{7\cancel{m}}{4}$

$4z + 4W = 7$

$$\begin{array}{l} 16z + 16W = 28 \quad ② \\ 16z - 16W = -21 \\ \hline 32z = 7 \\ z = 7/32 \end{array}$$

N.E.L : $\frac{z - W}{\frac{7}{4}} = -\frac{3}{4}$

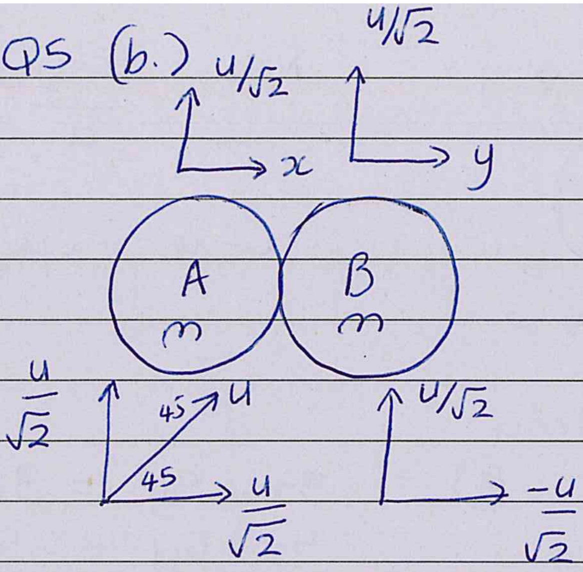
$4z - 4W = -\frac{21}{4}$

$16z - 16W = -21$

Since Vel of P = $1/4$ and Vel of Q = $7/32$

since $\frac{1}{4} > \frac{7}{32}$, P will strike Q again

005 Q5 (b.)



N.E.L. $\frac{x-y}{\frac{2u}{\sqrt{2}}} = -e$

② $x - y = -e.\frac{2u}{\sqrt{2}}$

P.C.M. $mx + my = \frac{mu}{\sqrt{2}} - \frac{mu}{\sqrt{2}}$

① $x + y = 0$

② $x - y = -e.2u/\sqrt{2}$

$2x = -2e\,u/\sqrt{2}$

$x = \frac{-ue}{\sqrt{2}}$

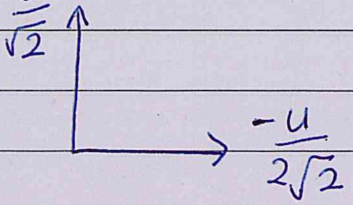
$x + y = 0$

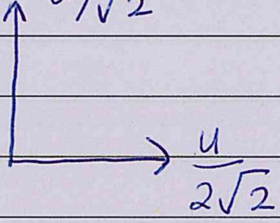
$-x + y = e.2u/\sqrt{2}$

$2y = 2eu/\sqrt{2}$

$y = \frac{ue}{\sqrt{2}}$

) $x = \frac{-u}{2\sqrt{2}}$, $y = \frac{u}{2\sqrt{2}}$ (when $e = \frac{1}{2}$)



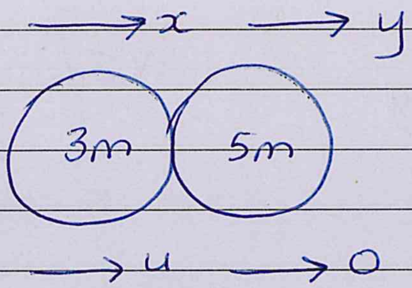


$\operatorname{Tan}\theta = -2$

$\operatorname{Tan}\alpha = 2.$

$$\operatorname{Tan}(\theta-\alpha) = \frac{\operatorname{Tan}\theta - \operatorname{Tan}\alpha}{1 + \operatorname{Tan}\theta\operatorname{Tan}\alpha} = \frac{-2-2}{1-2.2} = \frac{-4}{-3} = \frac{4}{3}$$

2004 Q5: (a)



NEL : $\frac{x-y}{u} = -e$

$x - y = -eu$ ①

P.C.M.

$3\cancel{m}x + 5\cancel{m}y = 3\cancel{m}u$

$3x + 5y = 3u$ ②

$-3\times$① $\underline{-3x + 3y = 3eu}$

$8y = 3u(1+e)$

$y = \frac{3u}{8}(1+e)$

$3x + 5y = 3u$

$\underline{5x - 5y = -5eu}$

$8x = u(3-5e)$

$x = \frac{u}{8}(3-5e)$

For spheres to move in opp directions :

since $0<e\leq 1$ y is $\oplus$

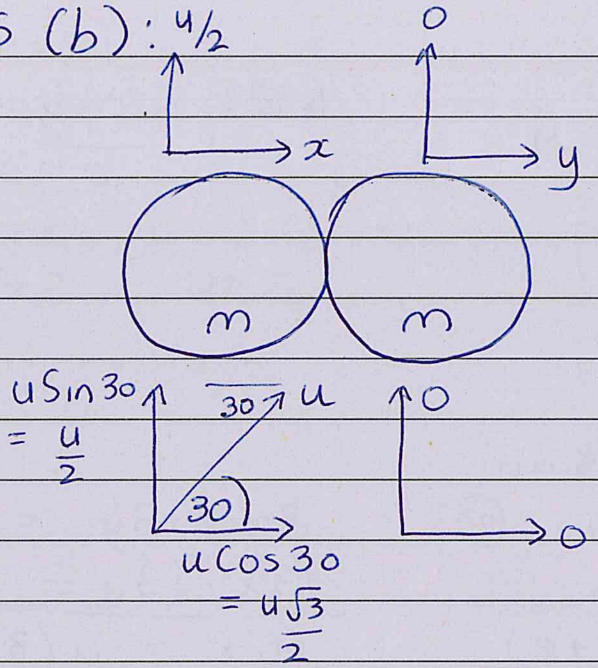
So we need to find condition for x to be $\ominus$

this is when $3 - 5e < 0$

$3 < 5e$

$\frac{3}{5} < e$

04 Q5 (b): 4/2



N.E.L.

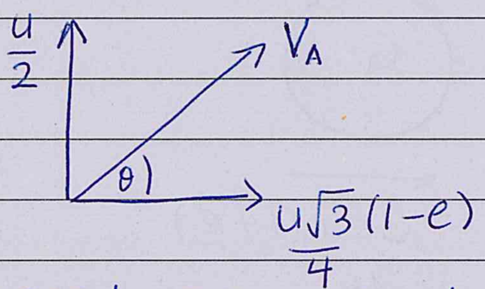
$$\frac{x-y}{\frac{u\sqrt{3}}{2}} = -e$$

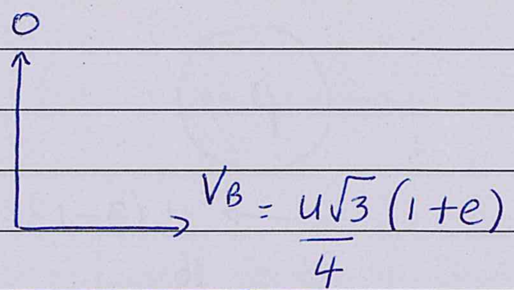
$$2x-2y = -eu\sqrt{3} \quad ①$$

P.C.M: $\cancel{m}x + \cancel{m}y = \cancel{m}u\sqrt{3}/2$

$$2x + 2y = u\sqrt{3} \quad ②$$
$$2x - 2y = -eu\sqrt{3}$$
$$4x = u\sqrt{3}(1-e)$$
$$x = \frac{u\sqrt{3}}{4}(1-e)$$

$$2x+2y = u\sqrt{3}$$
$$-2x+2y = eu\sqrt{3}$$
$$4y = u\sqrt{3}(1+e)$$
$$y = \frac{u\sqrt{3}}{4}(1+e)$$





speeds are equal $\Rightarrow \sqrt{\left(\frac{u}{2}\right)^2 + \left(\frac{u\sqrt{3}}{4}(1-e)\right)^2} = \frac{u\sqrt{3}}{4}(1+e)$

sqr both sides : $\frac{u^2}{4} + \frac{3u^2}{16}(1-2e+e^2) = \frac{3u^2}{16}(1+2e+e^2)$

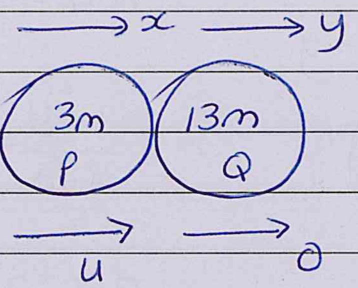
×16 : $4\cancel{u^2} + 3\cancel{u^2} - 6e\cancel{u^2} + 3e^2\cancel{u^2} = 3\cancel{u^2} + 6e\cancel{u^2} + 3\cancel{u^2}e^2$

$$4 + \cancel{3} - 6e + \cancel{3e^2} = \cancel{3} + 6e + \cancel{3e^2}$$

$$4 - 6e = 6e \Rightarrow 12e = 4 \Rightarrow e = \frac{1}{3}$$

$$\tan\theta = \frac{\frac{u}{2}}{\frac{u\sqrt{3}}{4}\left(1-\frac{1}{3}\right)} = \frac{\frac{u}{2}}{\frac{u}{2\sqrt{3}}} = \sqrt{3} \qquad \theta = \tan^{-1}\sqrt{3} = 60°$$

2003 Q5 (a)



N.E.L: $\frac{x-y}{u} = -e$

$x - y = -eu$ ①

P.C.M:

$3mx + 13my = 3mu$

$3x + 13y = 3u$ ②

① x3 $-3x + 3y = +3eu$

$16y = 3u(1+e)$

$y = \frac{3u}{16}(1+e)$

$3x + 13y = 3u$

$13x - 13y = -13eu$

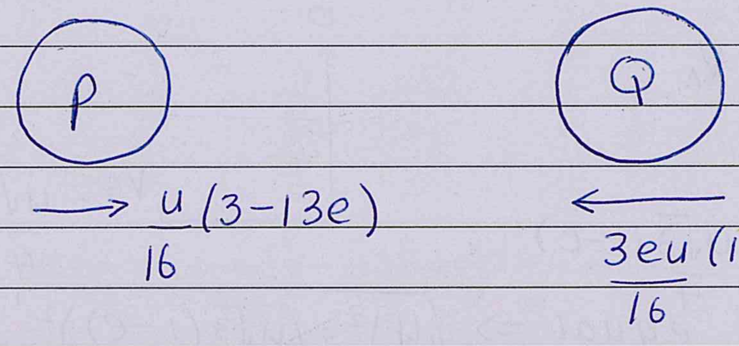
$16x = u(3-13e)$

$x = \frac{u}{16}(3-13e)$

Q then hits wall and rebounds back with speed

$Q_{back} = -\frac{3eu}{16}(1+e)$

10m



They <u>will</u> hit if V_P is ⊕ $3 - 13e > 0$

$3 > 13e$

$\frac{3}{13} > e$

<u>But</u> also,

They <u>will</u> hit if V_P is ⊖ but slower than V

so e can be > 3/13 so need to find upp

⊕ ←

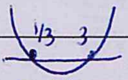
$+V_P < V_Q$

$-\frac{\cancel{u}}{\cancel{16}}(3-13e) < \frac{3e\cancel{u}}{\cancel{16}}(1+e)$

$-3 + 13e < 3e + 3e$

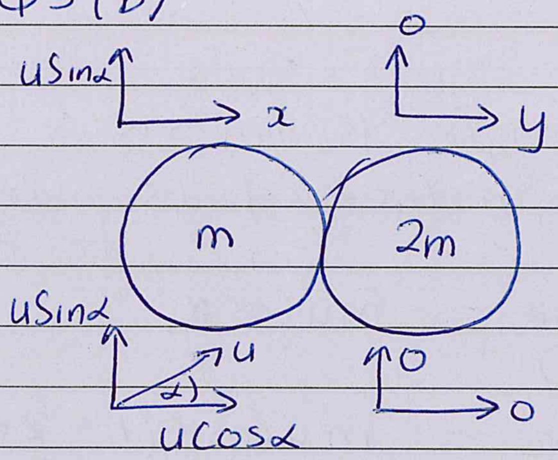
$3e^2 - 10e + 3 >$

$(3e-1)(e-3) > 0$



$e < 1/3$

2003 Q5 (b)



NEL: $\frac{x-y}{u\cos\alpha} = -e$

$x - y = -eu\cos\alpha$ ②

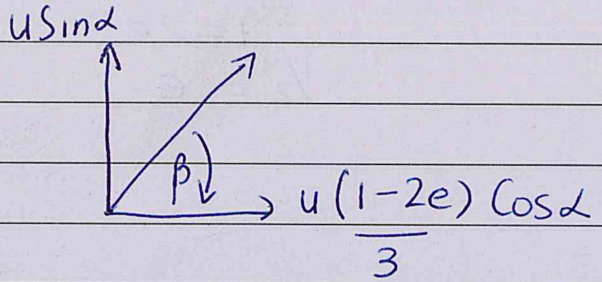
P.C.M:

$mx + 2my = mu\cos\alpha$

$x + 2y = u\cos\alpha$ ①

$$\begin{aligned} x + 2y &= u\cos\alpha \\ -x + y &= eu\cos\alpha \\ \hline 3y &= u\cos\alpha(1+e) \\ y &= \frac{u\cos\alpha(1+e)}{3} \end{aligned}$$

$$\begin{aligned} x + 2y &= u\cos\alpha \\ 2x - 2y &= -2eu\cos\alpha \\ \hline 3x &= u\cos\alpha(1-2e) \\ x &= \frac{u\cos\alpha(1-2e)}{3} \end{aligned}$$



$\tan\beta = \frac{u\sin\alpha}{\frac{u(1-2e)\cos\alpha}{3}}$

$\tan\beta = \frac{3u\sin\alpha}{u(1-2e)\cos\alpha}$

$\tan\beta = \frac{3}{1-2e}(\tan\alpha)$

but $\beta = \alpha + 90°$ So $\tan(\beta) = \frac{-1}{\tan\alpha}$

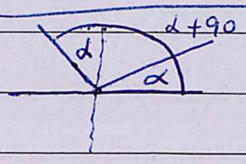
$\therefore \frac{3}{1-2e}\tan\alpha = \frac{1}{\tan\alpha} \Rightarrow \tan^2\alpha = \frac{2e-1}{3}$

$\tan\alpha = \sqrt{\frac{2e-1}{3}}$

$\tan(\alpha+90) = \frac{\tan\alpha + \tan 90}{1 - \tan\alpha\tan 90}$

$= \frac{\frac{\tan\alpha}{\tan 90} + 1}{\frac{1}{\tan 90} - \tan\alpha} = \frac{1}{-\tan\alpha}$

or



2003 Q5 (a.)(ii)

Impulse exerted by A on B is $mu\cos\alpha$

This means A loses $mu\cos\alpha$ of its momentum to B

momentum of A before = $mu\cos\alpha$
(in ⓧ direction)

momentum of A after = $mu\cos\alpha\left(\frac{1-2e}{3}\right)$

mom before = momentum after + $mu\cos\alpha$

$$\cancel{mu\cos\alpha} = mu\cos\alpha\left(\frac{1-2e}{3}\right) + \cancel{mu\cos\alpha}$$

$$\Rightarrow mu\cos\alpha\left(\frac{1-2e}{3}\right) = 0 \quad \Rightarrow 1-2e=0$$

$$1=2e$$

$$1/2 = e$$

<u>OR</u>

momentum gained by B = $mu\cos\alpha$

<u>before</u> B's momentum = 0

after B's momentum = $2mu\cos\alpha\left(\frac{1+e}{3}\right)$

$$\text{so} \quad \frac{2\cancel{mu\cos\alpha}(1+B)}{3} = \cancel{mu\cos\alpha}$$

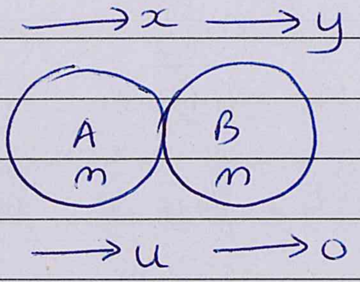
$$2(1+e) = 3$$

$$2+2e = 3$$

$$2e = 1$$

$$e = 1/2$$

2002 Q5 (a)

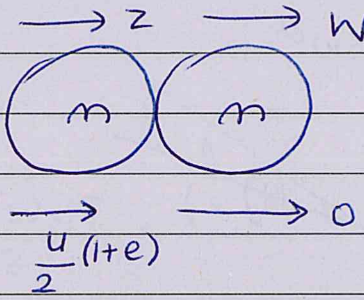


N.E.L: $\dfrac{x-y}{u} = -e$

$x - y = -eu$ ①

P.C.M: $\cancel{m}x + \cancel{m}y = \cancel{m}u$

$$\begin{aligned} x + y &= u \\ x - y &= -eu \\ \hline 2x &= u(1-e) \\ x &= \frac{u}{2}(1-e) \end{aligned} \qquad \begin{aligned} x + y &= u \\ -x + y &= eu \\ \hline 2y &= u(1+e) \\ y &= \frac{u}{2}(1+e) \end{aligned}$$



N.E.L.: $\dfrac{z-w}{\frac{u}{2}(1+e)} = -e$

$z - w = -\dfrac{eu(1+e)}{2}$

$2z - 2w = -eu(1+e)$. ①

P.C.M:

$mz + mw = \dfrac{mu}{2}(1+e)$

$z + w = \dfrac{u}{2}(1+e)$

$$\begin{aligned} 2z + 2w &= u(1+e) \\ 2z - 2w &= -eu(1+e) \\ \hline 4z &= u(1+e)(1-e) \\ z &= \frac{u(1+e)(1-e)}{4} \end{aligned} \qquad \begin{aligned} 2z + 2w &= u(1+e) \\ -2z + 2w &= +eu(1+e) \\ \hline 4w &= (1+e)^2 u \\ w &= (1+e)^2 \frac{u}{4} \end{aligned}$$

but $w = \dfrac{5u}{8} = (1+e)^2 \dfrac{u}{4}$

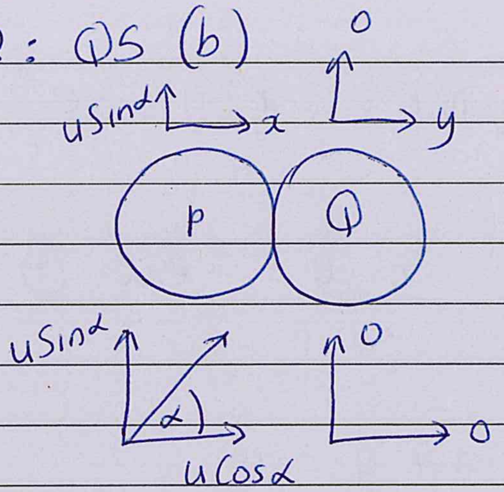
$5 = 2(1+e)^2 = 2(1+2e+e^2)$

$5 = 2 + 4e + 2e^2$

$2e^2 + 4e - 3 = 0 \qquad e = \dfrac{-4 \pm \sqrt{16+24}}{2(2)}$

$e = 0.58$

2002: Q5 (b)



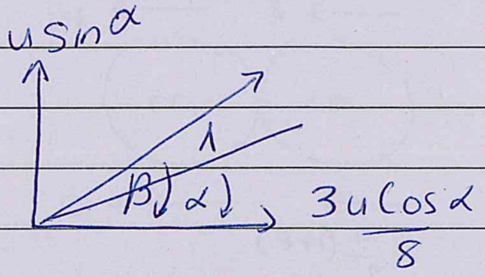
N.E.L.

$$\frac{x-y}{u\cos\alpha} = -\frac{1}{4}$$

$$4x-4y = -u\cos\alpha \quad (1$$

P.C.M:

$$mx + my = mu\cos\alpha$$
$$x + y = u\cos\alpha$$
$$4x + 4y = +4u\cos\alpha$$
$$4x - 4y = -u\cos\alpha$$
$$8x = 3u\cos\alpha$$
$$x = \frac{3u\cos\alpha}{8}$$



$$\mathrm{Tan}\,\beta = \frac{u\sin\alpha}{\frac{3u\cos\alpha}{8}} = \frac{8\,\mathrm{Tan}\,\alpha}{3}$$

$$\beta = \alpha + \lambda$$

$$\mathrm{Tan}\,\beta = \mathrm{Tan}(\alpha+\lambda) = \frac{\mathrm{Tan}\,\alpha + \mathrm{Tan}\,\lambda}{1 - \mathrm{Tan}\,\alpha\,\mathrm{Tan}\,\lambda}$$

$$\text{so} \quad \frac{\mathrm{Tan}\,\alpha + \mathrm{Tan}\,\lambda}{1 - \mathrm{Tan}\,\alpha\,\mathrm{Tan}\,\lambda} = \frac{8\,\mathrm{Tan}\,\alpha}{3}$$

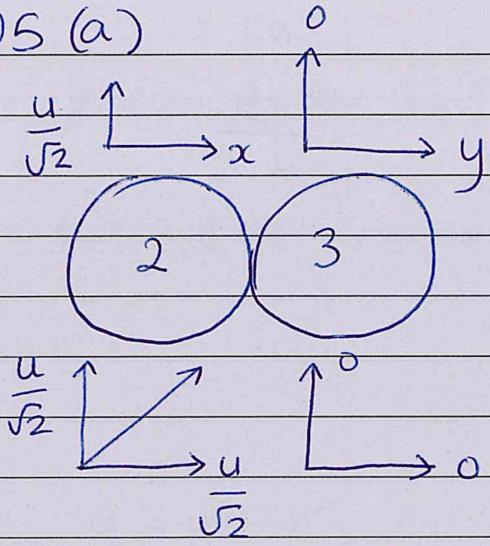
$$3\,\mathrm{Tan}\,\alpha + 3\,\mathrm{Tan}\,\lambda = 8\,\mathrm{Tan}\,\alpha - 8\,\mathrm{Tan}^2\alpha\,\mathrm{Tan}\,\lambda$$

$$+3\,\mathrm{Tan}\,\lambda + 8\,\mathrm{Tan}^2\alpha\,\mathrm{Tan}\,\lambda = 5\,\mathrm{Tan}\,\alpha$$

$$\mathrm{Tan}\,\lambda\,(3 + 8\,\mathrm{Tan}^2\alpha) = 5\,\mathrm{Tan}\,\alpha$$

$$\mathrm{Tan}\,\lambda = \frac{5\,\mathrm{Tan}\,\alpha}{3 + 8\,\mathrm{Tan}^2\alpha}$$

2001 Q5 (a)



N.E.L:

$$\frac{x-y}{\frac{u}{\sqrt{2}}} = -e$$

$$x-y = \frac{-eu}{\sqrt{2}} \quad ①$$

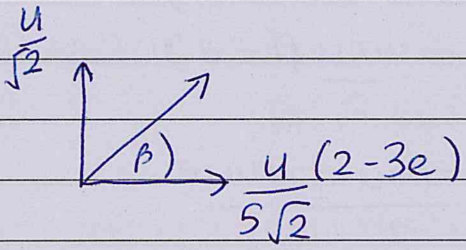
P.C.M:

$$② \quad 2x+3y = 2u/\sqrt{2}$$
$$-2x+2y = +2eu/\sqrt{2}$$
$$5y = \frac{2u}{\sqrt{2}}(1+e)$$

$$y = \frac{2u}{5\sqrt{2}}(1+e)$$

$$2x+3y = 2u/\sqrt{2}$$
$$3x-3y = -3eu/\sqrt{2}$$
$$5x = \frac{u}{\sqrt{2}}(2-3e)$$

$$x = \frac{u}{5\sqrt{2}}(2-3e)$$

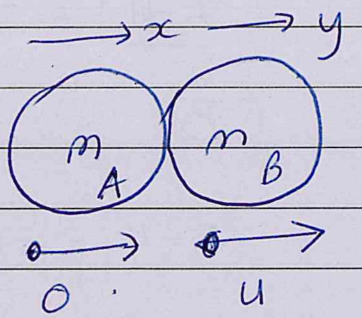


$$\text{Tan } \beta = \frac{u}{\sqrt{2}} \Big/ \frac{u(2-3e)}{5\sqrt{2}} = \frac{u}{\sqrt{2}} \times \frac{5\sqrt{2}}{u(2-3e)}$$

$$\text{Tan } \beta = \frac{5}{2-3e} = 10$$

$$5 = 20 - 30e$$
$$15 = 30e$$
$$\frac{1}{2} = e$$

2001 Q5 (b)



N.E.L.

$$\frac{x-y}{-u} = -e$$

$$x-y = eu \quad ①$$

P.C.M: $mx + my = mu$

$$x + y = u$$
$$x - y = eu$$
$$2x = u(1+e)$$
$$x = \frac{u}{2}(1+e)$$

$$x + y = u$$
$$-x + y = -eu$$
$$2y = u(1-e)$$
$$y = \frac{u}{2}(1-e)$$

A: Change in momentum $= m(0) - m(x)$

$$= -\frac{mu}{2}(1+e)$$

B: Change in momentum $= m(u) - m(y)$

$$= mu - \frac{mu}{2}(1-e)$$

$$= \frac{2mu - mu + mue}{2}$$

$$= \frac{mu + mue}{2} = \frac{mu}{2}(1+e)$$

loss in KE

KE Before $= \frac{1}{2}m(0)^2 + \frac{1}{2}mu^2$

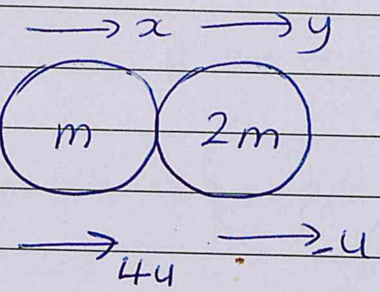
KE After $= \frac{1}{2}m\left(\frac{u^2(1+e)^2}{4}\right) + \frac{1}{2}m\left(\frac{u^2(1-e)^2}{4}\right)$

$$= \frac{1}{2}\frac{mu^2}{4}\left(1 + 2e + e^2 + 1 + e^2 - 2e\right)$$

$$= \frac{1}{2}\frac{mu^2}{4}(2 + 2e^2) = \frac{mu^2(1+e^2)}{4}$$

Loss in KE $= \frac{1}{2}mu^2 - \frac{mu^2}{4} - \frac{mu^2e^2}{4} = \frac{mu^2}{4}(1-e^2)$

2000 Q5 (a.)



N.E.L.

$$\frac{x-y}{5u} = -e$$

$x - y = -5eu$ ①

P.C.M.

$\cancel{m}x + 2\cancel{m}y = 4\cancel{m}u - 2\cancel{m}u$

$x + 2y = 4u - 2u$

$x + 2y = 2u$ ①

$-x + y = 5eu$

$3y = u(2+5e)$

$y = \frac{u(2+5e)}{3}$

$x + 2y = 2u$

$2x - 2y = -10eu$

$3x = 2u(1-5e)$

$x = \frac{2u}{3}(1-5e)$

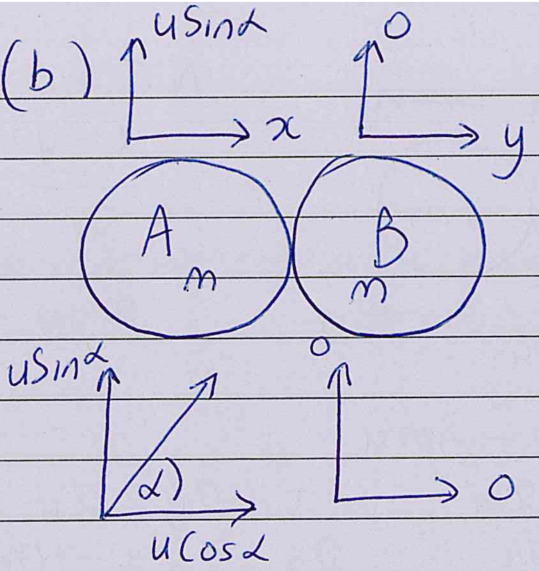
y = always ⊕ since $e>0$

We just need to find conditions for x to be ⊖ to let them move in opp. directions.

for x to be ⊖ $1-5e<0$ $1<5e$

$\frac{1}{5} < e$

2008 Q5 (b)



N.E.L.

$$\frac{x-y}{u\cos\alpha} = -\frac{1}{2}$$

$$2x-2y = -u\cos\alpha$$

P.C.M: $\cancel{m}x + \cancel{m}y = \cancel{m}u\cos\alpha$

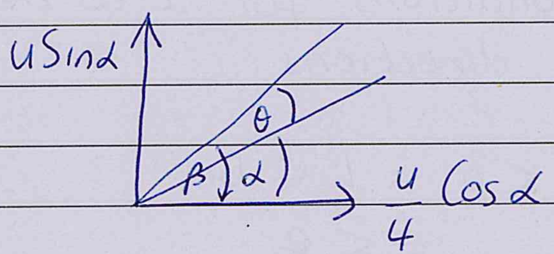
$$x+y = u\cos\alpha$$

$$2x+2y = 2u\cos\alpha$$

$$\underline{2x-2y = -u\cos\alpha}$$

$$4x = u\cos\alpha$$

$$x = \frac{u}{4}\cos\alpha$$



$$\tan\beta = \frac{u\sin\alpha}{\frac{u\cos\alpha}{4}} = 4\tan$$

$$\tan\theta = \tan(\beta-\alpha) = \frac{\tan\beta - \tan\alpha}{1+\tan\beta\tan\alpha}$$ but $\tan\beta = 4\tan\alpha$

$$\tan\theta = \frac{4\tan\alpha - \tan\alpha}{1+4\tan\alpha\tan\alpha} = \frac{3\tan\alpha}{1+4\tan^2\alpha}$$