MarkingScheme

5YMidtermWk

Question 1 (2017)

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$2\left(x^2-\frac{7}{2}x-5\right)$ $=2\left(\left(x-\frac{7}{4}\right)^2-\frac{129}{16}\right)$ $=2\left(\left(x-\frac{7}{4}\right)^2\right)-\frac{129}{8}$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • $a=2$ identified explicitly or as factor *Mid partial Credit:* • Completed square *High partial Credit:* • h or k identified from work
(b)	$\left(\frac{7}{4}, \quad \frac{-129}{8}\right)$	**Scale 10B (0, 4, 10)** *Partial Credit:* • One relevant co-ordinate identified

(c) (i)	$f(x)$ has min point as $a > 0$ y co-ordinate of min < 0 $\Rightarrow$graph must cut x-axis twice hence two real roots. **or** $b^2 - 4ac = 49 + 80 > 0$ Therefore real roots	**Scale 5B (0, 3, 5)** *Partial Credit:* • Mention of $a > 0$ • $b^2 - 4ac$ • Identifies location of one or two roots, e.g. between 4 and 5.
c (ii)	$2x^2 - 7x - 10 = 0$ $2\left(\left(x - \frac{7}{4}\right)^2\right) - \frac{129}{8} = 0$ $\left(x - \frac{7}{4}\right)^2 = \frac{129}{16}$ $x - \frac{7}{4} = \pm\frac{\sqrt{129}}{4}$ $x = \frac{7}{4} \pm \sqrt{\frac{129}{16}}$ **OR** $2x^2 - 7x - 10 = 0$ $x = \frac{7 \pm \sqrt{49 + 80}}{4}$ $= \frac{7 \pm \sqrt{129}}{4}$ $x = \frac{7}{4} \pm \sqrt{\frac{129}{16}}$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • Formula with some substitution • Equation rewritten with some transpose *High Partial Credit:* • $x - \frac{7}{4} = \pm\frac{\sqrt{129}}{4}$ or equivalent

Question 2 (2016)

Q5	Model Solution – 25 Marks	Marking Notes
(a) **(i)**	$(5x-9)^2 = (x-1)^2 + (4x)^2$ $8x^2 - 88x + 80 = 0$ $x^2 - 11x + 10 = 0$ $(x-1)(x-10) = 0$ $x = 1$ or $x = 10$ $x = 10$	Scale 10D (0, 2, 5, 8, 10) *Low Partial Credit* • any use of Pythagoras *Mid Partial Credit* • fully correct substitution *High Partial Credit* • both roots correct
(a) **(ii)**	Sides=9, 40, 41 $9^2 + 40^2 = 41^2$ $81 + 1600 = 1681$ $1681 = 1681$	Scale 5B (0, 2, 5) *Partial Credit* • 9 or 40 or 41 • using 1 or −10 from candidates work

Question 3 (2015)

$x = \sqrt{x+6}$

$\Rightarrow x^2 = x + 6$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x+2)(x-3) = 0$

$\Rightarrow x = -2, \quad x = 3$

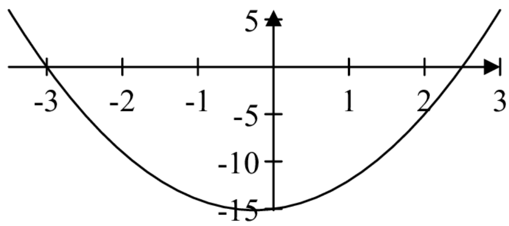
$x = -2: \quad -2 \neq \sqrt{-2+6} = \sqrt{4} = 2$ ✗

$x = 3: \quad 3 = \sqrt{3+6} = \sqrt{9} = 3$ ✓

$2x^2 + x - 15 = 0$

$\Rightarrow (2x-5)(x+3) = 0 \Rightarrow x = 2\tfrac{1}{2}$ or $x = -3$

$2x^2 + x - 15 \geq 0$ for $\{x \mid x \leq -3\} \cup \{x \mid x \geq 2\tfrac{1}{2}\}$



OR

$f(x) = 2x^2 + x - 15 = (2x-5)(x+3)$

$(2x-5)(x+3) = 0$

$\Rightarrow x = \dfrac{5}{2}$ or $x = -3$

(i): $x \geq -3$ and $x \geq \dfrac{5}{2} \Rightarrow x \geq \dfrac{5}{2}$

(ii): $x \leq -3$ and $x \leq \dfrac{5}{2} \Rightarrow x \leq -3$

Solution Set: $\{x \mid x \leq -3\} \cup \{x \mid x \geq \frac{5}{2}\}$

$$\begin{aligned} x + y + z &= 16 \\ \tfrac{5}{2}x + y + 10z &= 40 \end{aligned} \quad \Rightarrow \quad \begin{aligned} 2x + 2y + 2z &= 32 \\ 5x + 2y + 20z &= 80 \\ \hline 3x \qquad + 18z &= 48 \end{aligned}$$

$$\begin{aligned} x + y + z &= 16 \\ 4x + y + 8z &= 42 \\ \hline 3x \qquad + 7z &= 26 \end{aligned}$$

$$\begin{aligned} 3x + 18z &= 48 \\ 3x + 7z &= 26 \\ \hline 11z &= 22 \quad \Rightarrow \quad z = 2 \end{aligned}$$

$3x + 7z = 26 \Rightarrow 3x + 7(2) = 26 \Rightarrow 3x = 12 \Rightarrow x = 4$

$x + y + z = 16 \Rightarrow 4 + y + 2 = 16 \Rightarrow y = 10$

(a) If the ticket price was €18, how many people would be expected to attend?

$$12000 + (20 - 18)1000 = 14000$$

(b) Let x be the ticket price, where $x \leq 20$. Write down, in terms of x, the expected attendance at such an event.

$$12000 + (20 - x)1000 = 32000 - 1000x$$

(c) Write down a function f that gives the expected income from the sale of tickets for such an event.

$$f(x) = (32000 - 1000x)x$$

(d) Find the price at which tickets should be sold to give the maximum expected income.

$$f(x) = (32000 - 1000x)x$$

$$f'(x) = 32000 - 2000x = 0 \Rightarrow x = €16$$

(e) Find this maximum expected income.

$$f(x) = (32000 - 1000x)x$$

$$f(16) = (32000 - 16000)16 = €256\,000$$

(f) Suppose that tickets are instead priced at a value that is expected to give a full attendance at the stadium. Find the difference between the income from the sale of tickets at this price and the maximum income calculated at **(e)** above.

$$32000 - 1000x = 25000 \Rightarrow 1000x = 7000 \Rightarrow x = 7$$

$$f(x) = (32000 - 1000x)x \Rightarrow f(7) = (32000 - 7000)7 = 175\,000$$

Difference: €256 000 – €175 000 = €81 000

(g) The stadium was full for a recent special event. Two types of tickets were sold; a single ticket for €16 and a family ticket (2 adults and 2 children) for a certain amount. The income from this event was €365 000. If 1000 more family tickets had been sold the income from the event would have been reduces by €14 000. How many family tickets were sold?

Single ticket: €16; Family ticket €y

Number of single tickets: p; Number of family tickets: $\frac{25000-p}{4}$

$$16p + \frac{25000-p}{4}y = 365000$$

$$16(p-4000) + \left(\frac{25000-p}{4} + 1000\right)y = 351000 \Rightarrow 16p + \frac{29000-p}{4}y = 415000$$

$$\frac{29000-p}{4}y - \frac{25000-p}{4}y = 50000 \Rightarrow 4000y = 200000 \Rightarrow y = 50$$

$$16p + \frac{25000-p}{4}50 = 365000 \Rightarrow 7p = 105000 \Rightarrow p = 15000$$

Number of family tickets: $\frac{25000-p}{4} = \frac{25000-15000}{4} = 2500$

OR

x = number of single tickets
f = number of family tickets
y = cost of family ticket

$$x + 4f = 25000$$

$$16x + fy = 365000$$

$$16(x-4000) + (f+1000)y = 351000$$

$$\begin{array}{r} 16x - 64000 + fy + 1000y = 351000 \\ 16x \qquad\quad + fy \qquad\qquad = 365000 \\ \hline 1000y = 50000 \\ y = 50 \end{array}$$

Question 6 (2017)

(a)	$A(0,6) \rightarrow G\left(\frac{2}{3},\ \frac{4}{3}\right)$ $\rightarrow P\left(\frac{2}{3}+\frac{1}{2}\left(\frac{2}{3}\right), \frac{4}{3}+\frac{1}{2}\left(\frac{-14}{3}\right)\right)$ $= \left(\frac{3}{3},\ -\frac{3}{3}\right)$ $P = (1\,, -1)$ **or** $P = (x, y)$ $\left(\frac{2x+1(0)}{3}, \frac{2y+6}{3}\right) = \left(\frac{2}{3}, \frac{4}{3}\right)$ $x = 1, \quad y = -1$ **or** $P = (x, y)$ $\left(\frac{3(\frac{2}{3}) - 1(0)}{3-1}, \frac{3\left(\frac{4}{3}\right) - 1(6)}{3-1}\right)$ $= \left(\frac{2}{2}, \frac{-2}{2}\right) = (1, -1)$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • $P\left(\frac{4}{3},\ -\frac{10}{3}\right)$ or equivalent, i.e ratio 1:1 • $\frac{2}{3}$ or $\frac{1}{3}$ identified as part of change in x ordinate • $-\frac{14}{3}$ or $-\frac{7}{3}$ identified as part of change in y ordinate • Ratio formula with some substitution *High Partial Credit:* • one relevant co-ordinate of P found
(b)	$C(4,2) \rightarrow P(1,\ -1) \rightarrow B(1-3, -1-3)$ $= (-2,\ -4)$ $B(x,y) \rightarrow \left(\frac{4+x}{2}, \frac{2+y}{2}\right) = (1, -1)$ $x = -2, \quad y = -4$ $B = (-2\,, -4)$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • P as mid-point of BC *High Partial Credit:* • one relevant co-ordinate of B found ***Note:*** Accept (–2, –4) without work Accept correct graphical solution

(c)	$AC \perp BC$ $AC = \frac{2-6}{4-0} = -1$ $BC = \frac{2+4}{4+2} = 1$ $-1 \times 1 = -1$ lines are perpendicular **or** $\text{Slope AB} = 5.$ $\text{Altitude from C}: y - 2 = -\frac{1}{5}(x-4)$ $\rightarrow x + 5y = 14 \ldots. \text{(i)}.$ $\text{Slope AC} = -1.$ $\text{Altitude from B}:$ $y + 4 = 1(x+2)$ $\rightarrow x - y = 2 \ldots\ldots \text{(ii)}$ $\rightarrow \text{Solving (i)and (ii)}$ $x = 4$ $y = 2$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • Identifies significance of right-angled triangle • one equation of perpendicular from vertex to opposite side found *High Partial Credit:* • slope of AC and slope of BC found but no conclusion • two equations of perpendiculars from vertex to opposite side found

(a) The co-ordinates of two points are $A(4,\ -1)$ and $B(7,\ t)$.

The line $l_1: 3x-4y-12=0$ is perpendicular to AB. Find the value of t.

$$\text{Slope } AB = \frac{t+1}{7-4} = \frac{t+1}{3} \qquad \text{Slope } l_1 = \frac{3}{4}$$

$$AB \perp l_1 \Rightarrow \frac{t+1}{3} \times \frac{3}{4} = -1 \Rightarrow t+1 = -4 \Rightarrow t = -5$$

or

$$AB: 4x+3y+c=0$$
$$(4,-1) \in 4x+3y+c=0 \Rightarrow 16-3+c=0 \Rightarrow c=-13$$
$$\therefore 4(7)+3(t)-13=0 \Rightarrow t=-5$$

(b) Find, in terms of k, the distance between the point $P(10,\ k)$ and l_1.

$$d = \left|\frac{3(10)-4k-12}{\sqrt{3^2+4^2}}\right| = \left|\frac{18-4k}{5}\right|$$

(c) $P(10,\ k)$ is on a bisector of the angles between the lines l_1 and $l_2: 5x+12y-20=0$.

(i) Find the possible values of k.

$$\left|\frac{18-4k}{5}\right| = \left|\frac{50+12k-20}{\sqrt{5^2+12^2}}\right|$$
$$\Rightarrow \left|\frac{18-4k}{5}\right| = \left|\frac{30+12k}{13}\right|$$
$$\Rightarrow 13(18-4k) = \pm 5(30+12k)$$
$$\Rightarrow -112k = -84 \quad \text{or} \quad 8k = -384$$
$$\Rightarrow k = \tfrac{3}{4} \quad \text{or} \quad k = -48$$

(ii) If $k > 0$, find the distance from P to l_1.

$$k = \frac{3}{4} \Rightarrow d = \left|\frac{18-4\left(\frac{3}{4}\right)}{5}\right| = 3$$