Question 1

(a) Write the function $f(x) = 2x^2 - 7x - 10$, where $x \in \mathbb{R}$, in the form $a(x+h)^2 + k$, where a, h, and $k \in \mathbb{Q}$.

(b) Hence, write the minimum point of f.

(c) **(i)** Explain why f must have two real roots.

(ii) Write the roots of $f(x) = 0$ in the form $p \pm \sqrt{q}$, where p and $q \in \mathbb{Q}$.

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Question 5 **(25 marks)**

The function f is such that $f(x) = 2x^3 + 5x^2 - 4x - 3$, where $x \in \mathbb{R}$.

(a) Show that $x = -3$ is a root of $f(x)$ **and** find the other two roots.

(b) Find the co-ordinates of the local maximum point **and** the local minimum point of the function f.

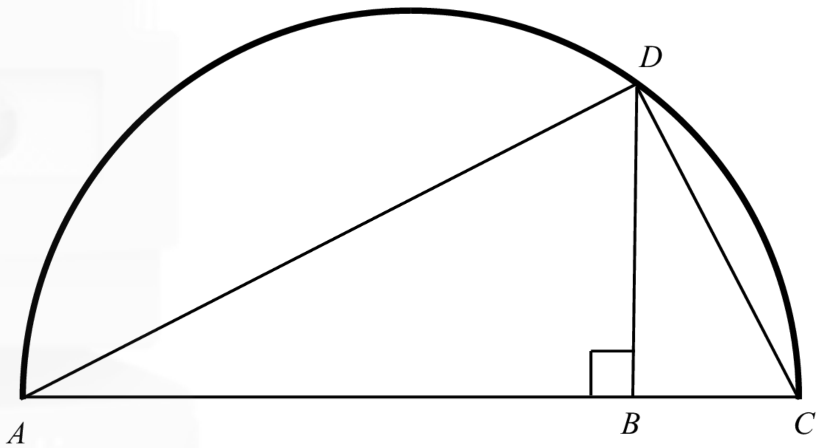
Local maximum point: Local minimum point:

(c) $f(x) + a$, where a is a constant, has only one real root.
Find the range of possible values of a.

Question 3

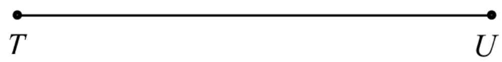
The diagram shows a semi-circle standing on a diameter $[AC]$, and $[BD] \perp [AC]$.

(a) **(i)** Prove that the triangles *ABD* and *DBC* are similar.



(ii) If $|AB| = x$, $|BC| = 1$, and $|BD| = y$, write y in terms of x.

(b) Use your result from part **(a)(ii)** to **construct** a line segment equal in length (in centimetres) to the square root of the length of the line segment $[TU]$ which is drawn below.



Question 4

(a) Find the range of values of x for which $|x - 4| \geq 2$, where $x \in \mathbb{R}$.

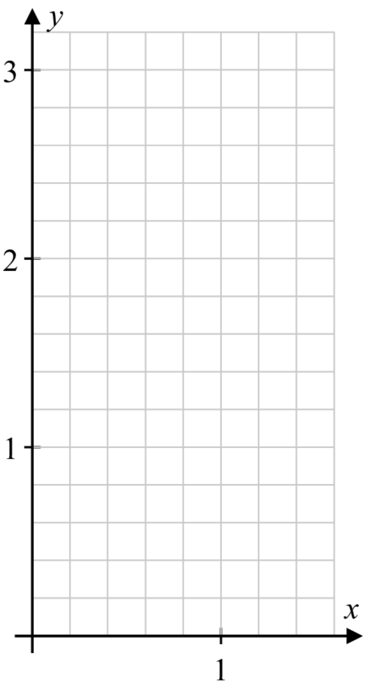
(b) Solve the simultaneous equations:

$$x^2 + xy + 2y^2 = 4$$
$$2x + 3y = -1.$$

(a) **(i)** $f(x) = \dfrac{2}{e^x}$ and $g(x) = e^x - 1$, where $x \in \mathbb{R}$.
Complete the table below. Write your values correct to two decimal places where necessary.

x	**0**	**0·5**	**1**	**ln(4)**
$f(x) = \dfrac{2}{e^x}$				
$g(x) = e^x - 1$				

(ii) In the grid on the right, use the table to draw the graphs of $f(x)$ and $g(x)$ in the domain $0 \leq x \leq \ln(4)$. Label each graph clearly.

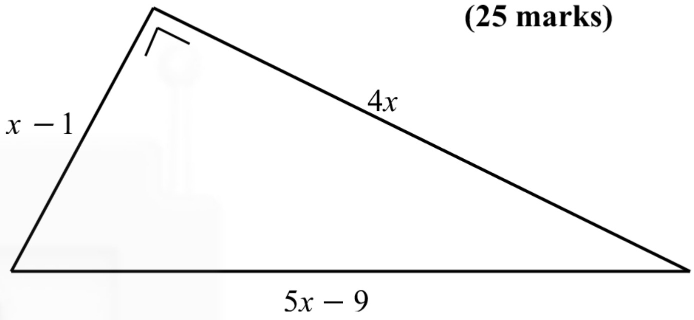


(iii) Use your graphs to estimate the value of x for which $f(x) = g(x)$.

(b) Solve $f(x) = g(x)$ using algebra.

Question 5 **(25 marks)**

(a) **(i)** The lengths of the sides of a right-angled triangle are given by the expressions $x - 1$, $4x$, and $5x - 9$, as shown in the diagram.
Find the value of x.



(ii) Verify, with this value of x, that the lengths of the sides of the triangle above form a pythagorean triple.

(b) The heptathlon is an Olympic competition. It consists of seven events including the 200 m race and the javelin. The scoring system uses formulas to calculate a score for each event. The table below shows the formulas for two of the events and the values of constants used in these formulas, where x is the time taken (in seconds) or distance achieved (in metres) by the competitor and y is the number of points scored in the event.

Event	x	**Formula**	a	b	c
200 m race	Time (s)	$y = a(b - x)^c$	4·99087	42·5	1·81
Javelin	Distance (m)	$y = a(x - b)^c$	15·9803	3·8	1·04

(i) In the heptathlon, Jessica ran 200 m in 23·8 s and threw the javelin 58·2 m. Use the formulas in the table to find the number of points she scored in each of these events, correct to the nearest point.

200 m	Javelin

(ii) The world record distance for the javelin, in the heptathlon, would merit a score of 1295 points. Find the world record distance for the javelin, in the heptathlon, correct to two decimal places.

(iii) The formula used to calculate the points for the 800 m race, in the heptathlon, is the same formula used for the 200 m race but with different constants.
Jessica ran the 800 m race in 2 minutes and 1·84 seconds which merited 1087 points.
If $a = 0{\cdot}11193$ and $b = 254$ for the 800 m race, find the value of c for this event, correct to two decimal places.

(b) A male bee comes from an unfertilised egg, i.e. he has a female parent but he does not have a male parent. A female bee comes from a fertilised egg, i.e. she has a female parent and a male parent.

(i) The following diagram shows the ancestors of a certain male bee. We identify his generation as G_1 and our diagram goes back to G_4. Continue the diagram to G_5.

G_1	G_2	G_3	G_4	G_5
			↗ Female	
		↗ Female		
Male →	Female		↘ Male	
		↘ Male →	Female	

(ii) The number of ancestors of this bee in each generation can be calculated by the formula

$$G_{n+2} = G_{n+1} + G_n,$$

where $G_1 = 1$ and $G_2 = 1$, as in the diagram.
Use this formula to calculate the number of ancestors in G_6 and in G_7.

G_6 | G_7

(iii) The number of ancestors in each generation can also be calculated by using the formula

$$G_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}.$$

Use this formula to verify the number of ancestors in G_3.

Question 2 **(25 marks)**

Solve the equation $x^3 - 3x^2 - 9x + 11 = 0$.

Write any irrational solution in the form $a + b\sqrt{c}$, where $a, b, c \in \mathbb{Z}$.

Question 10

(a) Solve the equation $x = \sqrt{x+6}$, $x \in \mathbb{R}$.

Question 3 **(25 marks)**

A cubic function f is defined for $x \in \mathbb{R}$ as

$$f : x \mapsto x^3 + (1 - k^2)x + k, \quad \text{where } k \text{ is a constant.}$$

(a) Show that $-k$ is a root of f.

(b) Find, in terms of k, the other two roots of f.

(c) Find the set of values of k for which f has exactly one real root.

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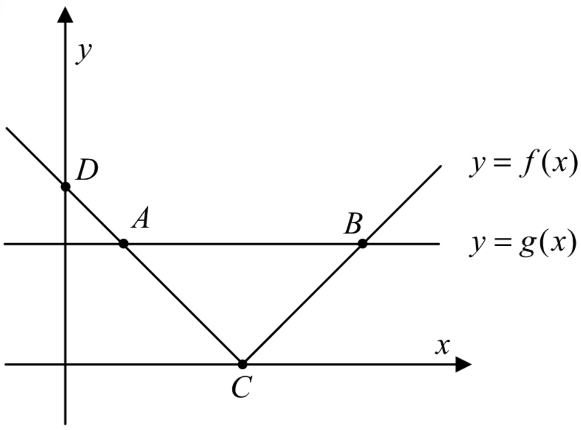
Question 4 **(25 marks)**

(a) Solve the simultaneous equations:

$$2x + 8y - 3z = -1$$
$$2x - 3y + 2z = 2$$
$$2x + y + z = 5.$$

(b) The graphs of the functions $f : x \mapsto |x - 3|$ and $g : x \mapsto 2$ are shown in the diagram.

(i) Find the co-ordinates of the points A, B, C and D.



$A = (\quad , \quad)$ $B = (\quad , \quad)$

$C = (\quad , \quad)$ $D = (\quad , \quad)$

(ii) Hence, or otherwise, solve the inequality $|x - 3| < 2$.

(a) Find the set of all real values of x for which $2x^2 + x - 15 \geq 0$.

(b) Solve the simultaneous equations;

$$x + y + z = 16$$
$$\frac{5}{2}x + y + 10z = 40$$
$$2x + \frac{1}{2}y + 4z = 21.$$

A stadium can hold 25 000 people. People attending a regular event at the stadium must purchase a ticket in advance. When the ticket price is €20, the expected attendance at an event is 12 000 people. The results of a survey carried out by the owners suggest that for every €1 reduction, from €20, in the ticket price, the expected attendance would increase by 1000 people.

(a) If the ticket price was €18, how many people would be expected to attend?

(b) Let x be the ticket price, where $x \leq 20$. Write down, in terms of x, the expected attendance at such an event.

(c) Write down a function f that gives the expected income from the sale of tickets for such an event.

(d) Find the price at which tickets should be sold to give the maximum expected income.

(e) Find this maximum expected income.

(f) Suppose that tickets are instead priced at a value that is expected to give a full attendance at the stadium. Find the difference between the income from the sale of tickets at this price and the maximum income calculated at **(e)** above.

(g) The stadium was full for a recent special event. Two types of tickets were sold, a single ticket for €16 and a family ticket (2 adults and 2 children) for a certain amount. The income from this event was €365 000. If 1000 more family tickets had been sold, the income from the event would have been reduced by €14 000. How many family tickets were sold?

Question 1 **(25 marks)**

(a) Solve the simultaneous equations:

$$a^2 - ab + b^2 = 3$$
$$a + 2b + 1 = 0$$

(b) Find the set of all real values of x for which $\dfrac{2x-5}{x-3} \leq \dfrac{5}{2}$.

Question 4 **(25 marks)**

(a) Solve the simultaneous equations,

$$2x + 8y - 3z = -1$$
$$2x - 3y + 2z = 2$$
$$2x + y + z = 5.$$