MarkingScheme

AlgebraH

Question 1 (2017)

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$2\left(x^2 - \frac{7}{2}x - 5\right)$ $= 2\left(\left(x - \frac{7}{4}\right)^2 - \frac{129}{16}\right)$ $= 2\left(\left(x - \frac{7}{4}\right)^2\right) - \frac{129}{8}$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • $a = 2$ identified explicitly or as factor *Mid partial Credit:* • Completed square *High partial Credit:* • h or k identified from work
(b)	$\left(\frac{7}{4}, \quad \frac{-129}{8}\right)$	**Scale 10B (0, 4, 10)** *Partial Credit:* • One relevant co-ordinate identified

(c) (i)	$f(x)$ has min point as $a > 0$ y co-ordinate of min < 0 ⇒graph must cut x-axis twice hence two real roots. **or** $b^2 - 4ac = 49 + 80 > 0$ Therefore real roots	**Scale 5B (0, 3, 5)** *Partial Credit:* • Mention of $a > 0$ • $b^2 - 4ac$ • Identifies location of one or two roots, e.g. between 4 and 5.
c (ii)	$2x^2 - 7x - 10 = 0$ $2\left(\left(x - \frac{7}{4}\right)^2\right) - \frac{129}{8} = 0$ $\left(x - \frac{7}{4}\right)^2 = \frac{129}{16}$ $x - \frac{7}{4} = \pm\frac{\sqrt{129}}{4}$ $x = \frac{7}{4} \pm \sqrt{\frac{129}{16}}$ **OR** $2x^2 - 7x - 10 = 0$ $x = \frac{7 \pm \sqrt{49 + 80}}{4}$ $= \frac{7 \pm \sqrt{129}}{4}$ $x = \frac{7}{4} \pm \sqrt{\frac{129}{16}}$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • Formula with some substitution • Equation rewritten with some transpose *High Partial Credit:* • $x - \frac{7}{4} = \pm\frac{\sqrt{129}}{4}$ or equivalent

Question 2 (2017)

(a)	$f(x) = 2x^3 + 5x^2 - 4x - 3$ $f(-3) = 2(-3)^3 + 5(-3)^2 - 4(-3) - 3$ $= -54 + 45 + 12 - 3$ $f(-3) = 0$ $\Rightarrow (x+3)$ is a factor $\begin{array}{r} 2x^2 - x - 1 \\ x+3 \overline{)\, 2x^3 + 5x^2 - 4x - 3} \\ \underline{2x^3 + 6x^2} \\ -x^2 - 4x \\ \underline{-x^2 - 3x} \\ -x - 3 \\ \underline{-x - 3} \end{array}$ $f(x) = (x+3)(2x^2 - x - 1)$ $f(x) = (x+3)(2x+1)(x-1)$ $x = -3 \quad x = -\frac{1}{2} \quad x = 1$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* • Shows $f(-3) = 0$ *High Partial Credit:* • quadratic factor of $f(x)$ found **Note**: No remainder in division may be stated as reason for $x = -3$ as root
(b)	$y = 2x^3 + 5x^2 - 4x - 3$ $\frac{dy}{dx} = 6x^2 + 10x - 4 = 0$ $3x^2 + 5x - 2 = 0$ $(x+2)(3x-1) = 0$ $3x - 1 = 0 \quad x + 2 = 0$ $x = \frac{1}{3} \quad x = -2$ $f\left(\frac{1}{3}\right) = \frac{-100}{27} \quad f(-2) = 9$ $Max = (-2, 9) \quad Min = \left(\frac{1}{3}, \frac{-100}{27}\right)$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • $\frac{dy}{dx}$ found (Some correct differentiation) *High Partial Credit* • roots and one y value found **Note**: One of Max/Min must be identified for full credit
(c)	$a > \frac{100}{27}$ **or** $a < -9$	**Scale 5B (0, 3, 5)** *Partial Credit:* • one value identified • no range identified (from 2 values)

<table>
<tr><th>Q4</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>$|\angle ABD| = |\angle CBD| = 90°$.......(i)
$|\angle BDC| + |\angle BCD| = 90°$ ...angles in triangle sum to $180°$
$|\angle ADB| + |\angle BDC| = 90°$ .... angle in semicircle
$|\angle ADB| + |\angle BDC| = |\angle BDC| + |\angle BCD|$
$|\angle ADB| = |\angle BCD|$........(ii)
$\therefore$ Triangles are equiangular (or similar)
or

$|\angle ABD| = |\angle CBD| = 90°$.......(i)
$|\angle DAB| = |\angle DAC|$ same angle $\Rightarrow |\angle ADB| = |\angle DCA|$ (reasons as above) which is also $\angle DCB$.........(ii)</td><td>Scale 15C (0, 5, 10, 15)
Low Partial Credit
• identifies one angle of same size in each triangle

High Partial Credit
• identifies second angle of same size in each triangle
• implies triangles are similar without justifying (ii) in model solution or equivalent</td></tr>
<tr><td>(a)
(ii)</td><td>$\frac{y}{1} = \frac{x}{y}$
$\Rightarrow y^2 = x$
$y = \sqrt{x}$
or
$|AD|^2 + |DC|^2 = |AC|^2$
$|AD| = \sqrt{x^2 + y^2}$
$|DC| = \sqrt{y^2 + 1}$
$x^2 + y^2 + y^2 + 1 = (x + 1)^2$
$2y^2 = 2x$
$y = \sqrt{x}$

Or

$\frac{\sqrt{x^2 + y^2}}{\sqrt{y^2 + 1}} = \frac{y}{1} \Rightarrow x^2 + y^2 = y^2(y^2 + 1)$
$y^4 = x^2 \Rightarrow y^2 = x \Rightarrow y = \sqrt{x}$</td><td>Scale 5C (0, 2, 4, 5)
Low Partial Credit
• one set of corresponding sides identified
• indicates relevant use of Pythagoras

High Partial Credit
• corresponding sides fully substituted
• expression in y^2 or y^4, i.e. fails to finish</td></tr>
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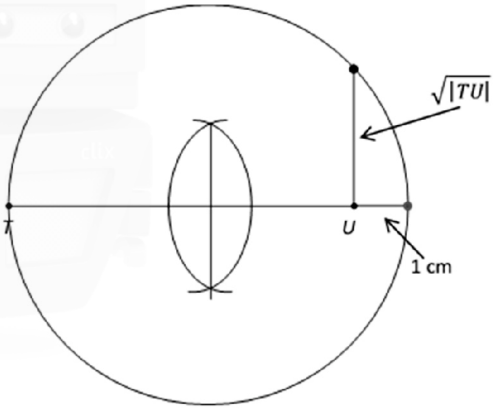
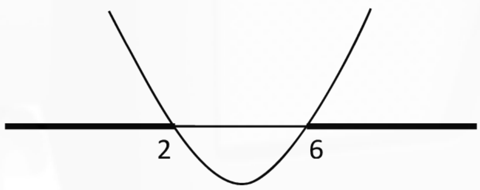
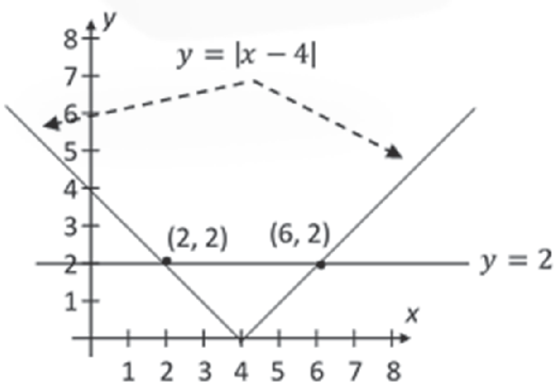
(b)	Construction 	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • perpendicular line drawn at *U* or *T* • relevant use of 1 cm length • mid point of incorrect extended segment constructed *High Partial Credit* • correct mid-point constructed

Question 4 (2016)

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$x^2 - 8x + 16 \geq 4$ $x^2 - 8x + 12 \geq 0$ $(x-2)(x-6) \geq 0$ $x = 2 \quad x = 6$ $\{x \mid x \leq 2\} \cup \{x \mid x \geq 6\}$ **Or** $x - 4 \geq 2 \ \cup x - 4 \leq -2$ $x \geq 6 \ \cup x \leq 2$ **Or** Graphical method (must indicate range on X-axis somehow)  **Or**  $x \leq 2 \cup x \geq 6$	Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • either side squared • one correct linear inequality written • stating range of natural numbers only *High Partial Credit:* • correct solutions to quadratic *Full Credit:* • correct answer without work **Note:** *use of natural numbers in range merits High Partial Credit at most* **Or** Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • any one straight line *High Partial Credit:* • three straight lines

(b)	$x = \frac{-3y-1}{2}$ $\left(\frac{-3y-1}{2}\right)^2 + \left(\frac{-3y-1}{2}\right)(y) + 2y^2 = 4$ $11y^2 + 4y - 15 = 0$ $(11y+15)(y-1) = 0$ $y = \frac{-15}{11} \ or \ y = 1$ $x = \frac{-3\left(\frac{-15}{11}\right) - 1}{2} \quad or \ x = \frac{-3(1)-1}{2}$ $x = \frac{17}{11} \quad or \ x = -2$ **or** $y = \frac{-2x-1}{3}$ $x^2 + x\left(\frac{-2x-1}{3}\right) + 2\left(\frac{-2x-1}{3}\right)^2 = 4$ $11x^2 + 5x - 34 = 0$ $(11x-17)(x+2) = 0$ $x = \frac{17}{11}$ or $x = -2$ $y = \frac{-15}{11}$ or $y = 1$	Scale 15C (0, 5, 10,15) *Low Partial Credit:* • effort to isolate x (or y) *High Partial Credit:* • fully correct substitution into quadratic

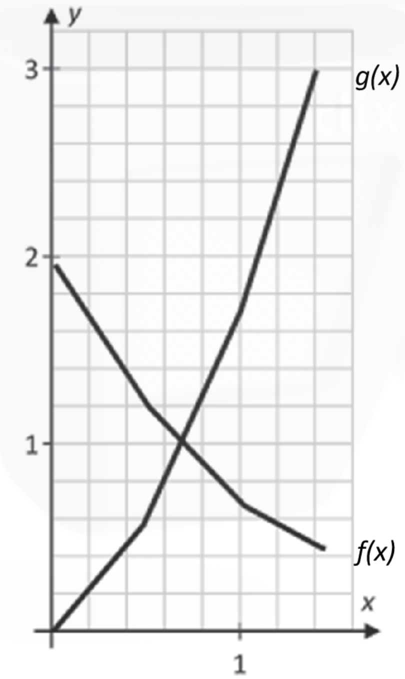
Question 5 (2016)

Q3	Model Solution – 25 Marks	Marking Notes
(a) (i)	see table below	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • one entry correct *High Partial Credit* • 5 entries correct
(ii)	see graph below	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • one plot correct *High Partial Credit* • 5 plots correct • one correct graph • no labelling **Notes:** - *straight lines NOT acceptable* - *one clear label merits full credit* - *one ambiguous label merits High Partial Credit at most*
(iii)	$f(x) = g(x)$ when $x \approx 0 \cdot 7$	Scale 5B (0, 2, 5) *Partial Credit* • point of intersection clearly indicated on graph, but value of x not stated

(a)(i)

x	0	0·5	1	$\ln(4)$
$f(x) = \dfrac{2}{e^x}$	2	1·21	0·74	0·5
$g(x) = e^x - 1$	0	0·65	1·72	3

(ii)



Q3	Model Solution – Continued	Marking Notes
(b)	$\frac{e^x - 1}{1} = \frac{2}{e^x}$ $e^{2x} - e^x = 2$ $(e^x)^2 - e^x - 2 = 0$ $(e^x - 2)(e^x + 1) = 0$ $e^x = 2$ **or** $e^x = -1$ $x = \ln 2$ or $x = 0{\cdot}693$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • substitution correct *High Partial Credit* • correct factors of quadratic • root formula correctly substituted $e^x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$ **Note:** *oversimplification of equation (i.e. not treating as quadratic) merits Low Partial Credit at most*
	Or	**Or**
	$(e^x)^2 - e^x - 2 = 0$ Let $y = e^x \Rightarrow y^2 - y - 2 = 0$ $y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$ $= \frac{1 \pm \sqrt{1+8}}{2}$ $= \frac{1 \pm 3}{2}$ $\Rightarrow y = 2$ or $y = -1$ (not possible) $y = e^x \Rightarrow e^x = 2$ $x = \ln 2$ or $x = 0{\cdot}693$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • substitution correct *High Partial Credit* • root formula correctly substituted $y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$ **Note:** *oversimplification of equation (i.e. not treating as quadratic) merits Low Partial Credit at most*

Question 6 (2016)

Q5	Model Solution – 25 Marks	Marking Notes
(a) (i)	$(5x - 9)^2 = (x - 1)^2 + (4x)^2$ $8x^2 - 88x + 80 = 0$ $x^2 - 11x + 10 = 0$ $(x - 1)(x - 10) = 0$ $x = 1$ or $x = 10$ $x = 10$	Scale 10D (0, 2, 5, 8, 10) *Low Partial Credit* • any use of Pythagoras *Mid Partial Credit* • fully correct substitution *High Partial Credit* • both roots correct
(a) (ii)	Sides=9, 40, 41 $9^2 + 40^2 = 41^2$ $81 + 1600 = 1681$ $1681 = 1681$	Scale 5B (0, 2, 5) *Partial Credit* • 9 or 40 or 41 • using 1 or −10 from candidates work

(b) **(i)**	200 m Race: $$y = a(b-x)^c$$ $$y = 4{\cdot}99087(42{\cdot}5 - 23{\cdot}8)^{1{\cdot}81}$$ $$y = 1000$$ Javelin: $$y = a(x-b)^c$$ $$y = 15{\cdot}9803(58{\cdot}2 - 3{\cdot}8)^{1{\cdot}04}$$ $$y = 1020$$	Scale 10D (0, 2, 5, 8, 10) *Low Partial Credit* • some relevant substitution into one formula *Mid Partial Credit* • one value of y found • some relevant substitution into both formulas *High Partial Credit* • one value correct and some relevant substitution into second formula • uses incorrect formula (once only)
(ii)	$$y = a(x-b)^c$$ $$1295 = 15{\cdot}9803(x - 3{\cdot}8)^{1{\cdot}04}$$ $$81{\cdot}0373 = (x - 3{\cdot}8)^{1{\cdot}04} = z^{1{\cdot}04}$$ $$\log z = \frac{\log 81{\cdot}0373}{1{\cdot}04}$$ $$z = 68{\cdot}4343 = (x - 3{\cdot}8)$$ $$x = 72{\cdot}2343 = 72{\cdot}23 \text{ m}$$	Scale 5B (0, 2, 5) *Partial Credit* • some relevant substitution into formula
(iii)	$$y = a(b-x)^c$$ $$1087 = 0{\cdot}11193(254 - 121{\cdot}84)^c$$ $$\frac{1087}{0{\cdot}11193} = (132{\cdot}16)^c$$ $$\log 9711{\cdot}426 = c \log 132{\cdot}16$$ $$c = \frac{\log 9711{\cdot}426}{\log 132{\cdot}16} = 1{\cdot}88$$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • some relevant substitution into formula *High Partial Credit* • fully correct substitution into formula

(b) (i)	G_5 =Female,Male,Female,Female,Male	Scale 5B (0, 2, 5) *Partial Credit* • one correct entry
(b) (ii)	$G_6 = G_5 + G_4 = 5 + 3 = 8$ $G_7 = G_6 + G_5 = 8 + 5 = 13$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • $G_6 = G_5 + G_4$ • $G_7 = G_6 + G_5$ • G_7 or G_6 correct • 8 and/or 13 without work *High Partial Credit* • correct substitution in both
(b) (iii)	$G_3 = \frac{(1+\sqrt{5})^3 - (1-\sqrt{5})^3}{2^3\sqrt{5}} = 2$ $(1+\sqrt{5})^3 = \left(1 + 3\sqrt{5} + 3\sqrt{5}^2 + \sqrt{5}^3\right)$ $= 16 + 8\sqrt{5}$ $(1-\sqrt{5})^3 = \left(1 - 3\sqrt{5} + 3\sqrt{5}^2 - \sqrt{5}^3\right)$ $= 16 - 8\sqrt{5}$ $G_3 = \frac{6\sqrt{5} + 2\sqrt{5}^3}{8\sqrt{5}}$ $= \frac{6 + 2\sqrt{5}^2}{8} = \frac{16}{8} = 2 \quad \text{Q.E.D.}$	Scale 5B (0, 2, 5) *Partial Credit* • some correct substitution • using approximate value for $\sqrt{5}$ • $G_3 = 2$ • some effort at cubing **Note:** *use of* $\sqrt{5}$ *as approximation, even if rounded off to* 2 *at end of work merits at most Partial Credit*

Question 2 **(25 marks)**

$$f(x) = x^3 - 3x^2 - 9x + 11$$
$$f(1) = 1^3 - 3(1)^2 - 9 + 11 = 0$$
$\Rightarrow x = 1$ is a solution.

$(x - 1)$ is a factor

$$\begin{array}{r|l} & x^2 - 2x \quad - 11 \\ \hline x - 1 & x^3 - 3x^2 - 9x + 11 \\ & x^3 - x^2 \\ \hline & \quad -2x^2 - 9x + 11 \\ & \quad -2x^2 + 2x \\ \hline & \qquad -11x + 11 \\ & \qquad -11x + 11 \\ \hline \end{array}$$

or

$$(x-1)(x^2 + Ax - 11) = x^3 - 3x^2 - 9x + 11$$
$$\Rightarrow x^3 + Ax^2 - x - x^2 - Ax + 1 = x^3 - 3x^2 - 9x + 11$$
$$\Rightarrow A - 1 = -3$$
$$\Rightarrow A = -2$$

or

	x^2	$-2x$	-11
x	x^3	$-2x^2$	$-11x$
-1	$-x^2$	$2x$	11

Hence, other factor is $x^2 - 2x - 11$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)} = \frac{2 \pm \sqrt{48}}{2} = \frac{2 \pm 4\sqrt{3}}{2} = 1 \pm 2\sqrt{3}$$

Solutions: $\{1,\ 1 + 2\sqrt{3},\ 1 - 2\sqrt{3}\}$

Question 10 (2015)

$$x = \sqrt{x + 6}$$
$$\Rightarrow x^2 = x + 6$$
$$\Rightarrow x^2 - x - 6 = 0$$
$$\Rightarrow (x + 2)(x - 3) = 0$$
$$\Rightarrow x = -2, \quad x = 3$$
$$x = -2: \quad -2 \neq \sqrt{-2 + 6} = \sqrt{4} = 2 \ ✗$$
$$x = 3: \quad 3 = \sqrt{3 + 6} = \sqrt{9} = 3 \ ✓$$

(a) Show that $-k$ is a root of f.

Substituting $-k$ for x we obtain

$$\begin{aligned} f(-k) &= (-k)^3 + (1-k^2)(-k) + k \\ &= -k^3 - k + k^3 + k \\ &= 0 \end{aligned}$$

Therefore $-k$ is a root of f.

(b) Find, in terms of k, the other two roots of f.

Since $-k$ is a root of f we know, by the Factor Theorem, that $(x+k)$ is a factor of $f(x)$. Now we carry out long division to find the other factor.

$$
\begin{array}{rrrrrrrr}
 & x^2 & - & kx & + & 1 & & \\
\cline{2-8}
x+k \,\big) & x^3 & + & 0x^2 & + & (1-k^2)x & + & k \\
 & x^3 & + & kx^2 & & & & \\
\cline{2-4}
 & & - & kx^2 & + & (1-k^2)x & & \\
 & & - & kx^2 & - & k^2x & & \\
\cline{3-6}
 & & & & & x & + & k \\
 & & & & & x & + & k \\
\cline{6-8}
 & & & & & & & 0
\end{array}
$$

So

$$x^3+(1-k^2)x+k=(x+k)(x^2-kx+1).$$

Therefore the other two roots of f are solutions of the equation

$$x^2-kx+1=0.$$

Using the quadratic formula we get

$$x=\frac{k\pm\sqrt{k^2-4}}{2}.$$

So the other two roots of f are

$$\frac{k+\sqrt{k^2-4}}{2} \text{ and } \frac{k-\sqrt{k^2-4}}{2}.$$

(c) Find the set of values of k for which f has exactly one real root.

From the solution to part (b), we see that f has exactly one real root if and only if $k^2-4<0$. This is equivalent to $k^2<4$ or

$$-2<k<2$$

We can subtract the second equation from the first:

$$\begin{aligned} 2x+8y-3z &= -1 \\ 2x-3y+2z &= 2 \\ \hline 11y-5z &= -3 \end{aligned}$$

Similarly, we subtract the third equation from the first:

$$\begin{aligned} 2x+8y-3z &= -1 \\ 2x+y+z &= 5 \\ \hline 7y-4z &= -6 \end{aligned}$$

Now we solve the simultaneous equations

$$\begin{aligned} 11y-5z &= -3 \\ 7y-4z &= -6 \end{aligned}$$

Multiply the first by 7, the second by 11 and subtract:

$$\begin{aligned} 77y-35z &= -21 \\ 77y-44z &= -66 \\ \hline 9z &= 45 \end{aligned}$$

Therefore $z = \frac{45}{9} = 5$. Now substitute $z = 5$ into $7y-4z = -6$ to get $7y-4(5) = -6$ or $7y = -6+20 = 14$ Therefore $y = 2$.
Finally substitute $y = 2$ and $z = 5$ into $2x+8y-3z = -1$ to get $2x+8(2)-3(5) = -1$ or $2x = -1-8(2)+3(5) = -2$ So $x = -1$.
So the solution is

$$x = -1, y = 2, z = 5.$$

Now we can check this by substituting into the original equations and verifying that they are all true:

$$\begin{aligned} 2(-1)+8(2)-3(5) &= -1 \\ 2(-1)-3(2)+2(5) &= 2 \\ 2(-1)+(2)+(5) &= 5. \end{aligned}$$

(b) The graphs of the functions $f : x \mapsto |x-3|$ and $g : x \mapsto 2$ are shown in the diagram.

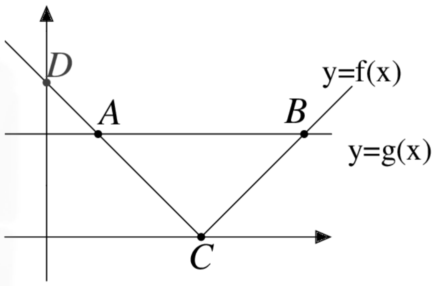
(i) Find the co-ordinates of the points A, B, C and D.

D is on the y-axis, so its x-co-ordinate is 0. Now $f(0) = |0-3| = |-3| = 3$. So $D = (0,3)$.
$C = (,0)$ (on the x-axis), so we solve $|x-3| = 0$ to find the x-co-ordinate. Now $|x-3| = 0 \Leftrightarrow x-3 = 0 \Leftrightarrow x = 3$. So $C = (3,0)$.
A and B both have y-co-ordinate 2, so we solve $|x-3| = 2$. Now $|x-3| = 2 \Leftrightarrow \pm(x-3) = 2$. So either

$$(x-3) = 2 \text{ or } -(x-3) = 2.$$

In the first case $x = 5$ and in the second case $-x+3 = 2$ or $x = 1$. So $A = (1,2)$ and $B = (5,2)$.

$$A = (1,2) \quad B = (5,2)$$
$$C = (3,0) \quad D = (0,3)$$



(ii) Hence, or otherwise, solve the inequality $|x-3| < 2$.

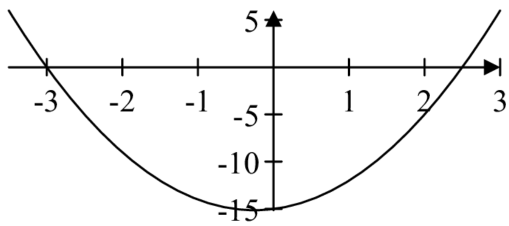
The solution set of the inequality corresponds to the values of x for which the graph of f is below the graph of g. From the diagram and calculations above, we see that the solution set is

$$1 < x < 5.$$

$2x^2 + x - 15 = 0$

$\Rightarrow (2x-5)(x+3) = 0 \Rightarrow x = 2\tfrac{1}{2}$ or $x = -3$

$2x^2 + x - 15 \geq 0$ for $\{x \mid x \leq -3\} \cup \{x \mid x \geq 2\tfrac{1}{2}\}$



OR

$f(x) = 2x^2 + x - 15 = (2x-5)(x+3)$

$(2x-5)(x+3) = 0$

$\Rightarrow x = \frac{5}{2}$ or $x = -3$

(i): $x \geq -3$ and $x \geq \frac{5}{2} \Rightarrow x \geq \frac{5}{2}$

(ii): $x \leq -3$ and $x \leq \frac{5}{2} \Rightarrow x \leq -3$

Solution Set: $\{x \mid x \leq -3\} \cup \{x \mid x \geq \frac{5}{2}\}$

$$\begin{aligned} x + y + z &= 16 \\ \tfrac{5}{2}x + y + 10z &= 40 \end{aligned} \quad \Rightarrow \quad \begin{aligned} 2x + 2y + 2z &= 32 \\ 5x + 2y + 20z &= 80 \\ \hline 3x \qquad + 18z &= 48 \end{aligned}$$

$$\begin{aligned} x + y + z &= 16 \\ 4x + y + 8z &= 42 \\ \hline 3x \qquad + 7z &= 26 \end{aligned}$$

$$\begin{aligned} 3x + 18z &= 48 \\ 3x + 7z &= 26 \\ \hline 11z &= 22 \quad \Rightarrow \quad z = 2 \end{aligned}$$

$3x + 7z = 26 \Rightarrow 3x + 7(2) = 26 \Rightarrow 3x = 12 \Rightarrow x = 4$

$x + y + z = 16 \Rightarrow 4 + y + 2 = 16 \Rightarrow y = 10$

(a) If the ticket price was €18, how many people would be expected to attend?

$$12000+(20-18)1000=14000$$

(b) Let x be the ticket price, where $x \leq 20$. Write down, in terms of x, the expected attendance at such an event.

$$12000+(20-x)1000=32000-1000x$$

(c) Write down a function f that gives the expected income from the sale of tickets for such an event.

$$f(x)=(32000-1000x)x$$

(d) Find the price at which tickets should be sold to give the maximum expected income.

$$f(x)=(32000-1000x)x$$

$$f'(x)=32000-2000x=0 \Rightarrow x=€16$$

(e) Find this maximum expected income.

$$f(x)=(32000-1000x)x$$

$$f(16)=(32000-16000)16=€256\,000$$

(f) Suppose that tickets are instead priced at a value that is expected to give a full attendance at the stadium. Find the difference between the income from the sale of tickets at this price and the maximum income calculated at **(e)** above.

$$32000 - 1000x = 25000 \Rightarrow 1000x = 7000 \Rightarrow x = 7$$

$$f(x) = (32000 - 1000x)x \Rightarrow f(7) = (32000 - 7000)7 = 175\,000$$

Difference: €256 000 – €175 000 = €81 000

(g) The stadium was full for a recent special event. Two types of tickets were sold; a single ticket for €16 and a family ticket (2 adults and 2 children) for a certain amount. The income from this event was €365 000. If 1000 more family tickets had been sold the income from the event would have been reduces by €14 000. How many family tickets were sold?

Single ticket: €16; Family ticket €y
Number of single tickets: p; Number of family tickets: $\frac{25000-p}{4}$

$$16p + \frac{25000-p}{4}y = 365000$$

$$16(p-4000) + \left(\frac{25000-p}{4} + 1000\right)y = 351000 \Rightarrow 16p + \frac{29000-p}{4}y = 415000$$

$$\frac{29000-p}{4}y - \frac{25000-p}{4}y = 50000 \Rightarrow 4000y = 200000 \Rightarrow y = 50$$

$$16p + \frac{25000-p}{4}50 = 365000 \Rightarrow 7p = 105000 \Rightarrow p = 15000$$

Number of family tickets: $\frac{25000-p}{4} = \frac{25000-15000}{4} = 2500$

OR

x = number of single tickets
f = number of family tickets
y = cost of family ticket

$$x + 4f = 25000$$
$$16x + fy = 365000$$
$$16(x - 4000) + (f + 1000)y = 351000$$

$$16x - 64000 + fy + 1000y = 351000$$
$$16x + fy = 365000$$
$$1000y = 50000$$
$$y = 50$$

$$a = -2b - 1$$

$$(-2b-1)^2 + (2b+1)b + b^2 = 3$$
$$7b^2 + 5b - 2 = 0$$
$$(7b-2)(b+1) = 0$$
$$b = \tfrac{2}{7} \quad \text{or } b = -1$$
$$a = \tfrac{-11}{7} \quad \text{or } a = 1$$

Solution: $\{b = \tfrac{2}{7} \text{ and } a = \tfrac{-11}{7}\}$ or $\{b = -1 \text{ and } a = 1\}$.

(b) Find the set of all real values of x for which $\dfrac{2x-5}{x-3} \le \dfrac{5}{2}$.

Multiply across by $2(x-3)^2$, which is non-negative:

$$2(x-3)(2x-5) \le 5(x-3)^2$$
$$4x^2 - 22x + 30 \le 5x^2 - 30x + 45$$
$$0 \le x^2 - 8x + 15$$
$$0 \le (x-5)(x-3)$$

$x \ge 5$ or $x < 3$.

OR

$$\frac{2x-5}{x-3} - \frac{5}{2} \le 0$$
$$\frac{2(2x-5) - 5(x-3)}{2(x-3)} \le 0$$
$$\frac{-x+5}{2(x-3)} \le 0$$

	$x < 3$	$3 < x < 5$	$x > 5$
$-x+5$	+	+	–
$x-3$	–	+	+
$\dfrac{-x+5}{2(x-3)}$	–	+	–

$x \ge 5$ or $x < 3$.