Question 1 (2017)

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$2\left(x^2 - \frac{7}{2}x - 5\right)$ $= 2\left(\left(x - \frac{7}{4}\right)^2 - \frac{129}{16}\right)$ $= 2\left(\left(x - \frac{7}{4}\right)^2\right) - \frac{129}{8}$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • $a = 2$ identified explicitly or as factor *Mid partial Credit:* • Completed square *High partial Credit:* • h or k identified from work
(b)	$\left(\frac{7}{4}, \quad \frac{-129}{8}\right)$	**Scale 10B (0, 4, 10)** *Partial Credit:* • One relevant co-ordinate identified

(c) (i)	$f(x)$ has min point as $a > 0$ y co-ordinate of min < 0 $\Rightarrow$graph must cut x-axis twice hence two real roots. **or** $b^2 - 4ac = 49 + 80 > 0$ Therefore real roots	**Scale 5B (0, 3, 5)** *Partial Credit:* • Mention of $a > 0$ • $b^2 - 4ac$ • Identifies location of one or two roots, e.g. between 4 and 5.
c (ii)	$2x^2 - 7x - 10 = 0$ $2\left(\left(x - \frac{7}{4}\right)^2\right) - \frac{129}{8} = 0$ $\left(x - \frac{7}{4}\right)^2 = \frac{129}{16}$ $x - \frac{7}{4} = \pm\frac{\sqrt{129}}{4}$ $x = \frac{7}{4} \pm \sqrt{\frac{129}{16}}$ **OR** $2x^2 - 7x - 10 = 0$ $x = \frac{7 \pm \sqrt{49 + 80}}{4}$ $= \frac{7 \pm \sqrt{129}}{4}$ $x = \frac{7}{4} \pm \sqrt{\frac{129}{16}}$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • Formula with some substitution • Equation rewritten with some transpose *High Partial Credit:* • $x - \frac{7}{4} = \pm\frac{\sqrt{129}}{4}$ or equivalent

(ii) Explain what is meant by the indefinite integral of a function f.

The indefinite integral of f is the general form of a function whose derivative is f.

Alternative answer: The indefinite integral of f is $F(x)+C$ where $F' = f$ and C is constant (the constant of integration).

(iii) Write down the indefinite integral of g, the function in part **(i)**.

Answer: $\int g(x)dx = \frac{1}{4}x^4 - x^3 + 3x + C.$

(b) **(i)** Let $h(x) = x\ln x$, for $x \in \mathbb{R}$, $x > 0$.
Find $h'(x)$.

Using the product rule we see that

$$h'(x) = (x)'\ln x + x(\ln x)'.$$

But $(x)' = 1$ and $(\ln x)' = \frac{1}{x}$. Therefore

$$\begin{aligned} h'(x) &= (1)\ln x + x\left(\frac{1}{x}\right) \\ &= \ln x + 1. \end{aligned}$$

(ii) Hence, find $\int \ln x dx$.

We know that $h'(x) = \ln x + 1$. Also, we know that $(x)' = 1$. So if $F(x) = h(x) - x$, then

$$F'(x) = h'(x) - (x)' = \ln x + 1 - 1 = \ln x.$$

Therefore $\int \ln x\, dx = F(x) + c$. But $F(x) = h(x) - x = x\ln x - x$. Therefore

$$\int \ln x\, dx = x\ln x - x + C.$$

Type of function	Function	First derivative	Second derivative
Quadratic	k	B	I
Cubic	f	D	II
Trigonometric	g	A	III
Exponential	h	C	IV

(b) For **one** row in the table, explain your choice of first derivative and second derivative.

A quadratic function differentiates to a line which differentiates to a constant.