

Question 1 (2017)

| Q1 | Model Solution – 25 Marks | Marking Notes |
|-----|---|--|
| (a) | $2\left(x^2 - \frac{7}{2}x - 5\right)$ $= 2\left(\left(x - \frac{7}{4}\right)^2 - \frac{129}{16}\right)$ $= 2\left(\left(x - \frac{7}{4}\right)^2\right) - \frac{129}{8}$ | <p>Scale 5D (0, 2, 3, 4, 5)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> • $a = 2$ identified explicitly or as factor <p><i>Mid partial Credit:</i></p> <ul style="list-style-type: none"> • Completed square <p><i>High partial Credit:</i></p> <ul style="list-style-type: none"> • h or k identified from work |
| (b) | $\left(\frac{7}{4}, \quad -\frac{129}{8}\right)$ | <p>Scale 10B (0, 4, 10)</p> <p><i>Partial Credit:</i></p> <ul style="list-style-type: none"> • One relevant co-ordinate identified |

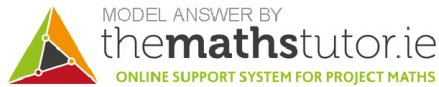
| | | |
|----------------------------------|--|---|
| <p>(c) (i)</p> | <p>$f(x)$ has min point as $a > 0$ y co-ordinate of min $< 0 \Rightarrow$ graph must cut x-axis twice hence two real roots.</p> <p style="text-align: center;">or</p> $b^2 - 4ac = 49 + 80 > 0$ <p>Therefore real roots</p> | <p>Scale 5B (0, 3, 5) <i>Partial Credit:</i></p> <ul style="list-style-type: none"> • Mention of $a > 0$ • $b^2 - 4ac$ • Identifies location of one or two roots, e.g. between 4 and 5. |
| <p>c (ii)</p> | $2x^2 - 7x - 10 = 0$ $2\left(\left(x - \frac{7}{4}\right)^2\right) - \frac{129}{8} = 0$ $\left(x - \frac{7}{4}\right)^2 = \frac{129}{16}$ $x - \frac{7}{4} = \pm \frac{\sqrt{129}}{4}$ $x = \frac{7}{4} \pm \sqrt{\frac{129}{16}}$ <p style="text-align: center;">OR</p> $2x^2 - 7x - 10 = 0$ $x = \frac{7 \pm \sqrt{49 + 80}}{4}$ $= \frac{7 \pm \sqrt{129}}{4}$ $x = \frac{7}{4} \pm \sqrt{\frac{129}{16}}$ | <p>Scale 5C (0, 3, 4, 5) <i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> • Formula with some substitution • Equation rewritten with some transpose <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> • $x - \frac{7}{4} = \pm \frac{\sqrt{129}}{4}$ or equivalent |

Question 2 (2014)

(ii) Explain what is meant by the indefinite integral of a function f .

The indefinite integral of f is the general form of a function whose derivative is f .

Alternative answer: The indefinite integral of f is $F(x) + C$ where $F' = f$ and C is constant (the constant of integration).



(iii) Write down the indefinite integral of g , the function in part (i).

Answer:
$$\int g(x)dx = \frac{1}{4}x^4 - x^3 + 3x + C.$$

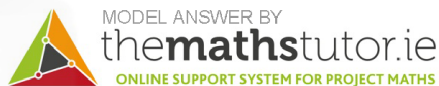
(b) (i) Let $h(x) = x \ln x$, for $x \in \mathbb{R}$, $x > 0$.
Find $h'(x)$.

Using the product rule we see that

$$h'(x) = (x)' \ln x + x(\ln x)'$$

But $(x)' = 1$ and $(\ln x)' = \frac{1}{x}$. Therefore

$$\begin{aligned} h'(x) &= (1) \ln x + x \left(\frac{1}{x} \right) \\ &= \ln x + 1. \end{aligned}$$



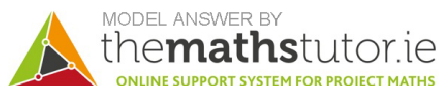
(ii) Hence, find $\int \ln x dx$.

We know that $h'(x) = \ln x + 1$. Also, we know that $(x)' = 1$. So if $F(x) = h(x) - x$, then

$$F'(x) = h'(x) - (x)' = \ln x + 1 - 1 = \ln x.$$

Therefore $\int \ln x dx = F(x) + c$. But $F(x) = h(x) - x = x \ln x - x$. Therefore

$$\int \ln x dx = x \ln x - x + C.$$



Question 3 (2013)

| Type of function | Function | First derivative | Second derivative |
|------------------|----------|------------------|-------------------|
| Quadratic | k | B | I |
| Cubic | f | D | II |
| Trigonometric | g | A | III |
| Exponential | h | C | IV |

(b) For **one** row in the table, explain your choice of first derivative and second derivative.

A quadratic function differentiates to a line which differentiates to a constant.