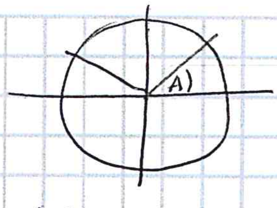
5Y HL Trigonometry Test: 90

Name: M.S.

1.) Find all the values of x for which $\sin(3x) = \frac{\sqrt{3}}{2}$, $0 \le x \le 360$, x in degrees.

$\sin(A) = \sqrt{3}/2$



$A = 60° + 360n$

$A = 120° + 360n$

$n = 0, 1, \ldots$

$3x = 60° + 360n$, $3x = 120° + 360n$

$x = 20° + 120n$. $x = 40° + 120n$

or

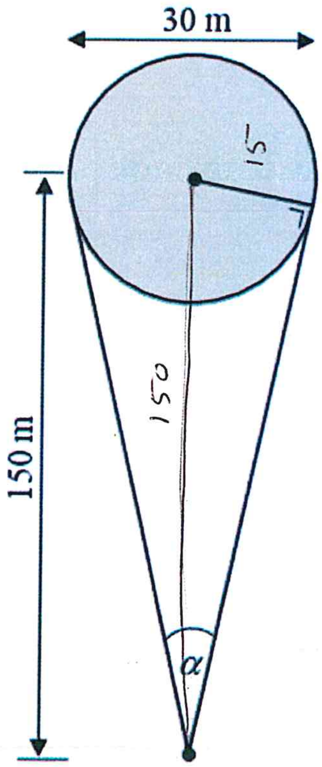
$x = 20°, 140°, 260°,$ $x = 40°, 160°, 280°$

15

(5, 10, 15) all correct

one correct 2 correct

2.)
(a.) Joan is playing golf. She is 150 m from the centre of a circular green of diameter 30 m. The diagram shows the range of directions in which Joan can hit the ball so that it could land on the green. Find α, the measure of the angle of this range of directions. Give your answer, in degrees, correct to one decimal place.



$\sin \alpha = \frac{15}{150} = \frac{1}{10}$

$\frac{\alpha}{2} = \sin^{-1} \frac{1}{10}$

$\frac{\alpha}{2} = 5.74°$

$\alpha = 11.48$

$11.5°$

10

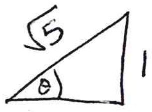
25

At a later hole, Joan's first shot lands at the point G, on ground that is sloping downwards, as shown. A vertical tree, $[CE]$, 25 metres high, stands between G and the hole. The distance, $|GC|$, from the ball to the bottom of the tree is also 25 metres.

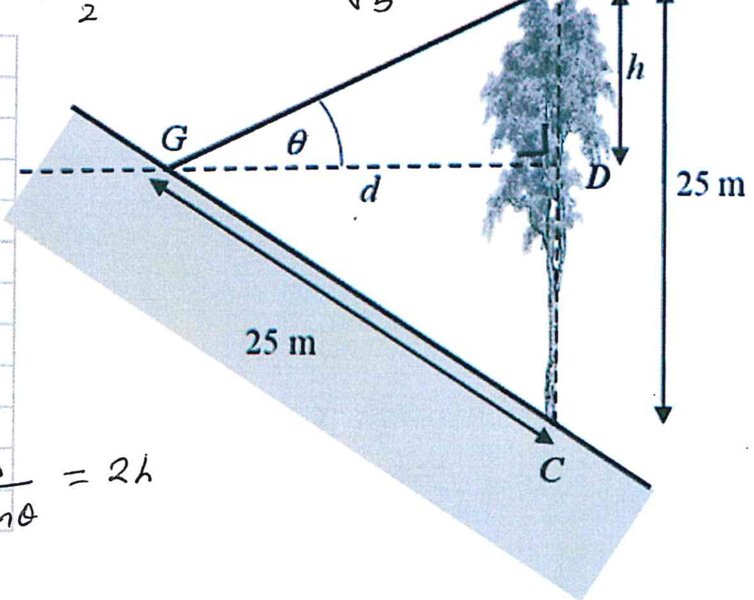
The angle of elevation at G to the top of the tree, E, is θ, where $\theta = \tan^{-1}\frac{1}{2}$.

The height of the top of the tree above the horizontal, GD, is h metres and $|GD| = d$ metres.

(i) Write d and $|CD|$ in terms of h.



$\cos\theta = \frac{2}{\sqrt{5}}$



$\tan\theta = \frac{h}{d} = \frac{1}{2}$

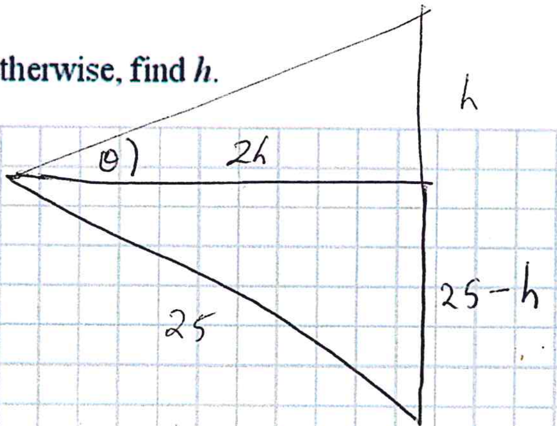
$d =$ $2h$

(10)

$|CD| =$ $25 - h$

$d = \frac{h}{\tan\theta} = 2h$

(ii) Hence, or otherwise, find h.



(10)

$(2h)^2 + (25-h)^2 = 625$

$4h^2 + \cancel{625} - 50h + h^2 = \cancel{625}$

$5h^2 - 50h = 0$

$h(5h - 50)$

$h(h-10) = 0$ $\quad h = 10.$

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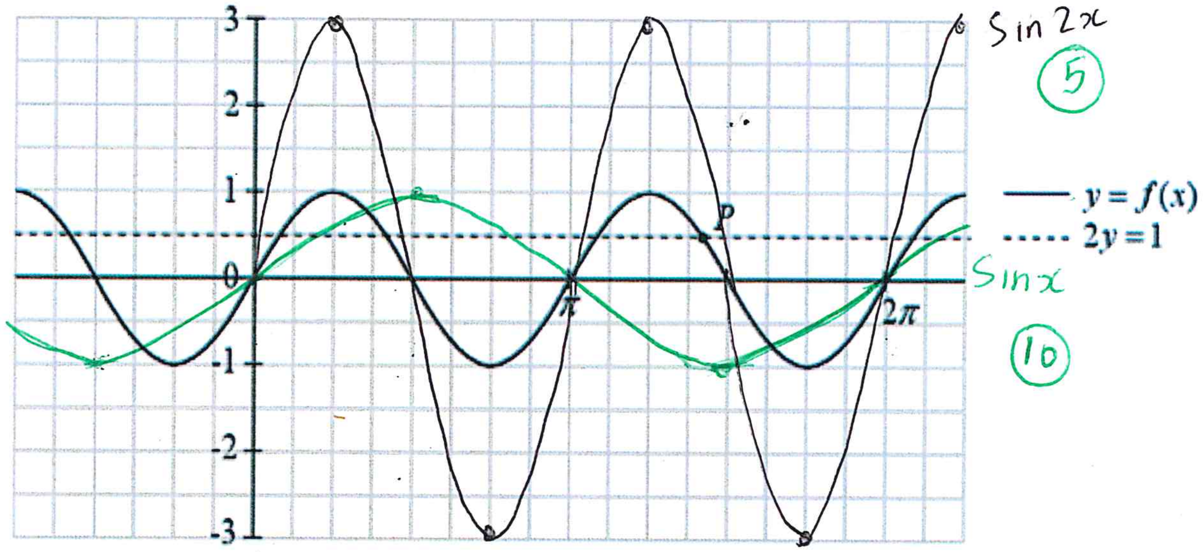
3.) The diagram below shows the graph of the function $f : x \mapsto \sin 2x$. The line $2y = 1$ is also show

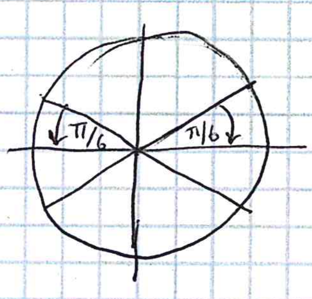


(a) On the same diagram above, sketch the graphs of $g : x \mapsto \sin x$ and $h : x \mapsto 3\sin 2x$. Indicate clearly which is g and which is h.

(b) Find the co-ordinates of the point P in the diagram.

$\sin 2x = \frac{1}{2}$

$\sin A = \frac{1}{2}$



$A = \frac{\pi}{6} + 2n\pi$

5

or $A = \frac{5\pi}{6} + 2n\pi$

$x = \frac{\pi}{12} + n\pi \quad = \frac{\pi}{12}, \frac{13\pi}{12} \ldots$

10

$x = \frac{5\pi}{12} + n\pi \quad \frac{5\pi}{12}, \frac{17\pi}{12} \ldots$

$P = \left(\frac{17\pi}{12}, \frac{1}{2}\right)$

15

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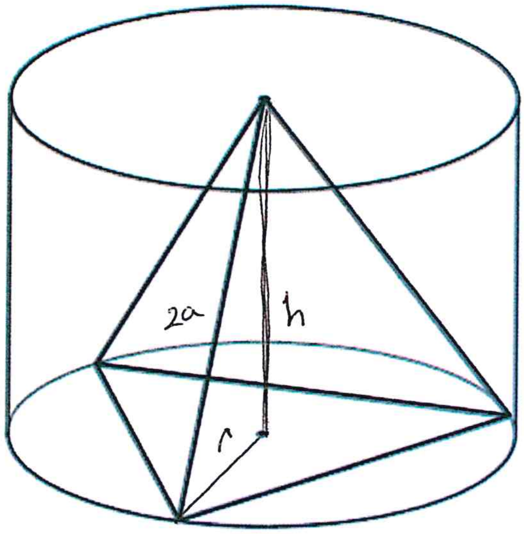
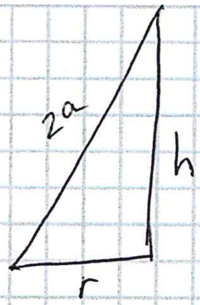
4.) A regular tetrahedron has four faces, each of which is an equilateral triangle.

A wooden puzzle consists of several pieces that can be assembled to make a regular tetrahedron. The manufacturer wants to package the assembled tetrahedron in a clear cylindrical container, with one face flat against the bottom.

If the length of one edge of the tetrahedron is $2a$, show that the volume of the smallest possible cylindrical container is $\left(\frac{8\sqrt{6}}{9}\right)\pi a^3$.



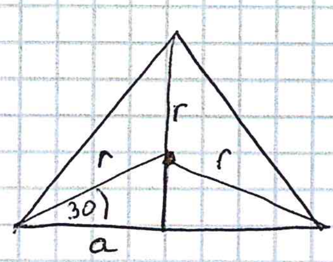
$V = \pi r^2 h$



$h^2 + r^2 = 4a^2$

$h^2 = 4a^2 - r^2$

find r :



$\cos 30 = \frac{a}{r}$

$r = \frac{a}{\cos 30}$

$r = \frac{a}{\sqrt{3}/2} = \frac{2a}{\sqrt{3}}$

$h^2 = 4a^2 - \left(\frac{2a}{\sqrt{3}}\right)^2 = 4a^2 - \frac{4a^2}{3} = \frac{8a^2}{3}$

$h = \sqrt{\frac{8a^2}{3}} = a\sqrt{\frac{8}{3}}$

$V = \pi r^2 h = \pi\left(\frac{4a^2}{3}\right)\left(a\sqrt{\frac{8}{\sqrt{3}}}\right) = \pi \cdot \frac{4a^3}{3}\frac{\sqrt{8}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \pi \frac{4a^3 \cdot 2\sqrt{2}\sqrt{3}}{9} = \pi a^3 \cdot \frac{8\sqrt{6}}{9}$

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