5th Year Functions Test

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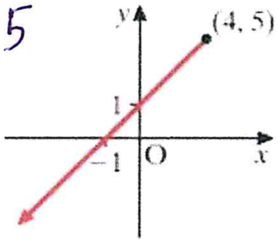
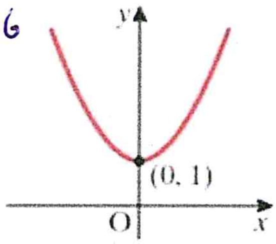
Name: M.S.

1.) The graphs and the ranges of six relations are given below.
Connect each graph to its correct range.

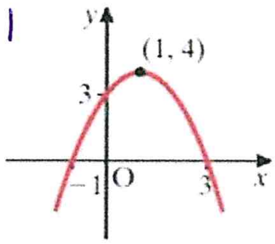
Ⓐ 5



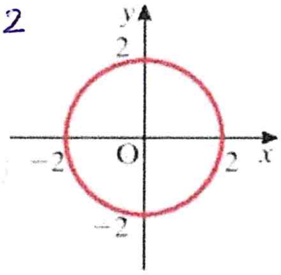
Ⓑ 6



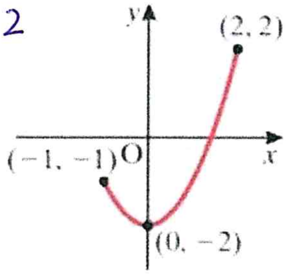
Ⓒ 1



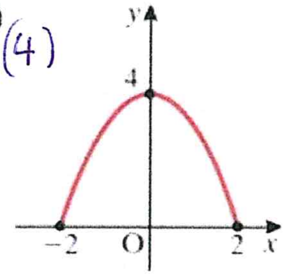
Ⓓ 2



Ⓔ 2



Ⓕ (4)



6×3

① Range = $(-\infty, 4]$ (C) ② Range = $[-2, 2]$ (D)(E) ③ Range = $(-\infty, 2]$

④ Range = $[0, 4]$ (F) ⑤ Range = $(-\infty, 5]$ (A) ⑥ Range = $[1, \infty)$ (B)

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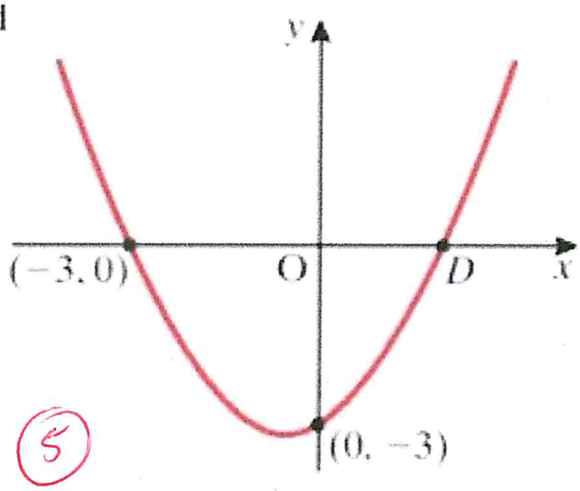
2.) The function $f(x) = x^2 + bx + c$ is graphed on the right.



Find b,c and D :

$f(0) = 0^2 + b(0) + c$

$f(0) = c$ but $(0, -3)$

$f(0) = -3$ $\quad c = -3$ $\quad$ 5

D = root, -3 = other root

$(x+3)(\quad) = x^2 + bx - 3$

must be $x - 1$ so $D = 1$ $\quad$ 5

so $(x+3)(x-1) = x^2 + 2x - 3$

$b = 2$ $\quad$ 5

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3.) The functions f and g are defined with their respective domains by

$f(x) = x^3$, for all real values of x

$g(x) = \dfrac{1}{x-3}$, for real values of x, $x \neq 3$

(a) State the range of f. $(-\infty, \infty)$ 5 or $\mathbb{R}$

(b) (i) Find $fg(x)$.
(ii) Solve the equation $fg(x) = 64$.

(i) $f\left(\frac{1}{x-3}\right) = \left(\frac{1}{(x-3)}\right)^3 = \frac{1}{(x-3)^3}$ 5

(ii) $\left(\frac{1}{(x-3)}\right)^3 = 64$ $\quad \left(\frac{1}{x-3}\right) = \sqrt[3]{64} = 4$ 10

So $1 = 4(x-3)$

$4x = 13$ $\quad x = \frac{13}{4} = 3\frac{1}{4}$

(c) (i) The inverse of g is g^{-1}. Find $g^{-1}(x)$.
(ii) State the range of g^{-1}.
(iii) Express in terms of x, $gg^{-1}(x)$.
(iv) State the value of x at which the graph of $y = g^{-1}(x)$ is not continuous.

$g = \frac{1}{x-3}$

$g(x-3) = 1$, $\quad gx - 3g = 1$,

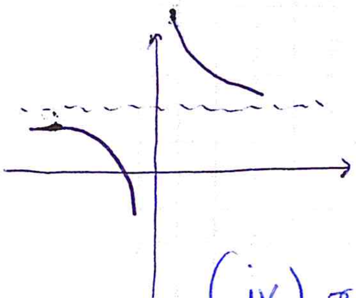
$gx = 1 + 3g$

$x = \frac{1+3g}{g}$ or $\frac{1}{g} + 3$

$g^{-1}(x) = \frac{1+3x}{x}$ 10

(ii) range of g^{-1}: $\frac{1}{x} + 3$ $\quad$ as $x \to \infty$ $\quad g^{-1} \to 3$

as $x \to 0$ $\quad g^{-1} \to \infty$



range $= \mathbb{R} / 3$? 5

(iii) $gg^{-1}(x) = \dfrac{1}{\frac{1+3x}{x} - 3} = \dfrac{1}{\frac{1}{x}} = x$ 10

(iv) $x = 0$ 5

35 + 15

4.) **(i)** **Find :** $\lim_{x\to 1} \dfrac{x^2 - 3x + 2}{x - 1}$

$$\frac{(x-1)(x-2)}{(x-1)} = x-2$$

$$= 1 - 2$$

$$= -1$$

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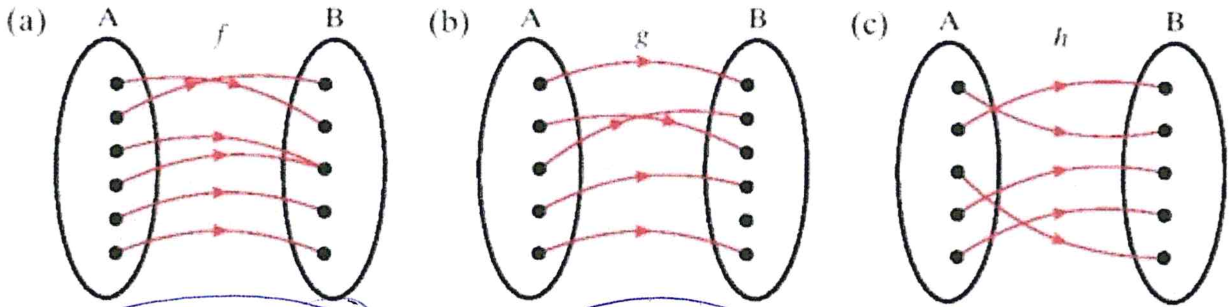
(ii) **Find:** $\lim_{n\to\infty} \dfrac{2n^2 - 3n + 2}{6n^2 + 5n - 6}$

$$\frac{2 - 3/n + 2/n^2}{6 + 5/n - 6/n^2} = \frac{1}{3}$$

10

5.) Of the three mapping diagrams shown below, one is injective, one is surjective and one is bijective.
Connect each mapping to one of these descriptions.



Surjective | injective | Bijective

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6.) A farmer accidentally spread a dangerous chemical on a paddock. The concentration of the chemical in the soil was initially measured at 5 kg/ha. One year later, the concentration was found to be 2.8 kg/ha.

It is known that the concentration, C, is given by

$$C = C_0 e^{-kt}$$

where C_0 and k are constants, and t is measured in years.

(i) Evaluate C_0 and k.

(ii) It is safe to use the paddock when the concentration is below 0.2 kg/ha? How long must the farmer wait after the accident before the paddock can be used? Give your answer in years, correct to one decimal place.

(i) $C_0 = C$ when $t = 0$ $\quad C_0 e^{-0} = C_0$.

so $C_0 = 5\text{kg/ha}$ $\quad$ 5

$2.8 = C_0 e^{-k(1)}$

$2.8 = 5e^{-k} \Rightarrow \frac{2.8}{5} = e^{-k}$

$\log_e\left(\frac{2.8}{5}\right) = \log_e e^{-k}$ $\quad$ 10

$\log_e\left(\frac{2.8}{5}\right) = -k \Rightarrow k = -\log_e\left(\frac{2.8}{5}\right)$ $\quad k = 0.5798$

(ii) $C_0 e^{-kt} < 0.2$

$e^{-kt} < \frac{0.2}{C_0}$

$\log_e e^{-kt} < \log_e \frac{0.2}{C_0}$ $\quad$ 10

$-kt < \ln\left(\frac{0.2}{C_0}\right)$

$t > \frac{-\ln\left(\frac{0.2}{C_0}\right)}{k}$ $\quad t > 5.55$ yrs $\quad$ 5.6 yrs

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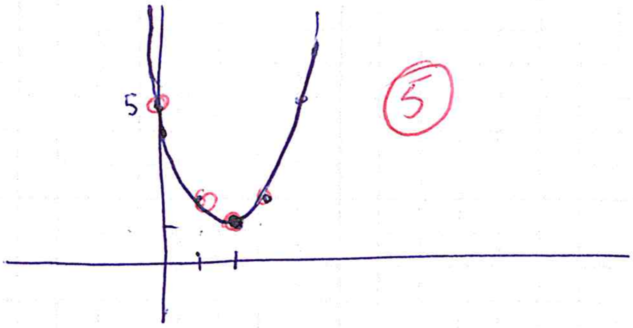
7.) The equation of a function is $y = x^2 - 4x + 5, x \in \mathbb{R}$.

(a) Express $x^2 - 4x + 5$ in the form $(x + h)^2 + k$, where $h, k \in \mathbb{Z}$.
Hence, write down the coordinates of the turning point of the function and draw a rough sketch of the graph.

(b) Use the result in (a) to find the inverse function of $y = x^2 - 4x + 5$.

(c) Using the inverse function found in part (b), or otherwise, add a sketch of this inverse function to the sketch drawn in (a) above.

(a) $(x-2)^2 + 1$ turning pt $= (2, 1)$ 10



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(b.)

$$y = (x-2)^2 + 1$$

$$y - 1 = (x-2)^2$$

$$\pm\sqrt{y-1} = x - 2$$

$$2 \pm \sqrt{y-1} = x$$

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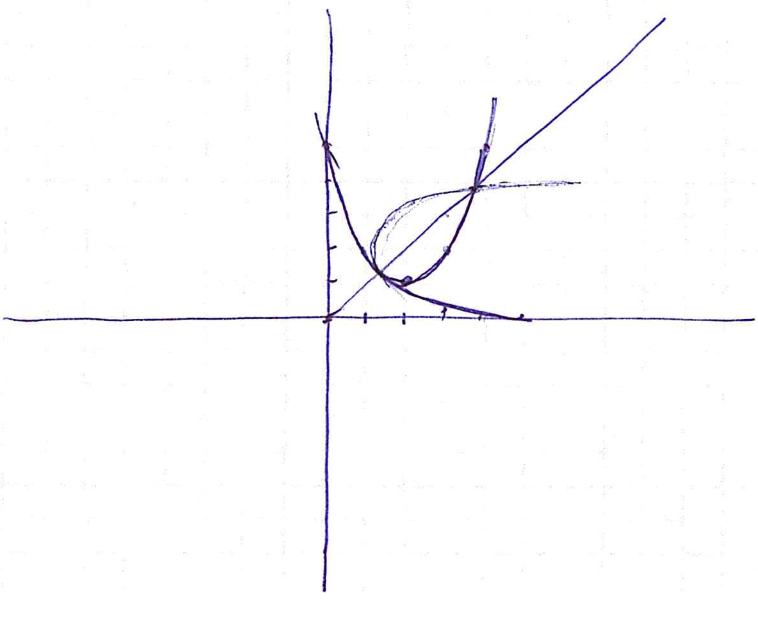
$$f^{-1}(x) = 2 \pm \sqrt{x-1}$$

(c.)



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8.) (a) Find the values of a and b such that the graph of $y = a\log_2(x - b)$ passes through the points (5, 2) and (7, 4).

(b) Which one of the following statements is not true of the graph of the function $f: R^+ \to R, f(x) = \log_5 x$?

(i) The domain is R^+.
(ii) The range is R.
(iii) It passes through the point (5, 0).
(iv) It has a vertical asymptote with equation $x = 0$.
(v) The slope of the tangent at any point on the graph is positive.

$2 = a\log_2(5-b)$

$4 = a\log_2(7-b)$

$\frac{2}{4} = \frac{\log_2(5-b)}{\log_2(7-b)} = \frac{1}{2}$

$2\log_2(5-b) = \log_2(7-b)$

$\log_2(5-b)^2 = \log_2(7-b)$

$(5-b)^2 = 7-b$ $\quad 25-10b+b^2 = 7-b$

$b^2 - 9b + 18 = 0$ $\quad (b-6)(b-3) \Rightarrow b = 3$

if $b = 6$ then $2 = a\log_2(-1)$

$\log(-\text{num})$ impossible

if $b = 3$

$2 = a\log_2(5-3)$

$2 = a\log_2 2$

$2 = a(1)$ $\quad a = 2$

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(b.) $f(x) = \log_5 x$ $\quad$ when $x = 5$, $f(x) = \log_5 5 = 1$

NOT 0!

Answer (iii) ✓ 5

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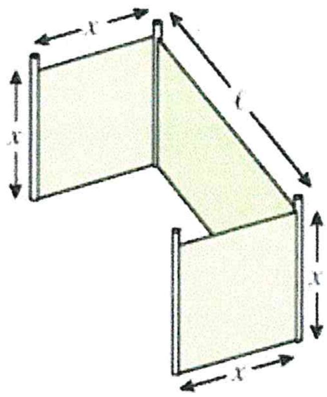
9.) A canvas wind-shelter has two square ends, each of side x metres, and a rectangular back of length ℓ metres. The area of canvas is $9\,\text{m}^2$.

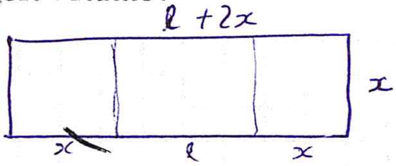


(i) Show that $\ell = \frac{9}{x} - 2x$, and hence show that the enclosed volume, $V\,\text{m}^3$, is $9x - 2x^3$.

(ii) Plot the graph of V against x for $0 \leqslant x \leqslant 3$.

(iii) (a) Use your graph to find the value of x that gives the largest possible volume.

(b) From your graph, what is this largest volume?



(i) Area $= (l + 2x)\,x$

$9 = (l + 2x)\,x$

$\frac{9}{x} = l + 2x \Rightarrow l = \frac{9}{x} - 2x$

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Volume $= l \times x \times x$

$= \left(\frac{9}{x} - 2x\right)x^2 = 9x - 2x^3$

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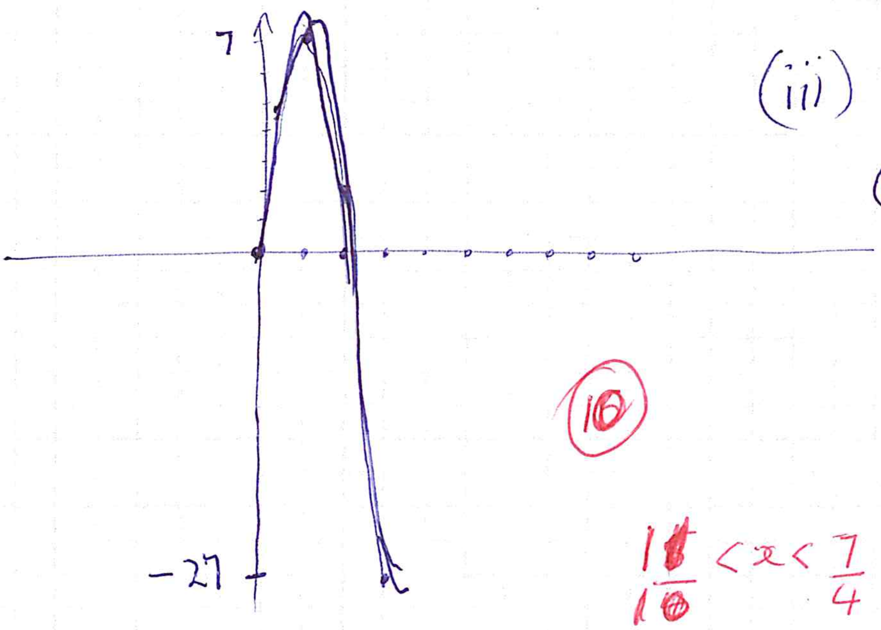
(ii)



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$\frac{11}{10} < x < \frac{7}{4}$

$1.1 < x < 1.4$

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(iii) largest volume

$(9 - 2x^2)\,x$

$\frac{dv}{dx} = 9 - 6x^2$

$9 - 6x^2 = 0$

$6x^2 = 9$

$2x^2 = 3$

$x^2 = \frac{3}{2}$ $\quad x = \pm\sqrt{\frac{3}{2}}$

$x = 1.22$

$y = \cancel{8.34}$

accept $7 < y < 8$

$\boxed{30}$

10.) The slope of a function $y = f(x)$ is given by slope $= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

Find the slope of each of these functions:

(i) $f(x) = 2x - 3$ (ii) $f(x) = x^2$ (iii) $f(x) = x^2 + 5$

(i) $f(x+h) = 2x + 2h - 3$

$\frac{f(x+h) - f(x)}{h} = \frac{2h}{h} = 2$

$\lim_{h \to 0} = 2$

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(ii) $f(x+h) = x^2 + 2hx + h^2$

$\frac{f(x+h) - f(x)}{h} = \frac{2hx + h^2}{h} = 2x + h$

$\lim_{h \to 0} = 2x$

5

(iii) $f(x+h) = x^2 + 2hx + h^2 + 5$

$\frac{f(x+h) - f(x)}{h} = \frac{2hx + h^2}{h}$

$\lim_{h \to 0} = 2x$

5

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