Question 1

$T^2 = 4\pi^2R^3/GM$ (4)

6.57×10^{23} kg *(–1 for omission of or incorrect units)* (3)

Question 2

6. (i) **State Newton's law of universal gravitation.**

force proportional to product of masses // $F \propto \frac{m_1 m_2}{d^2}$ / $F = \frac{Gm_1m_2}{d^2}$ 3

inversely proportional to square of distance // correct notation given 3

(ii) **Explain what is meant by angular velocity. Derive an equation for the angular velocity of an object in terms of its linear velocity when the object moves in a circle.**

rate of change of angle // $\frac{\theta}{t}$ and correct notation given 3

$\omega = \frac{\theta}{t}$ *(stated or implied)* 3

$\omega = \frac{vt}{rt}$ / $\frac{v}{r}$ 3

The International Space Station (ISS), shown in the photograph, functions as a research laboratory and a location for testing of equipment required for trips to the moon and to Mars. The ISS orbits the earth at an altitude of 4.13×10^5 m every 92 minutes 50 seconds.

(iii) **Calculate (*a*) the angular velocity, (*b*) the linear velocity, of the ISS.**

$\omega = \frac{2\pi}{T}$ 3

$\omega = \frac{2\pi}{5570}$ /1.1×10^{-3} s^{-1} *(–1 for omission of or incorrect units)* 3

$v = r\omega$ 3

$v = (6.783 \times 10^6) \times (1.1 \times 10^{-3}) = 7651.5$ m s^{-1} 3

(–1 for omission of or incorrect units)

(iv) **Name the type of acceleration that the ISS experiences as it travels in a circular orbit around the earth.**

centripetal / gravitational 3

What force provides this acceleration?

gravitational *(do <u>not</u> accept "gravity")* 3

(v) **Calculate the attractive force between the earth and the ISS.**

$F = \frac{mv^2}{r}$ // $F = mr\omega^2$ 3

$F = 3.884 \times 10^6$ N *(–1 for omission of or incorrect units)* 3

Hence or otherwise, calculate the mass of the earth.

$F = \frac{GmM}{r^2}$ // $T^2 = \frac{4\pi^2 r^3}{GM}$ // $g = \frac{GM}{r^2}$ 3

$M = \frac{Fr^2}{Gm}$ // $M = \frac{4\pi^2 r^3}{GT^2}$ // $M = \frac{(8.63)r^2}{G}$ 3

$M = 5.95 \times 10^{24}$ kg *(–1 for omission of or incorrect units)* 3

(vi) **If the value of the acceleration due to gravity on the ISS is 8.63 m s^{-2}, why do occupants of the ISS experience apparent weightlessness?**

they are in freefall // ISS accelerating at the same rate as occupants 3

(vii) **A geostationary communications satellite orbits the earth at a much higher altitude than the ISS. What is the period of a geostationary communications satellite?**

1 day 5

(mass of ISS = 4.5×10^5 kg; radius of the earth = 6.37×10^6 m)

Question 3

(a) **Define the moment of a force.**

($T =$) force × (perp) distance / $F \times d$ 6

When the toy is knocked over, it always returns to the upright position. Explain why this happens.

(toy non-vertical) c.g. has a (turning) moment about fulcrum / point of support/contact /
(c.g. has) zero turning moment when toy is in vertical position 6

(any valid reference, e.g. 'low c.g.', 'equilibrium', 'turning moment', … 3 marks)

(b) **State the conditions necessary for the equilibrium of a body under a set of co-planar forces.**

(vector/algebraic) sum of the forces = zero // forces up = forces down // $\Sigma F = 0$ 6+3

sum of the (turning) moments (about any point) = zero // CTM = ACTM // $\Sigma T = 0$

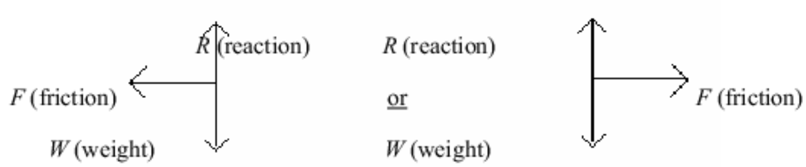
Three children position themselves on a uniform see-saw so that it is horizontal and in equilibrium. The fulcrum of the see-saw is at its centre of gravity. A child of mass 30 kg sits 1.8 m to the left of the fulcrum and another child of mass 40 kg sits 0.8 m to the right of the fulcrum. Where should the third child of mass 45 kg sit, in order to balance the see-saw?

$30g(1.8)$ / $40g(0.8)$ / $45g(x)$ 6

$30g(1.8) = 40g(0.8) + 45g(x)$ 3

$x =$ 0.488 m / 0.49 m / 49 cm **(–1 for omission of or incorrect unit)** 3

(c) **A simple merry-go-round consists of a flat disc that is rotated horizontally. A child of mass 32 kg stands at the edge of the merry-go-round, 2.2 metres from its centre. The force of friction acting on the child is 50 N. Draw a diagram showing the forces acting on the child as the merry-go-round rotates.**



(–1 per each unlabelled force; 3 marks per each correct force) 3×3

What is the maximum angular velocity of the merry-go-round so that the child will not fall from it, as it rotates?

$F = m\omega^2 r$ 3

$50 = (32)(\omega)^2(2.2)$ 3

$\omega = 0.842 \text{ rad s}^{-1}$ **(–1 for omission of or incorrect unit)** 3

If there was no force of friction between the child and the merry-go-round, in what direction would the child move as the merry-go-round starts to rotate?

child remains stationary / any appropriate answer, e.g. '*it depends on the frame of reference*' 5

Question 4

Question 6

Define (i) velocity, (ii) angular velocity. (12)

(i) rate of change // $\frac{dx}{dt}$ / $\frac{ds}{dt}$ 3

of displacement // where (x =) s = displacement 3

['*speed in a given direction*' … 3 marks]

(ii) change in <u>angular displacement</u> /<u>angle</u> // $\frac{d\theta}{dt}$ // $(\omega) = \theta / t$ 3

per sec // θ <u>angular displ</u>. / <u>angle</u> // notation 3

Derive the relationship between the velocity of a particle travelling in uniform circular motion and its angular velocity. (12)

$\theta = \frac{|arc|}{r} / \frac{vt}{r}$ 3

$\theta = \omega t$ 3

$\Rightarrow \frac{vt}{r} = \omega t$ 3

$v = \omega r$ 3

Alternative method:

$\theta = s / r$

$\theta / t = s / rt$

$\omega = v / r$

$v = \omega r$

A student swings a ball in a circle of radius 70 cm in the vertical plane as shown. The angular velocity of the ball is 10 rad s^{-1}.

What is the velocity of the ball? How long does the ball take to complete one revolution? (9)

v (= ωr) = (10)(0.70) / 7.0 m s^{-1} 3

$T = \frac{2\pi r}{v} / \frac{2\pi}{\omega}$ 3

$T = \frac{2\pi(0.70)}{7} / \frac{2\pi}{10}$ / 0.63 s **(-1 for omission of or incorrect unit)** 3

Draw a diagram to show the forces acting on the ball when it is at position A. (6)

weight (W)downwards; <u>force(F) /reaction</u> (R) upwards **(-1 if either force omitted)** 3

centripetal force to left (due to friction or curled fingers) 3

(description without diagram … -1)

The student releases the ball when it is at A, which is 130 cm above the ground, and the ball travels vertically upwards. Calculate

(i) the maximum height, above the ground, the ball will reach;

(ii) the time taken for the ball to hit the ground after its release from A. (17)

$v^2 = u^2 + 2as$ 3

$0 = (7)^2 + 2(-9.8)\, s$ / s = 2.5(0) m 3

=> max. height = 2.5 + 1.30 / 3.8 m 3

Overall: A => max. height =>ground:

$s = ut + {}^1/_2\, at^2$ 3

$-1.30 = 7t - {}^1/_2\,(9.8)t^2$ 3

(time =) t = 1.59 s **(no penalty applied for units here)** 2

Alternative method for *time taken*:

from point A to max. height: $v = u + at$ / $0 = 7 - 9.8t$ / t =0.71(43) s (3)

from max. to ground: [$s = ut + {}^1/_2\, at^2$ =>] $3.8 = 0(t) + 4.9\, t^2$ / t = 0.88 s (3)

total time = 0.71 + 0.88 / 1.59 s **(units not required)** (2)

Question 5

Define (i) angular velocity, (ii) centripetal force. (12)

(i) $\omega = \theta/t$ // angle traced out / angular displacement // rate of change 3

correct notation // per unit time / sec // of angle 3

(ii) $F = mv^2/r$ or $m w^2 r$ // force on body in circular motion 3

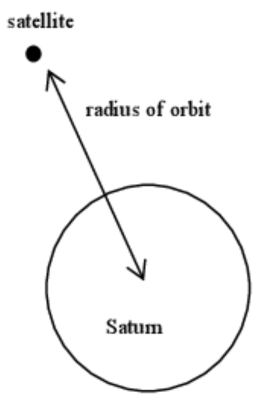
correct notation // towards the centre (of orbit) 3

State Newton's Universal Law of Gravitation. (6)

$F = \dfrac{Gm_1m_2}{d^2}$ / $F \propto \dfrac{m_1m_2}{d^2}$ // force proportional to product of masses 3

correct notation // inversely proportional to distance squared 3

A satellite is in a circular orbit around the planet Saturn. Derive the relationship between the period of the satellite, the mass of Saturn and the radius of the orbit. (15)



Gravitational force = centripetal force 3

$v^2 = GM/r$ or $w^2 = GM/r^3$ 3

$T = 2\pi r/v$ or $T = 2\pi/w$ 3

$T^2 = 4\pi^2 r^2/v^2$ or $T^2 = 4\pi^2/w^2$ 3

$T^2 = 4\pi^2 r^3/GM$ 3

(final formula presented without derivation .. 3 marks only)

The period of the satellite is 380 hours. Calculate the radius of the satellite's orbit around Saturn. (9)

$T = 380 \times 60 \times 60$ / 1.368×10^6 / 1.37×10^6 (s) 3

$(380 \times 3600)^2 = 4\pi^2 r^3/(6.7 \times 10^{-11})(5.7 \times 10^{26})$ 3

$r = 1.2 \times 10^9$ m (-1 for omission of or incorrect unit) 3

The satellite transmits radio signals to earth. At a particular time the satellite is 1.2×10^{12} m from earth. How long does it take the signal to travel to earth? (9)

$v = s/t$ 3

$(3.0 \times 10^8) = (1.2 \times 10^{12})/t$ 3

t = 4000 s / 1.1 h (-1 for omission of or incorrect unit) 3

It is noticed that the frequency of the received radio signal changes as the satellite orbits Saturn. Explain why. (5)

Doppler effect // satellite moves towards (earth) 3

due to relative motion between source (of signal) and detector // and away from earth / detector 2

(universal gravitational constant = 6.7×10^{-11} N m^2 kg^{-2} mass of Saturn = 5.7×10^{26} kg;
speed of light = 3.0×10^8 m s^{-1})

Question 6

(*b*) The moon orbits the earth. What is the relationship between the period of the moon and the radius of its orbit?

$T \propto$ // $T^2 \propto$ // period squared is proportional to // 4

$\sqrt{R^3}$ // R^3 // radius cubed 3

$T^2 = \frac{4\pi^2 R^3}{GM}$ 7 marks