5th Year Algebra 2 Test: **Name:............................**

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1.) Solve: $\left(y+\frac{4}{y}\right)^2-9\left(y+\frac{4}{y}\right)+20=0$

$y+\frac{4}{y}=x$

$x^2-9x+20=0$

$(x-5)(x-4)=0$

$x=5,\quad x=4$ (0,3,5)

$y+\frac{4}{y}=5$

$y^2+4-5y=0$

$y^2-5y+4=0$

$(y-4)(y-1)=0$

$y=4,\ y=1$ (0,2,3,5)

$y+\frac{4}{y}=4$

$y^2+4-4y=0$

$y^2-4y+4=0$

$(y-2)(y-2)=0$

$y=2.$ (0,2,3,5)

check $\left(\underset{\downarrow\ 5}{4+\frac{4}{4}}\right)^2-9\left(4+\frac{4}{4}\right)+20=0$ ✓

$\left(2+\frac{4}{2}\right)=4$

✓

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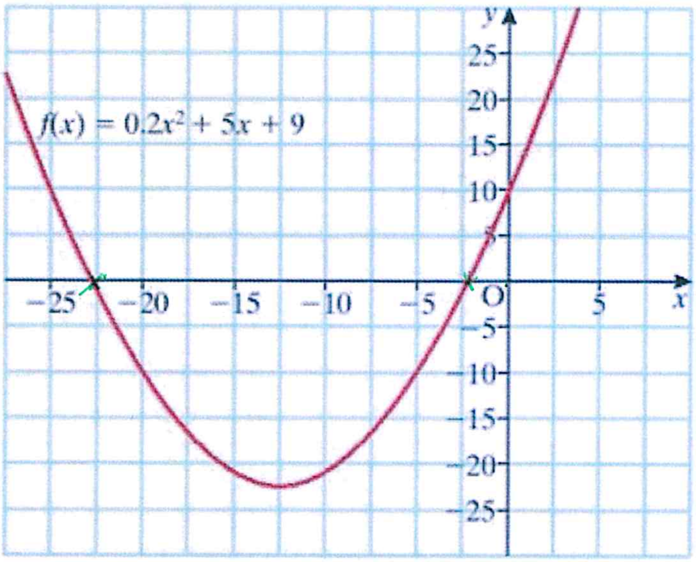
2.) If x_1 and x_2 are the roots of the equation $f(x) = 0.2x^2 + 5x + 9 = 0$ and $x_1 > x_2$, using the graph, find an approximate value for

(a) $(x_2 - x_1)$

(b) $(x_2 + x_1)$



$x_2 = -23$

$x_1 = -2$

$x_2 - x_1 = -21$ (0, 2.5, 3.5)

$x_2 + x_1 = -25$ (0, 2, 3.5)

3.) What is the discriminant of the equation : $ax^2 + bx + c$?

$b^2 - 4ac$ = discriminant 5

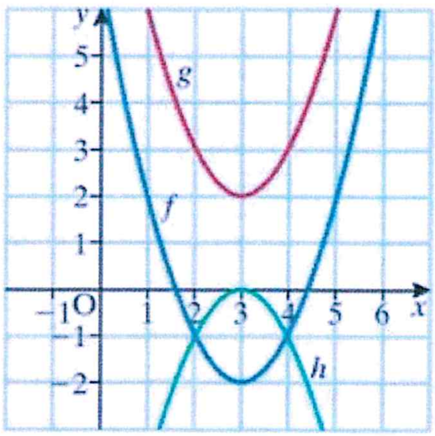
4.) By inspection, state which of the curves – f, g and h – have

(i) real and distinct roots f (0, 5)

(ii) real and equal roots h (0, 5)

(iii) imaginary roots. g (0, 5)

(iv) In the case of real roots, estimate from the graph



f : 1.6, 4.4

g : x none

h : 3, 3, (0, 2, 3, 5)

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5.) Find the value of k for which the equation $(k-2)x^2 + x(2k+1) + k = 0$ has equal roots.

$b^2 - 4ac = 0$

$(2k+1)(2k+1) - 4(k-2)(k) = 0$

$4k^2 + 4k + 1 - 4(k^2 - 2k) = 0$

$4k^2 + 4k + 1 - 4k^2 + 8k = 0$

$12k = -1$

$k = -\frac{1}{12}$

(0, 2, 5, 7, 10)

6.) **Solve to find x,y :**

$xy = 4$

$2x - y + 2 = 0$

$y = \frac{4}{x}$

$2x - \frac{4}{x} + 2 = 0$

$2x^2 - 4 + 2x = 0$

$2x^2 + 2x - 4 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2$ $\quad x = 1$

$y = -2$ $\quad y = 4$

(0, 2, 5, 7, 10)

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7.) **Solve to find s,t :**

$$s = 2t - 1$$
$$3t^2 - 2ts + s^2 = 9$$

$3t^2 - 2t(2t-1) + (2t-1)(2t-1) = 9$

$3t^2 \cancel{-4t^2} \cancel{+2t} \cancel{+4t^2} \cancel{-4t} + 1 - 9 = 0$

$3t^2 - 2t - 8 = 0$

$(3t+4)(t-2)$

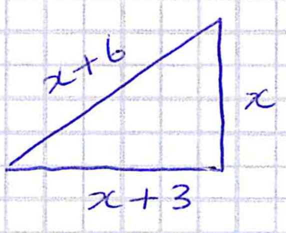
$t = \frac{-4}{3}$ $\quad t = 2$

$S = \frac{-8}{3} - \frac{3}{3}$ $\quad S = 2(2) - 1$

$S = 3$ $\Big\}$ $(0, 3, 5, 7, 10, 15)$

$S = \frac{-11}{3}$

8.) The hypotenuse of a right-angled triangle is 6 cm longer than the shortest side. The third side is 3 cm longer than the shortest side. Find the length of the shortest side.



$x^2 + (x+3)^2 = (x+6)^2$

$x^2 + \cancel{x^2} + \cancel{6x} + 9 = \cancel{x^2} + \cancel{12x} + 36$

$x^2 - 6x - 36 + 9 = 0$ $\quad (0, 3, 5, 7, 10, 12, 15)$

$x^2 - 6x - 27 = 0$

$(x+3)(x-9)$ $\quad x = 9$

$x \neq -3$

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9.) Express $2x^2 - 12x + 7$ in the form $a(x - b)^2 + c$.

$2\left(x^2 - 6x + \frac{7}{2}\right)$

$2\left((x-3)^2 - 9 + \frac{7}{2}\right)$

$2\left((x-3)^2 - \frac{11}{2}\right)$

$2(x-3)^2 - 11$ (0, 2, 5, 7, 10)

10.) (0, 2.5, 1)

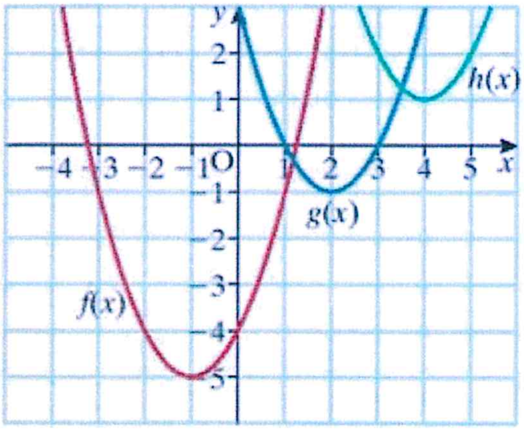
(i) Write down the coordinates (p, q) of the minimum point of each of these graphs. $f = (-1, -5)$, $g = (2, -1)$, $h = (4, 1)$

(ii) Write the equation of each graph in the form

(a) $y = (x - p)^2 + q$

(b) $y = ax^2 + bx + c$.

(iii) By picking a suitable point on each graph (other than the minimum point), verify each equation.



(ii) $f(x) = (x+1)^2 - 5$

(a) $g(x) = (x-2)^2 - 1$ (0, 3, 5, 7, 10)

$h(x) = (x-4)^2 + 1$

(iii) verify

(b) $f(x) = x^2 + 2x - 4$ (0, 2.5, 1)

$g(x) = x^2 - 4x + 3$ (0, 3, 5, 7, 10)

$h(x) = x^2 - 8x + 17$

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11.) Letting $X = \dfrac{4+\sqrt{3}}{\sqrt{2}}$ and $Y = \dfrac{4-\sqrt{3}}{\sqrt{2}}$, find in its simplest form: $\dfrac{X}{Y}$

$$\frac{\frac{4+\sqrt{3}}{\sqrt{2}}}{\frac{4-\sqrt{3}}{\sqrt{2}}} = \frac{4+\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{4-\sqrt{3}} = \frac{4+\sqrt{3}}{4-\sqrt{3}}$$

$$\frac{4+\sqrt{3}}{4-\sqrt{3}} \frac{(4+\sqrt{3})}{4+\sqrt{3}} = \frac{19+8\sqrt{3}}{13}$$

$$= \frac{19}{13} + \frac{8\sqrt{3}}{13}$$

(0, 2, 3, 5, 7, 10)

12.) **State the factor theorem:**

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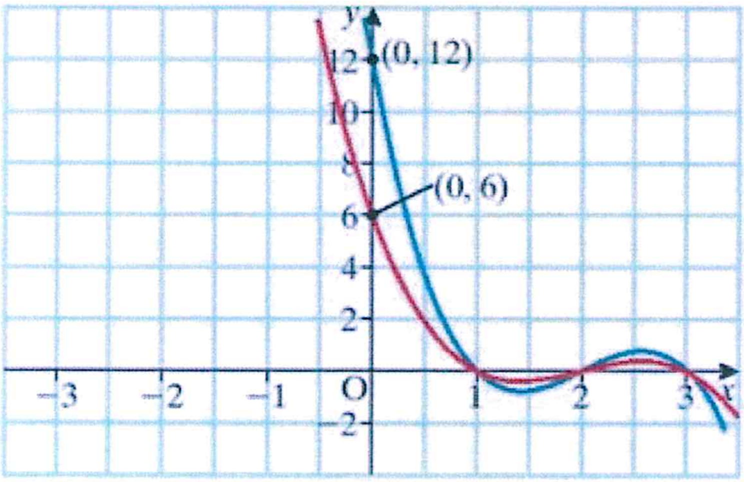
13.) Write a polynomial expression for each of the following cubic graphs.



blue : $f(x)$

$f(x) = a(x-1)(x-2)(x-3)$

$(0,12)$: $12 = a(-1)(-2)(-3)$

$12 = -6a$

$-2 = a$

$f(x) = -2(x-1)(x-2)(x-3)$

$f(x) = -2x^3 + 12x^2 - 22x + 12$

(0, 2, 3, 5, 7, 10, 12, 15)

red :

$g(x) = a(x-1)(x-2)(x-3)$

$(0,6)$ $6 = -6a$

$a = -1$

$g(x) = -1(x-1)(x-2)(x-3)$

$-(x^2-3x+2)(x-3)$

$-x^3+3x^2+3x^2-9x-2x+6$

$g(x)$ $-x^3+6x^2-11x+6$

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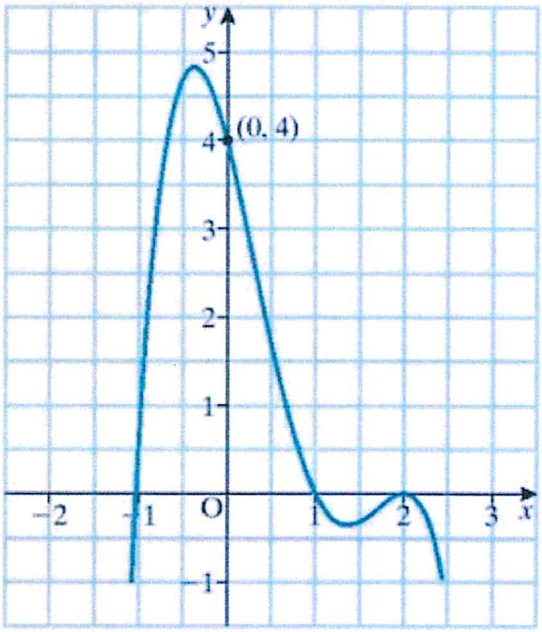
14.) The graph of a polynomial
$f(x) = ax^4 + bx^3 + cx^2 + dx + e$
is given in the diagram.

(i) Find the factors of the expression.

(ii) Hence find the values of a, b, c, d and e.



(i) $a(x+1)(x-1)(x-2)(x-2)$

(0, 5, 10)

(ii) $a(0+1)(0-1)(0-2)(0-2) = 4$

$a = -1$

$-(x+1)(x-1)(x-2)(x-2) = f(x)$

$-((x^2-1)(x^2-4x+4))$

$-(x^4 - 4x^3 + 4x^2 - x^2 + 4x - 4)$

$-(x^4 - 4x^3 + 3x^2 + 4x - 4) = -x^4 + 4x^3 - 3x^2 - 4x + 4$

$a = -1$ $\quad d = -4$

$b = +4$ $\quad e = +4$

$c = -3$

(0, 3, 5, 7, 10)

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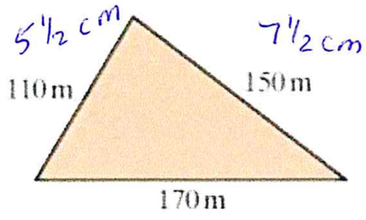
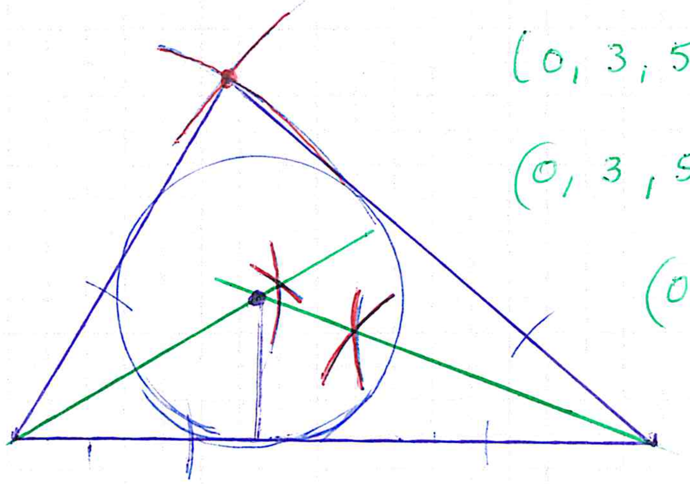
15.) A campsite is in the shape of a triangle with busy roads running along all three sides of the site.
The sides of the site are 110 m, 150 m and 170 m in length.
(i) Using 20 m = 1 cm, draw a scaled diagram of this site.
(ii) Show on the diagram the best position to pitch a tent so that it is as far away as possible from all three roads.
Show your construction lines.





(0, 3 5) for triangle

(0, 3, 5) for angle bisectors

(0, 3, 5) for locating incentre

(0, 3, 5) for selecting incentre as the correct point.

16.) A sculptor makes a small-scale clay model of a sculpture she is planning.
The model is 40 cm tall. The final sculpture will be 100 cm tall.

(i) Scale factor = 2.5 (0, 5)

(ii) Scale factor for volume = 2.5^3 (0, 5)

$$\begin{array}{r} 6.25 \\ \times\ 2.5 \\ \hline 3125 \\ 12500 \\ \hline 15.625 \end{array}$$

(iii) ~~(240 x 240 x 240)~~

~~or 2.4 x 2.4~~

240 x 15.625 = 37500$\cancel{00}$ cm³

3750 cm³

(0, 3, 5, 7, 10)

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