5th Year Test on Algebra 3 **Name:** MS

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1.) Solve the following inequality: $-4 \leq \frac{2}{5}(1-3x) \leq 1$

$-20 \leq 2-6x$

$-22 \leq -6x$

$22 \geq 6x$

$\frac{22}{6} \geq x$

$\frac{11}{3} \geq x$

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$2-6x \leq 5$

$-6x \leq 3$

$x \geq -\frac{1}{2}$

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2.) Find the range of values of x for which : $\frac{3+4x}{5x-1} > 3, x \neq \frac{1}{5}$

$(3+4x)(5x-1) > 3(5x-1)(5x-1)$

$\cancel{15x} - \cancel{3} + \cancel{20x^2} - \cancel{4x} > \cancel{75x^2} - \cancel{30x} + 3$

$0 > 55x^2 - 41x + 6$

$(11x-6)(5x-1)$

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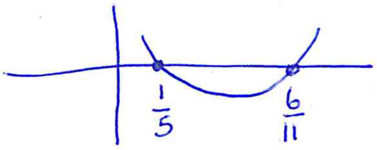
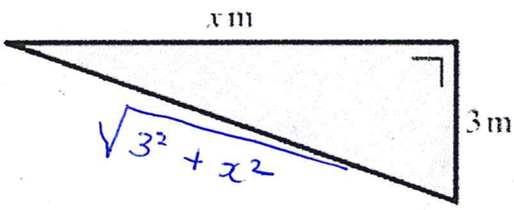


$x < \frac{6}{11}$ 5 $x > \frac{1}{5}$

3.) Find the range of values of x, $x \in Z$, so that the perimeter of this triangle is between 8 m and 12 m long.



$x + 3 + \sqrt{3^2 + x^2} > 8$

$\sqrt{3^2 + x^2} > 5 - x$

$9 + \cancel{x^2} > 25 - 10x + \cancel{x^2}$

$10x > 16 \Rightarrow x > 1.6$

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$x + 3 + \sqrt{9 + x^2} < 12$

$9 + x^2 < (9-x)(9-x)$

$9 + \cancel{x^2} < 81 - 18x + \cancel{x^2}$

$18x < 72$ $x < 4$

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x can only be 2m or 3m

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4.) The path of a ball is given by the expression $f(t) = -11 + 13t - 2t^2$, where t represents time.
Find the range of values of t that satisfies each of the following inequalities.

(i) $f(t) \leq 4$

(ii) $f(t) \geq 7$, and hence deduce the set of values of t that satisfies

(iii) $4 < f(t) < 7$.

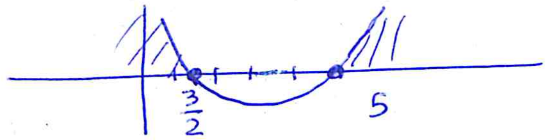
$-2t^2 + 13t - 11 \leq 4$

$-2t^2 + 13t - 15 \leq 0$

$2t^2 - 13t + 15 \geq 0$

$(2t - 3)(t - 5) \geq 0$

$t =$



$t \leq 3/2 \qquad t \geq 5$

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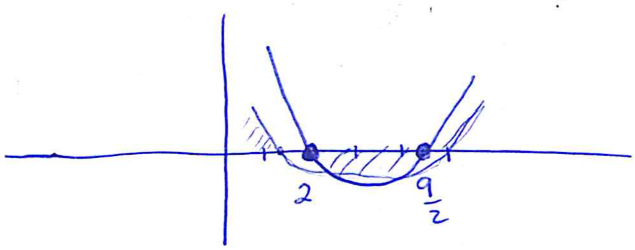
$-2t^2 + 13t - 11 \geq 7$

$2t^2 - 13t + 18 \leq 0$

$(2t - 9)(t - 2) \leq 0$

$t = 9/2 \qquad t = 2$



$t \geq 2 \qquad t \leq \frac{9}{2}$

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$t \leq 3/2$

$f(t) > 4$ and $f(t) < 7$

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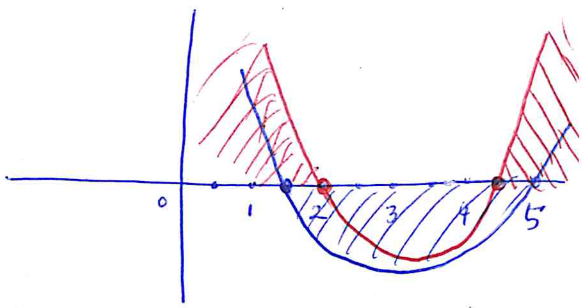
$f(t) > 4 \Rightarrow t \geq 3/2 \qquad t \leq 5$

$f(t) < 7 \Rightarrow t \leq 2 \qquad t \geq 9/2$



Ans: $\frac{3}{2} < t < 2$ and $\frac{9}{2} < t < 5$

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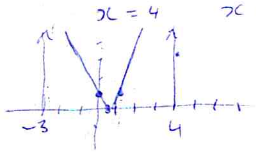
$y = 2x - 1$

5.) Solve the following inequality ($x \in R$) : $|2x - 1| \geq 7$

$|2x - 1| = 7$

$2x - 1 = 7$ or $2x - 1 =$

$x = 4$ $\quad$ $x = -3$

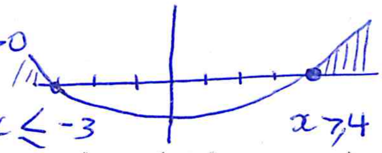


$4x^2 - 4x + 1 \geq 49$ $\qquad$ $2x - 1$

$4x^2 - 4x - 48 \geq 0$ $\qquad$ or

$x^2 - x - 12 \geq 0$

$(x + 3)(x - 4) \geq 0$



$x \leq -3$ $\qquad$ $x \geq 4$,

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$x \geq 4$
or $x \leq -3$

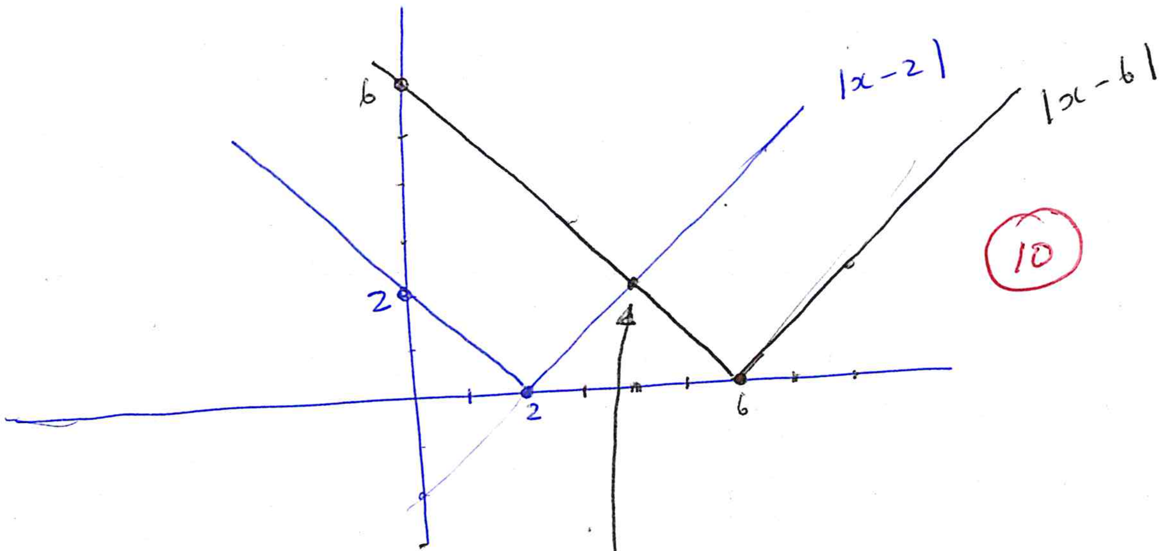
6.) On the same set of axes, sketch the graphs of the functions

$f : x \to |x - 2|$ and $g : x \to |x - 6|$.

$y = x - 2$ $\qquad$ $y = x - 6$

Hence solve the equation $|x - 2| = |x - 6|$.

Verify your answer algebraically.



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$|x - 2| = |x - 6|$

$\cancel{x^2} - 4x + 4 = \cancel{x^2} - 12x + 36$

$8x - 32 = 0$

$x = 4, \; y = 2$

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7.) The expression $\sqrt{3^{2n+1}} \times \sqrt[3]{3^{-3n}}$ can be written in the form 3^k; find k.

$$\left(3^{2n+1}\right)^{1/2} \times \left(3^{-3n}\right)^{1/3}$$

$$= 3^{n+1/2} \times 3^{-n}$$

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$$= 3^{n+1/2-n} = 3^{1/2}$$

$$k = \frac{1}{2}$$

8.) Given $f(n) = 3^n$, find expressions for (i) $f(n+3)$ (ii) $f(n+1)$.

Hence find the value of k such that $f(n+3) - f(n+1) = k\,f(n)$, where $k \in N$.

(i) $f(n+3) = 3^{n+3}$ 3 (ii) $f(n+1) = 3^{n+1}$ 3

(iii) $3^{n+3} - 3^{n+1} = k.3^n$

$$3^3.3^n - 3^1.3^n = k.3^n$$

$$27(3^n) - 3(3^n) = k(3^n)$$

$$24(3^n) = k(3^n)$$

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$$k = 24$$

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9.) Solve the exponential equation $3^x + 81(3^{-x}) - 30 = 0$.

$y + \frac{81}{y} - 30 = 0$

$y^2 + 81 - 30y = 0$

$y^2 - 30y + 81 = 0$ 5

$(y-27)(y-3)$ 5

$y = 27 \qquad y = 3$ 15

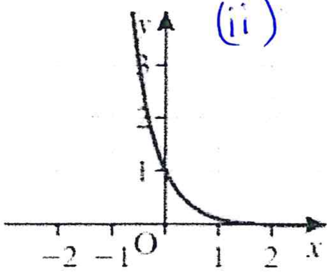
$3^x = 27 \qquad 3^x = 3$ 5

$x = 3 \qquad x = 1$

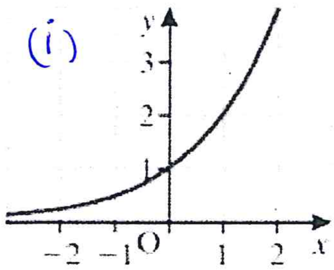
10.) Match each of the following exponential functions with one of the graphs.

(i) $y = 2^x$ (B)
(ii) $y = (0.1)^x$ (A)
(iii) $y = 10^x$ (D)
(iv) $y = (0.5)2^x$ (C)

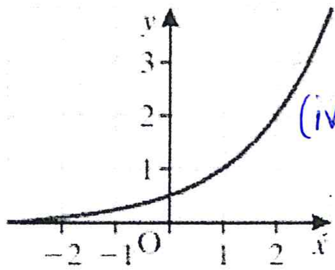
A



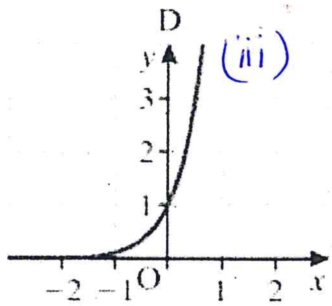
B



C



D



(0, 4, 8, 12, 15) 15

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11.) Let $y = 2^{x-1} + 3$.

(i) Express x in terms of y using common logarithms.

(ii) Hence find, correct to 4 decimal places, the value of x for which $y = 8$.

$y - 3 = 2^{x-1}$

$\log_2 y - 3 = x - 1$

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$\log_2(y-3) + 1 = x$

(ii) $\log_2(8-3) + 1 = x$

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$3.3219 = x$

12.) Show that $\log_b a = \dfrac{1}{\log_a b}$.

$\log_b a = \dfrac{\log_a a}{\log_a b} = \dfrac{1}{\log_a b}$

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13.) Use the change of base rule to solve the following equation:

$$2\log_4 x + 1 = \log_x 4.$$

$\log_x 4 = \dfrac{\log_4 4}{\log_4 x}$

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$2\log_4 x + 1 = \dfrac{1}{\log_4 x}$

$y = \log_4 x$

$2y + 1 = \dfrac{1}{y}$ 5

$2y^2 + y = 1$

$2y^2 + y - 1 = 0$

$(2y - 1)(y + 1)$

$y = \frac{1}{2}$ $y = -1$

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$\log_4 x = \frac{1}{2}$

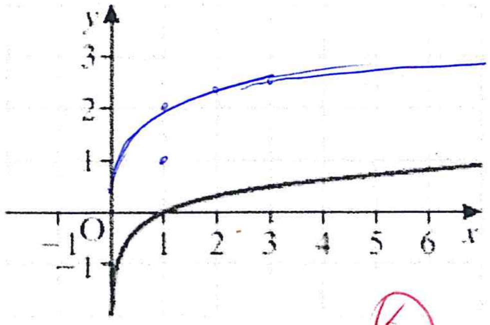
$x = 4^{1/2} = 2,$

$\log_4 x = -1$

$x = 4^{-1}$

or $x = \frac{1}{4}$

14.) In parts (i) ...(iv) you are given the graph of $y = \log_{10}(x)$. Sketch a graph of



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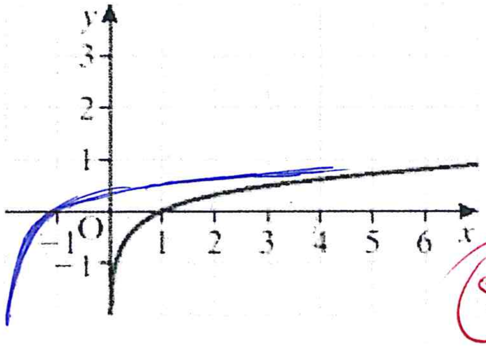
(i) $y = \log_{10}(x) + 2$



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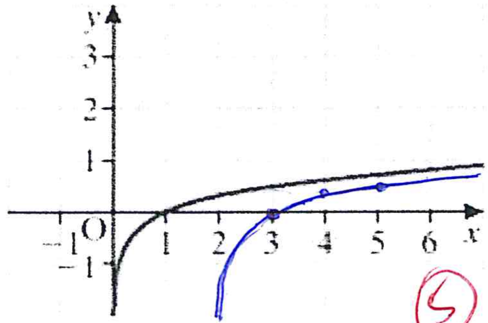
(ii) $y = \log_{10}(x + 2)$



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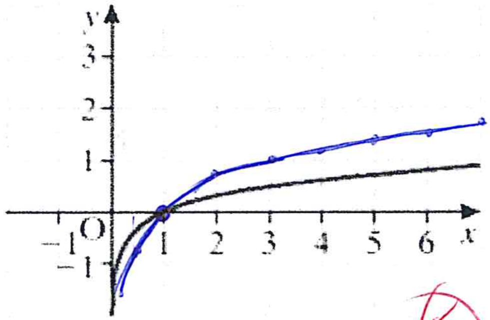
(iii) $y = \log_{10}(x - 2)$



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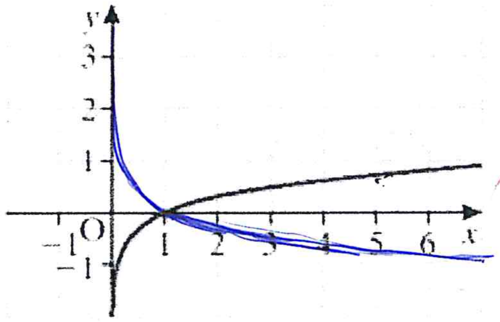
(iv) $y = 2\log_{10}(x)$



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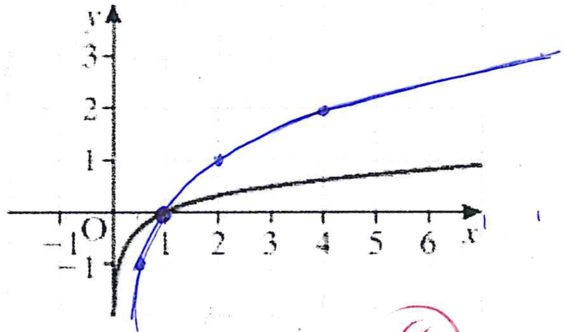
(v) $y = -\log_{10}(x)$



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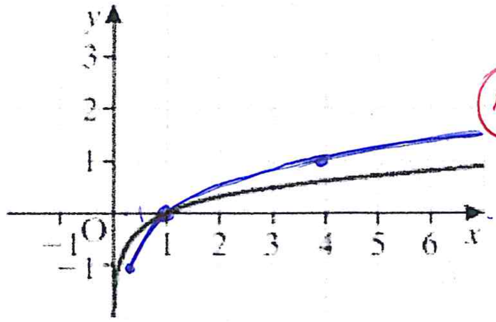
(vi) $y = \log_2(x)$



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(vii)) $y = \log_4(x)$

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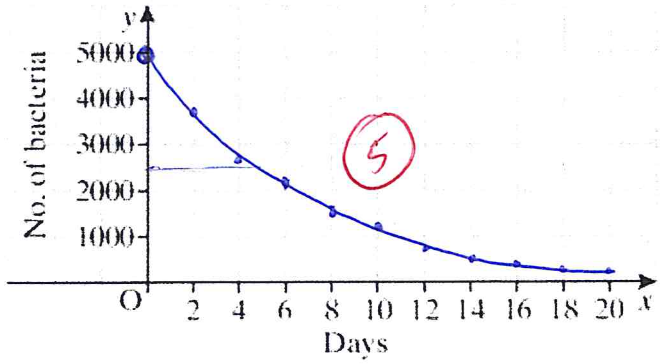
15.) The manufacturer of a facial cream *SPOTLESS* claims that the population of bacteria which create spots will be halved within five days of using their cream.
During a trial in his laboratory, Professor Snape finds that the number of bacteria N in the population is given by the formula $N = 5000e^{-0.15t}$, where t is time, measured in days.



(i) Using this formula, test the claim that the bacteria population will halve in five days.

(ii) According to Professor Snape's equation, what will the level of bacteria be after 10 days?

(iii) How many bacteria are present at the beginning of the trial?

(iv) After how many days will the population reduce to 100?

(v) Copy this grid and sketch a graph of the number of bacteria in the population over a 20-day trial.

(i)

$N = 5000e^{-0.15t}$

$N_0 = 5000e^0 = 5000$ 5

$\frac{\cancel{2500}}{\cancel{5000}} = e^{-0.15t}$

$\frac{1}{2} = e^{-0.15t}$

$\ln \frac{1}{2} = \ln e^{-0.15t}$

$\ln \frac{1}{2} = -0.15t$

$\frac{\ln 1/2}{-0.15} = t = 4.62$ days

(ii) $N = 5000e^{-1.5}$

$N = 1115.65$ 5

(iii) $N_0 = 5000$ 5

(iv) $100 = 5000e^{-0.15t}$

$\frac{10\cancel{0}}{500\cancel{0}} = e^{-0.15t}$

$\ln 1/50 = -0.15t$

$\frac{\ln 1/50}{-0.15} = t = 26.0$ days 10

(v) above

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