Question 1

(a) Write the function $f(x) = 2x^2 - 7x - 10$, where $x \in \mathbb{R}$, in the form $a(x+h)^2 + k$, where a, h, and $k \in \mathbb{Q}$.

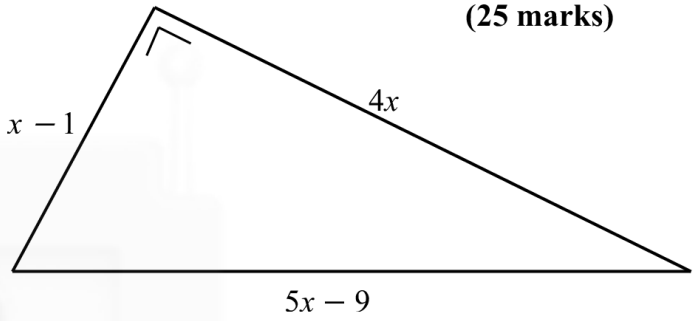
(b) Hence, write the minimum point of f.

(c) **(i)** Explain why f must have two real roots.

(ii) Write the roots of $f(x) = 0$ in the form $p \pm \sqrt{q}$, where p and $q \in \mathbb{Q}$.

Question 5 **(25 marks)**

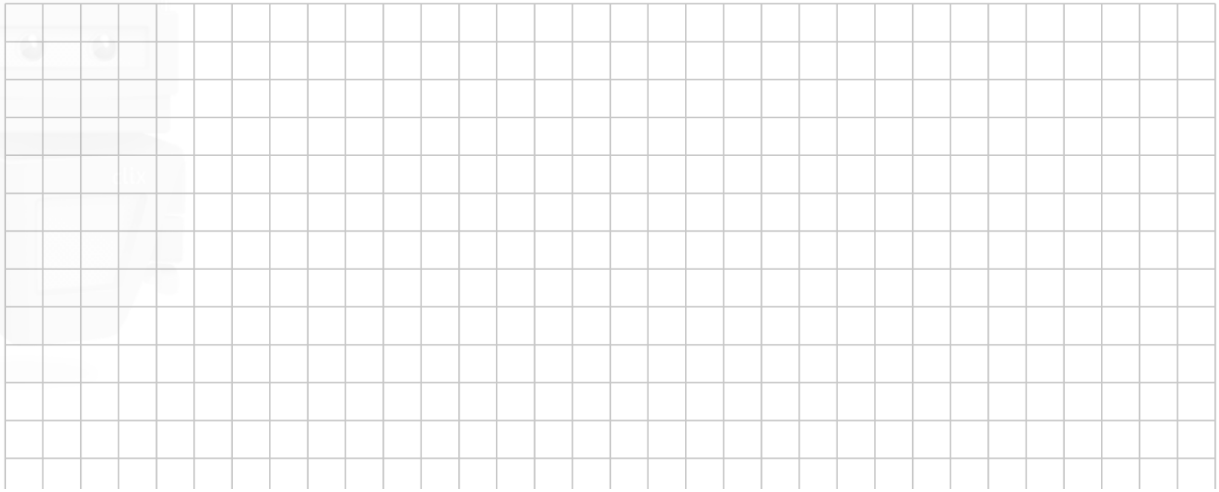
(a) **(i)** The lengths of the sides of a right-angled triangle are given by the expressions $x - 1$, $4x$, and $5x - 9$, as shown in the diagram. Find the value of x.



(ii) Verify, with this value of x, that the lengths of the sides of the triangle above form a pythagorean triple.

Question 3

(a) Solve the equation $x = \sqrt{x+6},\ x \in \mathbb{R}$.



Question 4

(a) Find the set of all real values of x for which $2x^2 + x - 15 \geq 0$.

(b) Solve the simultaneous equations;

$$x + y + z = 16$$
$$\frac{5}{2}x + y + 10z = 40$$
$$2x + \frac{1}{2}y + 4z = 21.$$

Question 5

A stadium can hold 25 000 people. People attending a regular event at the stadium must purchase a ticket in advance. When the ticket price is €20, the expected attendance at an event is 12 000 people. The results of a survey carried out by the owners suggest that for every €1 reduction, from €20, in the ticket price, the expected attendance would increase by 1000 people.

(a) If the ticket price was €18, how many people would be expected to attend?

(b) Let x be the ticket price, where $x \leq 20$. Write down, in terms of x, the expected attendance at such an event.

(c) Write down a function f that gives the expected income from the sale of tickets for such an event.

(d) Find the price at which tickets should be sold to give the maximum expected income.

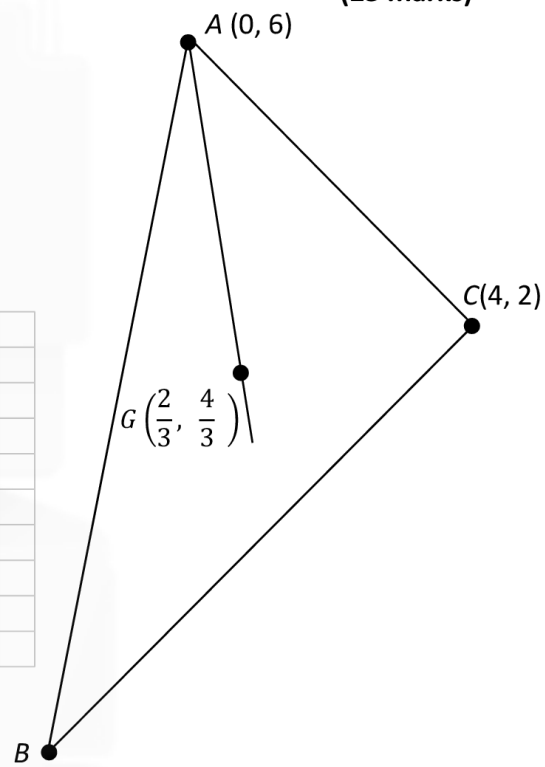
(e) Find this maximum expected income.

(f) Suppose that tickets are instead priced at a value that is expected to give a full attendance at the stadium. Find the difference between the income from the sale of tickets at this price and the maximum income calculated at **(e)** above.

(g) The stadium was full for a recent special event. Two types of tickets were sold, a single ticket for €16 and a family ticket (2 adults and 2 children) for a certain amount. The income from this event was €365 000. If 1000 more family tickets had been sold, the income from the event would have been reduced by €14 000. How many family tickets were sold?

Question 3 **(25 marks)**

ABC is a triangle where the co-ordinates of *A* and *C* are (0, 6) and (4, 2) respectively.
$G\left(\frac{2}{3}, \frac{4}{3}\right)$ is the centroid of the triangle *ABC*.
AG intersects *BC* at the point *P*.
$|AG| : |GP| = 2 : 1$.



(a) Find the co-ordinates of *P*.

(b) Find the co-ordinates of *B*.

(c) Prove that *C* is the orthocentre of the triangle *ABC*.

Question 3 **(25 marks)**

(a) The co-ordinates of two points are $A(4, -1)$ and $B(7, t)$.

The line $l_1 : 3x - 4y - 12 = 0$ is perpendicular to AB. Find the value of t.

(b) Find, in terms of k, the distance between the point $P(10, k)$ and l_1.

(c) $P(10, k)$ is on a bisector of the angles between the lines l_1 and $l_2 : 5x + 12y - 20 = 0$.

(i) Find the possible values of k.

(ii) If $k > 0$, find the distance from P to l_1.